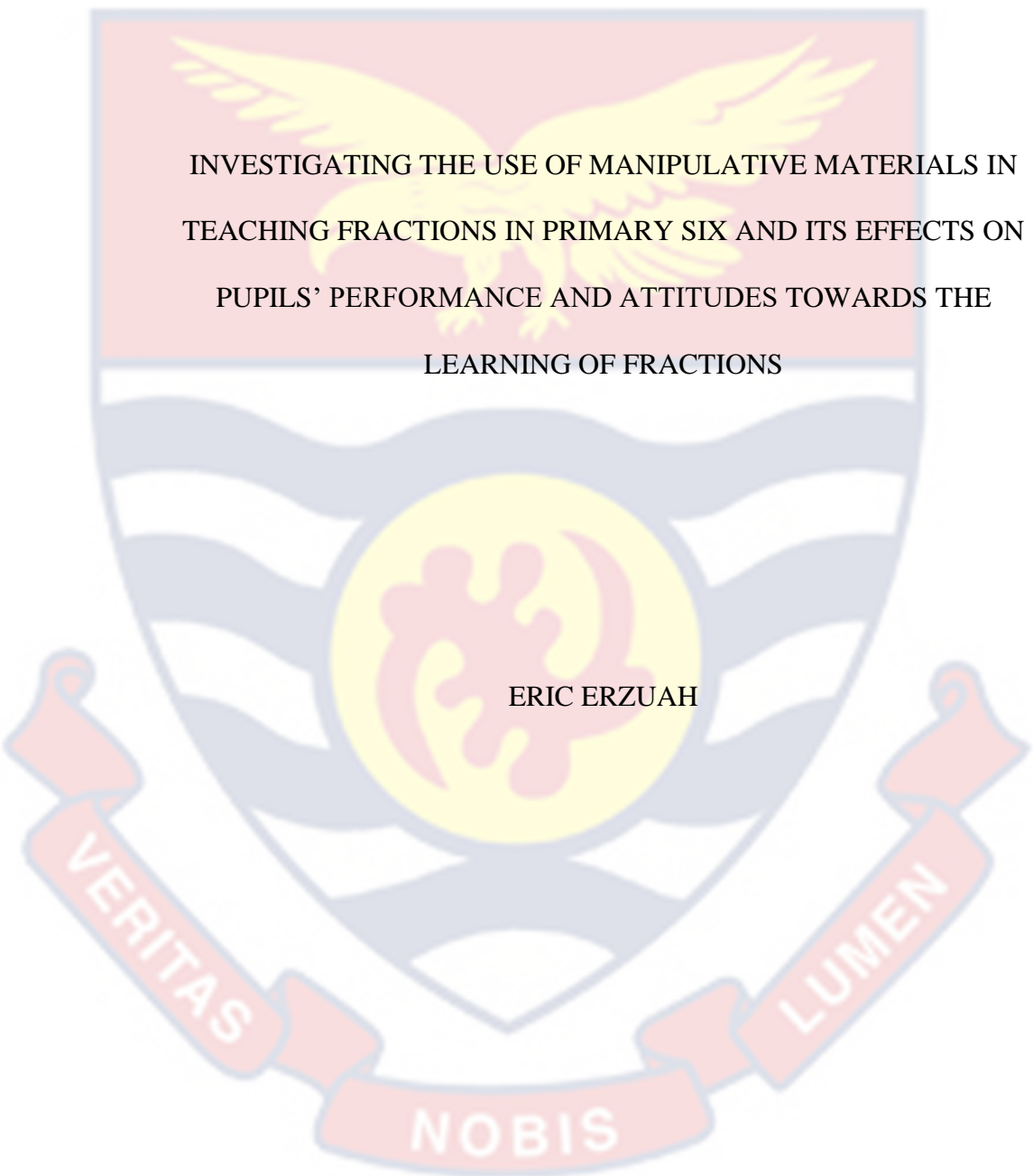


UNIVERSITY OF CAPE COAST



INVESTIGATING THE USE OF MANIPULATIVE MATERIALS IN  
TEACHING FRACTIONS IN PRIMARY SIX AND ITS EFFECTS ON  
PUPILS' PERFORMANCE AND ATTITUDES TOWARDS THE  
LEARNING OF FRACTIONS

ERIC ERZUAH

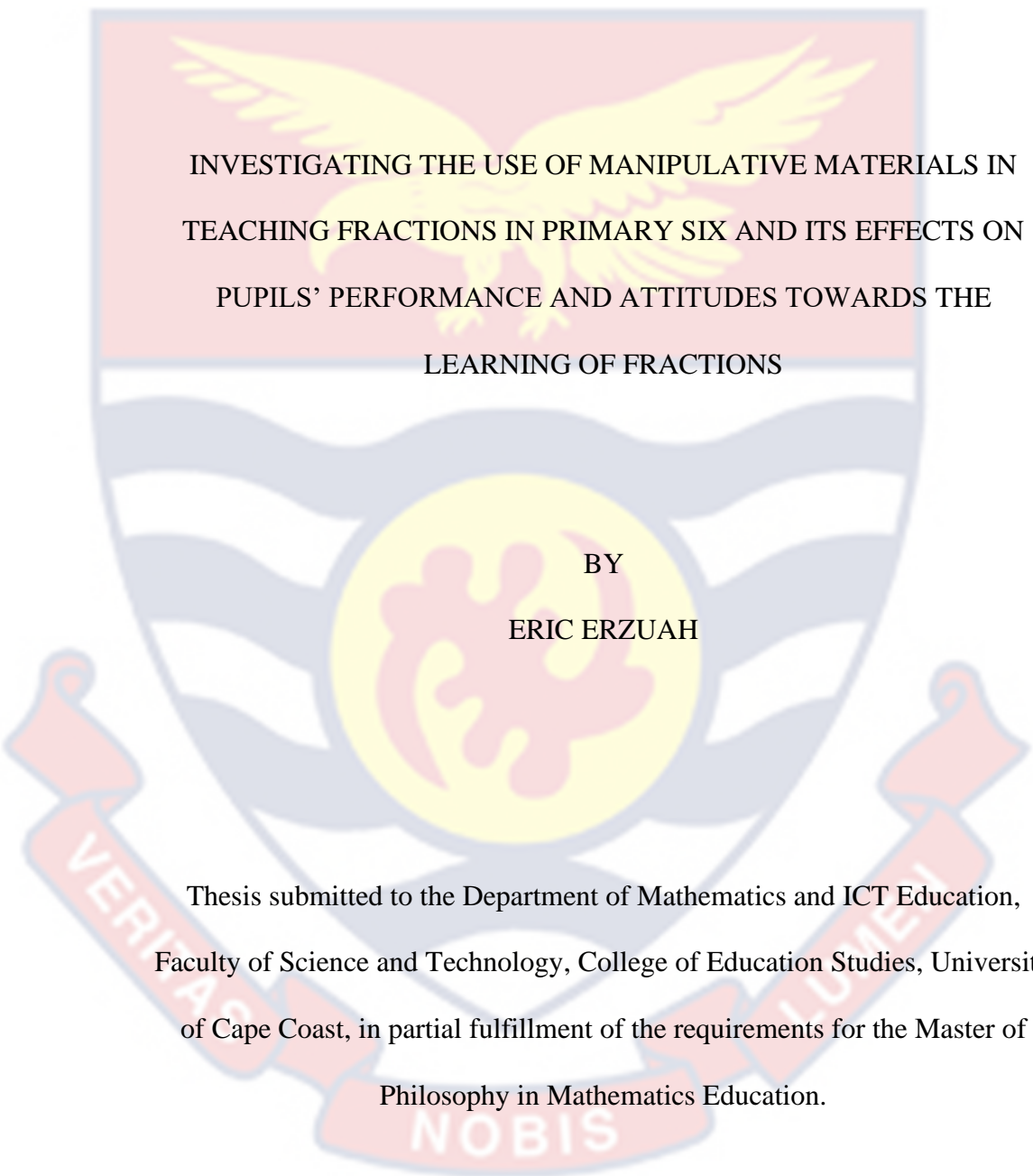
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BY  
ERIC ERZUAH

Thesis submitted to the Department of Mathematics and ICT Education,  
Faculty of Science and Technology, College of Education Studies, University  
of Cape Coast, in partial fulfillment of the requirements for the Master of  
Philosophy in Mathematics Education.

MAY 2023

## DECLARATION

### Candidate's Declaration

I declare that this thesis is the result of my own original research and that no part of it has been submitted for this University or any other degree.

Candidate's Signature.....

Date.....

Name: Eric Erzuah

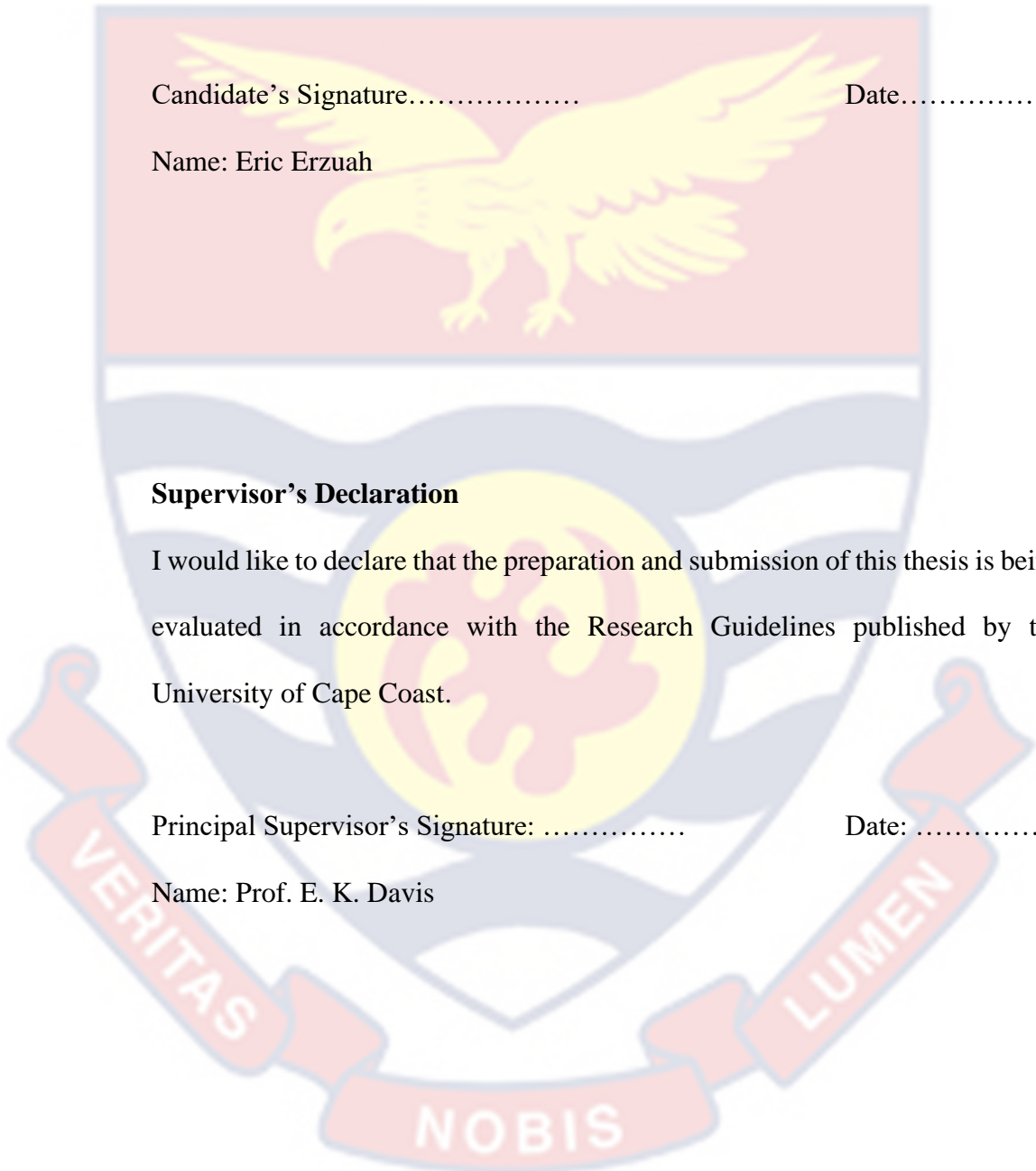
### Supervisor's Declaration

I would like to declare that the preparation and submission of this thesis is being evaluated in accordance with the Research Guidelines published by the University of Cape Coast.

Principal Supervisor's Signature: .....

Date: .....

Name: Prof. E. K. Davis



## ABSTRACT

The purpose of this study was to look into how teaching fractions with manipulatives influences the performance and attitudes of six primary school pupils in Aowin Municipality, Western North Region. Six teachers and elementary school pupils made up the population of the study, which employed a quasi-experimental research method. The study selected an 80-student sample using a quantitative sampling approach, of which 40 were given to the experimental group and 40 to the control group. 34 teachers of the sixth grade were also included in the sample. Data from the participants were gathered using a test and a questionnaire for the study. The study's research questions and guiding hypotheses were used to examine the results using descriptive and inferential data. The findings demonstrated a considerable improvement in the students' performance and attitude toward the lesson following the introduction of manipulatives to the study of passages. It is advised that the Ghana Education Services (GES) give public school teachers training and education to improve their capacity to choose and apply manipulative materials so as to improve pupils' performance and attitude toward fractions.

## ACKNOWLEDGEMENTS

I wish to express to sincere thanks to my main mentor, Professor E. K. Davis, God bless him abundantly. I want to thank the headmasters, teachers, and pupils of the various schools for their maximum cooperation and support when I was collecting the data. I also want to appreciate my sisters, wife and children for their support and prayers.



**DEDICATION**

To my family.



## TABLE OF CONTENTS

	Page
DECLARATION	ii
ABSTRACT	iii
ACKNOWLEDGEMENTS	iv
DEDICATION	v
TABLE OF CONTENTS	vi
LIST OF TABLES	xi
LIST OF FIGURES	xii
<b>CHAPTER ONE: INTRODUCTION</b>	
Overview	1
Background to the Study	1
Statement of the Problem	5
Purpose of the Study	7
Research Questions	8
Research Hypotheses	8
Significance of the Study	8
Delimitations	10
Limitations	10
Operational Definitions of Terms	10
Organisation of the Study	11
<b>CHAPTER TWO: LITERATURE REVIEW</b>	
Overview	13
Theoretical Framework	13
Conceptual Framework	19



The Place of Mathematics in National Development	20
The Concept of Fraction	23
Teaching Fractions in Primary Schools	24
Multiplication of Fractions	34
Multiplication of a Fraction by a Fraction	35
Multiplication of Mixed Fractions:	37
Division of Fractions	38
Fraction by a Fraction	39
Introducing Decimals Fractions	40
Converting terminating decimals to common fractions	40
Converting recurring decimals to proper fractions	41
Operations with Decimals	43
Addition and Subtraction of Decimals	43
Multiplication of Decimal Fractions	44
Pupils Attitude towards the Learning of Fractions	45
Gender Difference in Attitude towards the Learning of Fractions	46
The Concept of Manipulatives	47
Attitude of Teachers towards the use of Manipulative Materials	52
Impact of Manipulatives on the Attitude of Pupils	56
Impact of Manipulative Materials on the Performance of Pupils	58
Chapter Summary	72
<b>CHAPTER THREE: RESEARCH METHODS</b>	
Overview	73
Research Design	73
Population	74

Sample and Sampling Technique	74
Data Collection Instruments	75
Data Collection Procedure	76
Ethical Consideration	78
Data Analysis Procedure	78
<b>CHAPTER FOUR: RESULTS AND DISCUSSION</b>	
Overview	81
Biographical Data of Respondents	82
Main Results	85
The attitudes of primary six pupils towards the learning of fractions in the Aowin Municipality.	85
The attitudes of primary six classroom teachers towards the use of manipulative materials in teaching fractions.	89
There is no significant difference between the performance of pupils in the control group and experimental group when manipulative materials are used in teaching fractions.	93
There is no significant difference between the attitudes of pupils in the controls and experimental groups after the use of manipulative materials in teaching fractions.	97
There is no gender difference in attitudes towards fraction in the experimental group when manipulative materials are used in teaching fraction.	100
Discussion	102
The attitudes of primary six pupils towards the learning of fractions in the Aowin Municipality	102

The attitudes of primary six teachers towards the use of manipulative materials in teaching fractions.	103
Effect of manipulating materials on primary six pupil performance in fractions	105
Difference between the performance of pupils in the control group and experimental group when manipulative materials are used in teaching fractions.	105
There is no significant difference between the attitudes of pupils in the controls and experimental groups after the use of manipulative materials in teaching fractions.	108
There is no gender difference in attitudes towards fraction in the experimental group after the use of manipulative materials in teaching fraction.	109
<b>CHAPTER FIVE: SUMMARY, CONCLUSIONS AND RECOMMENDATIONS</b>	
Overview	111
Summary	111
Key Findings	112
Conclusions	113
Recommendations	113
Suggestions for Further Studies	114
<b>REFERENCES</b>	115
<b>APPENDICES</b>	131
APPENDIX A: Questionnaire for Teachers	131
APPENDIX B: Questionnaire for Pupils	133

APPENDIX C: Pre-Test Questions	135
APPENDIX D: Post-Test Questions	137
APPENDIX F: Lesson Notes	140



## LIST OF TABLES

Table	Page
1 Sex Distribution of Pupils	82
2 Sex Distribution of Teachers	83
3 Age Distribution of Teachers	83
4 Teaching Experience	84
5 Professional Qualification of Teachers	85
6 Attitude of Pupils towards Learning of Fractions	87
7 Attitude of Teachers towards the use of Manipulative Materials	90
8 Descriptive Statistics for Control and Experimental Groups	93
9 Independent t-test for Pretest for Experimental and Control Group	94
10 Paired Sample t-test for Experimental Group	95
11 Paired Sample t-test for Control Group	95
12 Independent couple t-test for Post-test for Experimental and <i>Control Group</i>	96
13 Descriptive Statistics on pupils attitude toward learning of fractions before treatment	98
14 Pretest Results for Experimental and Control Groups	98
15 Descriptive Statistics for Posttest	99
16 Posttest Results for Experimental and Control Group Groups	100
17 Descriptive Statistics of Male and Female Attitude towards fractions	101
18 Independent t-test for Posttest for male and female pupils	101

## LIST OF FIGURES

Figure		Page
1	Conceptual Framework	20
2	Representation of $\frac{1}{4}$	25
3	Cuisenaire rods	28
4	Adding two Fractions	30
5	Subtraction of fractions	31
6	Representation of $\frac{1}{3}$	33
7	Adding two like fractions	33
8	Shading two part of each whole.	34
9	Representation of five - thirds	35
10	Multiplying a fraction by a fraction	36
11	Mixed number multiply by mixed number	37
12	Representation one-thirds in four circular cuts-out papers.	38
13	Fractions divided by another fraction	40
14	Representation of a decimal fraction multiply by decimal fraction	44

## CHAPTER ONE

### INTRODUCTION

#### Overview

This chapter serves as an introduction and covers many different aspects including the background of the study, statement of the problem, objectives, research questions, hypotheses and significance. It also describes the delimitation of the study and limitations of the study, as well as how the study is organized

#### Background to the Study

The relevance of education in global development no longer seems to be a perception but a settled fact. According to Tefera (2014), education inculcate in the individual the capacity, skill and attitude necessary to solve problems in the environment and to blend seamlessly in society. This implies that, education performs a multi-faceted function in developing the whole individual. To Asare (2011), a sound education produces a well-informed world and the needed human resource which are ingredients for socio-economic growth and development. It therefore seems to be the case that, the role of education as the heartbeat of development is reason for countries and other stakeholders globally devoting massive resources into the education of its population.

Mathematics is one of the many disciplines in the school system that contributes to the overall realization of the goals of education. Mereku (2000) explains the discipline mathematics as a science of patterns that requires the pupil to solve problems which are related to their daily activities, look out for relationships, and employ their imaginative, creative, critical and logical thinking skills. According to Agbozo and Fletcher (2020), mathematics is the

fulcrum on which the advancement of the world revolves and as it is seen as a language that is understood by all globally. In fact, mathematics is one of the key components of all other subjects and professions. This means that, there is no profession of subject that does not make use of mathematics. This is corroborated by Legner (2013) who maintains that, it is highly difficult to identify an aspect of mathematics which has no real-life application to the individual. Legner further postulates that on a daily basis we use mathematics be it measurements of quantities, reading and interpreting information from journals or any printed material or business transactions. According to Mefor, (2014) as seen in Sa'ad, Adamu & Sadiq (2014) concludes that everything in this world ranging from the smallest to the biggest involve mathematics. It is against this background that Sa'ad, Adamu & Sadiq (2014) averred that mathematics and the life of humanity are two inseparable entities. This suggests that, for human life to function effectively and efficiently, mathematics must take centre stage. It is presumably this enviable place that mathematics occupies in the life and development of humanity that it is regarded as one of the prerequisites in almost every level of education in the Ghanaian system. In fact, it is one of the core subjects studied by all pupils in Ghana at the pre-tertiary level.

There are numerous topics in the Ghanaian mathematics curriculum among which is fractions. There is also enough evidence in literature to support the claim that fractions is a topic that teachers consider it tough teach and learners also find it hard to learn especially at the basic level (Davis, Bishop & Seah, 2010; Delaney, Charalambous, Hsu & Mesa, 2007; and Lamon, 2005). According to Tirosh (1998) as cited in Agbozo and Fletcher (2020), the



everyday applicability of whole numbers by children in their daily activities is not the same as that of fractions, and account for their difficulty to learn and understand fractions efficiently. Agbozo and Fletcher (2020) reiterated the ideas of Hiebert and Lefevre (1986) by demonstrating that young children struggle to recognize fractions as distinct numbers. Instead, they tend to view both the numerator and denominator as independent whole numbers, which can impede their ability to transition from whole numbers to fractions, especially when learning new methods of operation. The observation that young children view fractions as separate whole numbers rather than as distinct numerical entities can have consequences for their performance and attitudes towards learning fractions. Therefore, it is important to investigate these implications further.

Notwithstanding the challenges posed by young children's perception of fractions as separate whole numbers, a solid and conceptual understanding of fractions is crucial, as it lays the foundation for more advanced mathematical activities. In light of this, the use of manipulative materials in teaching and learning fractions is essential. Studies consistently show that using tangible manipulatives promotes a better understanding of mathematical concepts, including fractions. Manipulative materials are concrete materials that pupils can see and touch as they learn mathematical concepts. They are items planned to embody clearly and concretely mathematical concepts that are mental in nature. These materials can be seen and touched and as such the pupils can manipulate them with their hands as they discover mathematical ideas and facts (Moyer, 2001). Several manipulative materials are suitable for teaching and learning fractions, including Cuisenaire rods, strips of paper, fraction charts, fraction circles, and other similar tools. These materials can help students

develop a better understanding of fractions by providing a hands-on approach to learning, making the concepts more concrete and easier to visualize.

The use of manipulatives in teaching fractions is especially important because they allow teachers to clearly explain the concept of fractions and help students gradually move on to learning functions with fractions. The National Council of Teachers of Mathematics (1989) standards cited in Hougas (2003) emphasize the importance of providing students with activities that promote discovery and thinking, use multiple methods to solve problems, and build confidence in mathematics. By incorporating manipulatives into the lesson plan, teachers can facilitate these activities and create a more engaging and effective experience for students. It is in the light of this that Hougas (2003) identifies the teacher as a key ingredient for learning of mathematics effectively in our schools. This means that the mathematics teacher needs enough resources at his disposal so as to be able to arrange relevant activities necessary mathematical discovery and investigation. This the teacher can do through group exercise, individual tasks or whole-class discovery.

In recent times, it is argued that mathematical concepts are better understood when learners get involved actively in the learning process by manipulating objects within their environment. This presupposes that, when pupils are taken through hands-on experiences through the use manipulative materials, they will have a firm grasp of the concept of fractions through their exploration and the sharing of ideas among themselves. To this end, Yusof and Lusin (2013), argues strongly that teachers of mathematics should deploy manipulative materials in teaching as they make learning real and concrete.

Teachers have different attitudes towards the usage of manipulative materials among upper primary school pupils. Yusof and Lusin (2013) have identified two main reasons why teachers of mathematics are reluctant in using manipulative materials in their classrooms. These teachers believe that manipulative materials in teaching is not appropriate for learners beyond the fourth grade; and also, some mathematics teachers lack the requisite competence in the use of manipulative materials in teaching mathematical concepts. Other reasons why mathematics teachers do not use manipulative materials as identified by Sherman and Richardson (1995) include the unfamiliarity and uncomfotability of teachers in the usage of manipulative; apprehensions about time restrictions, likely difficulties in relation to classroom discipline; accessibility of manipulative materials and the cost involved in securing them for lessons.

From the above, fraction is an indispensable topic in mathematics without which pupils will have problems understanding related topics. It can also be deduced that learning and teaching fractions is tough. However, it can also be seen that manipulative resources can assist pupils develop a better understanding fractional concept. Notwithstanding, teachers, particularly in the upper grades, have differing perspectives on the use of manipulative devices. This study investigated manipulative devices use in the teaching of fractions in upper primary six schools in the Aowin Municipal in Ghana's Western-North Region.

### **Statement of the Problem**

The learning and teaching of mathematics in our schools and pupils' performance seem to gained attention of almost everyone in the country

(Schoenfeld 2002). It also seems to be the case that the proposition that manipulative materials use in mathematics teaching promotes adequate learning and easy understanding of mathematical concepts remains unchallenged (Roth 2020). As part of my classroom interaction with teacher trainees as a tutor, it was realised that most of the pupils were not comfortable with problems involving fractions. In fact, some of the trainees could solve problems involving fractions but only applied procedural knowledge rather than relational understanding.

Again, most of the pupils also demonstrated unawareness about the concrete materials I used to demonstrate how to teach concepts involving fractions. This confirms the assertion of Laurella (2017) and WAEC (2015) that most pupils shiver in their seats when fraction topics are to be taught or assessed. Pupils are having hard time understanding the concept of fractions as well as any of its application. They simply “hated” fractions. In an interaction with the teacher trainees, they claimed that they were not taught fractions using manipulatives. A study by Davis (2016) stated how pupils learn in out of classroom situations how to manipulate fractions. The observation that young children perceive fractions as separate whole numbers raises several questions, including the effectiveness of manipulative materials in improving students' understanding of fraction-related concepts and teachers' attitudes towards the use of such tools in the classroom. Further investigation is needed to explore the potential benefits of manipulative materials in enhancing students' conceptual understanding of fractions and to examine any barriers that may prevent teachers from utilizing these teaching and learning materials (TLMs).

Despite the concerning nature of this situation, there has been little or no research conducted to examine the effectiveness of manipulative materials in teaching fractions in the Ghanaian context. Most studies in this area have been conducted in other countries, and therefore, their findings may not be entirely applicable to the Ghanaian educational setting. Consequently, this study aims to address this research gap by investigating the use of manipulative materials in teaching fractions in primary six classrooms in the Aowin Municipality. This research will provide valuable insights into the effectiveness of manipulative materials, their role in shaping students' attitudes towards fractions, and the attitudes of teachers towards the use of these materials in teaching fractions at the primary six level in Ghana.

#### **Purpose of the Study**

The primary aim of this study was to investigate the impact of using manipulative materials on the performance and attitudes of primary six pupils towards fractions in schools located in the Aowin Municipality of the Western North Region. Specifically, the study aimed to:

1. Explore primary six teachers' attitudes towards the use of manipulative materials in teaching fractions.
2. Explore primary six pupils' attitudes towards learning fractions.
3. Determine the impact of using manipulative materials on the performance of primary six pupils in fractions.
4. Examine the influence of using manipulative materials on primary six pupils' attitudes towards learning fractions.
5. Determine whether there are any gender differences in attitudes towards learning fractions among primary six pupils.

### Research Questions

This study was guided by the following research questions:

1. What are the attitudes of primary six pupils towards the learning of fractions in the Aowin Municipality?
2. What are the attitudes of primary six teachers towards the use of manipulative materials in teaching fractions?
3. What is the effect of manipulative materials on primary six pupils' performance in fractions?

### Research Hypotheses

In addressing research question three, the following research hypotheses were formulated:

1. There is no statistically significant difference between the performance of pupils in the experimental group and control group when manipulative materials are used in teaching fractions.
2. There is no statistically significant difference between the attitudes of pupils in the experimental group and control group after the use of manipulative materials in teaching fractions.
3. There is no gender difference in attitudes towards fraction in the experimental group after the use of manipulative materials in teaching fraction.

### Significance of the Study

Overall, it is hoped that this study will contribute to the improvement of decentralized teaching and learning in primary schools in Aowin municipality, and potentially throughout the Ghanaian educational system. In addition, this study will provide teachers with information on the effectiveness of

modifications in teaching fractions and how to use them effectively in their classrooms. It will also help teachers make informed decisions about curriculum development and implementation and provide potential researchers with a foundation for further study in this area.

The results of the study can help teachers to understand how strategic resources play a role in teaching components, which in turn can improve their teaching strategies and shape their students' perspectives improved understanding of parts and functions of parts. By improving students' understanding of fractions, teachers can also improve their overall math performance and foster positive attitudes toward both fractions and math. The course will also provide teachers with new insights into the use of power to improve students' conceptual understanding of the concept of fractions and fractional activities. This will help improve students' performance and develop a positive attitude towards fractions as a subject and mathematics as a subject.

The results of the study can help teachers to understand how strategic resources play a role in teaching components, which in turn can improve their teaching strategies and shape their students' perspectives improved understanding of parts and functions of parts. By improving students' understanding of fractions, teachers can also improve their overall math performance and foster positive attitudes toward both fractions and math.

The study will contribute to the existing body of knowledge with valuable information for researchers interested in exploring the use of strategic objects in teaching and learning from different angles and perspectives in the relevant field.

### **Delimitations**

This study was restricted to class six pupils and teachers in Aowin Municipality in Western-North Region of Ghana. Its scope was limited to investigating the effectiveness of manipulatives in teaching fractions and did not include any other topics in the mathematics curriculum.

### **Limitations**

The study is limited in sample size, making it difficult to extend the results to the six primary classrooms in Ghana. However, the results still apply to primary pupils and teachers in Aowin Municipality. Another limitation was that pupils were not asked to write their names on the question papers, making it impossible to predict efficacy by gender.

### **Operational Definitions of Terms**

This study adopted the following definitions for the terms used in the study:

**Fractions:** Are parts of one whole: A whole is composed of a single or group of elements.

**Denominator:** The term "denominator" refers to the number of equal parts divided, meaning that the fraction is a half, a third, a quarter, a fifth, and so on.

**Numerator:** The number of equal parts under consideration.

**Equivalent Fractions:** These are fractions that represent the same quantity but have different names. They have different denominators but different values.

**Like Fractions:** Are fractions with the same denominator

**Unlike Fractions:** They are fractions with different denominators.



**Manipulatives:** These are specific objects that students can see and touch to learn mathematical concepts. They are objects intended to represent abstract mathematical concepts explicitly and specifically. They are visual and tactile and students are able to engage with them through practical experience (Moyer, 2001).

### **Organisation of the Study**

In the second chapter, the literature on the use of manipulatives in teaching and learning fractions is reviewed. This include theoretical underpinnings the use of manipulatives in teaching of fractions, an empirical review of the use of manipulatives in teaching mathematical concept, and conceptual review of the literature related to the use of manipulatives in the teaching and learning of fractions.

The third chapter explains the research methodology. It provides a detailed description of the methods used by the researcher to collect data and answer the research questions and hypotheses. The chapter discusses various aspects of research design, including population, sampling, and sampling methods, as well as data collection instruments and methods. Furthermore, it describes the methods used to analyze the data collected.

The fourth chapter presents the results and conclusions of the study. This includes a discussion of the results and an analysis of the data collected. The chapter concludes with a summary of the main conclusions of the study.

Chapter five provides a summary of the overall study, including a summary of the research questions, methodology, and results. The chapter also provides policy and management recommendations based on the study results, as well as recommendations for future research in the area.



## CHAPTER TWO

### LITERATURE REVIEW

#### Overview

This chapter reviews related literature to the topic of study. The literature review consists of three categories: conceptual analysis, theoretical analysis, and empirical analysis. Each section of the review examines relevant journals from a variety of sources, including academic journals, books, and other publications, to provide a comprehensive overview of the body of knowledge on the topic.

#### Theoretical Framework

In their comprehensive literature review, Yusof and Lusin (2013) argue that the teaching of fractions has become a major concern for teachers and mathematics educators around the world because of the challenges many students have with the concept of fractions. The authors suggest that active participation in learning is vital and suggest that students should be encouraged to engage with things in their environment to generate ideas.

The authors' recommendation is to give students the opportunity to explore fractional concepts through practical experiments, exchange ideas with peers, and above all to use fractional concepts as a tool for learning, teaching and learning fractions. They also suggest that in order to gain a deeper understanding of mathematical concepts, students should be actively involved in their learning and encouraged to interact with objects in their immediate environment.

The theoretical basis of this study is based on constructivism. This means that when teaching and learning about fractions, students participate in experiments, exchange ideas with their peers, and most importantly, often understand concepts before using them with the appropriate equipment. The constructivist approach assumes that students actively build their knowledge and understanding of mathematical concepts based on their experiences and interactions with the environment. As such, he recognizes the importance of providing students with a supportive learning environment that allows them to improve their existing knowledge and gain deeper understanding, mainly through collaboration and the use of the manipulatives.

Miles and Huberman (1998) define a theoretical framework as the molecular explanation of a complex entity or process. They explain that it is a tool that describes the main elements to be searched for, including factors, constructs, or variables. In the context of this study, the theoretical framework is based on a constructivist approach to education.

Constructivism is an educational theory that stresses the learner's involvement in building knowledge and understanding via experience and interaction with the outside world. This knowledge comprises all the pupil learns before formal teaching. Students actively develop their grasp of mathematical topics such as fractions in this setting by discovery, investigation, and reflection. Learning, according to the constructivist viewpoint, is a process of building meaning from experience rather than simply accumulating knowledge or abilities. This theoretical framework guides study design and implementation and provides a lens through which data can be analyzed and interpreted.

According to Elliott et al. (2000), constructivism is a learning approach that views the learner as an active participant who constructs his own knowledge and establishes meaning in his experiences. This strategy contrasts from traditional methods, which regard learners as passive recipients of information.

Constructivism holds that human learning is created and that humans acquire new information by building on existing knowledge. This prior knowledge is fresh or altered knowledge acquired through new learning experiences (Phillips, 1995).

Constructivists see learning as a dynamic and active process, as opposed to a passive approach where students are seen as empty vessels filled with knowledge. Learning through constructivism requires active engagement with the environment through experiments or solving real problems. Passively receiving information is not enough for understanding, because it requires meaningful connections between current information, new knowledge and the learning process.

According to Vygotsky (1978), who was a constructivist theorist, all knowledge is created through social interaction. Learning, rather than being an abstract concept, is clearly a communal activity originating through connections with people. Vygotsky (1978), in contrast to Dewey (1938), highlighted the role of community in the process of learning and growth. Vygotsky argued that a child's upbringing influenced their thoughts and perceptions. As a result, communicating and manipulating socially constructed knowledge is an important element of teaching and learning. According to Vygotsky (1978), knowledge is co-constructed by children and their peers through social

interactions, which provide direction and support in the early developmental period.

According to Driscoll (2000), constructivism holds that knowledge exists solely in the human mind and does not always reflect reality. Students strive to develop their own mental models of the world as they perceive new experiences, adjust their old mental models to accommodate new knowledge, and attempt to establish their own interpretation of reality. The major responsibility of a teacher is to promote a collaborative problem-solving atmosphere in which pupils are actively involved in their own learning. Rather than being an instructor, the teacher facilitates learning. The teacher ensures that he or she is aware of pupils' preconceived notions and provides guidance (Oliver, 2000).

Tam (2000) defines four major constructivist learning environment aspects that must be considered when applying constructivist teaching strategies.

1. An exchange of knowledge takes place between teachers and students.
2. In the context of constructivist studies, power is shared between them teachers and students.
3. In a constructivist approach, the instructor's role is that of a facilitator or guide
4. In constructivist teaching, a small number of different students consist.

Honebein (1996) provides a summary of seven educational goals of constructivist environmental studies:

1. It allows students to experience the process of knowledge building (allows them to determine how they learn).

2. Encourage students to be exposed to and understand different perspectives with alternative evaluations.
3. Students are encouraged to take ownership and actively participate in the learning process, highlighting student-centered learning.
4. Students' grasp of the process of knowledge formation will be enhanced, with a focus on reflection and metacognition.
5. The use of different presentation methods (such as video, audio and text) is encouraged to enhance learning.
6. To improve students' comprehension of the process of knowledge formation, with a focus on reflection and metacognition.

Brooks and Brooks (1993) identify twelve teaching behaviors associated with constructivist learning, including:

1. Encourage and accept students' initiative and autonomy in learning.
2. Incorporate hands-on, interactive experiences and tangible objects with raw data and primary sources
3. A framework of cognitive tasks using words such as 'classify', 'analyze', 'predict' and 'create'.
4. Create interactive and social learning opportunities.
5. Begin teaching activities by asking students to share their knowledge of the subject before sharing their own understanding.
6. Encourage students to discuss with their classmates and the teacher
7. Cultivate students' curiosity by asking open-ended, intelligent questions, and encourage them to ask individual questions.
8. Encourage students to expand on their initial comments or responses.

9. Students engage in activities that may lead to the development of contrasting hypotheses and then facilitate discussions about those hypotheses.
10. Make sure you have enough time to answer the questions.
11. Give students plenty of time to build relationships and develop metaphors.
12. Cultivate students' innate curiosity by regularly applying the learning cycle model.

Students are viewed as active knowledge and understanding through cognitive processes in both the social and cultural environments. They build new knowledge on top of their existing knowledge and develop metacognitive skills that allow them to regulate their learning. These teaching views have important implications for the teaching methods used by teachers, as suggested by Bruner (1985) and Vygotsky (1978) and as highlighted in the work of Piaget (1954), Greenfield (2009) and Bransford, Brown, & Cocking (2000).

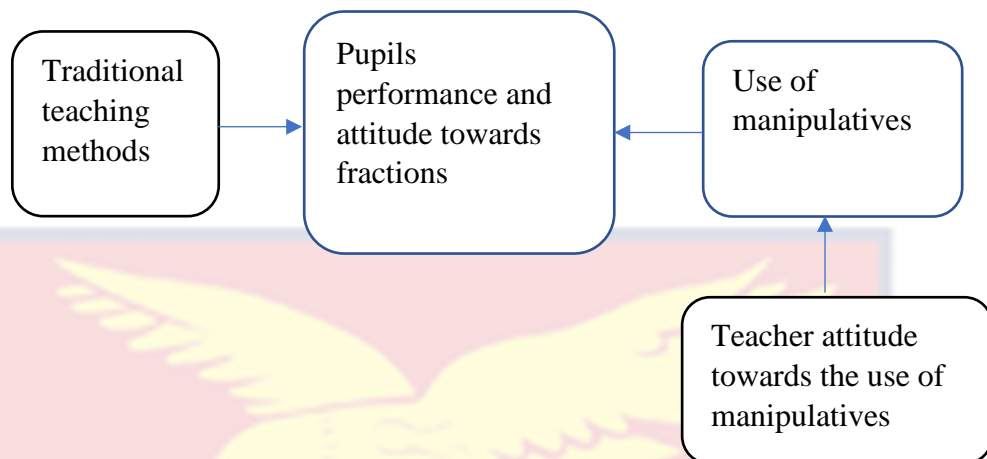
In support of the above, Jia (2010) explains that the theory of constructivism suggests that each learner enters the school with prior knowledge and learns, combining new knowledge with that already learned and new knowledge in the existing context without the need to change in the existing knowledge. This shows that the growth of learning fractions depends on the foundation of the previous steps. If fractions are taught well at the stage, understanding the concept will be very difficult to achieve at a higher level. Therefore, it is necessary to use appropriate manipulations to teach upper primary pupils, especially the concept of fractions.



Piaget, Bruner and Zoltan Dienes are other important contributors to the development of constructivism. Jean Piaget (1952) suggested that children aged seven to ten tend to think concretely and can grasp difficult abstract mathematical concepts without practical abilities. The Cuisenaire rods, invented by Caleb Gattegno and Georges Cuisenaire (1954), is based on this idea. Hieronymus Bruner (1966) further develops the idea of an active, virtual, symbolic mode of cognition, emphasizing the importance of using concrete and representative methods for the development of abstract thinking. According to constructivist philosophy, kids build their own understanding by connecting physical manipulatives to abstract symbols that have meaning for them.

### **Conceptual Framework**

The extent to which a learner will succeed or fail in learning is determined by cognitive, emotional, and behavioral control. Mutai (2010), on the other hand, contends that other elements, such as the amount of work done by students and, more crucially, the teaching strategy, particularly the inclusion or omission of manipulatives employed by the teacher, affect student performance, particularly in mathematics. As a result, the study's methodology supports whether the use of manipulatives enhances students' fraction performance and attitudes substantially more than the traditional chalk and talk technique to teaching fractions.



*Figure 1: Conceptual Framework*

The details in Figure 1 highlight that the teacher's pedagogical approach, which includes traditional and manual methods, is considered as an independent variable, while the dependent variable is the performance and attitude of the students. The attitude of the teacher towards the use of manipulatives in the classroom to some extent influences the extent they use manipulative materials in the classroom. In this study, the attitude of the teachers is investigated to inform the extent to which teachers in Aowin Municipality use manipulative materials in the classroom. Two classrooms are used to determine the extent to which the effect of manipulatives and traditional methods differ in influencing the performance and attitudes of pupils relative to fractions

### **The Place of Mathematics in National Development**

Mathematics is an important subject in education and nation building. It is widely recognized by students and other stakeholders and can be observed in the immediate and remote environment. With the knowledge of patterns, mathematical problem solving, finding relationships, imagination, creativity, critical thinking and logical thinking, as stated by Mereku (2000). Its usefulness in everyday life consists in solving problems. Mathematics is the foundation of

all things and all professions. Mathematics is taught at all levels of education in Ghana, from primary to university. This emphasizes the significance of mathematics in Ghana's educational system.

Mathematics is crucial in Ghana's technical and economic development because it provides students with the skills and knowledge, they need to pursue employment in disciplines such as engineering, finance, and information technology. Furthermore, mathematics is a subject that fosters critical thinking, problem-solving, and analytical skills, all of which are useful not only in academia but also in daily life. As a result, a strong mathematical foundation is required for pupils to flourish in various parts of life.

The applicability of mathematics to various fields of study and real-world situations cannot be overstated. Mathematics in fields such as science, work, finance, commerce, medicine, and even art. It equips students with the skills necessary for critical thinking, problem solving, and decision making. In today's world, the ability to evaluate data, spot trends, and make correct forecasts are all vital talents, and all of these skills rely on mathematics. It is therefore imperative that students in Ghana and around the world have a solid foundation in mathematics to prepare them for success in their future endeavours. According to Sa'ad, Adamu, and Sadiq (2014), mathematics is inextricably linked to daily living and long-term planning, making it a necessary subject in effective education and human life. Therefore, many countries, with the exception of Ghana, require all primary and secondary school students to study mathematics. The importance of mathematics in the development of people and nations cannot be disputed.

If students learn mathematics effectively and gain a deep understanding of the subject, it can pave the way for potential success in the world today and in the future. Not surprisingly, a strong mathematical foundation is required for many professions and jobs in today's society. According to a Ministry of Education report (2001), developing good mathematical abilities in lower secondary education is critical for future success in mathematics, science, business, industry, and a variety of other subjects, businesses, and vocations. According to Plato (2000), mathematics has a philosophical value and serves as a tool for expanding and sharpening the mind's reasoning abilities. This kind of thinking can aid the mind in comprehending the concept of good, which is the ultimate purpose of philosophy. Plato recognized the significance of mathematics in people's daily lives. He recognized, however, that the philosophical meaning of mathematics was more significant than honors and awards since it had the power to self-understand.

Mathematics is also associated with power, subjecting many innovations in society, and leading people who excel in the field to gain wealth. As a result, mathematicians serve as the gatekeepers of research all over the world, denying a substantial portion of the people access to its "power" (Ernest, Greer, and Sriraman, 2009; Secada, 1995). Although power and wealth may not appear to be immediately relevant to elementary school students, these formative years lay the groundwork for academic success. According to the Department of Education (2001), developing excellent mathematical abilities in high school is critical for pursuing courses in mathematics, science, commerce, industry, and a variety of other professions and employment. The prosperity of the rich and technologically advanced nations of the world is attributed to the development

of mathematics, which acts as a bridge between science and technology. It emphasizes the importance of mathematics education as a critical aspect of scientific and technological progress for any society. As a result, groups, especially academics, should do more to make mathematics instruction and learning more suitable and relevant.

Davies and Hersh (2012) go on to suggest that mathematics is not only required for academic achievement in high school or college, but also to prepare kids for the future, regardless of the job path they select. In fact, mathematics not only provides students with opportunities for employment, but also helps them develop skills in other fields. In Ghana, mathematics has become a compulsory subject for students to progress from one level to another. Failure in spring math increases the likelihood that a person will not be able to enroll in a post-secondary institution that will prepare them for a career. Knowing math can open up many opportunities for a child, including careers in teaching, engineering, and statistics careers,.

### **The Concept of Fraction**

Fracture is from the Latin word 'fractus' which means 'broken'. That fracture can be explained in many ways.

1. A fraction may be called a part of a whole. For example, when an orange is divided into three equal parts and you consider one part (one third).
2. A fraction can also be used as part of a group/total. For example, in a class of 20 students, if the number of girls is 13, the percentage of girls in the school can be expressed;  $\frac{13}{20}$
3. A partition can also be expressed as a ratio. A ratio is a mathematical expression that expresses a relative quantity. So it is better to think of it

as comparison index rather than the number. For example, in a class of 20 students, if the number of girls was 13, then the ratio of girls to boys would be 13:7, which can also be expressed as  $\frac{13}{7}$ .

4. Fractions can also be expressed using an operator. When expressed as an operator, it indicates one integer that divides another. For example,  $13 \div 7$  it can be expressed by  $\frac{13}{7}$ .

The denominator of the fraction is at the bottom, and signifies the amount or number of equal parts that make up the whole. Examples of denominations include halves, thirds, quarters, fifths, etc. On the other hand, the numerator of the fraction is at the top and represents the number of equal parts considered or counted. For example, in a fraction such as  $\frac{2}{5}$ , the denominator is 5, indicating that the whole is divided into five equal parts, while the numerator is 2, indicating the number of parts considered.

### **Teaching Fractions in Primary Schools**

#### *Resources / Teams for teaching Fractions*

You can identify fractions using counters, Cuisenaire rods, strips of papers, number lines, and more. It is the teacher's responsibility to choose the most suitable and available resources for the activity. The ultimate goal is for your students to understand the concept rather than memorizing or simply following the rules without understanding why the rules work.

#### *Modelling or representing fractions*

In order for students to understand and work easily with fractions, the child can represent and/or name the fractions. This requires the use of concrete materials. In order for children to have a good understanding of fractions, they must first understand how they are formed. as the mean of something is that

which is taken when we divide it equally into two parts. Similarly, when we divide something into three equal parts, four equal parts, etc., it becomes a third, a fourth, &c. To get two thirds, you divide something into three parts, and then choose two of those parts. It is impossible to point to one object and say that it represents the half, because fractions are not objects. There are activities. It must be taken to show half, a third, or a quarter of something. Only when we learn to use symbols to represent these actions can we treat them as objects. Therefore, children must first have practical experiences, dividing tangible things into equal parts, so that they can make parts. Examples of these tangible materials include paper wrappers, bottle caps, Cuisenaire rods, etc.

Children's first experiences with fractions should begin with simple fractions, such as halves, fourths, eighths, thirds, fifths, and tenths. Direct the child to fold the strip of paper into two equal parts. Tell the boy that the part is half. Ask the child to fold the paper into four equal parts, tell them that one part is a quarter. Use other materials to express and name fractions. If a paper is folded into four equal parts and considered as three parts, it will be a fraction of three quarters.



The shaded portion represents the fraction  $\frac{1}{4}$ .

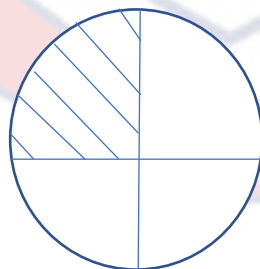


Figure 2: Representation of  $\frac{1}{4}$

Again, the shaded portion is  $\frac{1}{4}$

*Types of Fractions*

1. *Proper parts*: Parts are smaller than one. Their number is always less than the denominator. Examples include;  $\frac{1}{2}$ ,  $\frac{3}{7}$ ,  $\frac{2}{5}$ ,  $\frac{13}{28}$ , *etc.*
2. *Improper fractions*: These fractions are larger than one. Unlike proper fractions, the numerator is always greater than one. Examples include;  $\frac{3}{2}$ ,  $\frac{9}{7}$ ,  $\frac{7}{5}$ ,  $\frac{29}{28}$ , *etc.*
3. *Mixed fractions*: mixed fractions consist of a whole and a proper fraction. Examples include;  $3\frac{1}{2}$ ,  $1\frac{3}{7}$ ,  $2\frac{2}{5}$ ,  $5\frac{13}{28}$ , *etc.*

*Determine simple equivalent fractions*

When dealing with fractions, it's always a good idea to reduce your last answer (when it's in a fraction) to its simplest or lowest term. Three ways to do this include successive division by a common factor, division by a common factor, and division by prime factors.

**Method 1: successive division by a common factor**

In this way they successively divide the denominator and the numerator by the commons. Common items may or may not appear in this division. Here

is an example  $\frac{560}{960} = \frac{56}{96} = \frac{28}{48} = \frac{14}{24} = \frac{7}{12}$

**Method 2: Divide by the common sum of both a and b**

In this way, find the highest common denominator and numerator and complete the division. For example:  $\frac{560}{960}$  The highest common factor of 560 and

960 is 80. We divide the numerator and denominator by 80 so that in  $\frac{560 \div 80}{960 \div 80} =$

$$\frac{7}{12}$$



**Method 3: Dividing the common prime factorizations a and b.**

In this way, find the prime factor of the denominator and the numerator and complete the division.  $\frac{560}{960} = \frac{2^4 \times 5 \times 7}{2^6 \times 3 \times 5} = \frac{7}{2^2 \times 3} = \frac{7}{12}$

*Compare the fractions*

When children compare fractions, which is larger and which is smaller. Knowing this will help further sorting fractions (sorting fractions in ascending or descending order). There are several approaches to teaching fraction comparison. These include using equivalent paper bands, using equivalent fractions, and converting given fractions to percentages. However, it is recommended that children's first experience with comparing fractions begin with the use of concrete materials such as strips of paper and Cuisenaire rods.

We compare parts with the same numerator because dividing the whole into fewer parts means that each part is larger. Therefore  $\frac{5}{7}$  represents more of the whole than  $\frac{5}{9}$ . When comparing fractions with different denominators and numerators, we need to find the common denominator. For this we can use equivalent fractions, which are fractions with the same value, but a different numerator and denominator. Once we find the common denominator, we can compare the fractions by comparing their numerators. For example, for  $\frac{1}{3}$  and  $\frac{2}{5}$  we can compare them to equivalent fractions with a common denominator of 15, which gives us  $\frac{5}{15}$  and  $\frac{6}{15}$ . Therefore  $\frac{2}{5}$  is greater than  $\frac{1}{3}$ , because  $\frac{6}{15}$  is greater than  $\frac{5}{15}$ .

Let the writer compare now  $\frac{2}{3}$  and  $\frac{1}{2}$

1. *Using the strips of paper:*
  - a. Take two strips of paper of the same size

- b. Fold the first into three equal parts and shade two parts to represent  $\frac{2}{3}$
- c. Take the other paper and fold it into two equal parts and one part of it to represent  $\frac{1}{2}$
- d. Compare the shade portions of the two papers by placing them together to determine the fraction that is greater.
- e. The child will notice that  $\frac{2}{3}$  is greater than  $\frac{1}{2}$
- f. Introduce the  $>$  and  $<$  to pupils



Figure 3: Cuisenaire rods

*Using the Cuisenaire Rods*

- a. Select an appropriate whole (a rod that can exactly divide 2 and 3) which is dark green
- b. Identify a rod that exactly divides the dark green rod into three equal parts and select two of them to represent  $\frac{2}{3}$  (two red rods)
- c. Identify a rod that exactly divides the dark green rod into two equal parts and select one of them to represent  $\frac{1}{2}$  (one light green)
- d. Compare the lengths of the two red rods put together and the one light green rod by placing them side by side with their bases on a flat surface.
- e. You will notice that the two red rods are longer than the one light green hence  $\frac{2}{3} > \frac{1}{2}$

*Using percentages*

Percent means out of 100. It should be noted that any number expressed in a percentage is in itself a fraction. For instance, 30% means  $\frac{30}{100}$ . To convert a given fraction to a percentage, you multiply the given fraction by 100. In our previous example,  $\frac{2}{3}$  can be expressed as  $\frac{2}{3} \times 100 = 66.667\%$  and  $\frac{1}{2}$  expressed as  $\frac{1}{2} \times 100 = 50\%$ . By comparison 66.667% is greater than 50% hence  $\frac{2}{3} > \frac{1}{2}$

*Equivalent fractions:*

Two or more fractions are said to be equivalent if they have the same value and therefore can be simplified to the same form. We can use paper clothes to make fractions of given fractions. We can use tables/breaks. To create parts equivalent to the given fraction, multiply the denominator and numerator by the same amount. For example,  $\frac{2}{3} = \frac{2 \times 2}{3 \times 2} = \frac{4}{6}$ , this is just  $\frac{2}{3}$  equivalent  $\frac{4}{6}$ . Many more can be created using the same process.

Using equivalent parts to compare fractions and measures, we create as many equivalent fractions as possible for the given fractions. Then we determine where the two fractions have the same denominator. This makes comparison easier. This is depicted as

$$\frac{2}{3} = \frac{4}{6} = \frac{6}{9} = \frac{8}{12} = \frac{10}{15} = \frac{12}{18} = \frac{14}{21} = \frac{16}{24} = \frac{18}{27}$$

$$\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \frac{5}{10} = \frac{6}{12} = \frac{7}{14} = \frac{8}{16} = \frac{9}{18}$$

From the above,  $\frac{2}{3} = \frac{4}{6}$  and  $\frac{1}{2} = \frac{3}{6}$

By comparison,  $\frac{4}{6} > \frac{3}{6}$  hence  $\frac{2}{3} > \frac{1}{2}$

*Addition and Subtraction of Fraction*

Teachers often recommend starting by adding and subtracting as fractions because it is a simpler process than working with different fractions. Like fractions have the same denominators, while unlike fractions have different denominators. To help students with these concepts, many resources are available, such as paper packets, Cuisenaire rods, and number lines.

**Example 1:** find the sum  $\frac{1}{7}$  and  $\frac{3}{7}$

1. Fold the paper liner into 7 equal parts and 1 part to represent the color  $\frac{1}{7}$
2. Fold a strip of paper into 7 equal parts and color 3 parts to represent them  $\frac{3}{7}$
3. Put the two together like

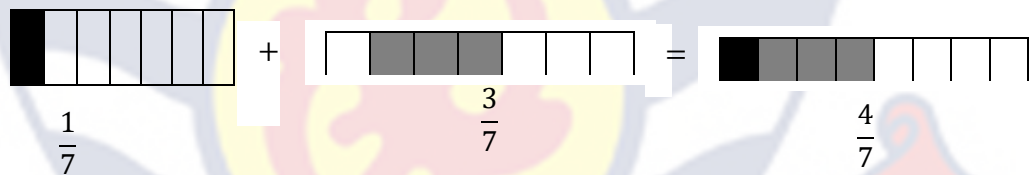


Figure 4: Adding two Fractions

**Example 2:** Find the sum of  $\frac{5}{8}$  and  $\frac{2}{8}$  using Cuisenaire rods

1. Select an appropriate whole that can be divided exactly into 8 parts which is brown
2. Note that when the brown rod is divided into eight, each part is equivalent to one white rod. So, select 5 white rods to represent  $\frac{5}{8}$  and 2 white rods to represent  $\frac{2}{8}$ .
3. Put the two together to get seven white rods.

4. Compare this with the brown rod (whole) and you will notice that it is  $\frac{7}{8}$  of the whole.

5. Hence,  $\frac{5}{8} + \frac{2}{8} = \frac{7}{8}$

When you have used several examples, it will be deduced that in adding like fractions we add the numerators and maintain the denominator. Generally,

addition of like fractions is given as  $\frac{a}{b} + \frac{c}{b} = \frac{a+c}{b}$

For example,  $\frac{3}{9} + \frac{4}{9} = \frac{3+4}{9} = \frac{7}{9}$

**Example 3:**  $\frac{3}{7} - \frac{1}{7}$

1. Fold a strip of paper into 7 equal parts and shade 3 part to represent  $\frac{3}{7}$
2. *Unshade* one part (meaning take away one-seventh)
3. Determine what is left as illustrated below

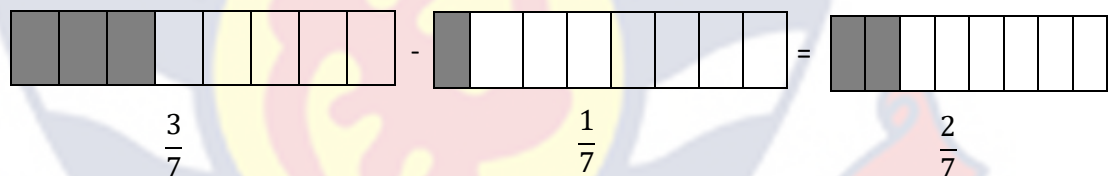


Figure 5: Subtraction of fractions

**Example 4:** Find the difference between  $\frac{5}{8}$  and  $\frac{2}{8}$  using the Cuisenaire rods

1. Choose a suitable place that can be divided into exactly 8 parts, which are brown
2. Note that when the brown bar is divided into eight, each part is equivalent to a white bar. So, choose 5 and bring the bars to represent it  $\frac{5}{8}$
3. Remove 2 white bars  $\frac{2}{8}$  to 5 bars to make 3 white bars
4. Compare it with the brown bar and you will know one of them  $\frac{3}{8}$ .

5. Yes,  $\frac{5}{8} - \frac{2}{8} = \frac{3}{8}$

Using many examples, it is gathered that in the subtraction of similar fractions, from the numerator of the reduction (of the first fraction) subtract the numerator of the reduction and the denominator. Deduction of similar fractions in

general  $\frac{a}{b} - \frac{c}{b} = \frac{a-c}{b}$

*For example,*  $\frac{7}{9} - \frac{5}{9} = \frac{7-5}{9} = \frac{2}{9}$

By adding and subtracting unequal fractions, you must convert unequal fractions into like fractions. This can be done using Cuisenaire rods, equivalent fractions or folding paper. Let's look at these examples:

**Example 1:**  $\frac{2}{3} + \frac{1}{2}$

1. Generate as many equivalent fractions as possible for both fractions as demonstrated below

$$\frac{2}{3} = \frac{4}{6} = \frac{6}{9} = \frac{8}{12} = \frac{10}{15} = \frac{12}{18} = \frac{14}{21} = \frac{16}{24} = \frac{18}{27}$$

$$\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \frac{5}{10} = \frac{6}{12} = \frac{7}{14} = \frac{8}{16} = \frac{9}{18}$$

2. Look out for a pair of fractions from both that are like as in  $\frac{4}{6}$  and  $\frac{3}{6}$ ,  $\frac{8}{12}$  and  $\frac{6}{12}$ ;  $\frac{12}{18}$  and  $\frac{9}{18}$  etc. if add any of these pairs, you will get the same results but it is advisable to use the smaller denominator.

3. This means that  $\frac{2}{3} + \frac{1}{2} = \frac{4}{6} + \frac{3}{6} = \frac{4+3}{6} = \frac{7}{6}$  or  $1\frac{1}{6}$

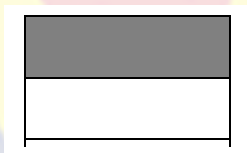
**Example 2:**  $\frac{2}{3} + \frac{1}{2}$  using Cuisenaire rods

1. Select an appropriate whole (rod that can exactly be divided into 3 and 2) which is dark green

2. Select a rod that exactly divides the dark green rod into three equal parts and select two of them to represent  $\frac{2}{3}$ . two red rods
3. Select a rod that exactly divides the dark green rod into two equal parts and select one of them to represent  $\frac{1}{2}$ , one light green rod
4. Exchange the two red rods for 4 white rods and the light green for 3 white rods.
5. Put them together to get 7 white rods
6. Compare the 7 white rods to the whole (dark green) and you will notice that it is  $\frac{7}{6}$  or one whole and a white rod which is same as  $1\frac{1}{6}$
7. Therefore  $\frac{2}{3} + \frac{1}{2} = \frac{7}{6}$  or  $1\frac{1}{6}$

**Example 3:**  $\frac{1}{3} + \frac{1}{5}$  using paper folding

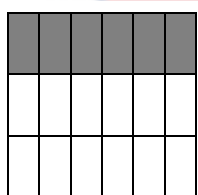
1. Fold your paper horizontally into three equal parts and shade 1 part to



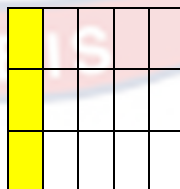
represent  $\frac{1}{3}$  as in

Figure 6: Representation of  $\frac{1}{3}$

1. On the same paper, fold your paper vertically into five equal parts and shade one part to represent  $\frac{1}{5}$  as in



+



=

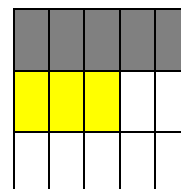
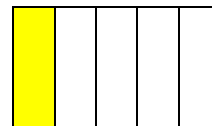


Figure 7: Adding two like fractions

2. Putting the two together we shall get

$$3. \text{ Therefore, } \frac{1}{3} + \frac{1}{5} = \frac{8}{15} \text{ or } \frac{1}{3} + \frac{1}{5} = \frac{5}{15} + \frac{3}{15} = \frac{5+3}{15} = \frac{8}{15}$$

Generally, for addition of unlike fractions,  $\frac{a}{b} + \frac{c}{d} = \frac{a \times d}{b \times d} + \frac{c \times b}{b \times d} = \frac{ad+cb}{bd}$

Example 3:  $\frac{2}{3} - \frac{1}{2}$

### Multiplication of Fractions

#### a. Whole by Fraction

In this method, multiplication is viewed as repeated addition. For instance,  $4 \times \frac{2}{3}$

is interpreted as 4 groups of two-thirds. By applying multiplication as repeated

$$\text{addition, } 4 \times \frac{2}{3} = \frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} = \frac{2+2+2+2}{3} = \frac{8}{3} \text{ or } 2\frac{2}{3}$$

This can be demonstrated using a concrete material like strips of paper.

1. Get four strips of paper of equal size
2. Fold each strip into three equal parts and shade two parts each
3. By rearrangement, you will notice that we will get two wholes and a two-third. This means that  $4 \times \frac{2}{3} = 2\frac{2}{3}$ . Alternatively, by shading each

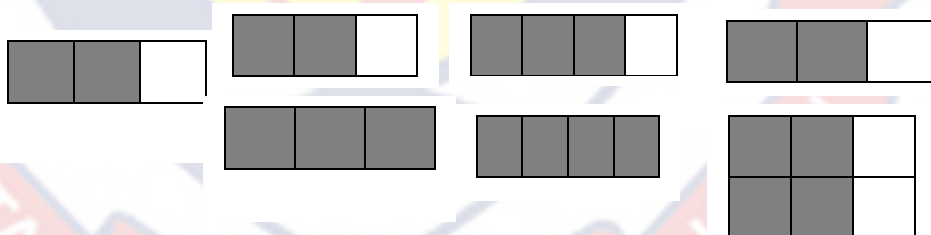


Figure: 8 Shading two part of each whole.

Of the four strips two parts out of three divisions will give us 8 parts being

shaded and each part is one-third meaning in all we will have  $\frac{8}{3}$  or  $2\frac{2}{3}$

This will be same as



When children have worked several examples, they are likely to discover that

$$a \times \frac{x}{y} = \frac{a \times x}{y}. \text{ For example, } 5 \times \frac{1}{6} = \frac{5 \times 1}{6} = \frac{5}{6}$$

**Fraction by a Whole**

This is interpreted as ‘fraction of a whole’. For instance,  $\frac{1}{3} \times 5$  can be interpreted as  $\frac{1}{3}$  of 5 wholes. This can be represented using strips of paper as illustrated below.

- Take five strips of papers of the same lengths and breadths.
- divide each strip into three parts and shade one part
- count the number of one-third in all the five strips
- the results is five-thirds

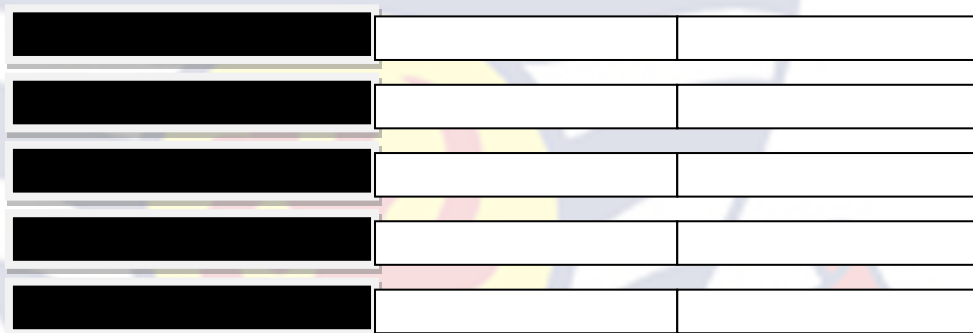


Figure 9: Representation of five - thirds

**Multiplication of a Fraction by a Fraction**

It is interpreted fraction by fraction. For example, it can be  $\frac{3}{4} \times \frac{4}{5}$  interpreted as  $\frac{3}{4}$  of  $\frac{4}{5}$ . This problem can be solved by using either Cuisenaire rods or folding paper. Let's take a look at Cuisenaire's rods first.

1. Choose a suitable whole (a rod that can be divided into exactly five) actually a yellow rod. This is because we need to express the multiplication (the fraction after the multiplication sign).

2. Divide this yellow rod into five (5 white lines) and choose 4 to represent  $\frac{4}{5}$ .
3. Divide these four white bars into four equal parts and get three parts. This will be in the three white bars
4. Compare to set (yellow bar). You will find what he wants  $\frac{3}{5}$

$$5. \text{ SO, } \frac{3}{4} \times \frac{4}{5} = \frac{3}{5}$$

Similarly, folded paper can also be used in the following way.

1. Fold the paper horizontally into four equal parts and three representative parts  $\frac{3}{4}$
2. Using the same paper, fold it vertically into five equal parts, and shade into four representative parts  $\frac{4}{5}$
3. Count the number of double shaded areas as the numerator and the number of areas (odd or double or shaded areas) as the denominator.

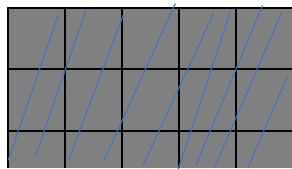


Figure 10: Multiplying a fraction by a fraction

4. The number of gray regions will be doubled to 12, and therefore the total number of regions will be 20.  $\frac{3}{4} \times \frac{4}{5} = \frac{12}{20} = \frac{3}{5}$

**Note:** When children have practiced a lot, they will infer that  $\frac{m}{n} \times \frac{x}{y} = \frac{m \times x}{n \times y}$ ,

$n$  and  $y \neq 0$ . For example,  $\frac{3}{5} \times \frac{4}{9} = \frac{3 \times 4}{5 \times 9} = \frac{12}{45} = \frac{4}{15}$

**Multiplication of Mixed Fractions:**

Example 1:  $1\frac{1}{2} \times 1\frac{1}{4}$

In this example, you need one whole sheet of paper and half of the sheet.

Notice that there are three halves in  $1\frac{1}{2}$  and five quarters in  $1\frac{1}{4}$

1. Fold your paper horizontally into three parts and label each division as one-half to get  $1\frac{1}{2}$ . Shade the entire region representing  $1\frac{1}{2}$
2. Fold that same paper vertically into five parts and label each part  $\frac{1}{4}$  to get  $1\frac{1}{4}$ . Shade the entire region representing  $1\frac{1}{4}$
3. The numerator is the entire double shaded region and the denominator is the region bounded by the first whole of both fractions.

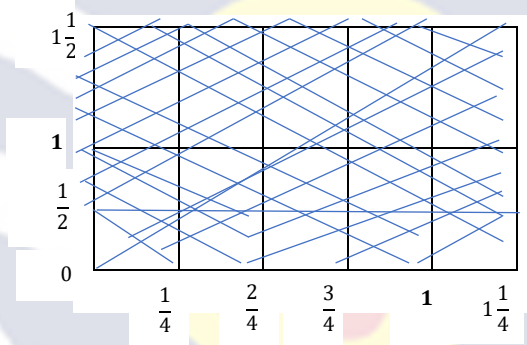


Figure 11: Mixed number multiply by mixed number

The doubled shaded regions are 15 and the regions bounded by the first whole of both fractions are 8. Therefore  $1\frac{1}{2} \times 1\frac{1}{4} = \frac{15}{8}$  or  $1\frac{7}{8}$

Note the following when multiplication involves mixed fractions

- a. The denominator is determined by the area over the first integer of the two fractions, regardless of the integers. For example, the denominator of the product  $2\frac{1}{3}$  and  $3\frac{2}{5}$  corresponds to terminating the first integral of two fractions.

b. When multiplying a mixed fraction by a proper fraction, the divisions of the proper fraction are extended to the first term to obtain the denominator. This extended area is not opaque and cannot be part of an abacus.

c. In general, when multiplying mixed fractions, students should change the mixed fractions into improper fractions, multiply the numerators, and do the same with the denominators. For example,  $3\frac{1}{4} \times 2\frac{2}{3} =$

$$\frac{(4 \times 3) + 1}{4} \times \frac{(3 \times 2) + 2}{3} = \frac{13}{4} \times \frac{8}{3} = \frac{13 \times 8}{4 \times 3} = \frac{104}{12} = \frac{26}{3} = 8\frac{2}{3}$$

### Division of Fractions

Generally, for  $\frac{a}{b} \div \frac{x}{y}$ , it is interpreted as how many  $\frac{x}{y}$  *th* are there in  $\frac{a}{b}$ . This is in direct opposite to multiplication. Division is therefore generally done by inversion.

### Whole by a Fraction

This is interpreted as how many of the fractional parts are in the number of wholes. For example,  $4 \div \frac{1}{3}$  can be interpreted as how many  $\frac{1}{3}$  are there in four wholes.

- take four circular cuts-out of strips of papers
- divide each strip into three parts
- count the number of one-thirds in all the four
- it will be discovered that there are 12 one-thirds

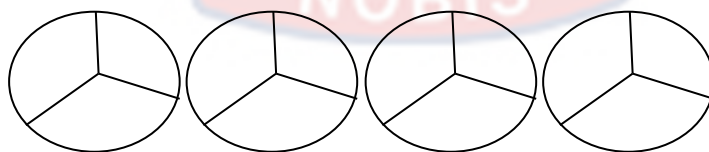


Figure 12: Representation one-thirds in four circular cuts-out papers.

### A Fraction by a Whole

This is interpreted as dividing the fraction into the number of wholes. For instance,  $\frac{1}{2} \div 4$  can be interpreted as 'if four people share one-half orange what will each person receive. This can be illustrated as follows

- take one strip of circular paper cut-out
- divide it into two equal parts and shade one part to represent  $\frac{1}{2}$
- further divide the shaded half into four parts and shade one part
- you will notice that each half is divided into four the total number of segments are eight representing the denominator and the double shaded portion is the numerator. Hence,  $\frac{1}{2} \div 4$

### Fraction by a Fraction

This is also interpreted as how many of the divisor (second fraction) for the dividend. To do this, we model the first fraction and then divide the same whole into bits of the second fraction. Then we count how many of the divisor form the dividend. Let us look at  $\frac{2}{3} \div \frac{1}{2}$

1. Fold a sheet of paper horizontally into three equal parts and shade two parts representing  $\frac{2}{3}$ .
2. Fold that same paper vertically into two parts
3. Compare the shaded region ( $\frac{2}{3}$ ) to half wholes by rearranging the shaded regions into halves of the whole sheet.
4. You will notice that we will get 1 half whole and  $\frac{1}{3}$  half whole. Hence

$$\frac{2}{3} \div \frac{1}{2} = 1 \frac{1}{3} \text{ or } \frac{4}{3}$$

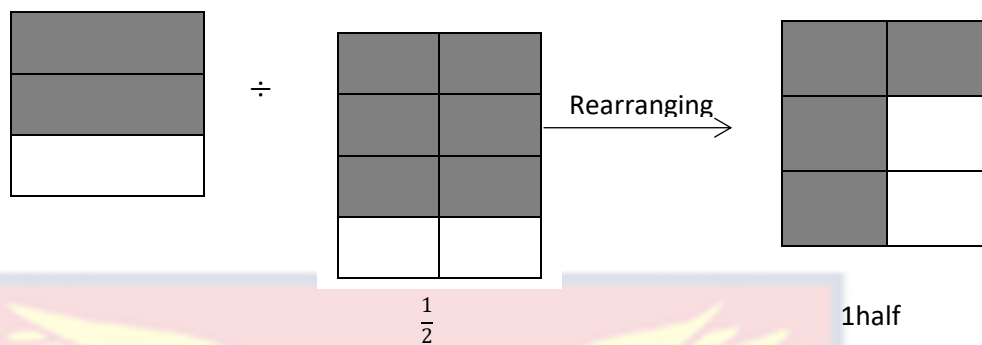


Figure 13: Fractions divided by another fraction

In general,  $\frac{a}{b} \div \frac{m}{n} = \frac{a}{b} \times \frac{n}{m}$ . For instance,  $\frac{3}{4} \div \frac{2}{7} = \frac{3}{4} \times \frac{7}{2} = \frac{21}{8}$  or  $2\frac{5}{8}$

### Introducing Decimals Fractions

The first decimal to introduce will be the tenth. Here, you divide your strip of paper into ten equal parts and shade a number of them. If you have shaded one portion, then that will be one-tenth. Tell your pupils that  $\frac{1}{10}$  is also written as 0.1. Similarly,  $\frac{4}{10}$  is also written 0.4 and pronounced 'zero-point-four'.

To introduce the hundredth, you take a 10by10 paper grid. Shade a number of them say 16. Represent this as a fraction which is  $\frac{16}{100}$

### Converting terminating decimals to common fractions

Decimals such as 0.5, 0.05, 0.25 0.125 and so on are called terminating or finite decimals. For terminating decimals such as 3.756, the place-values of the decimal digits of 3.756 means  $\frac{3}{1} + \frac{7}{10} + \frac{5}{100} + \frac{6}{1000}$ . Because all of the denominators are powers of ten, finding a common denominator makes adding these fractions a breeze. Put all the digits over the denominator that corresponds to the last decimal place value as a general rule.

When you convert from decimals to repeating common fractions, you cannot specify the number of digits after the decimal point. So, we create two equations where their decimals will be the same. First, we separate the terminal faces by multiplying them by the proper power of ten if they exist. We then multiply the direct equation by the appropriate power of ten to get our general equations. Then one equation is subtracted from the other. This is illustrated in the following examples

$$\text{Example 1: } 0.5 = \frac{0.5 \times 10}{1 \times 10} = \frac{5}{10} = \frac{1}{2}$$

$$\text{Example 2: } 0.25 = \frac{0.25 \times 100}{1 \times 100} = \frac{25}{100} = \frac{1}{4}$$

$$\text{Example 3: } 3.25 = \frac{3.25 \times 100}{1 \times 100} = \frac{325}{100} = \frac{13}{4} = 3 \frac{1}{4}$$

### Converting recurring decimals to proper fractions

Repeating/recurring decimals is decimal representation of a number whose digits are periodic and the infinitely repeated is not zero. Examples include  $0.\dot{3}$ ,  $0.\dot{1}2$ ,  $2.2\dot{1}5$ ,  $\ddot{0}.\overline{124}$ . Note that the dot or lines on top of the digits indicate that those digits are repeating. In converting recurring decimals to common fractions, you cannot determine the number of digits after the decimal point. So, we generate two equations where their decimal aspects will be the same. We first separate the terminating aspects by multiplying by the appropriate power of ten if it exists. Next, we then multiply the immediate equation with appropriate power of ten to have our two common equations. We then subtract one equation from the other. This is illustrated in the following examples





$$y = \frac{233}{990}$$

$$\text{Therefore } 0.2\dot{3}\dot{5} = \frac{233}{990}$$

### Operations with Decimals

Some of the manipulative materials / aids for addition, subtraction, multiplication and division of decimal fractions include base ten materials such as an abacus and multi-base number block (Day log) and folding paper.

#### Addition and Subtraction of Decimals

**Example 1:**  $2.34 - 1.85$ ,

1. Express 2.34 with 2 flat, 3 long and 4 square.
2. Subtract 1.85, 1 letter, 8 lengths and 5 squares from the reduction. When we subtract, we start at the centimeter column. Subtract 5 units/beds from 4 beds. But we don't have enough centimeters to 5 centimeters. So to compose himself. We change 1 long with 10 squares to 14 squares (centimeters). Now we can subtract 5 squares from 9 squares.
3. Let us now pass to the long years which the tithes represent. Subtract 8 lengths from the remaining 2 lengths. But we don't want enough to buy 8. Again, we buy 1 division for 10 longs (decimal) and now we get 12 longs. Now we can go from 12 long (tenths) to 4 long.
4. Now let's move on to the rooms. Subtract 1 step from the remaining step so that there is no mass.
5. This means that the remaining difference would be 4 lengths (decimals) and 9 cubic/units (centimeters) symbolically written as 0.49.
6. Therefore  $2.34 - 1.85 = 0.49$ .

$$2.56 + 1.7$$

1. Represent 2.56 with 2 flats (ones) 5 longs (tenths) 6 cubes (hundredths) and 1.7 with 1 flat 7 longs.
2. Put the two together to get 3 flats 12 longs 6 cubes.
3. Exchange the 10 of the longs for 1 flat to get 4 flats, 2 longs and 6 cubes which is symbolically expressed as 4.26,
4. Hence,  $2.56 + 1.7 = 4.26$

### Multiplication of Decimal Fractions

**Example 1:**  $0.6 \times 0.3$

1. Fold a sheet of paper horizontally into ten equal parts and shade 6 portions representing 0.6
2. Fold that same paper vertically into 10 equal parts and shade three portions to represent 0.3
3. Count the double shaded region as the numerator and entire regions as the denominator.
4. You will notice out of the 100 portions; it is 18 portions that are double shaded which can be written as 0.18.

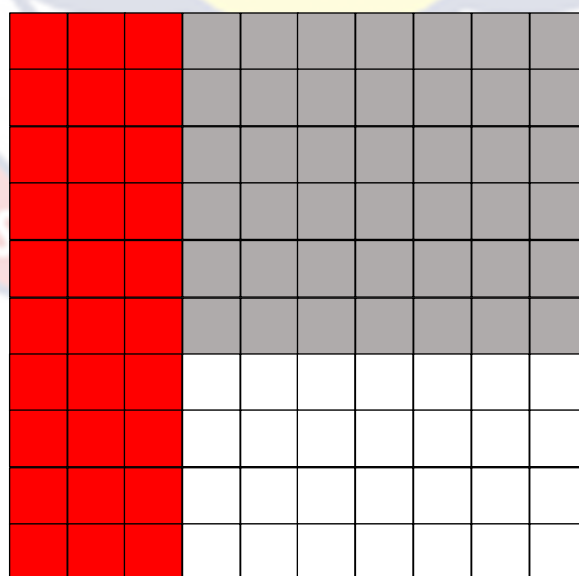


Figure 14: Representation of a decimal fraction multiply by decimal fraction

In general, when multiplying decimal fractions, multiply the numbers by whole numbers and move the decimal point according to the number of digits after the decimal point when multiplying two decimal fractions. For example,  $0.25 \times 0.5$  would be  $25 \times 5 = 125$ . There are two numbers after the decimal point in the first decimal fraction and one in the second, which make three numbers. So, we move three decimal places to the left to get 0.125.

### **Pupils Attitude towards the Learning of Fractions**

Different perceptions of attitude. According to Oppenheim (2001), attitude has to do with how and how a person responds to the stimuli he is exposed to. According to him, these attitudes are generally enhanced by the beliefs of individuals and are expressed in speech or behavior. In this study, students' attitudes were measured using a four-point Likert scale with the main purpose of identifying their attitudes towards learning roles. The research will further explore whether there is a gender difference in pupils' attitudes towards learning fractions. This is because attitude has been found to be positively correlated with student achievement. This means that a highly positive attitude is more likely to lead to higher achievement scores and vice versa.

There was a dearth of literature in the affections of study in the fractions of learning. Most studies have focused on attitudes toward learning mathematics in the classroom. A study of student behavior towards learning fractions was conducted by Raiman (2001) with Bruneian students. In his study, he found that about two-thirds of students said they approved of fractions, while 28% said the opposite, with the remaining 5% undecided. And he added that those who said that they liked fractions said that they did, because they found fractions easy

and useful for their daily activities. Those who have not done fractions say that working with fractions is not only difficult, but also boring.

Groff (1994) recounting his experience as a teacher claimed that most of his students disliked learning fractions because they saw fractional activities as "dead end activities." This means that the students disliked fractions because they felt that the study of fractions was not useful for them in their daily activities. These students, according to Groff, therefore experienced parts of their studies as madness and unnecessary. This study attempted to use Ghanaian students instead of Bruneian students if the same attitude exists.

Yusof (2003) undertook a longitudinal study to investigate Bruneian students' errors in mathematical fractions. As part of his focus, he explored P5 and P6 students' intentions to learn based on four constructs, namely enjoyment, understanding, engagement and confidence. His study found that four attitude substructures had higher mean scores ranging from 3.51 to 4.33 out of a possible five. This was observed in P5 and P6. He decided that more students had a better attitude towards learning in fractions.

### **Gender Difference in Attitude towards the Learning of Fractions**

As stated above, research has shown that students' attitudes can significantly affect their mathematical behavior. While many studies have looked at differences in achievement between boys and girls, few of them have looked at differences in attitudes towards learning mathematics. The results of the limited research on gender differences in attitudes toward mathematics have been inconsistent, with some studies reporting no significant differences and others reporting significant differences. For example, research by Relich (1996) shows that boys tend to have a higher attitude towards learning mathematics

than girls, which can lead to better academic performance. However, he noted that these findings may not be universally applicable and may vary by context.

Another study, by Marsh, Smith and Barnes (1985) with Bruneian students, found that while girls performed better than boys on standardized math tests, boys were more confident than girls in math. This is possible because girls tend to overestimate their own mathematical performance (Thomas and Costello, 1988), which makes boys more apt in mathematics. The researchers also wanted to see if there were gender differences in how students approached learning. Yusof (2003) found no statistically significant differences in attitudes toward learning fractions between male and female students ( $t=0.091$ ,  $df=394$ ,  $p>0.05$ ) in research examining differences in gender attitudes. This suggests that male and female students felt the same way about learning fractions. However, it is clear that it is an educational institution for Bruneian students, with the majority of teachers in Brunei being women. These female professors may serve as role models for female students. This study, however, employs kids from Ghana, where the majority of teachers are men.

### **The Concept of Manipulatives**

Manipulative materials have attracted a lot of definitions. Manipulatives, according to Moyer (2001), are tangible things that are utilized to demonstrate abstract mathematical principles in a concrete and visible manner. Students physically interact with them to grasp the content with their hands, and they engage by sight and touch. However, Moyer notes that students can sometimes use repetitive manipulatives without fully understanding the principles of mathematics. In order to use manipulatives effectively, it is important to know how to incorporate mathematical structures and understand their meaning to the

user. Students are generally seen as significant factors in the classroom and it is important to provide them with opportunities to learn and manipulate and connect with the manipulatives.

Moyer (2001) defines manipulatives as concrete objects that serve to explain abstract mathematical concepts. These tools are not only visually appealing, but also provide a tactile learning experience for students. Moyer also emphasizes the importance of familiarizing students with the manipulations to reduce cognitive load. If the student is not familiar with the features of the operator, he may have difficulty using them as a representation of some mathematical structure. In my classroom, I have found that students need a lot of practice with Cuisenaire rods, as well as a good understanding of the proportional relationships between rods of different colors, before they can use them to solve complex problems such as adding fractions.

Children's mental representations and their understanding of abstract ideas are strongly influenced by personal experience. Thus, children who have had more opportunities to observe and manipulate different objects tend to have a clearer mental representation and a better understanding of abstract concepts compared to those who have limited experiences (Kennedy, 1986). Manipulators have a long history in mathematical programming. For example, in the nineteenth century, Johann Pestalozzi (1746-1827) advocated for the use of manipulatives such as blocks to teach youngsters number sense (Saettler, 1990). Similarly, Maria Montessori emphasized the significance of hands-on exploration in the first Montessori school, which opened in 1907, employing things such as balls, puzzles, and wooden forms to aid children's learning.

Research has shown that using specific objects like slices, fraction circles, and Cuisenaire rods can help children better understand and develop mental representations of mathematical concepts like fractions. These manipulatives provide a real and visual depiction of abstract mathematical concepts, allowing students to experiment with and learn about mathematical principles through hands-on experiences. This is consistent with Piaget's constructivism theory, which emphasizes the necessity of active learning and discovery in knowledge production. Using manipulatives can definitely help develop a deeper understanding of mathematical concepts, especially when it comes to number concepts. By manipulating physical objects and observing their relationships with each other, children can create mental representations and develop a more intuitive sense of mathematical relationships. As you said, partial connections can be especially useful for illustrating the concept of part-whole relationships, which is a fundamental concept in many areas of mathematics.

Virtual manipulators are considered the most convenient and practical manipulative gear. As Moyer (2002) notes, visual representations are electronic devices that allow students to reproduce large amounts of material. According to Bouck and Flanagan (2010), several websites have been created to provide teachers with access to these virtual manipulatives for use with their students. These sites can complement educational institutions and broaden the scope of the assessment, as Johnson, Campet, Gaber and Zuidema (2012) demonstrate. It should be noted, however, that computer software is not the only source of information for students. According to the National Council of Teachers of Mathematics (NCTM), students learn mathematics by creating and using

representations to organize, document, and express concepts. They will be able to select, apply, and transfer various representations to solve issues and materials to demonstrate and comprehend physical, social, and mathematical processes. According to Moyer-Packenham, Ulmer, and Anderson (2012), it is critical for students to visualize concepts outside their expertise with computers.

Suh, Moyer and Heo (2005) discuss the positive side of using virtual manipulatives, namely that many of them are incorporated by teachers into their activities, allowing students to better understand mathematical concepts. This will allow teachers to use their time to improve the physical resources used in this interactive learning activity.

Moyer, Bolyard, and Spikell (2002) also noted other benefits of virtual manipulatives, such as ease of use in the classroom, students having access to computers, and older students' understanding that computers are age appropriate. In addition, Clements and McMillan (1996) highlighted various benefits of virtual manipulatives, including increased student motivation and engagement, ease of use, ability to monitor student progress, and assessment of competencies. Johnson, Campert, Gaber, and Zuidema (2012) recommend that teachers follow specific recommendations when using manipulative virtual assessment tools. They should consider the extent to which the virtual abuse target addresses the topic, how it uses technology, and how it elicits meaningful insights for student learning. Virtual manipulatives can be used for small group work in addition to individual assessments. They can also support interactive peer learning groups, as children generally prefer to work together rather than alone, as Clementis (2002) notes. Rosen and Hoffman (2009) observed that students who are able to use a virtual keyboard, approach study tasks with pride



and show more positive emotions and enthusiasm for the activity. This greater involvement allows children to share their ideas and collaborate with each other to develop them. In one example Mrs. Smith uses direct deformation to explore how shapes are represented and measured in Grade 1. and the pattern block.

To build first-grade math skills, students are required to build models of concrete materials and illustrate them with drawings, according to the National Council on Geometric Mathematics Education criteria. The effectiveness of these exercises was evaluated by the teacher's consideration of the student-student interactions that they will handle in the learning process. This is a good example of how virtual manipulatives can enhance the use of real materials in the classroom. According to Joyner (1990), teachers should allow students to freely explore manipulatives, arrange resources according to the lecture, make learning objectives clear, and shape the use of materials in order to use the materials effectively. Stein and Bovalino (2001) interviewed teachers who used manipulations to demonstrate good teaching practices. Teachers from Stein and Bovalino (2001) shared three things in common: they had extensive training in abusive practice, they planned their lessons and anticipated students' barriers, and they spent time preparing classrooms and equipment. However, several issues do not prevent the implementation of manipulatives in classrooms. Some professionals argue that using these facilities consumes too much time, but factories can also produce the same amount of time, according to Suydam (1986). Others do not know how and when to use concrete materials, even if they have the time and resources (McBride & Lamb, 1986). In addition, when teachers expect students to acquire skills too early or ask them to complete

activities gradually, the goals of using these materials can be undermined (Moch, 2001).

Research has shown that manipulatives have a higher purpose and significantly improve students' understanding of mathematical concepts. Therefore, teachers should consider manipulatives not only as a "fun" activity, but also as a powerful teaching tool. In addition, it is important for educators to recognize that manipulatives are not a substitute for good educational instruction, but rather an enhancement of it (Sowder, 2007). Finally, the success of trickery in the classroom depends on teachers' knowledge, skills, and abilities to effectively integrate the curriculum. Students are given access to these tools at the end of each lesson. Friday of the week or at the end of the school year when district assessments are completed (Moyer, 2001).

In summary, the use of manipulatives in mathematics education has been debated among educators. Some say that it is not necessary or too long to implement. However, research has shown that, when used effectively, manipulatives can significantly enhance students' learning experience and improve their motivation for mathematics. Teachers must thoroughly understand the mathematical concepts they teach and receive training on how to use manipulatives effectively in the classroom. With the right training and experience, teachers can create a positive and engaging learning environment that promotes student achievement in mathematics.

### **Attitude of Teachers towards the use of Manipulative Materials**

In fact, teachers' beliefs and attitudes toward the use of manipulatives greatly influence their implementation in the classroom. Some teachers may have misconceptions about the effectiveness of manipulative, or believe that

they are not necessary for teaching mathematics. However, research has shown that when used effectively, manipulatives can improve students' understanding and engagement in mathematics. Marshall and Swan (2008) concluded that the use of manipulative materials in teaching mathematics is effective when they are used in the long term rather than when they are used in short term. This highlights the importance of professional development opportunities for teachers to learn and teach the use of manipulatives in their teaching. In addition, it is important for managers to understand that using manipulatives should be a long-term and not a short-term solution. Consistent usage of manipulatives over time can help children build a deeper comprehension of mathematical ideas and excellent problem-solving skills.

Several factors, including teachers' increased confidence in their ability to teach mathematics without abuse, time restraints, or a lack of resources, could account for the decline in the use of manipulatives. However, it's crucial to point out that the use of manipulatives significantly improves students' comprehension and interest in mathematics. Teachers must receive appropriate training and support to effectively integrate abusive teaching practices and sustain their use over time. Additionally, Uribe-Florez and Wilkins (2010) suggest that ongoing professional development opportunities and access to high-quality manipulative tools can help teachers be more consistent throughout their learning careers.

Discrete fractions is indeed a difficult subject for elementary school children. Manipulatives can be especially useful for teaching fractions because they help children understand the concept of fractions as parts of a whole. Empson and Jacobs (2008) claim that manipulatives can aid children in

understanding fractional concepts like equivalent fractions, comparisons, and addition and subtraction. Teachers can help their pupils build a strong foundation in fraction concepts that will help them in later grades by using manipulations when teaching fractions.

Education and training of teachers in the use of manipulatives is vital to their effective integration into classroom instruction. Teachers who lack the knowledge and training on how to effectively use tools to understand their full potential in improving students' mathematical understanding. It is important that teachers carefully design and select manipulatives that are age appropriate and suitable for teaching mathematical concepts. In addition, teachers must create a classroom environment that encourages students to question, explore, and take risks in abusive ways. This can help students gain a deeper understanding of mathematical concepts and improve their problem-solving skills.

It is important to note that while these concerns are valid, research has shown that the benefits of using manipulatives in mathematics education outweigh the potential challenges. Teachers who receive appropriate training and support can effectively integrate their manipulatives into lessons and address these concerns. Indeed, as stated above, initial teacher training in the use of manipulatives has been shown to improve math skills and increase awareness of the value of manipulatives in the classroom (Green, Piel, & Flores, 2008). Additionally, there are numerous free or inexpensive treatment options available, including using commonplace items like beans or toothpicks. Planning carefully and including the use of manipulatives into routine education might help overcome time restrictions. Teachers can effectively employ

manipulatives to enhance students' math education with the right planning, support, and instruction.

Teachers in Moyer's (2001) study also mentioned employing manipulatives to make abstract mathematical topics more concrete for children to understand and to assist them learn concepts that are challenging. Teachers also used manipulatives to facilitate group work and engage students who were working with traditional pencil and paper methods. Overall, the study found that teachers who were trained in the use of manipulatives were more likely to use them in their classrooms and use a variety of methods to support student learning.

It is important for educators to understand that manipulatives are not just for play, but necessary tools for effective mathematics instruction. When teachers use props appropriately and intentionally, they provide students with visual and tactile experiences that help them understand abstract mathematical concepts. Teachers trained in the most effective use of mathematics are more likely to incorporate it into their teaching and help their students develop a deeper understanding of mathematical concepts. Using manipulatives, teachers can create an engaging, interactive and meaningful learning environment that leads to better student learning outcomes.

Teachers can employ manipulations as tools to better comprehend the mental processes of their pupils (Naiser, Wright, & Capraro, 2004). According to Naiser et al., the teacher can advise that the student's conceptual comprehension be evaluated by seeing how they engage with and interpret the manipulatives. According to Hatfield, teachers must possess the knowledge, expertise, and experience necessary to effectively address the requirements of

pupils while using manipulatives in mathematics instruction. According to studies, some teachers are less likely to use handouts because they feel they don't improve math instruction, and Hougas concurs that teachers need to be trained on how to use them properly. Teachers should be confident in using a variety of non-invasive materials in appropriate settings that are safe and engaging for both students and teachers.

According to the findings of a local researcher named Gyok, teachers often lacked the skills to use manipulatives effectively and relied only on demonstrating and explaining concepts. The students were also unable to get used to concrete materials due to the lack of supplies. Giok's research found that factors such as time constraints, class size, availability of faculty, and teachers' pedagogical skills influenced the choice of classroom manipulatives.

### **Impact of Manipulatives on the Attitude of Pupils**

Farooq and Shah taught that teachers believe that students' attitudes toward mathematics play an important role in their performance and are important steps toward promoting positive attitudes (2008). According to Slavin (1995), teaching specific materials in mathematics promotes cooperative learning environments that encourage active participation and participation. Suh, Moyer and Heo (2005) suggest that virtual manipulatives can help students overcome their fear of making mistakes in whole-class activities because they allow students to take risks without worrying about negative feedback. This in turn increases their willingness to participate and learn.

In a study by Baki, Kosa and Guven (2011), the effects of manipulations on learning outcomes were compared to those of standard teachers. The study found that students who received manipulative-based instruction as well as

computer-assisted instruction showed significant improvement in their final semester scores compared to those who received non-manipulative mathematics delivery. Students who had abusive instruction specifically displayed a higher rise in their scores from the start of the semester to the end.

In order to better understand how manipulatives (different types of learning materials) affect high school students' academic behavior and attitude toward mathematics, Kontas (2016) carried out a study. Pre-post-test control group experimental design, a type of experimental research methodology, was employed in the study. In the 2014–2015 academic year, 48 seventh graders from a public school in southern Turkey—24 in the experimental group and 24 in the control group—made up the study group. The study discovered that the math post-test caused the academic outcomes for the experimental and control groups to differ significantly, favoring the post-test. Additionally, the results of the post-test showed that the attitude of the experience group toward mathematics was significantly more favorable than that of the control group.

Ozgun-Koca and Edwards (2011) liked that students use manipulatives in class and thought that doing so was engaging and helpful for learning new ideas. Students have psychological demands to control the learning process, according to Deci, Koestner, and Ryan (1999). As a result, teachers work to encourage natural learning through the use of manipulatives because doing so produces good results. Students who are intrinsically driven typically just adhere to their teacher's norms and procedures, which reduces their ability to engage in more abstract cognitive processes and gain knowledge about academic achievement. In contrast, intrinsically motivated students are more likely to choose their own problem-solving tactics and tools.

### **Impact of Manipulative Materials on the Performance of Pupils**

A crucial component of a student's education is mastering math. For survival in the modern world, mathematical aptitude is necessary. Mathematical abilities are crucial in our daily lives as well as for academic objectives. For students to become adults, they must understand fundamental math concepts like fractions. One way to teach math concepts, including fractions, is to use manipulatives. The impact of manipulations on pupils' mathematical performance has received extensive research. A overview of the research on the efficacy of manipulatives in mathematics instruction was done by Suydam and Higgins and Sowell in 1976. Suydam and Higgins examined 40 research ranging from grades 1 to 8, finding that 24 of them supported the use of manipulations, 12 found no significant difference with or without procedures, and four found evidence against manipulation. Suydam and Higgins come to the conclusion that engaging materials should be used successfully in classes to increase the likelihood of improved arithmetic performance. Similar findings were reported in Sowell's recent analysis, demonstrating that the success of manipulative materials in many trials is attributable to extensive pedagogical preparation in the teaching of these materials. Sowell does not specify whether children can benefit from using manipulatives at a particular level or for a specified amount of time, though strong support for the use of manipulatives in mathematics instruction may be found in a meta-analysis by Carbonneau, Marley, and Selig (2013). Positive learning practices had a statistically significant impact on learning with small to moderate effect sizes, according on data collected from 55 research involving more than 7,000 students from kindergarten through middle school. Retention effects ranged in size from



moderate to substantial, whereas those for problem solving, transfer, and justification were much smaller. Despite the fact that certain students may benefit from manipulations, it should be remembered that not all pupils will respond well to them. When deciding whether or not to use manipulatives in their lessons, teachers must take into account the variety of students' learning needs and preferences. Maximizing the advantages of manipulatives in the classroom also requires providing teachers with the necessary training and assistance.

Manipulatives have been shown to promote hands-on, concrete learning experiences that can help students develop a deeper understanding of mathematical concepts. Because it places a strong emphasis on the use of visual and manipulative aids to assist students in making connections between abstract mathematical concepts and real-world situations, this method of teaching mathematics is frequently referred to as the "visual-spatial" approach. Manipulatives can assist kids in acquiring critical problem-solving abilities, boost math engagement, and motivate them to learn math in addition to increasing math-based learning understanding.

A meta-analysis of earlier studies on the use of manipulatives in mathematics instruction was undertaken by Swan and Marshall (2010), and they discovered that these tools were effective in enhancing student learning and accomplishment. The usage of manipulatives and the growth of students' conceptual knowledge were also found to be significantly correlated. In order to respond to your new inquiry, Belenky and Nokes' (2009) study looked into how manipulating problem-solving strategies affected students' understanding of mathematics. In contrast to students who used abstract manipulatives or

concrete manipulatives with structured issue prompts, the study indicated that students who used concrete manipulatives with metacognitive prompts demonstrated a higher level of procedural leadership transfer. The inclusion of manipulatives and prompts improved student engagement and recall, according to the study.

Interestingly, prompting through thinking with specific manipulations seems to be more effective for low-engagement students, while problem-based problems are more effective for high-engagement students. This suggests that teachers need to adapt their teaching to the needs of each student and provide different stimuli to support their learning. Moreover, the finding, which showed strong learning in all groups, encourages and supports the use of manipulations in teaching mathematics.

Researchers have found manipulative ways to improve children's understanding of concepts. Correlations were found to be significant and significant for both effective and ineffective manipulations involving students' mathematical communication. This suggests that manipulatives are useful for teaching and learning mathematics (Belenky & Nokes, 2009; Kosko & Wilkins, 2010). The easiest way for kids to learn is to explore their surroundings, frequently through play, where they can create mental images of the outside world. Toddlers need concrete experiences and abstract concepts to learn, grow, and reflect before the formal stage of Piaget's work. The use of manipulatives has been shown to facilitate the development of the arithmetic mind and improve operational and conceptual understanding compared to traditional delivery methods. A large body of evidence suggests that manipulatives help

students learn and achieve higher levels of mathematical proficiency (Alghazo, Alsawaie, & Al-Awidi, 2010; Swan & Marshall, 2010).

In the past, mathematics education systems focused on processes where students had to remember specific steps to reach the correct answer (McLeod, Vasinda & Dondlinger, 2012). However, recent studies show that learning mathematics takes more than just memorizing and completing worksheets. It requires the use of mathematical thinking and abstract reasoning to understand rules, and students to understand concepts to learn truth (Moyer, 2001). Belenky and Nokes (2009) argue that the use of specific materials, combined with metacognitive guidance from teachers, depends on students' ability to solve complex internal problems. Using manipulatives, students can relate new material to prior knowledge and key abstract features through reflection, which improves student performance.

Although the devices have been shown to have many benefits, not all research supports their use. McNeil, Uttal, Jarvin and Sternberg (2009) found that when students were given specific materials that were identified as real materials, such as coins and tickets to count money, more of them made arithmetic errors. They decided to provide these manipulatives with both advantages and disadvantages, as they were sensibly rich. Another problem is the lack of support for students as they move from concrete to abstract mathematical concepts (Clements & McMillen, 1996). Moyer (2001) argues that manipulatives can also interfere with student outcomes, as they add an extra level that some students may find difficult.

## **Empirical Review on the Use of Manipulative Materials Among Upper Primary Teachers in Teaching Fractions**

The growth of mathematical competence and knowledge requires an understanding of mathematical concepts. The importance of mathematics in practically every aspect of life development cannot be overstated. All technical progress in the world is built on the foundation of mathematics. Without proper and sufficient mathematical understanding, there can be no real advancement in this modern technology period. Mathematics improves one's comprehension of the world by using symbols and abstract representations to depict phenomena. It is a subject that is critical for people's academic success, regardless of their program of study; nevertheless, many pupils do not have access to instruction that leads to such success.

What comes to mind when thinking about the difficulty of mathematics is perhaps how it is taught or presented in our classrooms. Teachers are said to require "a comprehensive comprehension of fundamental mathematics" in order to effectively teach mathematics. Every mathematics instructor faces the challenge of making the subject relevant to pupils. Unfortunately, the majority of primary school instructors lack this expertise, and teacher professional development focusing on a strong understanding of mathematics is critical for teachers to provide outstanding educational experiences for their pupils. Educationalists have long promoted the use of manipulatives in the classroom, to the point where they are now ubiquitous in the teaching of fractions in primary schools. Yet many mathematics teachers use them. Manipulatives are physical tools that are used to depict a mathematical concept such as fractions. Manipulatives are instructional resources that improve pupils' conceptualization

and interpretation processes while facilitating teaching and learning (Tunç, Durmuş, & Akkaya, 2011). Several studies back up the use of manipulatives in mathematics as a means of assisting pupils in forming internal representations. Manipulatives are frequently recommended as an effective teaching approach for mathematics. Teachers said they don't use manipulatives in their courses because of the time commitment and poor results (Kontas, 2016).

Manipulatives are concrete learning materials that assist pupils to concretize abstract concepts (Boggan, Harper, & Whitmire, 2010; Cope, 2015; Laski, Jor'dan, Daoust, & Murray, 2015; Ojose & Sexton, 2009; White, 2012). As a result, by providing real experiences, they will be able to develop a link between manipulatives and abstract mathematical concepts, and their mathematical skills will be long-lasting (Holmes, 2013). It has also been articulated that those manipulatives also aid pupils fit in their knowledge and relate it with their opinions so that they can fully comprehend mathematical concepts (Boggan, Harper, & Whitmire, 2010); they help pupils communicate with their own mathematical thinking and bring their mathematical ideas to a higher cognitive level (Ojose & Sexton, 2009). By allowing both pupils and teachers to actively participate in the learning process, manipulative tools elicit amusement (Boggan, Harper, & Whitmire, 2010).

Many teachers believe that manipulatives will instantly help comprehension and that pupils only need to touch and look at manipulatives to understand. This Chapter offers a brief literature review of the study. This study seeks to examine the use of manipulatives materials in teaching fractions among upper primary six schools' teachers in Aowin Municipal, Ghana. The advantages of drawing on a range of research will be exemplified by considering

teachers perception and use of manipulative materials and what must be the most basic manipulative that is available.

UribeFlórez and Wilkins (2010) investigated the usage of manipulatives in primary schools. They looked at the association between teachers' background characteristics, beliefs about manipulatives, and the frequency with which manipulatives are used in mathematics education using data from 503 in-service elementary teachers. Findings from the study show that teachers' grade level and beliefs about manipulatives are important predictors of teachers' use of manipulatives in their mathematics instruction. Their study did not pay a particular attention to fractions which research has proven to be difficult to learn by pupils. This study investigates their attitude towards the use of manipulative materials in teaching fractions. This study further investigates the impact of the use of these manipulative materials on the performance of the pupils as compared to the traditional talk and chalk approach.

Brijlall and Niranjana (2015) investigated the role of manipulatives in the teaching and learning of trigonometric ratios among grade 10 pupils in another study. The strategy aims to address the three areas of intelligence identified by the Multiple Intelligence Theory (linguistic/verbal intelligence, logical/mathematical intelligence, and spatial intelligence). The interpretative paradigm was used to base this study on a case study involving five grade 10 mathematics pupils at a South African high school. The following procedures were used to acquire data: (1) an activity sheet with written responses from pupils; (2) observations; and (3) semi-structured interviews. The data was analysed, and it was discovered that using manipulatives in mathematics teaching and learning helped pupils understand trigonometric ratios. Generally,

their findings confirm the findings of previous that argue that using manipulative materials to back an embodied approach to learning promote the development of conceptual and procedural understanding of mathematical concepts. This study however uses the experimental design to measure the extent to which manipulative materials influence the performance of pupils as compared to other approaches to teaching fractions. It also draws its respondents from the Ghanaian context with a special focus on fractions.

Sandir (2016) investigated the manipulative material design processes of preservice mathematics teachers in a study. Interviews and questionnaires were used to gain information about how preservice instructors come up with novel manipulatives. It was discovered that preservice mathematics teachers are having difficulty coming up with new concepts for manipulative material design. When they tried to turn their thoughts into physical models, they came across structural problems. Furthermore, their concepts and designs may differ significantly from those created by specialists.

Kablan (2016) also investigated the impact of manipulatives when used in conjunction with traditional mathematics instruction, as well as how varied amounts of time spent manipulating affect pupil progress across different learning styles. Three learning environments with varied amounts of traditional teaching methods and manipulative methods were designed. In one of the learning contexts that is more conducive to abstract learning, the teacher primarily used lecture and exercise-based teaching modalities. Abstract learners outperformed concrete learners academically in a context where only traditional methods were used. In the other two contexts, which included a combination of manipulative tools and traditional procedures, the differences in mathematics

achievement levels among youngsters of different learning styles were not statistically significant. The study also discovered that when manipulatives were used, concrete learners performed better in mathematics than their counterparts in the environment where only abstract activities were used; however, increasing the amount of manipulative use in the third learning environment did not provide concrete learners with any additional benefit.

Kontas (2016) looked at the impact of manipulatives (concrete learning materials) on secondary school pupils' academic achievement as well as their attitudes toward mathematics. The study used a pretest-posttest control group experimental model, which is a type of quasi-experimental research methodology. In the 2014-2015 school year, the study group comprised of 48 seventh grade children (24 in the experiment group and 24 in the control group) who attended a state school in Turkey's Southeastern Region. From his study, the experimental and control groups' posttest mathematics academic achievement scores were shown to differ significantly from the pre-test scores in favour of posttests in both groups. In posttests, the results of the experiment and control groups' views about mathematics were considerably different in favour of the experimental group.

Sarama and Clements (2016) constructed a theoretical framework for the use of manipulatives in mathematics learning and teaching from early childhood through primary school, as well as a review of empirical evidence to support that paradigm. The researchers discovered that manipulatives are only valuable for learning when they are used in conjunction with learners' actions and thoughts, and that both physical and virtual manipulatives can be helpful. When used in full, well-planned instructional contexts, both physical and virtual



manipulatives can help pupils make their information explicit, allowing them to develop Integrated-Concrete knowledge. In this age of technology and standardised testing, Furner and Worrell (2017) undertook a study to investigate the use of manipulative materials in the teaching of mathematics. The paper examines the disadvantages of using math manipulatives in the classroom and cautions educators. It also looks at certain cognitive issues that arise when a teacher uses math manipulatives to educate. The study examines a variety of regularly used math manipulatives in today's schools and compares them to some of the Common Core Math Standards that are taught in US and international classrooms.

Ubah and Bansilal (2018) investigated elementary teachers' knowledge of fraction addition and subtraction using the Action-Process-Object-Schema framework. Pre-service pupils' written responses to two assignments using fraction operations were used to collect data. Ten pupils agreed to be interviewed, and three of them are included in this piece. According to the study, many pre-service teachers fared well with addition and subtraction of common fractions with the same denominator. More than 52% of respondents, on the other hand, struggled to conduct comparable operations on common fractions with different denominators, showing that their concepts had not yet evolved into object-level structures. The study concluded that the incorrect processes had been embedded in the pupil's mental paradigm. As a result, it's critical that pre-service teacher development programs include opportunities for instructors to test their own understandings of fundamental mathematical concepts.

Saka and Roberts (2018) published a study that detailed some of the differences in early mathematics learning between manipulatives, mental images, and structured and unstructured external representations. This theoretical framework sets the stage for a closer examination of the situation in Malawian elementary schools, with a focus on the most common objects. The bow abacus, an indigenous Malawian object commonly found in rural Malawian children's classrooms and homes, is investigated. It has been used in schools, but the amount to which it has been used has been assessed to be limited. As a result of the theoretical conceptualization of the utility of structured representations, recommendations are given to improve how structured representations are employed.

Based on literature on using manipulatives to improve learners' performance in mathematics, Ndlovu and Chiromo (2019) conducted a study that recounted foundation phase pre-service teachers' perceptions of using manipulatives to enhance their competencies and reasoning skills to model the solution in number operations. The study relied on participants' written work (e.g., classroom activities, homework, quizzes, and examinations) as well as class talks. Additionally, several pupils were questioned. Pre-service instructors had a better understanding of how to use manipulatives. According to the findings, the vast majority of pupils have an action conception in which they utilize manipulatives to express or model a solution. However, most pupils display process or object conception in the second semester, as described in the genetic breakdown. The change in education in the second semester, when we taught utilizing the APOS theory, was responsible for the improvement. A multitude of factors clearly influence pre-service teachers' perceptions of

mathematical concepts, and teacher educators must pay special attention to these in order to help pre-service teachers understand the concepts they will teach in the classroom.

Lafay, Osana, and Valat (2019) discovered that manipulatives were particularly helpful in learning, maintenance, and transfer in a range of mathematical fields. According to the authors, manipulative-based therapies have been demonstrated to be beneficial for a range of learning outcomes, including conceptual comprehension and computational fluency.

Davis and Ampiah (2009) investigated the problems in teaching and learning of fractions at the primary school level in Ghana. The study focused on pre-service teacher trainees' conception about addition of two unlike fractions. The authors analyzed the data using percentages and by looking at trainees' explanation of processes involved in solving the problem of addition of two unlike fractions. Results from the study revealed that quite a number the prospective teachers had weak conception on the addition of two unlike fractions. This suggest that these prospective basic school teachers are likely to start teaching with weak knowledge of addition of two unlike fractions.

Larbi and Mavis (2016) assessed the usefulness of manipulative materials in boosting junior high school pupils' performance in Ghana. The participants in the study were 56 pupils from two schools in the Komenda Edina Eguafo Abirem municipality, who were chosen at random from two towns. Experimental and control groups of pupils were formed. Over the course of four weeks, each group was taught the same algebra modules. The experimental group, on the other hand, was taught with algebra tile manipulatives, while the control group was taught with the traditional 'talk and chalk' method. The pretest

and posttest for mathematics achievement were utilised to collect data. Percentages, mean, standard deviation, and the independent t-test were used to analyze pupils' posttest results. The study found that individuals who were taught using a large number of algebra tiles did much better. While a result, the study indicated that using manipulative materials to teach and learn algebra was a very effective and promising strategy, and that it also improved pupils' cognitive processes as they solved algebra problems.

Agbozo and Fletcher (2020) looked at prospective teachers' comprehension of fraction concepts after the Institute of Education's chief examiners' reports expressed concerns regarding pre-service teachers' appalling performance on fractions items in mathematics tests in Ghana's institutions of education. A total of 26 pre-service teachers from one college of education in Ghana's Central Region participated in the case study. The study employed a mixed method sequential explanatory design approach. While nearly all of the potential teachers had strong computational skills, none of them could explain why the procedure for dividing a fraction by a fraction works. Almost half of the participants were unable to demonstrate effective Pedagogical Content Knowledge (PCK) of fractions by demonstrating any technique for teaching fraction division. The authors suggested that mathematics tutors in colleges of education adopt a novel approach to teaching fractions in which linkages are made between topics relevant to fractions to help potential teachers have a richer conceptual knowledge of fractions and their applications.

Gaetano conducted studies to determine the effectiveness of manipulatives in the teaching of fractions. According to the data, pupils who did not use manipulatives during the learning process showed only minimal

growth ( $M = 16.17$ ,  $SD = 10.4$ ). On the other hand, pupils who used manipulatives throughout the learning process made significant development ( $M = 52$ ,  $SD = 15.33$ ). Pupils who learn with concrete, hands-on manipulatives are more likely to understand and internalize topics. These findings suggest that if educators employ manipulatives when teaching fractions, children will be able to comprehend fractional concepts and demonstrate considerable pupil improvement. They also demonstrate that employing manipulatives while teaching fractions is more effective than teaching fractions using the paper pencil method.

Hougas (2003) used a quasi-experimental method to see if using manipulatives when teaching fraction addition and subtraction affected sixth-grade math pupils' achievement and attitudes. After employing manipulatives to teach addition and subtraction of fractions, the researcher discovered that there was no significant difference in achievement between the experimental and control groups. However, when it came to pupils' views regarding fraction notions, the researcher discovered a substantial difference between the experimental and control groups. Hougas found that allowing pupils to use things to aid their learning increased their enthusiasm for the mathematic principles being taught.

From the above, it can be concluded that the impact manipulative materials on the performance seems to be inconclusive. However, majority of the findings agree that the use of manipulative materials improves the performance of pupils. It is in the light of this that a study of similar nature is conducted to ascertain the impact of manipulative materials on the performance and attitudes relative to the teaching and learning of fraction,

## Chapter Summary

Manipulatives are objects that students can see, feel, handle and move them because they are real. By manipulating and rearranging them into different orders and categories, students can stimulate their senses and close the gap between their sensory experiences and more sophisticated mathematical concepts. Dry models, dimensional models, and spatial models are common types of manipulations that use representations of objects or real objects, numerical bars or lines, and tiles or graphs. The relationship between teachers' mathematical beliefs and their instructional strategies has been the subject of numerous studies, with teachers' beliefs serving as the primary predictor of instructors' instructional strategies. According to Uribe-Flórez and Wilkins (2010), one of the reasons teachers employ practical exercises is because they appeal to pupils more than abstract and symbolic mathematics. When teachers use flexible strategies in their classes, students tend to be active, engaged, and interested. However, some teachers prefer to use tricks as a reward for good behavior rather than a useful learning tool, as they are fun for both teachers and students. Some teachers also prefer to use adaptive strategies at the end of the lesson, as long as there is enough time.

## CHAPTER THREE

### RESEARCH METHODS

#### Overview

The techniques and methods for gathering data utilized to answer the research questions and test the hypotheses will be covered in this chapter. The study design, demographic, sample size, sampling technique, data collection tools, validity and reliability, and data collection processes will all be covered in this chapter. data and methods for data analysis.

#### Research Design

A study by Burton and Bartlett (2005) shows that the study design serves as a guide for the researcher to keep a clear direction and focus on the objectives of the study. Punch (2006) supports this idea by stating that the study design is the overall plan for conducting the study. The study design, according to Henning, Van Rensburg, and Smith (2004), is the fundamental framework that keeps the researcher on track. The experimental research design was utilized to investigate the impact of manipulative tools on teaching of fractions in primary six schools in Aowin Municipality. The quasi-experiment design was used. According to Gall et al. (2005), this design is one of the most potent for establishing the existence of cause-and-effect links among variables. This approach has provided researchers with more accurate results regarding the causal relationship between two variables, as highlighted by Marsden and Thorgerson (2012). The primary objective of this study was to determine how manipulative materials affect student performance and behavior, and to assess the appropriateness of the chosen study design.

White and Sabarwal (2014) identified that the main problem with semi-experimental methods is the risk of getting a bad fit. To minimize this, the comparison group should be as comparable as possible to the treatment group before the intervention. Another potential problem is the tendency to prioritize statistically significant results over unimportant results, which was addressed in the study by presenting all results and limiting discussion to statistically significant results. Because quasi-experimental methods rely on assumptions, White and Sabarwal (2014) argue that conclusions about causation drawn from these studies are less compelling than those from well-conducted randomized controlled trials. To mitigate this, the study explained in detail its limitations and how they affected the results.

### **Population**

Population is described by Polit and Hungler (1999) as a collection of issues satisfying a particular set of requirements refers to the group that the researcher is most interested in studying, from which he gathers data and draws conclusions. The population of this study comprises of all primary six school instructors and pupils in Aowin Municipality.

### **Sample and Sampling Technique**

The researcher employed a multistage sampling strategy to pick the sample for this investigation. Due to the researcher's familiarity with the area and proximity to schools, the municipality of Aowin in the western-north region was conveniently selected. The control and experimental groups were chosen using a stratified sample. In order to choose one school at random from each zone, schools were first divided into east and west zones. The main reason of using this method was to avoid any interaction between the groups and to ensure



that each school had the same probability of being selected for teaching. The experimental and control groups were randomly assigned. The researcher wrote the letters E for experiment and C for control on sheets of paper, then folded them. Then, a representative of one of the two groups was selected to determine their participation in the control or experimental group. The representative who picked E was assigned experimental group and the representative who picked C was considered control group. Both schools' intact classroom was used in the study.

The study used all classes from both schools, with 40 pupils in each class. The study's goal was to determine how primary six school teachers felt about using manipulative techniques to teach fractions in the classroom. Therefore, the instructors for the basic six were chosen using a purposive sampling. The schools were chosen using a simple random selection method. For the study, a total of 34 teachers from 107 primary schools were chosen for the study.

### **Data Collection Instruments**

The data for the study was collected using two instruments. A test and a questionnaire were developed and used. Both the control and experimental groups of pupils were given the test. A pre-test and a post-test were given to both groups. The items covered representation of fraction, comparing fractions, ordering fraction, operations on fractions and word problems involving fractions. This is in line with the new mathematics curriculum for basic six pupils. The test consisted multiple choice items and supply questions. The pupils were required to answer all the questions.

The questionnaire was administered to both teachers and pupils. The questionnaire was administered to teachers once. However, the questionnaire was administered to the pupils in both groups twice (before and after the treatment). The questionnaire was in two sections. Section A elicited the biodata of the respondents and Section B elicited their attitude towards the use of manipulatives in teaching fractions (teachers) and their attitude towards the learning of fractions (pupils). The items on the questionnaire were on a four-point Likert scale ranging from strongly disagree to strongly agree. See Appendix A for further clarification.

Given that all respondents can read and respond effectively to the questions in the questionnaire, it was chosen over other instruments. It also enabled the researcher to produce numerical values for hypothesis testing. Questionnaires have a number of advantages, including the fact that they are inexpensive and may be used to collect data from a large number of people. Responders may skip parts of the questions or refuse to return them, which is one flaw. Some of the respondents may misunderstand some of the questions, which could have an impact on the study's findings. To make up for this deficiency, the researcher discussed key issues with respondents and incorporated their candid comments.

#### **Data Collection Procedure**

Before collecting data, permission was obtained from the municipal education directorate. The shortlisted schools were visited for a familiarization visit. Before the instructors are informed about the study's purpose, permission from the head teachers was obtained. The data collection process lasted for six weeks. The teachers' questionnaire was administered to teachers once and the

pupils' questionnaire administered to both groups to ascertain their attitude towards the use of manipulatives in teaching fractions at the primary six classrooms and their attitudes towards the teaching and learning of fractions respectively. Pupils from both groups were administered the pre-test in the first week. The items were the same for both groups. This was to ensure that the differences in performance between the pre-test and post-test scores was not attributed to the differences in the test items.

The experimental group was taught using manipulative materials such as strips of paper, Cuisenaire rods among others for six weeks. Each week has four periods with each period being an hour. The teaching was done after normal classes' hours. Appendix F shows the sample for the six weeks lesson plan used for the teaching of both groups. The control group was taught using the conventional method which reflects the normal practices of the teachers. The posttest was done on the 5<sup>th</sup> day of the 6<sup>th</sup> week after the pretest ( Embretson, Susan, Reise & Steven, 2000). This makes pupils to have forgotten nearly all of the pretest questions and answers, therefore will not repeat the same answers in the posttest (Embretson, Susan, Reise & Steven, 2000). Hence making the same items in the pre-test and post-test to have no significant consequences on the outcome of the study.

The test lasted for 45 minutes for both pre-test and posttest.

Again, pupils from the experimental group and control group responded to the questionnaire involving their attitude towards the learning of fractions before the treatment and after the treatment. The questionnaire was given to the pupils during the first week before the teaching periods. After the five weeks of teaching and learning the researcher gave the pupils the same questionnaires.

### **Validity and Reliability**

The instruments were validated using expert judgment. My supervisor was given the instrument to guarantee that they measure exactly what they should and that they are error-free and the instrument was also validated by some of my colleague teaching staff as well as the departmental head of my department. In the Suaman District, the instrument was pilot tested with a group of Pupils and instructors. This district was chosen due to cultural similarities. The pilot testing also helped to ensure that the data collection device was effective and efficient. The Cronbach alpha co-efficient was used to measure the questionnaire's reliability. The correlation coefficients for the pupils and instructor questionnaires were determined to be 0.734 and 0.763, respectively.

### **Ethical Consideration**

To guard against ethical lapses, the study took many precautions. The study's Appendix E includes the ethical clearance that was first obtained after consulting with the University of Cape Coast Institutional Review Board. In order to safeguard their anonymity, each respondent was also given the assurance that their identities would be kept secret and that any information gathered would only be applied for scientific research. All authors cited in the study also have the proper citations provided for them. Last but not least, to ensure the data's legitimacy, it has not been modified in any way.

### **Data Analysis Procedure**

Data analysis was guided by research questions and research hypotheses. Questionnaire responses from teachers and students were studied to gather their opinions on the use of manipulative resources for faction teaching, as well as their attitudes toward faction teaching. The schoolchildren's

responses were also assessed before treatment to determine their baseline attitudes towards the study of fractures. Descriptive statistics such as replicates, percentages, means, and standard deviations were used to analyze the study questions. Responses to questionnaire questions classified as "agree" or "strongly agree" were considered as agreement, and responses as "disagree" or "disagree" as disagreement. Decisions were made by simple majority of responses.

The first hypothesis was tested, and the results showed that when manipulating fractions to teach skills, there was no discernible difference in student performance between the experimental group and the control group. We employed paired and independent t-tests. An independent t-test was conducted before the start of the treatment to see if there were any notable changes between the two groups. As a result, it may be concluded that the two groups' performance traits both before and after therapy were comparable. If each group significantly improved following various treatments, a validated t-test was performed to assess this. In order to evaluate whether there was adequate data to refute the null hypothesis that there was no significant difference in student performance between the two groups, an independent t-test was conducted.

It is possible to assess whether there is a significant difference in learning fractions between the experimental and control groups using the independent t-test, a statistical test appropriate for comparing the means of two groups. The answers' numerical values will make it possible to compare the means numerically. Any significant difference between the groups was found using a dependent t-test. The learning of fractions by manipulation between

boys and girls in the experimental groups was compared using an independent t-test to see if there was a significant difference.



## CHAPTER FOUR

### RESULTS AND DISCUSSION

#### Overview

In this chapter, the study's findings are presented and assessed in further depth. The findings are contrasted with the research's initial questions and hypotheses. The study's major goal was to find out how manipulative materials usage affected the attitudes and academic performance of class six school pupils who attended schools in the western-north Region's precisely Aowin Municipality. The study aims to accomplish the following particular goals:

1. Explore the attitudes of primary six school teachers to the use of manipulative materials in teaching fractions.
2. Explore class six school pupils' attitudes toward learning of fractions.
3. Determine the effect of manipulatives on pupil performance in fractions.
4. Examine the effect of using manipulative materials on pupils' attitudes toward learning fractions.
5. Determine if there are sex differences in attitudes toward fractions when using manipulative materials.

To achieve these goals, pupils underwent tests to assess the impact of manipulative material on their academic performance. In addition, a questionnaire was distributed to pupils to assess their attitudes toward learning fractions, as well as the effects of using manipulative materials on their attitudes toward learning fractions. The study also included surveying six primary school teachers to verify their attitudes toward using manipulative materials to teach fractions. Both descriptive and inferential statistics, including t-tests, were used to present the study's findings. Descriptive statistics included frequency counts,

percentages, means, and standard deviations. The results are presented based on the research questions and hypotheses that guided the study and are preceded by the biographical data on respondents.

### Biographical Data of Respondents

**Table 1: Sex Distribution of Pupils**

Sex/Group	Control		Experimental		Total	
	Freq	%	Freq	%	Freq	%
Male	25	31.25	24	30.0	49	61.25
Female	15	18.75	16	20.0	31	38.75
Total	40	50.0	40	50.0	80	100.0

The gender distribution of the study participants' pupils is cross-tabulated in Table 1's data. Table 1 shows that of the 40 pupils in the control group, 25 were male and 15 were female. The findings in Table 1 also show that there were 24 males and 16 females in the experimental group. This indicates that 31 female students and 49 male students both took part in the study. Based on the data in Table 1, it can be determined that the experimental group (24) and the control group (25) each had roughly the same number of male pupils.

A similar conclusion can be drawn in the case of female pupils. This indicates that the study covered both sexes, so the results can be applied to both male and female populations. Therefore, the results of the study can be considered generalizable to both sexes. This also indicates that the respondents of both groups have similar characteristics in terms of gender distribution, which is essential for empirical research. In order to make decisions about how to administer the intervention, Tannenbaum, Greaves, and Graham (2016) claim



that it is essential to decide on the gender distribution for the study. This is because there is enough data in the literature to support this claim. In addition to evaluating student gender, the study looked at how gender was distributed among the participating teachers. The purpose of this was to provide light on how the study's representation of the two genders is different. The examination of the distribution of teachers by gender is presented in Table 2.

**Table 2: Sex Distribution of Teachers**

Gender	Frequency	Percentage (%)
Male	23	67.6
Female	11	32.4
Total	34	100

According to Table 2, out of the 34 teachers that took part in the study, 23 were men. This is 67.6% of the teachers chosen for the study. On the other side, there were 11 women, or 32.4% of the total number of teachers chosen for the study. This seems to imply that there are more male instructors than female teachers at the class six level in the research area. The age of teachers was a background factor that was investigated. The findings are shown in Table 3.

**Table 3: Age Distribution of Teachers**

Age Range	Frequency	Percentage (%)
21 – 30	14	41.2
31 – 40	16	47.1
41-50	3	8.8
51-60	1	2.9
Total	34	100

Table 3's data show that 16.1% of the sample, or 16 teachers, were between the ages of 31 and 40. This represents 47.1% of the entire sample. In the meantime, 14.2% of the sample, or 14 teachers, were between the ages of 21 and 30. Furthermore, only one teacher was between the ages of 51 and 60. This suggests that the average age of the participating teachers—nearly 90% of them—was under 41. Rahida-Aini, Rozita, and Zakaria (2018) claim that these kind of teachers are quite successful. The investigation also questioned the teachers' prior classroom experience. The results are summarized in Table 4.

**Table 4: Teaching Experience**

Experience (Years)	Frequency	Percentage (%)
Less than 4	10	29.4
4-8	10	29.4
9-12	8	23.5
13-16	5	14.7
Above 16	1	2.9
Total	34	100

Table 4 shows that 10 teachers (or 29.4%) have been teaching for less than four years. This means that 70.6 percent of the remaining 24 teachers have taught for four years or longer. 17.6% teachers had more than 12 years of teaching.

Another demographic variable that was considered was teachers' professional qualification. Teachers who have passed through formal training in teaching are expected to know the place of manipulative materials in the teaching and learning process. Table 5 presents the results.

**Table 5: Professional Qualification of Teachers**

Qualification	Frequency	Percentage
Diploma in Education	14	41.2
Bachelor of Education	18	52.9
Master's in Education	2	5.9
Total	34	100

Table 5 results show that 18 of the 52.9% teachers had a bachelor's degree, with two having a master's degree in education. All the participant were professional teachers, however, very few (5.9%) has master's degree. The bulk of them (94.1%) had either diploma or degree.

### **Main Results**

#### **The attitudes of primary six pupils towards the learning of fractions in the Aowin Municipality.**

Pupils' attitudes about learning fractions vary, according to the literature, which provides proof for this claim. Their knowledge of fractions or the manner in which teachers introduce the idea could be to blame for this. As a result, the focus of this study is on the attitudes of pupils (both in the experimental group and the control group) about fractional learning. Prior to the differentiating treatment administered to each group, the experimental group and the control group each received a questionnaire measuring the children's perspectives on fractional learning. The survey used a Likert scale with four options, from strongly agree to strongly disagree. For each item in the poll, the respondent was given a choice between four possibilities. A scale of 1 to 4 was used to score items 1-6, with 1 denoting "strongly disagree," 2 "agree," 3 "agree," and

4 "strongly agree." Instead, a different grading system was applied, with 1 denoting "strongly agree," 2 "agree," 3 "disagree," and 4 denoting "strongly disagree." Mean scores were used to gauge participants' attitudes toward fractions. A score of 3.00 or more indicates a favorable attitude, a score of 2.5–2.9 suggests a positive attitude, and a score of less than 2.5 indicates a negative attitude. Table 6 displays the perspectives of the pupils on learning fractions.



**Table 6: Attitude of Pupils towards Learning of Fractions**

Statement	SA(%)	A(%)	D(%)	SD(%)	Mean	Dev
Learning fractions is very difficult	50(62.5)	16(20)	12(15)	2(2.5)	1.58	0.84
I wish I do not meet any problem relating to fractions	41(51.3)	24(30)	12(15)	3(3.8)	1.71	0.86
I am always have fear when I am to learn or solve problems relating to fractions	42(52.5)	25(31.3)	11(13.8)	2(2.5)	1.66	0.81
Fractions is not related to real life situations	35(43.8)	33(41.3)	9(11.3)	3(3.8)	1.75	0.80
Fractions is too abstract.	32(40)	33(41.3)	10(12.5)	5(6.3)	1.85	0.87
To learn fractions, you have to memorise procedures and formulae.	23(28.8)	39(48.8)	12(15.0)	6(7.5)	2.01	0.86
I feel enthusiastic when learning fractions.	4(5.0)	11(13.8)	40(50.0)	25(31.3)	1.93	0.81
I enjoy learning fractions	4(5.0)	3(3.8)	21(26.3)	52(65.0)	1.49	0.80

Mean of means: 1.75

Results from Table 6 shows generally , pupils attitudes towards fraction were not favourable or good . They agreed or strongly agreed to all the negative framed items and disagreed or strongly disagreed to all positive framed items. For instance about 82.5% strongly agreed or agreed that fraction is a difficult topic to learn whereas 17.5% either strongly disagreed or disagreed. This means that, approximately eight in every ten pupils see fraction as difficult to learn. A look at the means and the standard deviations show that the majority of the scores clustered around the means. This supports the observation made about pupils poor attitude towards fraction.

Results from Table 6 show that 50 pupils representing 62.5% strongly agreed that fractions is a difficult topic to learn whereas only two representing 2.5% strongly disagreed. Again, 16 pupils representing 20% agreed that fraction is difficult with 12 (15%) disagreed. This suggests that, 82.5% affirms that fraction as a topic is very difficult to learn with the remaining 17.5% disagreeing. This means that, approximately eight in every ten pupils see fraction as difficult to learn. A mean score of 1.58 out 4 with a standard deviation of 0.84 confirms that most of the pupils agreed that fraction is difficult to learn.

Similarly, 42 pupils representing 52.5% strongly agreed that they always have fears when they are to learn or solve problems relating to fractions while 25 (31.3%) of them agreed. However, only two representing 2.5% strongly disagreed to the above assertion. A mean score of 1.66 showed, the pupils agreed that they entertain fears when confronted with problems involving fractions. Again, Table 6 reveals that 68 pupils representing 85.1% of the total approved that the concept of fractions not related to real life situations whereas

the remaining 15.1% responded otherwise. A mean score of 1.75 with a standard deviation of 0.80 further shows that on the average the pupils agreed that the idea of fractions is not connected to real life situations.

Results from Table 6 again indicate that 52 pupils representing 65.0% strongly disagreed that they enjoy learning fractions. From Table 6, it can be elicited that only seven pupils out of the eighty pupils agreed that they enjoy learning fractions. This illustrates that majority of the pupils do not enjoy learning fractions. It is in the light of this that any attempt to investigate strategies that can help pupils enjoy learning any mathematical concept such as fractions is welcome. Similar responses were recorded for items including “I wish I do not meet any problem relating to fractions” (Mean = 1.71 SD = 0.86); “Fractions is too abstract” (M = 1.85 SD = 0.87); “To learn fractions, you have to memorise procedures and formulae” (M = 2.01 SD = 0.86) and “I feel enthusiastic when learning fractions” (M = 1.93 SD = 0.81). Finally, the attitude of the pupils towards the learning of fractions recorded an overall mean score of 1.75.

This presupposes that most the pupils do not have positive attitude towards the learning of fractions. In other words, primary six pupils have negative attitude towards the learning of fractions. This has implications on their performances in fractions and other concepts that involve fractions.

### **The attitudes of primary six classroom teachers towards the use of manipulative materials in teaching fractions.**

The study's second area of inquiry was six primary school teachers' opinions about the use of manipulatives in fractional instruction and learning. The data required to address this study issue was obtained via a questionnaire

given to teachers. Each of the questionnaire's four questions required teachers to make a selection. Each respondent was given one of the following scores: Strongly Agree=1, Agree=2, Disagree=3, and Strongly Disagree=4. Table 7 displays the outcomes.

**Table 7: Attitude of Teachers towards the use of Manipulative Materials**

Statement	SA (%)	A (%)	D (%)	SD(%)	Mean	Dev
The use of manipulative materials is for children in the lower primary.	13(38.2)	9(26.5)	6(17.6)	6(17.6)	2.15	1.13
Using manipulatives is time wasting	3(8.8)	16(47.1)	10(29.4)	5(14.7)	2.50	0.86
I need support in order to select the most appropriate manipulative for a concept	11(32.4)	17(50.0)	6(17.6)	0(0)	1.85	0.70
It is very difficult to get access to appropriate materials.	10(29.4)	13(38.2)	5(14.7)	6(17.6)	2.21	1.07
Using manipulative materials slows down the pace of work	7(20.6)	16(47.1)	7(20.6)	4(11.8)	2.24	0.92
Using manipulatives makes the class boring	6(17.6)	11(32.4)	9(26.5)	8(13.5)	2.56	1.05
I will use manipulative materials as a means to reward my pupils	3(8.8)	10(29.4)	16(47.1)	5(14.7)	2.68	0.84
The use of manipulative materials is not effective in large classes	11(32.4)	11(32.4)	7(20.6)	5(14.7)	2.18	1.06
Manipulatives are used more for fun than instruction	4(11.8)	10(29.4)	12(35.3)	8(23.5)	2.71	0.97
Manipulatives for instruction is very costly	10(29.4)	11(32.4)	7(20.6)	6(17.6)	2.26	1.08

Mean of means: 2.33

SD = Strongly Disagree, D = Disagree, A = Agree, SA = Strongly Agree



Table 7 results show that generally, teachers attitudes towards the use of manipulative are not desirable or not appreciable. They agreed or strongly agreed to almost all the negative framed items and disagreed or strongly disagreed to almost all the positive framed items. For example, looking at the means and standard deviations show that the majority of their responses clustered around the means. It appears that teachers hold negative attitudes towards the use of manipulatives for teaching fractions. Table 7 presents the results, revealing that 38.2% of the primary six schools' instructors in the study strongly believe that manipulative resources are only suitable for lower primary students, with an additional 26.5% agreeing with this view. Nonetheless, six teachers (17.6%) disagreed or strongly disagreed with the idea that manipulative materials should be limited to lower primary students.

This implies that roughly 65% of teachers support the use of manipulative materials for lower primary pupils, while approximately 35% do not. The mean score of 2.15 and a standard deviation of 1.13 suggest that most instructors believe manipulative materials are appropriate for younger pupils. Additionally, Table 7 illustrates that 11 teachers (32.4%) strongly agreed that they require assistance in selecting the most appropriate manipulative for a particular concept, with another 17 (50.0%) agreeing. In contrast, six teachers (17.6%) disagreed that they need help selecting the best manipulative for a specific topic. This indicates that 82.4% of teachers acknowledged the need for assistance in selecting the appropriate manipulative, while 17.6% did not. It seems that the majority of teachers agree that they require assistance in selecting the best manipulative for a particular subject, as evidenced by a mean score of 1.85 and a standard deviation of 0.70. This indicates that many teachers feel

they lack the necessary skills to choose the most suitable manipulative when teaching mathematical concepts, such as fractions. This could have an impact on their willingness to use and their actual use of manipulative materials in the classroom.

According to Table 7, 29.4% of the teachers (10 individuals) strongly agree that it is challenging to acquire appropriate manipulative materials, while 17.6% of the teachers (6 individuals) strongly disagree with this statement. The majority of teachers (as indicated by a mean score of 2.21) agree that obtaining suitable manipulative materials is difficult. Comparable outcomes were achieved for the statements "the use of manipulative materials slows down the pace of work" (Mean=2.24, SD=0.92), "manipulative materials are not effective in large classes" (Mean=2.18, SD=1.06), and "instructional manipulatives are costly" (Mean = 2.26, SD = 1.08).

However, results from Table 7 revealed that 16 teachers representing 47.1% disagreed to the assertion that they would use manipulative materials as a means of rewarding their pupils with a further five representing 14.7% strongly disagreeing. However, 10 of the teachers representing 29.4% agreed to the assertion with another three representing 8.8% strongly agreeing to the assertion. According to the data, 62% of the teachers disagreed with the idea of using manipulative materials as a reward for their students, whereas 38% supported it. The mean score of 2.68 suggests that, on average, most teachers were not in favor of using manipulative materials as a form of reward for their pupils. The statement "manipulatives are used more for fun than instruction" had a similar outcome, with a mean score of 2.71 and a standard deviation of 0.97. Table 7 indicates that the overall mean score for teachers' attitudes towards

using manipulative devices to teach fractions in primary six schools is 2.33. This suggests that the majority of primary six instructors have a negative attitude towards the use of manipulative devices in fraction instruction.

**There is no significant difference between the performance of pupils in the control group and experimental group when manipulative materials are used in teaching fractions.**

Examining the influence of manipulative materials on the instructional strategies used in primary schools serving class six is the goal of this study. In order to ascertain these materials' impacts, the researchers compared the outcomes of two groups. They also compared the pre-test and post-test outcomes of both treatment groups, as well as the post-test outcomes of the two groups. The statistics for the experimental and control groups' pre- and post-test outcomes are shown in Table 8.

**Table 8: Descriptive Statistics for Control and Experimental Groups**

	Control Group		Experimental Group	
	Pre-test	Posttest	Pre-test	Posttest
Mean Score	2.58	7.85	2.68	11.15
Standard Dev	1.17	1.63	0.27	1.98
Number	40	40	40	40

They had a pre-test and a post-test. The average pretest score out of 16 possible scores in the control group was 2.58, but this average increased to an average posttest score of 7.85 out of 16. On the contrary, the average pretest score was 2.68 and the posttest 11.15 out of 16 in the experimental group. The mean of both groups was compared between the pre-test and post-test using an independent sample t-test to see whether this difference is statistically

significant. Table 9 displays the outcomes of the independent t-test for the control and experimental groups' pretest results.

According to Table 8, the pre-test and post-test populations for the control and experimental groups each consist of 40 students. In both the pretest and the posttest, the control group reported an average score of 2.58 out of 16 and 7.85 out of 16. The experimental group's typical posttest score was 11.15 out of 16, and their typical pretest score was 2.68. The averages of the two groups were contrasted using an independent t-test. Pre-test and post-test to determine whether or not this difference is statistically significant. Table 9 displays the outcomes of the independent t-test for the pretest for the control and experimental groups.

**Table 9: Independent t-test for Pretest for Experimental and Control**

Group	Lavene's test for equality of variance		Test for equality of means		
	F	Sig	t	df	Sig(2-tailed)
Equal variances assumed	7.352	0.008	0.304	78	0.762
Equal variances not assumed			0.304	68.967	0.762

Results from the pretest showed a degree of freedom (df) of 68.967 and a 0.304 outcome. A substantial difference between the control and experimental groups' first test scores was shown by the p value, which was larger than 0.05. Since there was no discernible difference in pupils performance between the experimental and control groups prior to the use of the fractions teaching tools, the null hypothesis that there was no such difference cannot be disproved. Both

groups performed equally prior to the resource manipulation, and Table 8's differences may simply be the result of sampling error. As a result, any notable difference that is discovered after the experiment may be the result of how the two groups were handled differently. Results of the paired sample t-test for the experimental group are displayed in Table 10 to further support the relative effect of therapy on pupils performance.

**Table 10: Paired Sample t-test for Experimental Group**

	t	df	Sig (2-tailed)
Pretest-posttest scores	71.406	39	.000

The significance level in Table 10 is  $p=0.001$  and the t statistic is 71.406. We can infer that there is a significant difference between the mean scores on the pretest and posttest on  $t(39) = 71.406, p=0.001$ , since  $p = 0.001$  is smaller than the alpha threshold of 0.05. The pretest mean scores were  $M=2.675, SD=1.716$ , while the posttest mean scores were  $M=11.15, SD=1.981$ . According to the null hypothesis, the estimated value and the estimated value do not differ significantly in the experimental group. In order to compare pre-test and post-test procedures, paired t-tests were utilized, as shown in Table 11, which aims to investigate how traditional teaching approaches affect pupils performance. The outcomes demonstrate a statistically significant performance difference between the scores from the experimental group and those from the control group.

**Table 11: Paired Sample t-test for Control Group**

	t	df	Sig (2-tailed)
Pretest-posttest scores	52.127	39	.000

The pre-test and post-test mean scores differ significantly from one another, according to statistical analysis. It is concluded that traditional teaching approaches do not significantly affect students' performance, rejecting the null hypothesis. A statistically significant difference is evident between the pre-test and post-test scores of the control group, which have a t-value of 52.127 and a p-value of .001. These findings demonstrate the power of conventional teaching techniques to significantly affect pupils' academic performance. It appears that the study wants to compare the outcomes of using conventional teaching techniques with pupils work tools. They might have done this by comparing the results of the experimental and control groups' pre- and post-tests using a paired t-test. The results of this test can demonstrate whether there is a substantial difference between each group's performance before and after the intervention. The results is presented in table 12.

**Table 12: Independent couple t-test for Post-test for Experimental and Control Group**

	Lavene's test for equality of variance		Test for equality of means		
	F	Sig	t	df	Sig(2- tailed)
Equal variances assumed	0.501	0.481	8.143	78	0.000
Equal variances not assumed			8.143	75.138	0.000

A Lavene's test for equality of variances got a sig value of 0.481, according to Table 10. The sig value of 0.481 was greater than the alpha threshold of 0.05, indicating that there was no statistically significant difference in variances between the two groups. This explains why the two groups' variances were considered to be equal. The test for equality of means yielded a t-statistic of 8.143 and a sig value of 0.000 as a result. The difference between the posttest scores for both groups was statistically significant because the sig value of p.000 was smaller than the alpha level of 0.05. As a result, we reject the null hypothesis that there is no significant difference in performance across the groups. This means that the mean score for the experimental group during the posttest was higher than the mean score for the control group during the posttest as seen in Table 8.

**There is no significant difference between the attitudes of pupils in the controls and experimental groups after the use of manipulative materials in teaching fractions.**

A study was conducted to determine the effect of manipulated materials on pupils' attitudes towards learning fractions. This study compared the questionnaire responses of two groups of pupils before and after treatment. The fractional learning settings of both groups were compared before treatment. Descriptive statistics before the test are presented in Table 13. The aim of this study was to determine whether the use of manipulative materials has a significant impact on pupils' attitudes towards fractional learning. Table 13 presents the descriptive statistics for the pre-test.

**Table 13: Descriptive Statistics on pupils' attitude toward learning of fractions before treatment**

Group	Number	Mean (out of 4)	Standard dev
Control	40	1.82	0.60
Experimental	40	1.67	0.46

From Table 13, the control group recorded a mean score of 1.82 out of four whereas the experimental group recorded a mean score of 1.67 out of 4. This means that, the pupils in the control group had a better attitude towards the learning of fractions than their counterparts in the experimental group. As to whether this difference in attitude was statistically significant, the independent t-test was used to compare their means. Table 14 presents the results of the independent t-test for the pre-test.

**Table 14: Pretest Results for Experimental and Control Groups**

	Lavene's test for equality of variance		Test for equality of means		
	F	Sig	t	df	Sig(2-tailed)
Equal variances assumed	4.659	0.034	1.255	78	0.213
Equal variances not assumed			1.255	73.36	0.213

From Table 14, the Lavene's test for equality of variances yielded an F statistic of 4.659 and a sig value of 0.034. The variations in variances were statistically significant because the sig value of 0.034 was less than the alpha threshold of 0.05. As a result, during the pretest, the variances for both groups



were considered to be unequal. As a result, with  $t(73.360) = 1.255$ ,  $p > .05$ ., the difference in averages between the control and experimental groups was not statistically significant. This indicates that the two groups had similar views toward learning fractions before manipulative techniques were used. This therefore, suggests that, any significant difference observed during the posttest can be attributed to the different treatments given to the two groups.

The posttest responses of the pupil were compared using the independent sample t-test to determine whether there was any significant difference in attitude after using the manipulative materials. The descriptive statistics results for both groups during the posttest are presented in Table 15.

**Table 15: Descriptive Statistics for Posttest**

Group	Number	Mean	Standard Dev
Control	40	2.04	0.40
Experimental	40	2.90	0.69

Results from Table 15 show that, both the experimental and control groups still had the same number of pupils responding to the questionnaire on attitude towards the learning of fractions. The control group recorded a mean score of 2.04 out of 4 whereas the experimental group recorded a mean score of 2.90 out of four. This suggests that the experimental group's pupils had a more positive attitude about learning fractions than the control group's pupils. The independent t-test was used to compare their mean scores to see if this difference in attitude was statistically significant. The results of the independent t-test for the posttest are presented in Table 16.

**Table 16: Posttest Results for Experimental and Control Group Groups**

	Lavene’s test for equality of variance		Test for equality of means of		
	F	Sig	t	Df	Sig(2-tailed)
Equal variances assumed	8.640	0.004	6.750	78	0.000
Equal variances not assumed			6.750	62.878	0.000

A Lavene's test for equality of variances of a sig value of 0.004 as shown in Table 16. Because the sig value of 0.004 was smaller than the alpha level of 0.05, it was deemed that the variances for both groups were statistically significant. As a result, it was considered that equal variances were not equal. The test for equality of means yielded a t-statistic of 6.750 and a sig value of 0.000 as a result. The difference between the posttest scores for both groups was statistically significant because the sig value of p.000 was smaller than the alpha level of 0.05. As a result, we reject the null hypothesis that there is no significant difference in pupil’s attitudes in the control group.

**There is no gender difference in attitudes towards fraction in the experimental group when manipulative materials are used in teaching fraction.**

This study also contrasted the posttest results of male and female students in the experimental group in order to look into how the materials utilized affected the learning process. The descriptive statistics for gender disparities and attitudes toward the study groups in the experimental group are shown in Table 15. The outcomes show that employing the treatment materials

had no behavioral changes between men and women in the experimental group. Although the differences were not identical, the statistical analysis of the data revealed a significant behavioral difference between the experimental and control groups in the sections, and the null hypothesis was rejected. depends on rejection. These findings suggest that the use of fictitious materials in fractions instruction may have a favorable impact on students' attitudes toward learning fractions.

**Table 17- Descriptive Statistics of Male and Female Attitude towards fractions**

Group	Number	Mean	Standard dev
Female	16	2.617	0.825
Male	24	3.083	0.517

The experimental group had 24 male and 16 female students, as shown in Table 17. The average score for males was 3.083, compared to 2.617 for females. This shows that after employing manipulative materials to teach fractions, males had a better attitude about learning them than their female counterparts. A t-test was used to compare the means of the two groups in order to ascertain whether this difference is statistically significant. Table 18 lists the findings of the t-test.

**Table 18: Independent t-test for Posttest for male and female pupils**

	Lavene's test for equality of variance		Test for equality of means		
	F	Sig	t	df	Sig(2-tailed)
Equal variances assumed	6.893	0.012	2.202	38	0.034
Equal variances not assumed			2.013	22.864	0.056

A Lavene's test for equality of variances generated a F statistic of 6.893 with a sig value of 0.012, as shown in Table 18. Because the sig value of 0.012 was smaller than the alpha threshold of 0.05, it was deemed that the variances for both groups were statistically significant. This explains why the two groups' variances were considered to be unequal. As a consequence, the t-statistic for equality of means was 2.013 and the sig value was 0.056. The difference between the posttest scores for both groups was not statistically significant because the sig value of  $p > .000$  was bigger than the alpha threshold of 0.05. At  $t(22.864) = 2.013, p > .05.$ , we fail to reject the null hypothesis that there is no gender difference in pupils' attitudes in the experimental groups after manipulative materials are used to teach fractions.

### **Discussion**

To analyze the study's findings, the researcher turned to the research questions and hypotheses. The analysis and discussion of the findings were organized around these topics and hypotheses. The researcher was able to reach findings and offer suggestions based on the information gathered by making reference to the study questions and hypotheses.

### **The attitudes of primary six pupils towards the learning of fractions in the Aowin Municipality**

Pupils' success in studying mathematics is dependent on their attitudes toward the topic, according to Farooq and Shah (2008), who conducted a study to assess the impact of pupil attitude on their performance in mathematics. As a result, they advocate for concerted efforts to cultivate such attitudes. It is in the light of this that this study elicited the attitude of pupils towards the learning of fractions. From the results, it can be seen that all the items measuring the attitude

of the pupils towards the learning of fractions recorded a mean score less than 2.5. This shows that, the pupils had negative attitude towards the learning of fractions. The argument that students in certain schools in Aowin Municipality showed a lack of interest in learning fractions is supported by the fact that their overall mean score was only 1.75 out of 4. This contradicts the results of a study conducted by Raiman in 2001, which found that around 66% of Bruneian students surveyed liked fractions, while 28% did not and 5% were undecided. These findings demonstrate a stark contrast in attitudes towards fractions among students from different regions. The findings of this study is also different from the findings of Yusof (2003) who undertook a longitudinal study to investigate Bruneian pupils' mathematical errors in fraction and revealed that all the four sub-constructs of attitude had higher mean scores ranging from 3.51 to 4.33 from a possible five meaning most of the pupils had better attitude towards learning of fractions. The study is however in tandem with Groff (1994) who in recounting his experience as a teacher claimed that most of his pupils do not enjoy learning fractions because they see operations with fractions as a “dead-end activity”. This should be a worry to educators and parents in some selected schools in Aowin Municipality. According to Mutai (2010), attitudes have a long-term impact on the child and if these attitudes are favorable, they tend to help pupils learn mathematics well.

**The attitudes of primary six teachers towards the use of manipulative materials in teaching fractions.**

The role of the teacher in creating a conducive mathematics environment for effective learning is a settled issued. They provide an environment through the use of manipulative materials that will enhance the

thinking of learners in a mathematics situation. It is however believed that the extent to which they use manipulatives is influenced by several factors including their attitude. Marshall and Swan (2008) opined that teacher who lacked the conviction about the effectiveness of manipulative materials are less likely going to use them in their lessons.

In this study, around 65 percent of teachers felt that manipulative tools are appropriate for children in the lower primary grades, whereas approximately 35 percent disagreed. With a mean score of 2.15 and a standard deviation of 1.13, most teachers believed that manipulative materials are appropriate for children in lower primary. This confirms the findings of Uribe-Florez and Wilkins, (2010) Swan & Marshall (2010) in their various studies identified that teacher's belief about manipulative materials and its applicability in the grade levels is one of the major predictors of teacher's attitude at the primary school level and their use of manipulative materials. For instance, Uribe-Florez and Wilkins (2010) reported that, the use of manipulative materials decreases with an increase in grade level.

The use of manipulative materials needs thoughtful planning from the teachers so as to make it effective. One intriguing revelation from this study indicate that a whopping 82.4% of the teachers accepted the assertion that they need support in order to select the most appropriate manipulative for a concept. This means that, most of them believe they lack the competence to select the most appropriate manipulative for teaching a mathematical concept. This is in tandem with the findings of Uribe-Florez and Wilkins (2010) who among other reasons discovered that teachers believe that they lack adequate competence in the use of manipulative materials in teaching mathematics. This has

implications on their willingness to use and use of manipulative materials in teaching concepts like fractions in their respective classrooms. Meanwhile, the use of manipulative materials has been reported to have a positive impact to conceptual learning and performance (Tooke, Hyatt, Leigh, Snyder, & Borda, 1992). The study affirms the findings of Sherman and Richardson (1995) who said teachers assign the availability and cost concerns as reasons for not using manipulative materials.

The overall mean score of 2.33 for teachers' attitudes about the use of manipulative materials in teaching fractions in upper primary six schools indicates that the majority of instructors had a negative attitude toward manipulative materials in upper primary six classes. According to Young-Loveridge, Bicknell, and Mills (2012), this has an impact on kids' attitudes and performance in fractions. The attitude of the teacher can influence their classroom practices. Recognising the impact of the teacher on the pupil, Deringöl (2019), reported that one of the major reasons why pupils do not do well in fractions is the instructional approach adopted by the teacher.

### **Effect of manipulating materials on primary six pupil performance in fractions**

The Research question above addresses the hypothesis below:

#### **Difference between the performance of pupils in the control group and experimental group when manipulative materials are used in teaching fractions.**

It is unanimous that mathematics is key driver to national development. One of its content is fraction which has real life connotations. For it to be understood by pupils, it should be well taught by teachers. Improved student

knowledge of mathematical ideas like fractions can be achieved by using manipulative materials. But studies on how these resources affect students' math performance have yielded a variety of outcomes. It is essential to note that teachers' attitudes towards innovative teaching methods and materials can significantly affect their instructional approach and students' performance in fractions. Therefore, having a positive attitude towards such materials can enhance students' learning outcomes.

Studies on the effectiveness of manipulative materials in teaching fractions have yielded mixed results. According to Suydam and Higgins' (1976) review of 40 studies, 24 studies reported an improvement in performance when manipulative materials were used, while 12 studies found no significant difference. The use of manipulation materials was not supported by the other four investigations, though. It is significant to remember that the teachers' attitudes on contemporary teaching techniques and materials have an impact on how well the students perform in groups. As a result, encouraging children to use manipulatives can help them learn more effectively. In light of current controversy regarding the efficacy of such tools, the aim of this study is to investigate how the usage of manipulative materials may alter students' performance when answering fraction problems.

Prior to employing the manipulation materials in the learning portions, there was no discernible difference between the experimental group's performance and that of the control group, according to the research. The outcomes demonstrated that the p-value was higher than 0.05, indicating that the difference was not statistically significant. In light of this, it is not possible to reject the null hypothesis, which states that there is no discernible difference



in the two groups' student performance. Prior to employing the modified materials in the learning segments, the study revealed no appreciable difference between the performance of the control and experimental groups. The null hypothesis could not be rejected because the P value was higher than 0.05, indicating that the difference was not statistically significant. This implies that the two groups were functioning about at the same level prior to the administration of the manipulation material and that any variations that may have been noticed are the result of sampling error. The fact that the two groups received different treatments further suggests that any significant difference seen at the post-test is a result of that.

Following treatment, the performance of both intact groups significantly improved, demonstrating the efficacy of both conventional methods and the use of manipulatives in raising student achievement. But when manipulatives were used, performance improved more, indicating that they are more effective than conventional methods for teaching fractions. To determine whether there was a noticeable difference between the control and experimental groups' performance, the post-test scores of the students were compared. The t-statistic for equality of means in the post-test was 8.143 with a value of 0.000, showing a significant difference in performance between the two groups.

When manipulative tools are employed in the classroom, the null hypothesis states that there is no discernible difference in the performance of the students between the control and experimental groups. So, it was anticipated that the experimental group's mean post-test score would be comparable to the control group's post-test score. The null hypothesis was refuted by the study's findings, which showed that the mean post-test score of the experimental group

was greater than the mean post-test score of the control group. These results refute Moyer's (2001) assertion that manipulative materials can impair student learning. The findings of Kontes (2016) concur with those of the aforementioned study. He looked into how manipulative content affected the academic performance and attitudes toward mathematics of high school students.

Kontas discovered similarities between the experimental and control group findings. It demonstrates that the usage of manipulatives in the classroom has little impact on how well pupils achieve academically in mathematics. Promoting understanding-based mathematics instruction is the aim of educational mathematics. According to Uribe-Florez and Wilkins (2010) and Burns and Hamm (2011), the preference for manipulative materials can be ascribed to the fact that they enable practical learning through concrete things. The use of manipulatives versus education based solely on abstract mathematical symbols was contrasted in a 2013 study by Carbonneau, Marley, and Selig. They discovered statistically substantial evidence that the alteration improved learning with only minor to moderate effects. Manipulations have improved student learning and math performance in comparison to conventional math teaching methods.

**There is no significant difference between the attitudes of pupils in the controls and experimental groups after the use of manipulative materials in teaching fractions.**

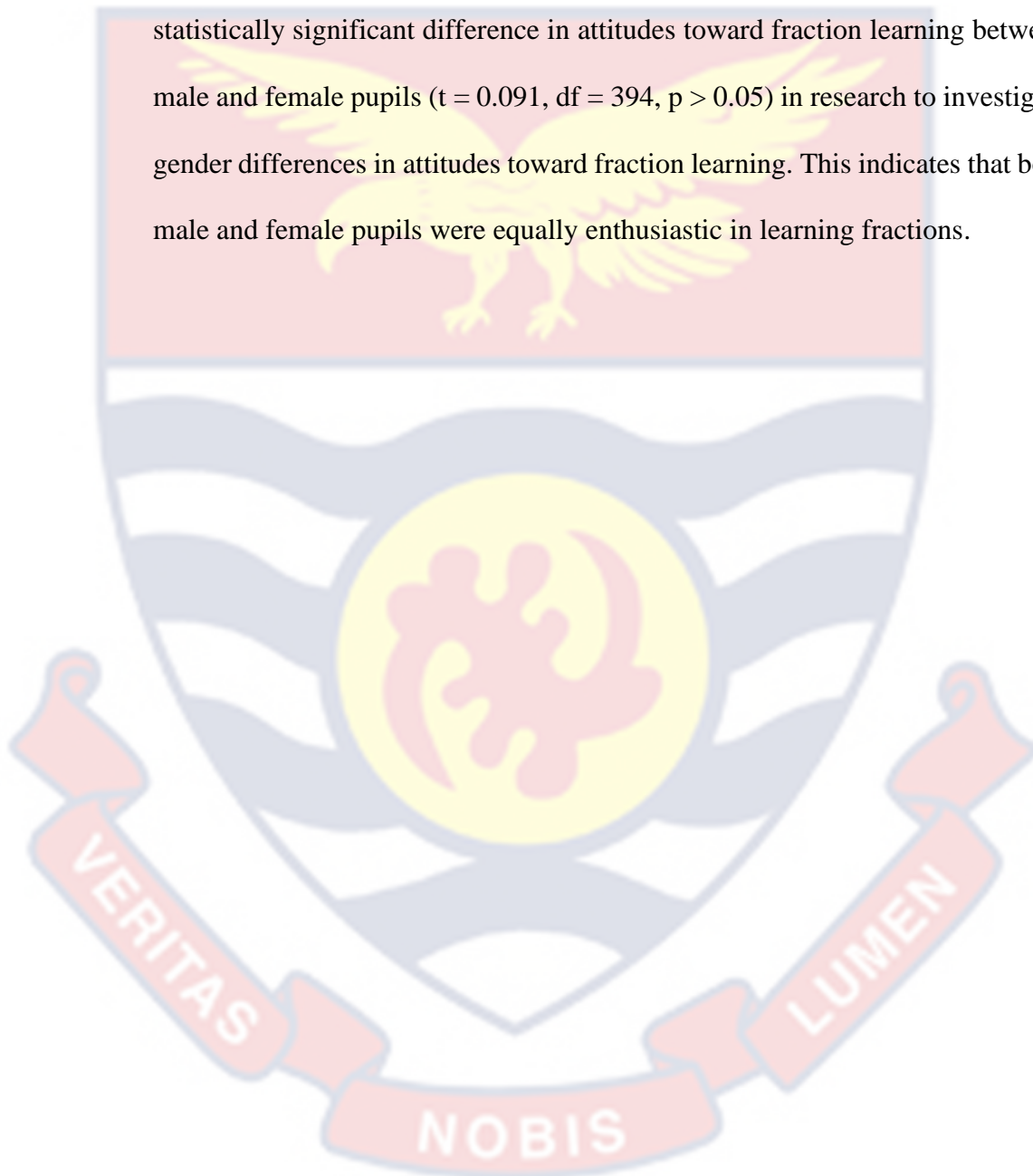
The use of manipulative materials do not only have an impact on the performance of pupils but also the attitude of the pupil towards fractions. This study compared the attitude of the experimental group and control to determine

if was any significant difference after the treatment. The test for equality of means resulted in a t-statistic of 6.750 and a sig value of 0.000. This means that, the difference between the posttest scores for both groups was statistically significant. We therefore reject the null hypothesis that there is no significant difference between the attitude of pupils in the control group and experimental group after the use of manipulative materials in teaching fractions at  $t(62.878) = 6.750, p < 0.05$ . The descriptive statistics as seen in Table 15 shows that, those who were taught using the manipulative materials had a better attitude towards the learning of fractions than those taught without the manipulative materials. This is in line with the findings of Kontas (2016), who looked at the impact of manipulatives on secondary school pupils' academic achievement in mathematics as well as their attitudes toward mathematics, and found that the scores of attitudes toward mathematics for the experiment and control groups were significantly different in posttests, favouring the experiment group.

**There is no gender difference in attitudes towards fraction in the experimental group after the use of manipulative materials in teaching fraction.**

This study explored the gender differences in attitude after the use of manipulative materials. The t-test results produced a sig value of 0.056 which is greater than the alpha level of 0.05. This means that the difference between the posttest scores for both sex groups was not statistically significant. We therefore fail to reject the null hypothesis that there is no significant difference between the attitude of pupils in the control group and experimental group after the use of manipulative materials in teaching fractions at  $t(22.864) = 2.013, p > .05$ . This also implies that using manipulative tools in the classroom bridges

any disparities in attitudes toward fractions between male and female pupils. This contradicts the findings of Relich (1996) and Marsh, Smith, and Barnes (1985), who claimed that guys have a better attitude toward mathematics learning than girls. It does, however, coincide with Yusof (2003), who found no statistically significant difference in attitudes toward fraction learning between male and female pupils ( $t = 0.091$ ,  $df = 394$ ,  $p > 0.05$ ) in research to investigate gender differences in attitudes toward fraction learning. This indicates that both male and female pupils were equally enthusiastic in learning fractions.



## CHAPTER FIVE

### SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

#### Overview

This chapter offers a summary of the research, along with inferences drawn from the results and suggestions based on them.

#### Summary

The purpose of this study was to investigate the effects of using manipulative materials on primary six pupils' performance and attitudes towards fractions in the Aowin Municipality of the Western North Region. Specifically, the study focused on the attitude of pupils towards the learning of fractions, the attitude of teachers towards the use of manipulative materials in teaching fractions at the upper primary level, the impact of manipulative materials on the performance and attitude of the pupils towards fractions and gender differences in attitude towards fractions when manipulative materials are used.

The study used a quasi-experimental approach to collect primary data from the respondents, two instruments were used: a questionnaire and a test. The study's participants were all primary six teachers and pupils in the Aowin Municipality. The study included 80 pupil (40 in each of the experimental and control groups) and 34 teachers. They were chosen using a multi-stage sampling method. The study's findings were presented using descriptive and inferential statistics. The attitudes of pupils toward learning fractions and the attitudes of teachers regarding the use of manipulative tools were reported using descriptive statistics such as frequency counts, percentages, means, and standard deviations. The t-test (inferential statistics) was used to ascertain the impact of

the manipulatives on the attitude and performance of the pupils as well as gender differences in the attitude of pupils after the use of the manipulatives.

### **Key Findings**

According to the study, a majority of the pupils had an unfavorable attitude towards learning fractions. Specifically, they believed that fractions were difficult to learn, not applicable to real-life situations, and that they did not enjoy learning them. Nevertheless, the experimental group showed a trend towards positive attitudes after the interventions, indicating that the use of TLM improved their attitude towards learning fractions. Based on the results of the survey, the majority of teachers (82.4%) think that manipulative materials are suitable for children in lower primary grades. However, most teachers also expressed the need for guidance in selecting the most appropriate manipulative material for teaching a specific concept. In addition, finding appropriate manipulative materials was also reported as a challenging task by most teachers.

In conclusion, the initial t-test performed prior to the use of manipulative materials revealed no significant difference in the mean scores between the experimental and control groups, while both groups shown improvement in performance from pre-test to post-test. But following the use of manipulatives, a significant attitude difference between the experimental and control groups was seen, demonstrating the beneficial effects of manipulatives on attitudes toward learning fractions. There was no discernible gender difference in the pupils' attitudes following use of the manipulative items.

## Conclusions

In accordance to the findings, the study give the following conclusions:

1. Pupils had negative attitude towards the learning of fractions. This has implications on their performance since your attitude can influence your willingness to learn.
2. The usage of manipulative materials by upper primary teachers can positively affect the academic achievement of their students.
3. Pupils in the experimental group
4. . In contrast to the conventional talk-chalk method, the study discovered that the use of manipulative materials had a favorable impact on both the performance and attitude of pupils toward the learning of fractions.
5. The use of manipulative materials bridges the gap in attitude between boys and girls towards fractions. This will help bridge the gap in performance between boys and girls and further demystify the perception that mathematics is male-domain

## Recommendations

On the basis of the study's findings, the following recommendations are made:

1. Teacher training programs should prioritize the incorporation of manipulative materials into their curriculum, to help prepare teachers for effective use of manipulative materials in the classroom. This can improve teachers' confidence and competence in teaching fractions, and ultimately lead to better academic performance and positive attitudes towards learning fractions among their pupils.
2. It is further recommended that teachers should have positive attitude towards the use of manipulative materials in the classrooms. This will

influence their decision to use them in teaching mathematical concepts such as fraction.

3. Since pupils from the experimental group were found to have performed better than pupils in the control group, it is recommended that management of GES and head teachers encourage teachers to use manipulatives in teaching mathematics in whichever level that they teach at the primary level.
4. Providing seminars and in-service training on the use of manipulative materials for primary six teachers can be a great initiative. This can help equip teachers with the necessary skills and knowledge to effectively integrate manipulative materials into their teaching practice, which can ultimately improve the performance and attitude of pupils towards learning fractions.
5. Parents and teachers should commit some resources to provide pupils with relevant manipulatives for self-learning. This would help maintain the current attitudes of learners.

### **Suggestions for Further Studies**

Similar studies in different contexts and with different age groups would provide a broader perspective on the effectiveness of manipulative materials in teaching mathematics. It would also enable researchers to identify any variations in the impact of manipulative materials in different settings and for different pupils.



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## APPENDICES

## APPENDIX A

## QUESTIONNAIRE FOR TEACHERS

## UNIVERSITY OF CAPE COAST

## COLLEGE OF EDUCATION STUDIES

EXAMINING THE USE OF MANIPULATIVE MATERIALS IN  
TEACHING FRACTIONS IN UPPER PRIMARY SCHOOLS IN THE  
AOWIN MUNICIPALITY OF THE WESTERN NORTH REGION  
QUESTIONNAIRE FOR TEACHERS

Dear Sir/Madam,

This questionnaire is being used to gather information on examining the use of manipulative materials in teaching fractions in upper primary schools in the Aowin Municipality of the Western North Region. I will be grateful to have you take part in the study by answering the questions as honestly as possible.

Please be assured that every information you provide will be kept confidential.

Instruction: Tick the appropriate bracket [] or column or write your response in the blank spaces where necessary.

**Background Data**

1. Sex: Female [] Male []
2. Which class do you teach? .....
3. Which of the following age range has your age?  
21-30 [] 31-40 [] 41-50 [] 51-60 [] 60 []
4. What is your professional qualification?  
Cert 'A' [] DBE [] B. Ed [] M. Ed./MPhil (Education) []

5. How many years have you been teaching?

Less than 4 years [ ] 4-8years [ ] 9-12 years [ ] 12-16 years [ ]

above 16 years [ ]

### Section B: Attitude towards the use of manipulative materials

For each item below, tick a box which indicates your level of endorsement of the attitude of teachers towards the use of manipulative materials.

No.	Statement	SD	D	A	SA
6.	The use of manipulative materials is for children in the lower primary.				
7.	Using manipulatives is time wasting				
8.	I need support in order to select the most appropriate manipulative for a concept				
9	It is very difficult to get access to appropriate materials.				
10	Using manipulative materials slows down the pace of work				
11	Using manipulatives makes the class boring				
12	I will use manipulative materials as a means to reward my pupils				
13	The use of manipulative materials is not effective in large classes				
14	Manipulatives are used more for fun than instruction				
15	Manipulatives for instruction is very costly				

## APPENDIX B

## QUESTIONNAIRE FOR PUPILS

## UNIVERSITY OF CAPE COAST

## COLLEGE OF EDUCATION STUDIES

## QUESTIONNAIRE FOR PUPILS

Dear pupil,

This questionnaire is being used to gather information on examining the use of manipulative materials in teaching fractions in upper primary schools in the Aowin Municipality of the Western North Region. I will be grateful to have you take part in the study by answering the questions as honestly as possible. Please be assured that every information you provide will be kept confidential.

Instruction: Tick the appropriate bracket [] or column or write your response in the blank spaces where necessary.

**Background Data**

1. Sex: Female [] Male []

**Section B: Attitude towards the learning Fractions**

For each item below, tick a box which indicates your level of endorsement of the attitude of teachers towards the teaching and learning of fractions.

No.	Statement	SD	D	A	SA
3.	Learning fractions is very difficult				
4.	I wish I do not meet any problem relating to fractions				
5.	I am always have fear when I am to learn or solve problems relating to fractions				
6	Fractions is not related to real life situations				

7	Fractions is too abstract.				
8	To learn fractions, you have to memorise procedures and formulae.				
9	I feel enthusiastic when learning fractions.				
10	I enjoy learning fractions				



## APPENDIX C

## PRE-TEST QUESTIONS

Answer ALL questions

1. Which of the following two fractions are equivalent?

A.  $\frac{5}{2}$  and  $\frac{2}{5}$

B.  $\frac{4}{3}$  and  $\frac{8}{6}$

C.  $\frac{1}{4}$  and  $\frac{2}{4}$

D.  $\frac{2}{3}$  and  $\frac{1}{3}$

2. How many minutes are there in two-thirds an hour?

A. 20 minutes

B. 40 minutes

C. 60 minutes

D. 100 minutes

3. Write  $2\frac{1}{3}$  as a fraction.

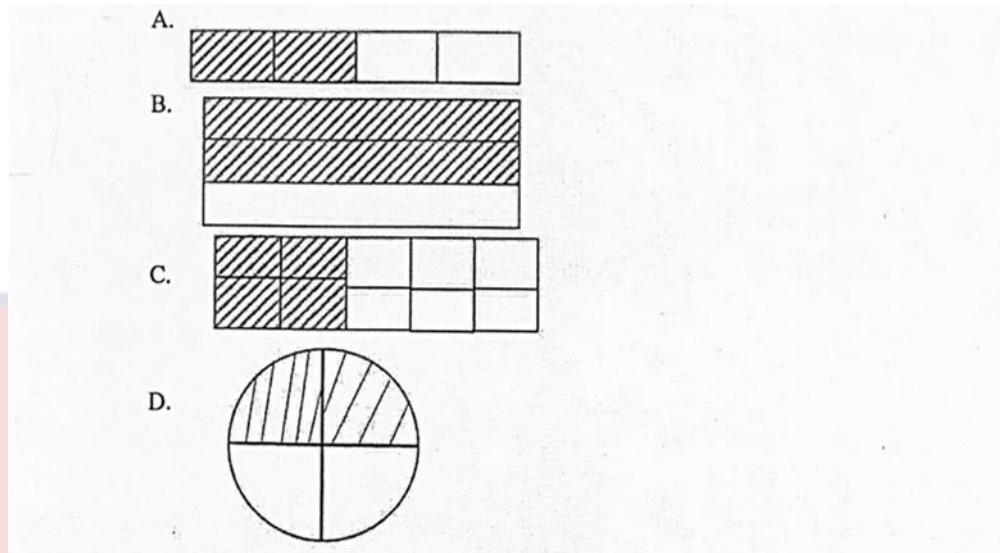
A.  $\frac{2}{3}$

B.  $\frac{7}{3}$

C.  $\frac{1}{3}$

D. 6

4. Which of the figures is shaded to show a fraction equal to  $\frac{2}{5}$  of its whole?



5. If the fraction  $\frac{N}{6}$  and  $\frac{2}{3}$  are equivalent, what is the value of N?

- A. 1
- B. 2
- C. 3
- D. 4

6. Eric ate  $1\frac{1}{4}$  pizzas and Lawrence ate  $1\frac{2}{3}$  pizzas. How much more pizza did Lawrence ate than Eric?

7. Simplify  $6 \div \frac{1}{2}$

8. Order from the least to the greatest the fraction  $\frac{3}{5}, \frac{7}{6}, \frac{1}{3}$  and  $\frac{4}{9}$

9. Write  $\frac{31}{7}$  as a mixed number

10. Simplify  $\frac{1}{5} + \frac{1}{2}$

11. Find the sum of  $\frac{1}{3}$  and  $\frac{2}{5}$

12. Simplify  $\frac{3}{4} - \frac{1}{2}$

13. Simplify  $\frac{1}{2} \div 6$

## APPENDIX D

## POST-TEST QUESTIONS

Answer ALL questions

1. Which of the following two fractions are equivalent?

E.  $\frac{5}{2}$  and  $\frac{2}{5}$

F.  $\frac{4}{3}$  and  $\frac{8}{6}$

G.  $\frac{1}{4}$  and  $\frac{2}{4}$

H.  $\frac{2}{3}$  and  $\frac{1}{3}$

2. How many minutes are there in two-thirds an hour?

E. 20 minutes

F. 40 minutes

G. 60 minutes

H. 100 minutes

3. Write  $2\frac{1}{3}$  as a fraction.

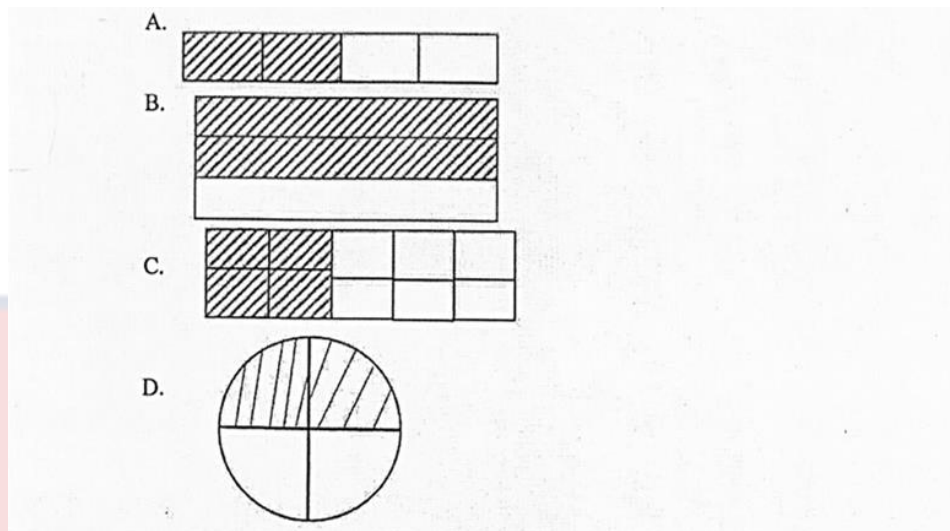
E.  $\frac{2}{3}$

F.  $\frac{7}{3}$

G.  $\frac{1}{3}$

H. 6

4. Which of the figures is shaded to show a fraction equal to  $\frac{2}{5}$  of its whole?



5. If the fraction  $\frac{N}{6}$  and  $\frac{2}{3}$  are equivalent, what is the value of N?

E. 1

F. 2

G. 3

H. 4

6. Eric ate  $1\frac{1}{4}$  pizzas and Lawrence ate  $1\frac{2}{3}$  pizzas. How much more pizza did Lawrence ate than Eric?

7. Simplify  $6 \div \frac{1}{2}$

8. Order from the least to the greatest the fraction  $\frac{3}{5}, \frac{7}{6}, \frac{1}{3}$  and  $\frac{4}{9}$

9. Write  $\frac{31}{7}$  as a mixed number

10. Simplify  $\frac{1}{5} + \frac{1}{2}$

11. Find the sum of  $\frac{1}{3}$  and  $\frac{2}{5}$

12. Simplify  $\frac{3}{4} - \frac{1}{2}$

13. Simplify  $\frac{1}{2} \div 6$



## APPENDIX E: ETHICAL CLEARANCE

## UNIVERSITY OF CAPE COAST

## INSTITUTIONAL REVIEW BOARD SECRETARIAT

TEL: 0558093143 / 0508878309  
 E-MAIL: [irb@ucc.edu.gh](mailto:irb@ucc.edu.gh)  
 OUR REF: UCC/IRB/A/2016/1303  
 YOUR REF:  
 OMB NO: 0990-0279  
 IORG #: IORG0009096

1<sup>ST</sup> APRIL, 2022

Mr. Eric Erzuah  
 Department of Mathematics and ICT Education  
 University of Cape Coast

Dear Mr. Erzuah,

**ETHICAL CLEARANCE – ID (UCCIRB/CES/2021/172)**

The University of Cape Coast Institutional Review Board (UCCIRB) has granted Provisional Approval for the implementation of your research **Effects of using Manipulative Materials on Primary Six Pupils Performance and Attitudes towards Fraction in Aowin Municipal**. This approval is valid from 1<sup>st</sup> April, 2022 to 30<sup>th</sup> March, 2023. You may apply for a renewal subject to submission of all the required documents that will be prescribed by the UCCIRB.

Please note that any modification to the project must be submitted to the UCCIRB for review and approval before its implementation. You are required to submit periodic review of the protocol to the Board and a final full review to the UCCIRB on completion of the research. The UCCIRB may observe or cause to be observed procedures and records of the research during and after implementation.

You are also required to report all serious adverse events related to this study to the UCCIRB within seven days verbally and fourteen days in writing.

Always quote the protocol identification number in all future correspondence with us in relation to this protocol.

Yours faithfully,

Samuel Asiedu Owusu, PhD

UCCIRB Administrator

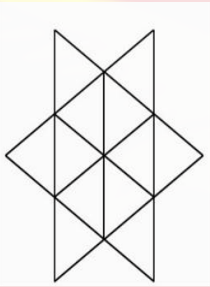
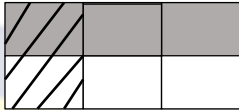
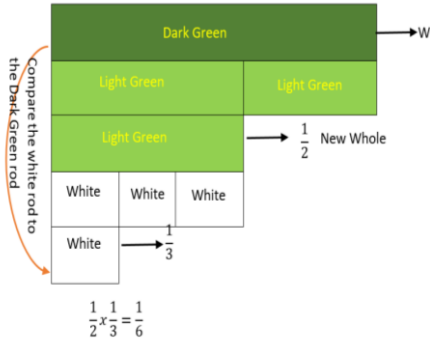
ADMINISTRATOR  
 INSTITUTIONAL REVIEW BOARD  
 UNIVERSITY OF CAPE COAST

## APPENDIX F

## LESSON NOTES

(Experimental group)

<b>Week Ending</b>			
<b>Class</b>	Six		
<b>Subject</b>	MATHEMATICS		
<b>Reference</b>	Mathematics curriculum Page 28-29		
<b>Learning Indicator(s)</b>	B6.1.5.1.1-2		
<b>Performance Indicator</b>	Learners can add two common fractions Learners can compare and order common		
<b>Strand</b>	Number		
<b>Sub strand</b>	Fractions		
<b>Teaching/Learning Resources</b>	Cuisenaire rod and paper folding		
Core Competencies: Problem solving skills; Critical Thinking; Justification of Ideas; Collaborative Learning; Personal Development and Leadership Attention to Precision			
<b>Days</b>	<b>PHASE 1: STARTER 10 MINS (Preparing The Brain For Learning)</b>	<b>PHASE 2: MAIN 40MINS (New Learning Including Assessment)</b>	<b>PHASE 3: REFLECTION 10MINS (Learner and Teacher)</b>

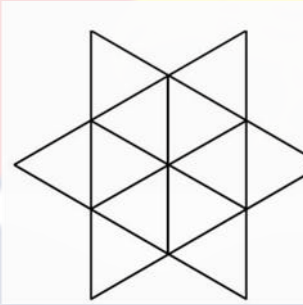
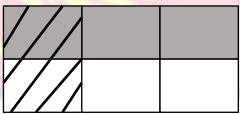
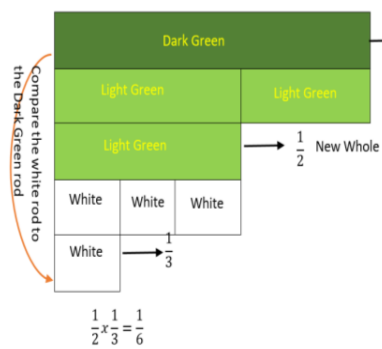
<p><b>Monday</b></p>	<p>How many triangles can you see in this picture?</p> 	<p>Use paper folding and Cuisenaire rod to introduce multiplication of two common fractions. <math>\frac{1}{2} \times \frac{1}{3}</math></p> <p>Select a rectangle sheet of paper</p> <p>Fold the paper in two equal parts horizontally and shade one portion to represent one half</p>  <p>The double shaded portion becomes the numerator of the fractions and the total division on the paper becomes the denominator.</p> 	<p>Give learners examples to solve using the paper foldings and the Cuisenaire rod.</p> <p>Teacher guide those who will have problems.</p> <p>Give remedial learning to those who need special help.</p>
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		Give learners assignments on using the paper folding and the Cuisenaire rod.	
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**Lesson notes**

**(Control group)**

<b>Week Ending</b>	
<b>Class</b>	Six
<b>Subject</b>	MATHEMATICS
<b>Reference</b>	Mathematics curriculum Page 28-29
<b>Learning Indicator(s)</b>	B6.1.5.1.1-2
<b>Performance Indicator</b>	Learners can add two common fractions Learners can compare and order common
<b>Strand</b>	Number
<b>Sub strand</b>	Fractions
<b>Teaching/Learning Resources</b>	Blackboard and Chalk illustrations
Core Competencies: Problem-solving skills; Critical Thinking; Justification of Ideas; Collaborative Learning; Personal Development and Leadership Attention to Precision	

Days	<b>PHASE 1:</b> <b>STARTER 10 MINS</b> <b>(Preparing The Brain For Learning)</b>	<b>PHASE 2: MAIN 40MINS</b> <b>(New Learning Including Assessment)</b>	<b>PHASE 3:</b> <b>REFLECTI ON 10MINS</b> <b>(Learner and Teacher)</b>
<b>Monday</b>	<p>How many triangles can you see in this picture?</p> 	<p>Use blackboard and chalk illustrations to introduce the multiplication of two common fractions. <math>\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}</math></p>  <p>The double-shaded portion becomes the numerator of the fractions and the total division on the paper becomes the denominator.</p>  <p>Give learners assignments on multiplication of fractions.</p>	<p>Give learners examples to solve using the paper folding and the Cuisenaire rod illustrations. Teacher guide those who will have problems. Give remedial learning to those who need special help.</p>