University of Cape Coast https://ir.ucc.edu.gh/xmlui

UNIVERSITY OF CAPE COAST

IN-SERVICE TEACHERS' KNOWLEDGE OF FRACTION MAGNITUDE: A CASE OF MATHEMATICS TEACHERS IN THE TAMALE METROPOLIS

FUSEINI ABUBAKARI

2023

Digitized by Sam Jonah Library

University of Cape Coast https://ir.ucc.edu.gh/xmlui

UNIVERSITY OF CAPE COAST

IN-SERVICE TEACHERS' KNOWLEDGE OF FRACTION MAGNITUDE: A CASE OF MATHEMATICS TEACHERS IN TAMALE METROPOLIS

BY

FUSEINI ABUBAKARI

Thesis submitted to the Department of Basic Education of the Faculty of Educational Foundations, College of Education Studies, University of Cape Coast, in partial fulfillment of the requirements for the award of Master of Philosophy Degree in Basic Education.

SEPTEMBER 2023

Digitized by Sam Jonah Library

University of Cape Coast https://ir.ucc.edu.gh/xmlui

DECLARATION

Candidate's Declaration

I hereby declare that this thesis is the result of my own original research and that no part of it has been presented for another degree in this university or elsewhere.

Candidate's Signature: …………………………………… Date: ………………...

Name: Fuseini Abubakari

Supervisor's Declaration

I hereby declare that the preparation and presentation of the thesis were supervised in accordance with the guidelines on supervision of thesis laid down by the University of Cape Coast.

Supervisor's Signature: …………………………. Date: ……………......

Name: Dr. Foster D. Ntow

Digitized by Sam Jonah Library

ABSTRACT

The premise of this research was that teachers' knowledge significantly influences students' mathematical achievement, particularly in their understanding of fractions. The aim was to explore if teachers' conceptual deficiencies in this area could explain students struggle with fractions, especially in understanding their magnitudes. The study employed an integrated theory of numerical development which posits that a teacher's ability to place a specific fraction on a number line reflects a comprehensive understanding of fraction magnitude. It adopted a cross-sectional survey design, involving a sample of 134 mathematics teachers from public junior high schools. Descriptive statistics revealed that mathematics teachers possessed an average level of knowledge concerning the magnitude of fractions. Furthermore, inferential statistics indicated a positive correlation between teachers' years of teaching experience and their knowledge of fraction magnitude. Moreover, the study discovered a significant difference in the fraction magnitude knowledge of in-service teachers and the specific classes they teach mathematics. In light of the findings, the study recommends that teacher training institutions improve on mathematical courses that incorporate the content and pedagogical elements of fractions. Additionally, stakeholders in education are encouraged to frequently organise periodic in-service training programmes, specifically targeting mathematics teachers, with the aim of enhancing their understanding of fraction magnitude and overall proficiency in teaching the subject.

Digitized by Sam Jonah Library

University of Cape Coast https://ir.ucc.edu.gh/xmlui

KEY WORDS

Rational Numbers

Fractions

Fraction magnitude

In-service teachers

Teaching experience

Digitized by Sam Jonah Library

University of Cape Coast https://ir.ucc.edu.gh/xmlui

ACKNOWLEDGEMENTS

All praises and thanks to almighty Allah for all his grace and favours in a form of health, knowledge and wisdom that translated to the materialisation of this thesis. More so, I extend a profound gratitude to my research Supervisor, Dr. Forster Danso Ntow from the Mathematics and ICT Education Department of the University of Cape Coast. His guidance, tremendous motivation, encouragement and timely response during the thesis project did not go unnoticed. May the Almighty bless you.

I equally extend my gratitude to all the members in the Department of Basic Education and that of Mathematics and ICT Education of the University of Cape Coast for their exceptional contribution towards the betterment of this work. I am especially indebted to my entire family especially my Aunt, Mrs. Aminatu Iddrisu and my two Uncles, Mr. Alhassan Iddrisu Abu and Mr. Fuseini Iddrisu. The support you rendered to me financially and physically cannot go unrecognized. May the Almighty Allah reciprocate your efforts with goodness. I will be ungrateful if I fail to thank my friends who have contributed on so many ways to the success of this work, especially Mr. Abdul Majeed Rufai. May Allah bless your efforts.

Digitized by Sam Jonah Library

University of Cape Coast https://ir.ucc.edu.gh/xmlui

DEDICATION

To my dear Aunt, Aminatu Iddrisu

Digitized by Sam Jonah Library

University of Cape Coast https://ir.ucc.edu.gh/xmlui

TABLE OF CONTENTS

Digitized by Sam Jonah Library

University of Cape Coast https://ir.ucc.edu.gh/xmlui

Digitized by Sam Jonah Library

University of Cape Coast https://ir.ucc.edu.gh/xmlui

Digitized by Sam Jonah Library

University of Cape Coast https://ir.ucc.edu.gh/xmlui

Digitized by Sam Jonah Library

University of Cape Coast https://ir.ucc.edu.gh/xmlui

LIST OF TABLES

Digitized by Sam Jonah Library

University of Cape Coast https://ir.ucc.edu.gh/xmlui

Digitized by Sam Jonah Library

University of Cape Coast https://ir.ucc.edu.gh/xmlui

LIST OF FIGURES

Digitized by Sam Jonah Library

University of Cape Coast https://ir.ucc.edu.gh/xmlui

CHAPTER ONE

INTRODUCTION

The role that fractions play in mathematics education cannot be underestimated. It forms the foundation for other mathematical concepts, such as Algebra, Geometry, Trigonometry, and Probability. Nevertheless, both students and teachers struggle with the concept, particularly, the magnitudes of fractions. Teachers require a strong grasp of fraction magnitude to effectively support students in developing a comprehensive understanding of fractions. However, research on teachers' understanding of the concept remains limited, especially among in-service teachers. Consequently, there is a need to investigate the knowledge of fraction magnitude among in-service teachers. This assessment will provide evidence on the current state of teachers' mathematical understanding of fractions. The findings will inform professional development programs, ensuring that teachers are better equipped to teach fractions effectively.

Background to the Study

When it comes to the sciences, mathematics is unquestionably indispensable. The need for mathematical literacy among individuals is not less than that of a Society. Nothing is conceivable without a strong grasp of mathematics. Many other branches of science rely heavily on mathematics, including physics, engineering, and statistics. Therefore, we cannot imagine modern life without mathematics. For individual citizens' future success in the workforce and in life, mathematics is also essential (Bed, 2017; Pacinello, 2018).

Digitized by Sam Jonah Library

University of Cape Coast https://ir.ucc.edu.gh/xmlui

Nonetheless, students' performance in mathematics has been consistently poor on a global scale for decades. In the 2015 edition of the Trends in International Mathematics and Science Study (TIMSS), Mullis, Martin, Foy, and Hooper (2016) highlighted the results: 48.8% of participating countries scored at the low benchmark, 39.53% fell within the intermediate benchmark, and only 11.63% reached the high benchmark. Notably, none of the participating countries achieved the advanced benchmark level.

An area of concern in mathematics is the concept of fractions. Etymologically, the word fraction emanates from the Latin word "*fractio*" which means to break. (New World Encyclopedia, 2022). Hence, the word fraction connotes a part, a quantity or a portion of something. Fractions became necessary due to the dynamic nature of the world and the need for humans to adjust. Therefore, the origin and use of fractions date back to man's development. One of the earliest known uses of fractions can be traced back to ancient Egypt where they were used to compute quantities of commodities in transactions. Today, fractions are an integral part of mathematics curriculums (Nikita, 2019).

Fractions according to Pienaar (2014) are numbers that convey part of a whole in a form of a quotient of integers and the divisor is a non-zero digit. It is in a form of division where both numerator and the denominator are integers and the denominator is not a zero. Moreover, the Australian Association of Mathematics Teachers (2013) conceptualized fractions as a multiplicative relationship between discrete or continuous quantities. In other words, it is a rational number that

Digitized by Sam Jonah Library

expresses quantities like countable items (discrete) or measurements like area and length.

Despite the significant relationship between the knowledge of fractions and general mathematics achievement (Siegler & Pyke, 2013; Lortie-Forques, Tian, & Siegler, 2015; Gabriel, 2016), the concept has become a hurdle for students over the years. It is not only students who struggle with fractions but teachers themselves find it difficult to comprehend (Tzur & Hunt 2015; Yeong, Dougherty & Berkaliev, 2015; Fitzsimmons, Thompson, & Sidney, 2020; Copur-Gencturk, 2022). Hence, students and their teachers struggle with fractions in the process of learning. For instance, Eichhorn (2018) explored the mathematical thinking of Indian primary students about fractions. It was revealed that many students portray misconceptions of fractions, especially fraction equivalence. Lester (as cited in Lopez, 2020) maintained that children's misconceptions are rooted in their teachers' lack of knowledge of the concept.

Moreover, Agbozo (2020) explored Ghanaian pre-service teachers' attitude towards teaching fractions and some concepts of fractions. It was discovered that participants found the addition of fractions easy, yet algorithms of fractions, in general, were difficult and appeared abstract to them. Also, Amuah, Davis and Fletchert (2019) investigated students' ideology of fractions. Participants were junior high school students in the Cape Coast Metropolis in Ghana. The study revealed that fractions greater than one were generally difficult for all the students who participated. The students could not comprehend that a

Digitized by Sam Jonah Library

fraction can be greater than a whole. This means that their ideology of fractions is limited to part-whole construct.

The misconceptions about fractions stem from several factors. To start with, research suggests that fractions have become an obstacle to many students which is often attributed to their weakness in fractions magnitudes (Gabriel, 2016; Namkung & Fochs, 2019; Dyson, Jordan, Rodreigues, Barbiere & Rinne, 2020). More so, a fraction is a compound concept with multiple meanings and interpretations. It can be regarded as part of a whole, a quotient, a ratio, a measure and an operator (Hackenberg & Lee, 2015; Getenet & Callingham, 2017). For instance a fraction such as $\frac{4}{5}$ has so many interpretations. It can be regarded as 4 out of 5 parts (Part-Whole), 4 parts to 5 parts (Ratio), $\frac{4}{5}$ of a quantity (Operator), 4 divide by 5 (Quotient) and $\frac{4}{5}$ as a point on a number line (Measure). This implies that a robust understanding fraction concept requires comprehending all meanings that fractions can be represented which is difficult to achieve.

Furthermore, students misconceive the property of whole numbers as fractions and apply the rule of whole numbers when dealing with fractions. For example, the concept of multiplication as repeated addition only works with whole numbers and becomes dysfunctional when applied to fractions (Yeong et al., 2015). Moreover, students overgeneralize the notion that, multiplication increases the value of a product, which does not entirely work with fractions. Also, many students are unable to view the numerator and the numerator of fractions as a single number with a magnitude (Cramer & White, 2010). Hence, in

Digitized by Sam Jonah Library

problem-solving with fractions, some students treat both (the numerator and the denominator) separately which leads to errors in fractions computations.

The crucial function of fractions in learning mathematics hass made it less fortunate that students as well as teachers have challenges grasping it. It is a foundation upon which other mathematical concepts are built. More so, Hannich (as cited in Ntow, 2022) maintains that one of the topics in mathematics at the basic level that learners' first encounter and experience beyond the basic arithmetic skills is fractions. For instance, in Ghana, fractions are first introduced to learners at Basic One through partitioning (Ministry of Education, 2019). A fractions skill does not only help in high mathematics achievement but applicable in other areas of Science, Sociology and Psychology (Lortie-Forques et al, 2015; Gabriel, 2016; Amalina, Fuad, & Agustina, 2018). On the other hand, problems with fractions can hinder further opportunities for high mathematics and other related scientific disciplines (Gabriel, 2016). Therefore, the hierarchical nature of mathematics curriculums makes the skill of fractions critical for understanding mathematics.

A fundamental skill that learners must be grounded with in order to gain mastery of other aspects of the concept is fraction magnitude (Amalina et al., 2018). According to Lopez (2020), "a fraction magnitude refers to the amount of a given unit that a fraction represents" (p.15). Furthermore, Gabriel (2016) also opined that the ability to accurately order, compare and place fractions precisely on number lines reflects fraction magnitude knowledge. Hence, the magnitude of a fraction is the size or value of a fraction in relation to other fractions. This

Digitized by Sam Jonah Library

implies that a teacher or a student with a good understanding of fraction magnitude will be able to comprehend, determine, compare or sort fractions in terms of the bigger or the smaller.

The fraction magnitude skill is considered a foundation for understanding fractions (Amalina et al., 2018). In other words, fraction magnitude is an integral part of fractional thinking. This was substantiated by Siegler, Thompson and Schneider (2011) when they mentioned that a student with a mastery of fraction magnitude can proficiently deal with fraction arithmetic. Furthermore, fractions are also considered an integral part of the numerical development theory propounded by Siegler and colleagues (2011). This theory exposes children to thinking beyond the idea of whole numbers. By virtue of this theory, children come to realise that the thinking that all numbers can be represented by a single value and that numbers never increase with the division are not true in the case of fractions (Fazio, Dewolf & Siegler, 2016).

Fraction magnitude knowledge has been documented in the literature as an indicator of mathematical success of students in the future. Siegler and Pyke (2013) substantiated this assertion when an investigation was done on 8th-grade and community college students. They were required to estimate and compared fractions and the accuracy of their estimations were about 70%. Also, magnitude knowledge of both fractions and whole numbers at age four was associated to mathematical achievement at later grades (Siegler et al., 2012; Watts, Duncan, Siegler & Davis-Kean, 2014). These studies indicated that a strong fraction

Digitized by Sam Jonah Library

magnitude skill at early age reflects a good mathematics achievement at high and tertiary institutions.

Fraction magnitude, though a key to mathematics, is difficult and complex for students and teachers. For instance, Mou et al. (2016) investigated grades 8th and 9th students' comprehension of fractions on a number line. It was discovered that the precision of students' location of whole numbers on a number line was very high yet considerably low with fractions, depicting difficulties of fraction magnitude. Furthermore, Fazio et al. (2016) after examining fraction magnitude comparison strategies of college mathematics students revealed that a significant number of college students lack fraction magnitude understanding. Moreover, a study was conducted by Copur-Gencturk, (2022) to explore the knowledge of fraction magnitude possessed by in-service mathematics teachers. It was revealed among other things that even though majority of the teachers employed strategies that were aligned with fraction magnitude, their estimation for both addition and division problems were partially accurate. A similar case was discovered by Toledo, Rosenberg-Lee & Abreu-Mendoza (2022) with Brazillian Elementary and Secondary mathematics teachers. For instance, majority of the participants the study used flawed Gap strategy in processing the magnitude of fractions which was attributed to flaws in their reasoning.

Notwithstanding the difficulties students and teachers face when encountering fraction magnitude, researchers' mostly tackled the problem through students (Amalina et al., 2018; Mou et al., 2016; Ntow, 2022), relegating that of teachers, especially in-service teachers. This is the reason behind the persistence

Digitized by Sam Jonah Library

of students' difficulty in fractions conception (Siegler, Fazio, Bailey, & Zhou, 2013; Lortie-Forgues, et al., 2015) despite efforts constantly being made and strategies suggested by mathematics educators to improve mathematics teaching. Hence, it is necessary to tackle the problem from the teachers' side as literature has proven a positive relationship between mathematical knowledge of teachers and students' performance (Depaepe et al., 2015; Copur-Genturk, 2015; 2021).

There are several factors influencing the level of knowledge of a teacher aside taking educational courses. Among those factors is the grade/class level that a teacher teaches. For instance, Copur-Genturk (2022) found that teachers at the high grades possess more fraction magnitude knowledge as compared to teachers at the lower grades. A study by Depaepe et al. (2015) discovered that Secondary teachers demonstrated high mathematical content knowledge as compared to their counterparts in the elementary grades. Therefore, the level and the kind of students that teachers encounter influence their level of mathematical content and pedagogical knowledge.

Furthermore, literature has shown that the level of a teacher's experience in teaching a particular subject also determines his knowledge on it. For instance, Harris and Sass (2011) study found teachers who have more years of experience in teaching more competent than their peers with less years of teaching experience. Thus, they improve in knowledge in the process of teaching. The study concluded that the productivity and efficiency of teachers increase as they teach. Moreover, Copur-Genturk (2021) discovered teachers with greater years of experience in teaching demonstrated robust conceptual understanding in their

Digitized by Sam Jonah Library

explanations of fraction concepts as compared to their peers with few years of teaching experience. Against this background, this particular study examined the level of accuracy of in-service teachers' estimation of fraction magnitude.

Statement of the Problem

Despite critical role that fractions play in mathematics education and other related sciences, the concept is still daunting and challenging for students and teachers worldwide (Getenet & Callingham, 2017). For instance, the fraction concept has been identified as one, if not the most complex concepts for students in Indonesia, America and Australia (Siegler et al., 2013; Amalina et al., 2016; Copur-Gencturk, 2021). This assertion was supported by a study of Kor et al. (2018) in which a Fraction Sense Test (FST) was conducted on a group of 198 primary 4 pupils in Malaysia. It was found that many students who participated lack fraction sense.

Turning to the African continent, a study by Ubah and Bansilal (2018) in South Africa discovered that prospective teachers are comfortable with addition and subtraction of fractions in which denominators are the same, yet struggle with fractions with different denominators. Moreover, Odigun (2018) examined the errors committed by senior secondary students in Nigeria, identifying errors such as incorrect operations, missing steps and fact errors of Least Common Multiple (LCM). This suggests that, African students struggle with fractions just like students in developed countries like America and Australia.

The situation is not less different in Ghana. For instance, Chief Examiners' Report in 2015 and 2017 at the BECE level mentioned word problems

Digitized by Sam Jonah Library

on fractions as one of the areas students performed miserably. In addition, Baah-Duodu et al. (2019) investigation of Ghanaian pre-service teachers' conception of fractions found that pre-service teachers' ideology of fractions, both content and pedagogical were generally low. Furthermore, the study of Agbozo (2020) on pre-service teachers revealed that the fraction algorithm appeared complex to them which was attributed to the weak foundations carried from the basic level. Also, Yakubu (2013) mentioned that teacher trainees perform miserably during their teaching practice on campus. They only stick to algorithms to add and subtract fractions. This means that these prospective teachers are likely to transfer these problems to their students.

The situation has attracted the attention of a lot of researchers. However, issues that are mostly tackled are the difficulties students encounter with the fraction concept (Tzur, & Hunt, 2015; Gabriel., 2016; Kor et al., 2018; Amalina et al., 2018; Odigun, 2018; Ntow, 2022). Studies on teachers are scarce. Generally, few studies on teachers' knowledge of fractions have focused on pre-service teachers (Yakubu, 2013; Baah-Doudu et al., 2019; Lee & Boyadzhiev, 2020; Agbozo, 2020), neglecting in-service teachers. It is imperative that in-service teachers' ideas and understanding of the concept be examined as they are responsible for guiding students in the learning process.

The few research on in-service teachers has only focused on the general concept of fractions (Copur-Gencturk, 2021). Focusing on one aspect of fractions like magnitude will reveal in detail the nature of teachers' knowledge on that area. Also, most studies in measuring teachers' knowledge of fraction magnitude have

Digitized by Sam Jonah Library

used fractions with same numerators or denominators (Lemonidis, Tsakiridou, & Meliopoulou, 2018). Such fractions can easily be worked out using algorithms and do not require fraction magnitude knowledge (Lortie-Forgues et al., 2015). Furthermore, most studies have used ordering or comparison of fractions to measure teachers' fraction magnitude knowledge (Whitacre & Nickerson, 2016; Lemonidis et al., 2018; Toledo et al., 2022). There are other ways of measuring magnitude, like determining the position of a given fraction on the number line (Siegler et al., 2011).

There is no denying the fact that previous research has delved into the understanding of in-service teachers' knowledge concerning fraction magnitude as indicated above. Alongside the gaps highlighted in these earlier studies, it appears that no such investigation has been conducted in Ghana, specifically in the Northern Sector. Hence, this study aims to explore the knowledge of fraction magnitude among in-service mathematics teachers within the Tamale Metropolis, located in the Northern Region of Ghana

Purpose of the Study

The pivotal role that fractions play in building the foundation of mathematics and the crucial role of teachers' knowledge on the success of students in mathematics classrooms have been spell out. Hence, it is necessary to determine in-service mathematics teachers' knowledge of fractions. Therefore, the study explored in-service teachers' knowledge of fractions focusing on the fraction magnitudes and strategies employed in solving fraction problems. It also investigated whether in-service teachers' understanding of fraction magnitude

Digitized by Sam Jonah Library

improves as they teach. Finally, it investigated whether the class that teachers teach has an influence on their fraction magnitude knowledge.

Research Objectives

The study aimed to:

1. Determine the level of accuracy of in-service teachers' estimation of fraction magnitude.
2. Identify the strategies used by in-service teachers in the estimation of fraction magnitude.
3. Examine the relationship between the level of accuracy of in-service teachers' estimation of fraction magnitude and their years of teaching experience.
4. Determine whether there is difference between the level of accuracy of in-service teachers' estimation of fraction magnitude and the class that they teach.

Research Questions

The following research questions guided the study:

1. What is the level of accuracy of in-service teachers' estimation of fraction magnitude?

2. What are the strategies used by in-service teachers in the estimation of fraction magnitude?

Digitized by Sam Jonah Library

Research Hypotheses

The following research hypotheses were tested:

1. There is no statistically relationship between the level of accuracy of in-service teachers' estimation of fraction magnitude and their years of teaching experience.
2. There is no statistically significant difference between the level of accuracy of in-service teachers' estimation of fraction magnitude and the class that they teach.

Significance of the Study

The constant struggle of students with fractions has persisted for decades. It is not certain whether the difficulties of students with fractions are as a result of teachers' weakness of the fraction concept. This piece of work may reveal the level of fraction knowledge of mathematics teachers at the junior high schools. It may bring to view whether in-service mathematics teachers have the needed knowledge to teach fractions.

Furthermore, it would aid stakeholders and policymakers in education to understand the kind of learning opportunities in-service mathematics teachers create for students to learn fractions in teaching and learning process. As teachers' understanding of fraction magnitude will be revealed in this study, institutions responsible for training teachers would get to know the kind of teachers they produce. This in a way would guide them in making the necessary adjustments to their programmes if the need be. This may bring about a significant impact on

Digitized by Sam Jonah Library

teachers' knowledge which would subsequently translate into mathematical success of students.

Moreover, Ghana Education Service (GES) and other Stakeholders in education can properly structure professional development programmes based on the outcome of this research to adequately address in-service teachers' deficiencies. In all, it will contribute more knowledge to the existing literature and serves as a referencing document for future researchers to assess the understanding of in-service teachers in other areas of mathematics which will eventually bring improvement of students in the mathematics.

Delimitations

This piece of work was designed to investigate the level accuracy of in-service mathematics teachers' estimation of fraction magnitude. The study engaged only in-service mathematics teachers in Tamale Metropolis in the Northern Region of Ghana. No other mathematics teachers in any other district in the Region were involved in the study.

The focus of the study was on fractions. No any other concept in mathematics was measured. Even though there are other ways of measuring fraction magnitude knowledge like comparing and ordering fractions, the study employed specifically the location of fractions on number lines.

Teacher-related variables such as the class level and teaching experience and their influence on teachers' knowledge of fraction magnitude were considered in this study. However, the relationship between gender and faction magnitude knowledge was not covered. Other variables like whether the teacher has

Digitized by Sam Jonah Library

background training in mathematics in relation to fraction magnitude understanding was excluded due to lack of resources and time factor.

Limitations

The study was affected by the small sample size. The study used few mathematics teachers in the junior high schools in the Tamale Metropolis due to limited time. There is a possibility of getting different results if all the teachers in the District were engaged. Also, it was not possible to include all other districts in the Region because of financial constraint. Moreover, the design used in the study provides a snapshot of a population at a specific moment, which might not be representative of other times. These factors limited the generalization of the result of the study. The inability of the design to establish causality and its proneness to biases can affect the validity and reliability of the results of the study.

Definition of Terms

The following terms are defined for the purpose of the study:

1. Rational number: For the purpose of this research, rational numbers are fractions of various forms.
2. Magnitude: this is the size or value of a fraction
3. In-service teachers: These are mathematics instructors at the junior high school.
4. Teaching experience: A total number of years a teacher has been teaching mathematics at the junior high school.

Digitized by Sam Jonah Library

Organisation of the Study

This study is organised into five chapters. Chapter One presents the introduction of the entire study, including the Background to the Study, Statement of the Problem, Research Purpose, Objectives, Questions and Hypothesis. Significance of the Study, Delimitations, Limitations and Definition of Terms used in the study were followed. The Chapter concluded with how the study is organised.

Chapter Two elaborated on relevant, existing and related literature. The literature was categorised into Theoretical, Conceptual and Empirical review. Issues discussed under the Empirical Review include; Teachers' Knowledge of Fractions, Teaching experience and Teachers' Mathematical Knowledge and Grade/class level teachers teach and their mathematical knowledge.

Chapter Three outlines the methodology used to conduct the study. It captured the Research Design, the Area of the Study, Population of the Study, Sampling Techniques used and the Instrument. It concluded with the Data Collection procedure, Data processing and Analysis.

Chapter Four presents the Results of the Study. Tables, diagrams and charts used to throw more light on the results. The Chapter concludes with the discussion of the results.

Finally, a Summary of the whole study, Conclusion, Findings, Recommendations and Suggestions for Future Research were presented in Chapter Five.

Digitized by Sam Jonah Library

University of Cape Coast https://ir.ucc.edu.gh/xmlui

CHAPTER TWO

LITERATURE REVIEW

Teachers' mathematical knowledge on the students' academic progress and the critical role of fraction magnitude in fractions understanding and mathematics as a whole have made it essential for a particular attention. Over the years, fractions have become a hurdle for students and mathematics educators. Researchers have endeavoured to identify the reasons for this hurdle and the possible remedies to overcome it.

The study aimed to explore the level of in-service teachers' knowledge of fraction magnitude in terms of accuracy and strategies they use when dealing with fraction magnitude problems. It also sought to investigate whether the fraction magnitude estimation accuracy of in-service teachers increases as they teach and whether there are significant differences between the level of accuracy of their estimation of fraction magnitude and the class that they teach.

The theories that guided the study have been thoroughly explained to fit the objectives of investigating the nature of teachers' knowledge of fraction magnitude. The chapter also reviews studies already conducted on teachers' knowledge of fractions, the relationship between years of teaching experience and teachers' mathematics knowledge or students' achievement, and teachers' class level and mathematical knowledge. The findings or conclusions drawn from these studies are also explored to get an in-depth understanding of what has been done so far regarding the topic to have a picture of the possible findings of the study.

Digitized by Sam Jonah Library

Theoretical Review

Integrated Theory of Numerical Development

The study employs Siegler et al. (2011) theory of integrated numerical development as its foundational framework. According to this theory, an in-depth comprehension of numerical magnitude holds pivotal importance. It posits that skills of numerical magnitude significantly influence proficiency in various mathematical domains, including fraction arithmetic. Therefore, a solid understanding of magnitude is considered crucial in cultivating an intuitive grasp of rational numbers, particularly fractions.

Having a good knowledge of the magnitude of fractions allows one to determine whether or not an answer is correct. The process and answer of $\frac{5}{7}$, for instance, that may be produced by direct summing the numerator and denominator of a task $\frac{1}{2}+\frac{4}{5}$, would be detected and rejected by someone with a thorough understanding of magnitudes. This is due to the fact that it is impossible for the sum of any two positive numbers to be smaller than the original numbers.

This theory differs from previous theories such as evolutionary and conceptual change theories in that it places a strong emphasis on learning about numerical magnitudes as a fundamental step in advancing comprehension of all real numbers. Also, whiles previous theories have focused on the distinctions between whole numbers and fractions and have postulated that students' have trouble learning fraction concepts because of the characteristics of whole numbers (DeWolf & Vosniadou, 2015), the similarities and differences between whole numbers and rational numbers received additional attention in the integrated

Digitized by Sam Jonah Library

theory of numerical development. The theory states that whole numbers have unique predecessors, are countable, include a finite number of entities within a given interval, can be represented by a single symbol, increase or remain contestant with addition and multiplication, and decrease or remain contestant with subtraction and division, but they are similar to fractions in that they all have magnitudes that can be arranged and giving particular positions on a number line (Siegler et al. 2011; Copur-Genturk, 2022). Students' understanding of this fact makes it less daunting for them to grasp the idea of fractions.

Being able to accurately locate or represent a particular fraction on a given number line depicts an understanding of fractions (Copur-Gencturk, 2022), because "all real numbers have magnitudes that can be ordered on a number line" (Siegler, 2016, p. 343). It means that teachers' understanding of fraction magnitude can therefore be explored by examining how correctly teachers can estimate and place fractions on number lines. Just like this theory, mathematics educators have unanimously agreed that locating the position of a fraction with good precision on a number line, ordering fractions and calculating operations with fractions correctly are indicators of knowledge of fraction magnitude (Behr, Lesh, Post, & Silver, 1983; Copur-Genturk, 2022). Therefore, in this study, it is hypothesized that in-service teachers' ability to estimate operations with fractions and place them exactly on a number line is an indication of fraction magnitude knowledge.

Digitized by Sam Jonah Library

University of Cape Coast https://ir.ucc.edu.gh/xmlui

Theory of Strategic Variability

The study was also informed by the theory of strategic variability, a central concept in Siegler (1995) overlapping wave theory. This theory posits that individuals possess unique traits, perspectives, and approaches when tackling problems. Moreover, how an individual approaches a task is influenced by various factors. These factors include their prior knowledge, the nature of the problem at hand, and whether they have previously encountered similar problems.

The way people approach problems and reason about fractions depends on the prior knowledge, different tasks, and cognitive abilities of such an individual (Siegler, 1995, 2016; Fitzsimmons, et al, 2020). In other words, the way and manner people approach tasks depends on the level of knowledge and the experiences they possessed. Therefore, a given strategy a teacher employs in estimating fraction magnitude determines his thinking and level of knowledge. For instance, to correctly estimate the magnitude of fractions, the numerator, denominator and the relationship between them should be simultaneously coordinated to determine its correct size (Behr et al., 1983). Therefore, a teacher referring a familiar fraction with a similar magnitude such as $\frac{1}{2}$ for $\frac{20}{37}$ or $\frac{2}{3}$ for $\frac{43}{67}$ is regarded as a sign of understanding of fraction magnitude. Conversely, a teacher dealing with the numerator or the denominator separately such as finding a common denominator to compare, add or subtract fractions is a reflection of low or weak understanding of fraction magnitude. More so, being conversant with variety of strategies enables people to choose the best one for a particular activity and adapt to various issues. For instance, a teacher who is familiar with fraction

Digitized by Sam Jonah Library

University of Cape Coast https://ir.ucc.edu.gh/xmlui

magnitude estimation strategies such as benchmarking, converting fractions to decimals, using a number line, and comparing the numerator and the denominator will perform better than a teacher who relies on one method. Such teachers could make mistakes or have trouble solving challenges that call for different approaches.

Fraction Constructs

Fractions have different meanings and interpretations. This implies that a comprehensive grasp of fractions necessitates understanding all these varied interpretations and their interrelationships. Fraction can be a Part-whole, a ratio, an operator, a Measure and a quotient (Getenet & Callingham, 2017; Kieren, 1976; Behr et al., 1983).

Part-Whole Construct

This concept, as the name suggests is part of something created when the whole is divided or partitioned into equal parts. It can be a continuous quantity or a set of discrete objects (Siemon et al., 2015; Getenet & Callingham, 2017). Moreover, Behr et al. (1983) further mentioned that the part-whole concept is fundamental to the study of other constructs. It is the most common construct in the curriculums and the starting point in the learning of fractions. This is because students' experiences originate from fair-sharing (Siemon et al., 2015; Getenet & Callingham, 2017). In this construct, a fraction is viewed as an object or a quantity partitioned into equal parts. According to Pienaar (2014) "The fraction represents an object cut into equal pieces and the numerator refers to how parts of the partitioned unit there are, whereas the denominator refers to the size of the

Digitized by Sam Jonah Library

pieces (parts) in which the unit is partitioned" (p. 24). This means that a robust comprehension of the size of a fraction requires understanding the part and the whole. Thus, the value of the numerator and the denominator and the kind of relationship that exists between them must be fully understood. The larger the denominator, the smaller the size of the fraction and vice versa. For example, $\frac{4}{5}$ indicates that an object or a quantity (whole) has been portioned into five equal parts and four out of those parts are considered.

In some curriculums, unfortunately, this is where fraction learning stops and this has a serious consequence on the learners' future progress in mathematics. For instance, this construct of fractions becomes dysfunctional when dealing with fractions greater than one (Siemon et al., 2015). For example, a student whose concept of a fraction is limited to part-whole construct will find fractions greater than one such as $\frac{7}{5}$ incomprehensible. This means that other constructs of fractions are critical for a better understanding of the fraction concept.

Fraction as a Quotient

The quotient construct interprets fractions as result from dividing a quantity by another quantity (Charalambous & Pitta-Pantazi, 2007; Pack, Gulcer & McCory, 2013; Pienaar, 2014) That is, dividing 1 by 5 gives $\frac{1}{5}$. Even though it involves partitioning, it is not a part-whole conceptualization. The part-whole construct is only limited to fractions greater than one but the quotient construct on the other hand includes fractions less or greater than one. Furthermore, Pienaar mentioned that the quotient is produced as a result of equal-sharing. It can be

Digitized by Sam Jonah Library

bigger or smaller or equal to a whole unit. This construct is critical to the understanding of the division and subtraction of fractions (Ubah, 2021). It also serves as the basis for development of the skill of renaming and comparing fractions in decimal form (Behr et al., 1983; Siemon et al., 2015). It means that every construct develops a specific skill in learners which is critical for problem-solving. Nonetheless, the quotient construct is often ignored in the teaching process (Park et al., 2013).

Fraction as a Ratio

The concept of ratio is another way fractions can be interpreted. This is the relationship between quantities (Charalambous & Pitta-Pantazi, 2007). In this concept, the emphasis is on comparison instead of numbers. It does not involve dividing or partitioning of an object. It can be a part-part ratio or part-whole. For example, the fraction $\frac{6}{7}$ could be interpreted as students wearing uniforms (part) to students not wearing uniforms (part) or those wearing pink (part) uniforms to those in the class (whole).

The ratio construct of fractions as viewed by Behr et al. (1983) is the most naturally method of helping students understand the equivalence of fractions. Thus, students viewed fractions as a proportional relation existing between two quantities. For instance, the fraction $\frac{1}{3}$ is equivalent to any other fraction in which the denominator is thrice as much as the numerator.

The ratio should always be in its simplest form and can contain two or more numbers. Most often, two numbers are compared, though it can be more. It

Digitized by Sam Jonah Library

University of Cape Coast https://ir.ucc.edu.gh/xmlui

can be represented in three different forms (Piennar, 2014). For example, in comparing 3 and 4, it can be in a form of 3 to 4, 3:4 or $\frac{3}{4}$.

Fraction as an Operator

A fraction can also be used as an operator (Charalambous and Pitta-Pantazi 2007). Getenet and Callingham (2017) states "the operator concept results from the combination of two multiplicative operations or two discrete, but related functions that are applied consecutively" (p. 4). For example, $\frac{2}{3}$ of 12 square feet or $\frac{5}{6}$ = 5×[$\frac{1}{6}$] of a unit]. This construct has been referred to as a "stretcher/shrinker" or a duplicator/partition-reducer (Pedersen & Bjerre, 2021: p. 146). The term depends on whether an operation is on a discrete set or a continuous object. When an object or a quantity stretches or shrinks, an entirely different object or quantity is formed. When an operator is performed on a discrete set it changes it to another set. For example, $\frac{2}{3}$ of 12 candies is equal to 8 candies. You either multiply 12 by the numerator (2) and divide by the denominator (3) or you divide 12 by the denominator (3) and then multiply by the numerator (2). When it is performed on a continuous quantity, it enlarges or reduces it. For example, a rectangle, with dimensions 4 × 6 units, when transformed with the scale factor $\frac{1}{2}$ reduces the object to 2×3 units. The operator construct helps in developing the skill of multiplication, division and equivalence of fractions (Behr et al., 1983; Piennar, 2014; Pedersen & Bjerre, 2021).

Digitized by Sam Jonah Library

Fraction as a Measure

A fraction can also be interpreted as a number or a measure of anything like size or distance according to the measurement construct (Charalambous and Pitta-Pantazi 2007). It incorporates knowledge of unit fraction as a scale of measurement (Kieren 1980).

This component has been explained as the measured distance between two points, and as a result, frequently associated with the display of numbers on number lines (Charalambous and Pitta-Pantazi 2007). Thus, fractions are numbers that can be arranged on a number line. It involves the identification of a length and using that piece to determine the whole distance. For example, being able to realize that, it takes 7 times of $\frac{1}{9}$ to reach $\frac{7}{9}$.

This construct has frequently been overlooked, but significant it is because of its connection with the cardinal size (Ubah, 2021). For learners to be successful in mathematics in the future, a solid grasp of a fraction as a measure is essential (Siegler et al., 2011; Ubah, 2021). The measurement interpretation of fractions helps in a deeper comprehension of the magnitude of fractions.

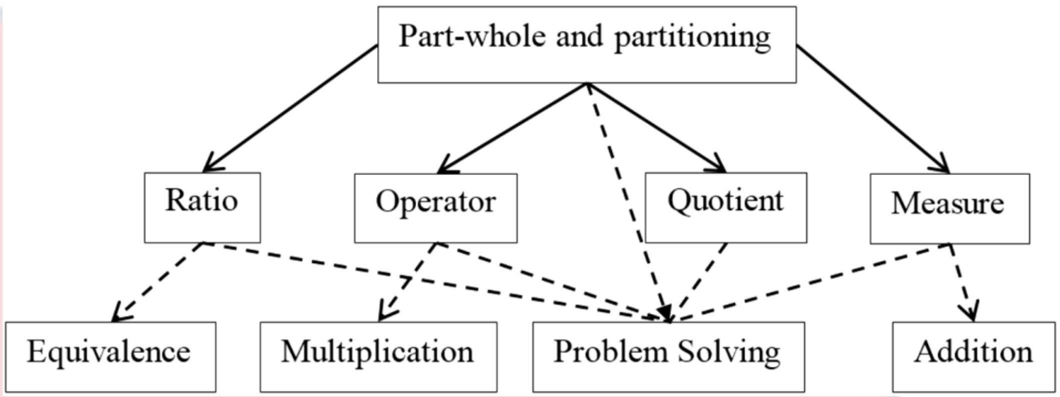
Interrelatedness of fraction constructs

The conceptual understanding of fractions requires a solid grasp of all the constructs that fractions can be represented and interrelatedness among them. Part-whole construct has been paired with partitioning and described as the foundation for learning other constructs (Behr et al, 1983). They have linked the various constructs with basic operations and problem-solving as presented in Figure 1.

Digitized by Sam Jonah Library

Figure 1

Interrelatedness of fraction constructs (Behr et al., 1983)



From the figure, it can be seen that the ratio construct helps in developing the understanding of fraction equivalence. The Operator construct is considered necessary for building the skill of fraction multiplication. Measure concept is viewed as necessary for proficiency in dealing with addition of fractions and the Quotient construct helps in developing understanding of subtraction and division of fractions. Also, comprehending all the constructs is a prerequisite for solving problems on fractions.

Forms of Fractions

There are two main forms of fractions which include;

1. **Fractions Less than one**

These are fractions in which the denominator is greater or larger than the numerator (National Council of Teachers of Mathematics, 2000). Simply put, it is a fraction whose magnitude lies between 0 and 1. These particular forms of fractions are always less than the whole and the numerator is less than the

Digitized by Sam Jonah Library

denominator. For instance, $\frac{3}{5}$, $\frac{2}{7}$ and $\frac{4}{9}$ are all fractions that represent values less than one.

2. **Fractions Greater than one**

These are fractions whose numerators are bigger than their denominators (NCTM, 2000). Thus, it represents fractions whose magnitudes are greater than one and the denominator is always smaller than the denominator. For example, $\frac{4}{3}$, $\frac{7}{5}$, and $\frac{9}{4}$ are fractions with values greater than one.

These particular forms of fractions can also be written in a mixed number form. Thus, a whole number part and a fractional part. For example, $\frac{4}{3}$ can also be written as $1\frac{1}{3}$. 1 represents the whole and $\frac{1}{3}$ is the fractional part.

Fraction Language Mistakes Among Teachers

If a teacher wants to have a great impact on his or her students, then the language that is utilised in the teaching and learning process is crucial. According to William (1986), mathematical language should be employed with attention and accuracy from the start. Using vague language or vocabulary in teaching fractions could make some concepts incomprehensible to students (Tobias, 2013). When a language use in fraction description is not transparent and precise, students find it difficult to comprehend. For instance, in a cross-national study, Paik and Mix (2003) examined the effects of language on the performance of fraction identification test among Korean and U.S. children. The children were tasked to identify and circle a correct fraction after observing and listening to a set of fractions names being mentioned. It was discovered that the Korean children

Digitized by Sam Jonah Library

outperform the U.S. children. In the second face of the study, the cultural difference of language was removed. This was achieved by translating the Korean naming of fractions into English. In the Korean naming, the whole comes first before the part. For example, the fraction $\frac{1}{3}$ is described as *three parts, one*. At this time, the U.S. children performed much better as compared to the Korean children. The performance of Korean children was attributed to the language transparency and clarity in its fraction naming. This evidence suggests a crucial role language plays in mathematics education. However, some of the languages used by most mathematics teachers in teaching especially fractions do not explicitly capture the meaning of mathematical concepts. The following are some of the inappropriate languages used in teaching fractions.

1. Proper and Improper Fractions

The terms *proper* and *improper* are not appropriate for discussing fractions. This may hinder pupils' understanding of fractions as they learn them by evoking feelings of hierarchy or judgment. Fractions naming should be neutral and descriptive, with no connotation that some fractions are superior to others (National Council of Teachers of Mathematics, 2000). In a similar vein, using words that can lead to misunderstandings or unfavorable attitudes about fractions should be avoided (Common Core State Standards Initiative, 2010). For instance, using *proper* and *improper* to name fractions gets students into thinking that, some fractions are better than others which can lead to negative perceptions of some fractions.

Digitized by Sam Jonah Library

Describing the fractions accurately based on their mathematical properties, such as fractions in which the numerator is less than the denominator and fractions in which the numerator is greater than or equal to the denominator is one of the surest ways of helping children learn the concept of fractions. Hence, Beckmann (2014) categorized fractions as a fraction whose numerator is less than their denominator and a fraction whose numerator is greater than or equal to their denominator. Therefore, using neutral terminology in naming fractions can help students concentrate on fractions' mathematical qualities.

2. Reducing Fractions

The word *reduce* as it is often used in mathematics classrooms when students are asked to work out some fraction tasks may confuse students and leads to negative thinking. Karp, Bush and Dougherty (2015) proposed the word *simplify* instead of *reduce*. Students tend to have the notion that the fraction size is getting smaller if the word *reduce* is used. Instead, teachers must educate pupils to write the fraction in its simplest form or lowest terms or use the term *simplify*.

3. Top and Bottom Number

Another inappropriate language used when dealing with fractions is describing the fractions as two separate digits. Fraction is a single digit and the use of *top and bottom number* has no mathematical basis (Karp et al., 2015). The location of each digit in a fraction should be described using the numerator and denominator. The use of *Top and bottom* have no mathematical significance and could inadvertently infer that a fraction includes multiple numbers. More so, Tobias (2013) asserted that using the *top and bottom* to describe fractions has the

Digitized by Sam Jonah Library

University of Cape Coast https://ir.ucc.edu.gh/xmlui

tendency of limiting the students because those words do not adequately describe fractions.

4. Using *Out of* and *Over* Language to Describe Fractions

Describing fractions such as $\frac{2}{5}$ as two over five or two out of seven can be problematic and lead to misunderstanding or ambiguity in interpretation, especially when the context is unclear. The use of *over* should be used cautiously, as it may be misleading for some pupils (NCTM, 2000). When describing fractions the word *over* can imply a spatial relationship between the numerator and denominator which can easily mislead students. Also, using the *out of* terminology frequently leads pupils to believe that a portion of the total is being deducted (Karp, Bush, & Dougherty, 2014). As a result, it is recommended to use straightforward wording, such as *two-sevenths* or *three-fifths* when describing fractions. By doing so, confusion is reduced and the fraction's meaning is guaranteed to be unmistakable and clear.

Role of Fractions in Mathematics Education

Knowledge of fractions is indispensable component of mathematics education as many other concepts in mathematics depend on the knowledge of fractions. Hence, proficiency in mathematics in general is dependent on fractions, especially the concept of fraction magnitude.

There is no denying the fact that algebra is a key concept and a foundation to other mathematical concepts. It is one of the concepts children first encounter in the high School and has been recognised as one of the concepts that plays a crucial role in mathematics curriculum (NCTM, 2000). More so, Kunuth et al. as

Digitized by Sam Jonah Library

cited in Osei (2020) asserts that, a good foundation in algebra will become a passage through which students can comfortable transitioned into higher mathematics. Nonetheless, algebra learning is not independent on fractions skills. For instance, some of the algebraic concepts have their roots in fractions. For example, Grouping *like terms* is a concept used when studying fractions arithmetic. Also, in order to do away with a denominator of an equation by multiplying with a constant, an idea of fractions is employed and proportional equations too have their foundations in the concept of fraction equivalence (Wu, 2001; Piennar, 2014).

In the area of probability, idea of fractions is significant. Probability is the likelihood of an occurrence of an event. Even though it can be expressed in percentages or decimals, it is much easier when expressed in a fraction form (Piennar, 2014). It is usually expressed using the formula:

$$\mathbf{P(E)} = \frac{\mathbf{n(E)}}{\mathbf{n(S)}}.$$

Where **P** denotes the probability, **E** represents the number of successful outcomes and **S** is the total number of outcomes. For example, if a box contains 3 blue, 5 red and 7 white balls, then the probability of picking a red ball is 5/15 or 1/3.

Trigonometry is yet another concept that requires the concept of fraction to comprehend. Trigonometry involves the measurement of angles and distance in many fields including surveying (Piennar, 2014). All the trigonometric ratios such as Sine, Cosine and Tangent as well as their inverses such as Cosecant, Secant and Cotangent have their basis in the fraction concept.

Digitized by Sam Jonah Library

The above role of fractions confirms the studies that found a correlation between skills of fractions and general mathematics achievements. For instance, Bailey et al (2014) discovered that a skill of fractions correlated consistently with mathematics achievement in American and Chinese learners. Moreover, Sigler et al (2013) found that knowledge of fractions in elementary school was a predictor of mathematics and algebra performance in high school. This therefore means that the skill of fraction and mathematical competency are inseparable and learning of mathematics will not be possible without the knowledge of fractions.

Factors that Contribute to Fractions Difficulties

Fractions have received a particular attention in studies over the years due to the function it plays in the mathematics learning. However, the topic has been complex and difficult for both students and mathematics teachers (Siemon et al., 2015). Fractions difficulty is not limited to a particular continent but it is a global challenge. For instance, literature revealed that the concept of fraction is complex for students in Indonesia, America and Australia (Siegler et al., 2013; Amalina et al., 2016; Copur-Gencturk, 2021). Studies in Ghana and Africa revealed similar results (Odigun, 2018; Ubah & Bansilal, 2018; Agbozo, 2020).

The complex nature of fractions stems from several factors. Apart from the compound nature of fraction concept as highlighted above, it is also difficult due to *natural number bias* (also called whole number bias). The *Natural number bias,* according to Van Hoof, Engelen, and Van Dooren, (2021,) is “a phenomenon in which learners inappropriately apply natural number characteristics in the rational number domain" (p.2). Thus, due to the conflicting

Digitized by Sam Jonah Library

nature of natural and rational numbers (fractions) students mistakenly apply rules of natural numbers when dealing with fractions.

For a meaningful learning to take place, students' prior knowledge should not be much different from the new information that is being provided. However, in the context of fractions, students' prior knowledge is always that of natural number. This is because natural numbers usually precedes rational numbers in curriculums. Hence, a conceptual change is required for the adjustment of the natural number ideology to fit in to rational number ideas (Vamvakoussi, Christou ,& Vosniadou, 2018; Van Hoof et al. 2021). Adjusting the concept of natural number to accommodate that of rational numbers (fractions) is not always easy and as a result once fractions are introduced in classrooms students tend to exhibit misconceptions known as *whole number bias*.

The whole number bias has been explored extensively by researchers. It includes representations, density, size and operations (Reinhold, Obersteiner, Hoch, Hofer & Reiss, 2020). With regards to the representations, all natural numbers has unique symbolic representation and conversely, all rational numbers have infinite symbolic representations. For example, $\frac{3}{5} = \frac{6}{10} = \frac{9}{15} = 0.6$ and so on. Students get confused with this many symbolic representations associated with fractions.

Regarding operations, learners misconceive multiplication as an operation that always increases a number and division reduces it. This is not generally the case with fractions (Christou, 2019). For instance, when a number is multiplied

Digitized by Sam Jonah Library

by a fraction less than one, it rather reduces the number. And when a number is divided by a fraction less than one, it rather increases instead of reducing.

In the case of the density, whilst natural numbers have unique successors, rational numbers do not have (Reinhold et al., 2020). In between two fractions, there are an infinite number of fractions. More so, Cramer and Whitney (2010) maintain that another major source of fraction difficulty is the problem of seeing fractions as one single digit. Treating the numerator and denominator separately can lead to errors.

Another source of fraction difficulty is the inappropriate language used in teaching and learning of fractions. Some languages such as *bottom and top number* tend to mislead students. It does not accurately describe the mathematical meaning of fractions (Tobias, 2013). Furthermore, students sometimes think that the magnitude of fractions increases when the numerator, denominator or both increases. However, this is not true in the case of fractions

Empirical Review

Teachers' Knowledge of Fractions

The level of teachers' knowledge is very crucial in mathematics education. This is because, teachers' mathematical knowledge has been found to have a critical function on the quality of instruction and the performance of students (Depaepe et al., 2015; Copur-Gencturk, 2015; 2021). Effective mathematics teaching requires both content and pedagogical knowledge. Thus, educators of mathematics need a strong foundation and mastery of the mathematical content as well as the methods and procedures of teaching it. Based on the above assertion,

Digitized by Sam Jonah Library

University of Cape Coast https://ir.ucc.edu.gh/xmlui

Junior high school mathematics teachers need a solid grasp of fractions especially the magnitude of fractions in order to be productive in the classrooms as mathematics educators.

Unfortunately, literature on teachers' knowledge of mathematics especially fractions have revealed that both pre-service as well as in-service teachers have weak knowledge in fractions. For instance, Lee and Boyadzhiev (2020) study examined the types of fractions misunderstandings held by the 22 prospective teachers admitted into a remedial course in mathematics in the US. It revealed that participants have misunderstandings of the lowest common multiples /lowest common denominator and a lack of comprehension of the order of operations. A misunderstanding of the negative sign was also discovered in students' answers. Moreover, Students' lack of number sense, misconceptions of fractions and calculation abilities with whole numbers, and their overreliance on algorithms without conceptual knowledge were all cited as contributing factors to their errors.

Furthermore, Ubah and Bansilal (2018) examined the answers given to questions about the fundamentals of fractions by 60 pre-service teachers in South Africa. The study employed the Action-Process-Object-Schema theory, involving operations on fractions with the different denominators as well as fractions with the same denominators to examine their level of fractions conceptions. It was discovered that many pre-service teachers were able to add and subtract common fractions with the same denominators without any problems. More than 52%, however, found it difficult to perform similar operations on fractions with

Digitized by Sam Jonah Library

different denominators, indicating that their ideas had not yet evolved into object-level structures. It means that these pre-service teachers only rely on procedures that are possible to deal with fraction tasks with the same denominators. When they encounter problems with different denominators which is not possible to be solved by procedures they struggle. These teachers' understanding of fractions is limited to algorithms they have memorized and begin to struggle when the rule becomes dysfunctional.

The same results were found in Ghana when studies were done on pre-service teachers. For instance, Agbozo (2020) investigated Ghanaian pre-service teachers' attitudes toward teaching the concept fractions. 26 pre-service teachers from a Ghanaian education college were used in a qualitative case study. The results showed that while participants regarded fraction addition to be simple, they found algorithms involving fractions to be challenging and abstract. They could not comprehend why algorithms work. They only memorised rules which worked out for them when applied in solving problems involving fractions. This type of understanding is dangerous as answers cannot be justified. These teachers will only teach their students how to apply rules to solve problems and when this happens, understanding the interrelatedness among concepts which is the essence of mathematics is forfeited.

Furthermore, Baah-Duodu et al. (2019) used a fractional knowledge test to determine pre-service teachers' mathematical proficiency in teaching fractions using a mixed-methods design. The main aim was to evaluate their content as well as their pedagogical knowledge of fractions. The findings of the problem-solving

Digitized by Sam Jonah Library

University of Cape Coast https://ir.ucc.edu.gh/xmlui

skills test indicated that the knowledge of the pre-service teachers on fractions in terms of content and pedagogical knowledge was poor. According to the study, pre-service teachers do not have sufficient training to facilitate learning of mathematics through problem-solving. They demonstrated more procedural knowledge of fractions than conceptual. This is sad because if people who are supposed to guide learners do not have the necessary skills then more cannot be expected from the trainees.

Not only are teachers unable to solve questions involving fractions, but struggle with formulating fraction questions. Being able to formulate a mathematical task is a reflection of mathematical understanding, yet this is challenging for many teachers. For instance, in Turkey, Doğan-Coşkun (2018) investigated whether pre-service elementary teachers could create problems adequate for teaching fraction subtraction, and if not, what common mistakes they were making. When their issues were reviewed in light of the meanings that they highlighted, it became clear that the majority of them concentrated on the distinct meaning of the subtraction operation. Yet, more than half of the respondents $(\frac{68}{83})$ were unable to submit a suitable issue. 34 of them were unable to even pose a challenge for the given expression.

Even though these teachers in the studies above were still studying in college, they have learnt fractions in high school or taken courses related to mathematics in college yet they still struggle with fractions. Even though literature suggests the pre-service teachers' difficulties of fractions are carried from the basic level (Agbozo, 2020) the teacher trainers and the curriculum

Digitized by Sam Jonah Library

designers cannot be excluded from the blame. Hence, the teacher trainers themselves need to change their pedagogical practices and the mathematics curriculum designers for mathematics educators need restructuring to bring out the desired goals.

Although the studies above were done on pre-service teachers, results from studies on in-service teachers revealed much similar results. For instance, using 103 fourth or fifth-grade instructors from around the United States, Copur-Gencturk (2021) explored the conceptual comprehension of fraction operations among teachers. It was discovered that only half of the teachers offered explanations focusing on the mathematical meaning of the operation, even for the algorithm that was relatively less sophisticated (i.e., the addition technique when dealing with fractions). Additional analysis of these findings demonstrated that teachers do not fully comprehend the role of the denominator in fractions representations.

Moreover, strategies used by Brazilian mathematics teachers when comparing the size of fractions were examined in a study by Toledo et al. (2022). The researchers collected quantitative data from a fraction comparison task to gauge the teachers' implicit knowledge. It was discovered that participants employed a range of fraction comparison techniques, from manipulating fraction components to thinking about fractions from a part-to-whole viewpoint. The Gap strategy, which was frequently employed despite not being appropriate for all challenges, was one strategy that distinguished itself from the others. In conclusion, the results of the qualitative and quantitative tests revealed that

Digitized by Sam Jonah Library

mathematics teachers do not approach fractions from a component perspective. Instead, they employ a variety of approaches that are not always aligned with a holistic understanding of fractions.

Furthermore, the accuracy, reasonableness, and alignment of the techniques in-service teachers employed in response to problems involving fraction magnitude were the main areas of attention for Copur-Genturk (2022) examination of in-service teachers' comprehension of fraction magnitude. The participants were asked to estimate both divisions of $\frac{41}{66}$ by $\frac{19}{35}$ and the sum of $\frac{19}{35}$ and $\frac{41}{66}$ and placed their estimates on a number line. The number line ranges from 1 to 3. Teachers found their answers on the number lines with the help of a computer mouse. At the period of the study, 603 elementary and middle school instructors in the United States were teaching students in Grades 3–7 on fractions. It was discovered that teachers' estimates particularly for activities requiring division, were only moderately accurate.

The studies above proved the fact that in-service mathematics teachers' understanding of fractions especially fraction magnitude is superficial. Therefore, a teacher cannot have quality instruction if his understanding of fractions is incomplete. Students are only exposed to the limited knowledge of teachers because no one can give more than what he/she possesses especially in the teaching and learning process. This could be one of the contributory factors causing students' difficulties with fractions.

Digitized by Sam Jonah Library

Teaching Experience and Teachers' Mathematical Knowledge

Every profession takes into account an employee's year of experience. The fundamental premise is that knowledge, skill, and productivity all improve with experience and the teaching profession is not an exception. It is expected of mathematics teachers who have been in the profession for a while to get better due to their encounter with various situations, categories of students, and new concepts, there ought to be ongoing progress every day. Therefore, teachers gain knowledge through their classroom experiences (Rice, 2010; Entsie, 2021).

Research suggests that teachers have spent many years in teaching are more productive in classrooms as compared to teachers with less years or no teaching experience (Darling-Harmmond, 2000; Klecker, 2002; Adeyemi, 2008; Harris & Sass, 2011). Through teaching, teachers come to learn new things that may not be possible to learn in class. Many things learned through educational programs are not exactly the same when teachers are exposed to realities in the classroom. Hence, teachers' knowledge improves through exposure to classroom realities and struggling to overcome obstacles it presents.

There are divergent views among studies on the relationship between teachers' teaching experience and mathematical knowledge. For instance, the association between years of teacher's experience and mathematics achievement of students was investigated in a study by (Klecker, 2002) in the US. The eighth-grade National Assessment of Educational Progress (NAEP) mathematics test scores of the students were taken into account, as well as mathematics teachers' experience in teaching in terms of years as measured in five distinct groups: 2

Digitized by Sam Jonah Library

years or under, 3-5 years of experience, 6-10 year experience 11-24 years of experience, and 25 or above years of experience. The findings indicated that learners who were handle by teachers with more classroom experience performed much better or earned higher grades than students of teachers with less instructional experiences.

Furthermore, Adeyemi (2008) performed a correlational survey in Nigeria to explore the relation between teaching experience and students' performance. 180 schools in the state of Ondo were selected by random stratification. It was shown that teachers' teaching experience had a substantial impact on students' learning as determined by how well they performed on the senior secondary certificate (SSC) examinations. A teacher can only have an impact on students especially the academic performance only when he has a solid understanding of mathematical concepts. The findings of the study are in agreement with that of Harris and Sass (2011) who concluded that a teacher with greater experience is more successful at teaching arithmetic and reading in elementary and middle schools.

Furthermore, a study by Copur-Genturk (2021) found more accuracy in the explanation of fractions concepts by teachers with more years of teaching as compared to their colleagues with less years of teaching. Also, in a current longitudinal study by Copur-Genturk and Li (2023), the extent to which experience in teaching can offer conducive environment for teachers to improve in the most pertinent elements of Pedagogical Content Knowledge (PCK): Knowledge of Students Mathematical Thinking (KSMT) and Knowledge of

Digitized by Sam Jonah Library

Mathematics Teaching (KMT) were examined. Data was collected from 207 in-service mathematics teachers for a period of three tears. It was discovered among other things that teachers increased in both components. Even though the increment in knowledge was visible in both groups, teachers with robust understanding of mathematics improved faster than their counterparts with less partial understanding. This means that teaching has a potential of increasing the level of mathematical knowledge as the grasp of new concept is facilitated with teaching experience.

Other studies discovered no correlation between teaching experience and mathematical teachers' knowledge. A current research conducted in the US which aimed at the three aspects of teachers' comprehension of fraction magnitude were examined in a study by Corpur-Genturk (2022). The correctness of teachers' estimations as well as its reasonableness and the alignment of the techniques they adopt in dealing with fraction tasks were examined using 603 in-service teachers. The study found no relationship between knowledge of fraction magnitude and teachers' years of teaching experience.

Furthermore, a study by Yarkwah (2017) investigated the mathematical knowledge of senior high school teachers in algebra. The study involved teachers from 40 senior high schools across three regions in Ghana participated. The findings showed a significant difference across these teacher categories—inexperience teachers, less experience, and experienced teachers. However, the degree of the difference was very small. More so, Osei (2020) investigated among other things, how basic school mathematics teachers' knowledge of algebra

Digitized by Sam Jonah Library

University of Cape Coast https://ir.ucc.edu.gh/xmlui

increases as their number of years of teaching experience rises. The results indicated that basic school mathematics instructors' competence in teaching algebra did not significantly improve with increasing experience in teaching, whether they earned their teaching credentials from educational colleges or through distance learning programs.

Several factors could contribute to the stagnated nature of in-service teachers' knowledge in the above studies despite teaching for many years. One of the things that boast the knowledge of mathematical teachers is in-service training programmes. However, many teachers in the studies that found no relationship between teaching experience and years of teaching do not participate in the in-service training programmes. For instance, about 81.82% of the in-service teachers who were respondents in the study by Osei (2020) said they have not received any in-service professional training programmes regarding mathematics education.

Teamwork is another opportunity of learning for the in-service teachers in the teaching field. This is where a group of teachers work in collaborations to assist each other in their difficult areas. When teachers work independently, it becomes difficult for them to learn new things thereby increasing their mathematical knowledge. This assertion is evident in the studies that claimed that teachers' knowledge does not increase as they teach. For instance, Osei (2020) discovered that basic school teachers do not help each other but everybody work in isolation.

Digitized by Sam Jonah Library

University of Cape Coast https://ir.ucc.edu.gh/xmlui

Teachers' Grade/Class Level and Mathematical Knowledge

The grade/class level of a teacher is the specific class or a particular group of students that they are in charge of instructing. This is more often than not, based on their training and experience. While other factors such as teacher's training and experience in teaching and teacher professional development programs impact teachers' knowledge (Hill, Sleep, Lewis, & Ball, 2007), there is evidence suggesting that there is a considerable association between teachers' mathematical knowledge and the grade level at which they teach (Hill, 2010). A study by Hill (2010) supported this argument when it found correlation between mathematical competency and the grade level of teachers. The study revealed that teachers in the lower grades possess a low level of mathematical knowledge. A study conducted by Copur-Gencturk (2022) also found similar results when she investigated 603 US grades 3-7 elementary and middle in-service mathematics teachers' knowledge of fraction magnitude. It focused on the accuracy, reasonableness and alignment of the in-service teachers' estimation strategies with fraction magnitude concept. It was discovered that teachers teaching high grades performed better as compared to the lower grades teachers. This suggests that the higher the grade that a teacher teaches, the higher the score on the fraction magnitude scale and vice-versa. The difference in mathematical knowledge may be associated to the fact that teachers at higher class levels encounter advanced and more sophisticated mathematical concepts due to the spiral nature of mathematics curriculums.

Digitized by Sam Jonah Library

Moreover, the study of Wilkin (2008) explored among other things, the mathematical knowledge and attitudes of 481 upper and lower elementary mathematics teachers. The study found many differences between the two categories of teachers. It revealed that teachers at the upper elementary (3-5) demonstrate high knowledge of the mathematical content knowledge as compared to the teachers at the lower elementary (k-2). In terms of attitude towards mathematics, teachers at the upper elementary were found to have a more positive attitude than their colleagues at the lower level. The study concluded that the positive attitude of the teachers at the upper elementary level partly contributed to their high performance.

Even though the studies above used only elementary teachers, other studies done with elementary and secondary teachers found similar results. For instance, Depaepe et al. (2015) examined the differences in knowledge between pre-service elementary and secondary school teachers' pedagogical and content knowledge of mathematics. The number of elementary and secondary teachers was 158 and 34 respectively. The test focused on rational numbers. It revealed that secondary school teachers performed better in the content knowledge but there was no difference in their pedagogical knowledge. The difference in the mathematical content knowledge between the two groups of teachers was attributed to the structure of teacher education programmes. It is evident that the scope, depth and complexity of content in mathematics differ among different grades thereby influencing the level of the mathematical content as well as the Pedagogical knowledge of teachers.

Digitized by Sam Jonah Library

Other studies disagree with the assertion that significant difference of mathematical knowledge exist among the class/grade levels of teachers. They believe that teachers' mathematical knowledge comes from their experience, the nature of teacher training programmes and the in-service training programmes. Hence, a teachers' mathematical expertise has nothing to do with the grade level that they teach mathematics. A practical example was a study by Copur-Genturk (2021) which required mathematics teachers to justify why a common denominator is needed in solving addition of fractions and why the algorithm of fraction multiplication works. The participants were 303 in-service teachers that were teaching fourth and fifth grades. She discovered that the highest grade at which teachers instructs mathematics was not related to the accuracy of their explanations. In other words, teachers at the high grades did not demonstrate better mathematical knowledge in their explanations as compared to their counterparts at the lower grades. The disagreement of the results of this recent study from the previous researches may be as a result of the number of grades the study used. The study used only grade 5 and 6 teachers. The mathematical content in grade 5 is not much different from grade 6, hence no difference in their explanation of the mathematical concepts.

Therefore, the class that a teacher teaches mathematics has an impact on the mathematical knowledge. The higher the grade/class that a teacher teaches mathematics, the higher the depth of the mathematics curriculum, which will ultimately lead to differences in mathematical knowledge among teachers of different grades.

Digitized by Sam Jonah Library

University of Cape Coast https://ir.ucc.edu.gh/xmlui

Summary of Literature Review

The concept of fractions and for that matter the skill of fraction magnitude has been identified as a better predictor of students' performance in mathematics (Siegler et al., 2012). More so, another important predictor is the level of mathematical of the teachers of mathematics (Depaepe et al., 2015). Therefore, teachers who have an in-depth mathematical knowledge tend to have more influence on the performance of students and the opposite is true.

Teachers with weak knowledge of fraction concept will only transfer same to students. This will result in learners' difficulties or misconceptions of fractions. Hence, junior high school teachers' knowledge of fractions should be assessed to determine whether their fraction knowledge level is up to the standard. This will guide education policymakers to effect the needed change in the knowledge level of the mathematics teachers if it is found below the standard.

Digitized by Sam Jonah Library

CHAPTER THREE

RESEARCH METHODS

There are many philosophical foundations guiding research methodologies. This study aligns with the positivism paradigm due to the researcher's belief that the accuracy of in-service teachers' estimation of fraction magnitude can be objectively measured through tests. It went further to investigate the potential association between in-service teachers' teaching experience and their accuracy in estimating fraction magnitudes. Additionally, the study sought to explore potential differences in the accuracy of fraction magnitude estimation based on the specific classes taught by teachers.

Research Design

The study used a cross-sectional survey design. It was considered appropriate because of its capability of gathering data or information about the participants by describing their characteristics within a short period (Creswell, 2012). It attempts to determine or describe the nature and state of the situation through classification, measurement and comparison. Orodho (cited in Isaboke, 2018 p.29) states, "descriptive aspect provided an opportunity for the researcher to probe deep and obtain precise and concise information about the target organization, which enabled the researcher to gather information about the present and existing condition of a phenomena understudy".

Furthermore, survey provides a higher level of anonymity. It also gives consistent and uniform measures and respondents are not influence by the researchers' presence or attitude. Indeed many researchers have discovered that

Digitized by Sam Jonah Library

survey designs have the potential to cover a large sample size which increases the generalizability of the findings.

This particular design does not consider the future happening due to the fact that it provides ‘snapshot’ of Participants’ characteristics (Levin, 2006). For instance, the data gathered on in-service teachers’ understanding of fraction magnitude was within a specified time. Changes may occur when the same information is gathered after the study and this design cannot account for these changes. The design is also criticised because of its inability to ask further questions or seek for further clarifications and its inconsiderate of the participants’ condition under which the responses were given (Sarantakos, 2013).

Even though the design is not free from limitations, it was deemed suitable for this work due to the fact that, it ensures representativeness of the population. It is less expensive compared to other designs and its ability to study many variables in a short period of time. Also, Cohen, Manion, and Marison (as cited in Yarkwah, 2017) contended that with cross-sectional survey design, both descriptive and inferential evidences are possible to establish an association between variables.

Study Area

This study was conducted in the Tamale Metropolis in the Northern Region of Ghana. The region has Tamale as the Metropolitan and Regional Capital City. It is found in the Region’s central part. The district shares boundaries with Mion District to the East, East Gonja to the South, Central Gonja to the South-West and Sagnarigu District to the West. About 646.90180^{2}km is the

Digitized by Sam Jonah Library

estimated land size of the Metropolis. Its Geographical location is between latitudes 9º16 and 9º 34 North and longitudes 0º 36 and 0º 57 West (Ghana Statistical service, 2014).

Population

The target population of this research was all mathematics teachers teaching the junior high schools in the Tamale metropolis. According to Tamale Metro Education Directorate (TMED, 2023), the Metropolis has a total of 167 mathematics teachers and 96 junior high schools. The area was chosen because it is densely populated with a lot of junior high schools and only Metropolis in the Region. Moreover, the researcher lives in the District, hence familiar with the location of the junior high schools and the participants. The familiar area was opted for to allow the researcher to be able to persuade the participants to respond to the questionnaire. The population of the study consists of 167 mathematics teachers from the 96 junior high schools in the metropolis, of which mathematics teachers from 76 schools were accessible.

Sampling Procedure

The target of the research was on mathematics teachers at the junior high schools. Multi-stage sampling was employed to select the respondents that constitute the sample. It encompasses the utilization of multiple sampling methods within a single research endeavour (Yarkwah, 2017). In the first stage, the lists of all junior high schools in the Metropolis were obtained and simple random sampling was employed to select 76 junior high schools from all the junior high schools in the study area. This technique ensures equitable opportunity

Digitized by Sam Jonah Library

University of Cape Coast https://ir.ucc.edu.gh/xmlui

for all the schools to be used for the study. This method mitigates bias, leading to a representative sample that mirrors the characteristics of the entire subjects (Saunders, Lewis & Thornhill, 2012).

In the second stage, the census method was use to select all teachers teaching mathematics in the selected junior high schools. A total 134 mathematics teachers which consist of 115 male and 19 female mathematics teachers were selected to participate in the study.

Data Collection Instrument

The research instrument used in this particular study was adapted from Siegler et al. (2011) and Copur-Genturk (2022). The instrument is divided in to three sections. Section One solicited demographic characteristics of the respondents, the Form/class level they teach as junior high schools consists of three classes and their years of teaching experience. Section Two contained number lines that captured teachers' fraction magnitude knowledge. This Section was further divided in to parts I and II. The parts I and II contain Addition and Subtraction tasks respectively. To properly measure each construct, each part contains three questions on the knowledge of fraction magnitudes. There were number lines below each task and teachers were expected to estimate answers to these arithmetic tasks and placed their estimations on the number lines where the think the answer would be located. Section Three contains two items seeking the strategies teachers used in estimating fraction magnitude problems. In this section, the respondents were expected to circle strategy/strategies from the list of strategies which they used in estimating each addition and subtraction fraction

Digitized by Sam Jonah Library

problems. The strategies were the *Benchmark/Common denominator strategy, Selecting a fraction with similar size/Using the number line, Rounding and Other strategies* (Copur-Genturk, 2022, Siegler et al., 2011).

Benchmark/Common denominator strategy involves selecting some numbers such as $\frac{1}{2}$ and 1 to serve us benchmarks in order to estimate. Using this technique means that attention is not paid to the magnitude of the fractions. Moreover, finding a common denominator to be able to estimate is also included in this category. Finding common denominator means that an absolutely little attention is given to the numerators. It is evident that, this is a rule based strategy which can even be used to perform operations with fraction even without understanding.

Selecting a fraction with similar size/Using the number line strategy involves selecting a fraction that is visually similar in size to estimate the magnitude. For instance, a teacher realizing that $\frac{2}{3}$ is similar in size as $\frac{43}{67}$ and therefore using the former to estimate the magnitude of the latter. Also included in this strategy is the number line segmentation in which a teacher divides the line in to segments to be able to locate the position of a faction. These are conceptual based strategy that suggests a robust understanding of fraction magnitude.

Rounding strategy or Other strategies deals with rounding strategy involves rounding the numerator or the denominator to the nearest 10 (Copur-Genturk, 2022) before estimating the magnitudes. The use of this strategy means that the teacher is not dealing with the numerator and the denominator simultaneously, whereas this is a requisite of correct estimating of the size of a

Digitized by Sam Jonah Library

fraction (Behr et al., 1983). With the *Other strategies*, the teacher may be guessing or using faulty strategy such as adding or Subtraction both the numerator and the denominator. Therefore, those teachers who used these two strategies have no understanding of fractions especially the magnitude of fractions

The advantage of the instrument is that it combines several ways of measuring fraction magnitude knowledge. For instance, literature suggests that comparing of fractions, fraction arithmetic and locating a given fraction on a number line are effective strategies of capturing knowledge of fraction magnitude (Siegler et al., 2011; Copur-Genturk, 2022). The combination of these strategies in this instrument makes it possible to avoid the problems of using only one of the strategies. Employing one strategy for instance, can easily be worked out using fraction algorithms or rules even without the knowledge of fraction magnitude. For instance, when a teacher is encountered with a problem of the nature $\frac{3}{5} + \frac{1}{5}$. With the use of the common denominator strategy, the teacher can easily add the numerators whiles maintaining the denominator ($\frac{4}{5}$). Even though this is the correct answer, it does not guarantee that the teacher understands fraction magnitude. However, in this instrument, an algorithm can be used to correctly estimate an operation with fractions. But locating the position of that particular estimate on a number line will demand the application of fraction magnitude knowledge.

Moreover, it captures the estimation techniques used. Thus, exposes the various strategies employed in the estimation of fraction magnitude and whether it

Digitized by Sam Jonah Library

reflects robust understanding of fraction magnitude. The disadvantage of the instrument is that it is difficult to construct and consumes much time.

Validity

The instrument's content validity was established by presenting to the research Supervisor who studied and modified it to ensure that the items examine the in-service mathematics teachers' knowledge of fraction magnitude. It is clear that similar studies on fraction magnitude understanding used similar instruments (Siegler et al., 2011; Copur-Genturk, 2022). This further gave the researcher a confidence on the research instrument's validity.

The instrument was first tested at Sagnarigu Municipality with a group ten public junior high school in-service mathematics teachers. After the exercise, modifications were done on the instrument. There was a careful review and refinement on some of the words and phrases used in the test items after this exercise. This was done to ensure the final instrument of the study is obtained.

Pilot Testing

After improvement on the instrument was done upon the outcome of the first testing as well as the suggestions from the research supervisor from the Mathematics and I.C.T Education Department of the University of Cape Coast and other professionals, it was field tested. The test was administered in the Tolon District in order to determine its validity and reliability. The participants in the pilot test were 30 mathematics teachers that were teaching at the junior high schools.

Digitized by Sam Jonah Library

Reliability

Following the pilot exercise on a group of 30 junior high school mathematics teachers, the accuracy score obtained by each participant was calculated. Even though, a score of 0% indicates highest level of accuracy on the instrument, the participants' accuracy levels ranges from 26% to 50%. It took them approximately 20 minutes complete the items. A scale was created for the accuracy scores of the participants and Cronbach's alpha was calculated. A reliability coefficient of 0.7 according to Mugenda and Mugenda (2003) is regarded as a rule of thumb that indicates an acceptable reliability and coefficient of 0.8 or high is an indication of good reliability. In light of this, the computed coefficient for the instrument found to be 0.864. This figure substantiated the instrument's robust reliability. To further establish the validity of the instrument, the difficulty indices of the items were computed. The coefficient of the final instrument was 0.867.

Data Collection Procedure

The focus of this work was to investigate the fraction magnitude estimation accuracy level of the in-service mathematics teachers of the public junior high schools. Consequently, a fraction magnitude knowledge test was developed to determine the fraction magnitude knowledge level of the mathematics teachers and the strategies they employ in the estimation of fraction magnitudes. It also explored to find out whether the estimation accuracy of teachers regarding fractions increases with their years of teaching. It went further

Digitized by Sam Jonah Library

to determine whether there are significant differences in the accuracy of in-service teachers estimation of fraction magnitude and the class that they teach.

Letters were acquired from the supervisor as well as the Department of Basic Education of the University of Cape Coast for ethical clearance letter. A way was paved for the data collection after the ethical clearance letter was received from the Institutional Review Board of the University of the University of Cape Cost. In a written letter, the purpose of the study was explain to the Tamale Metropolitan Education Directorate seeking permission to conduct a study in public junior high schools under its jurisdiction. Subsequently, permission was granted by the Tamale Education Directorate, commencing data collection process.

The researcher visited the schools selected and administered the instrument. To ensure the confidentiality and anonymity of the participants, their names and school names were not recorded in the instrument. The data collection was done in the month of June and July, 2023 during the third term of the school calendar.

The administration of the instrument was done after the arrangements that the researcher makes with the Head teachers of the selected schools and the respondents on a convenient time and date for the exercise. The researcher discussed with the Head teachers of the schools and then the participants. On a visit to each of the selected schools, the purpose of the study, its duration and its anticipated benefits and harms were explained to the heads as well as the respondents. After the purpose of the study was made known to the heads and the

Digitized by Sam Jonah Library

teachers of the various junior high schools, their consent was sought to allow the exercise to take place in their school. Few junior high schools opted not to take part in the study and they were allowed.

The schools were visited accordingly and the instrument was administered on every visit. To ensure that the instructions are strictly adhered to, the researcher observes the respondents while they respond to the questions. However, the researcher was at a distant to reduce pressure on the respondents. With this, any confusion with regarding the instructions was cleared by the researcher and the necessary guidance was also provided. Some of the teachers declined to take part in the study and they were allowed. The researcher collected the instruments right after the responses were provided.

Data Processing and Analysis

The responses were thoroughly examined before processing to avoid inconsistency. The data was entered into spreadsheet. The entry data was analyzed using the Statistical Package for Social Sciences (SPSS) version 27.

Analysis of data for research question one

In answering research question one which states, “What is the level of accuracy of in-service teachers’ estimation of fraction magnitude?”, the fraction magnitude test scores of the in-service teachers were used. The responses of the participants were computed to obtain their fraction magnitude test scores using the Percent Absolute Error (PAE) technique (Siegler et al., 2011). PAE is the absolute difference between the location of the teacher’s estimate and the precise location of the answer, divided by the numerical range of the number line. For

Digitized by Sam Jonah Library

instance, if a teacher is presented with a problem in which he is supposed to locate the position of $\frac{7}{4}$ on a 0-4 number line, and his mark on the number line corresponds to $\frac{3}{4}$. Then, the accuracy of the estimate is computed as;

PAE = [(|Teacher's answer − Correct answer|)/Numerical range]

$= [(|\frac{3}{4} - \frac{7}{4}|)/4]$

$= \frac{1}{4}$

= 25%

There is an inverse relationship between PAE and level of accuracy. Therefore, a small PAE value means that the estimate is close to the answer which indicates a high level of accuracy and a high PAE value implies the estimate is far away from the answer which depicts a less accuracy. This strategy was used to calculate for each Addition and Subtraction scores. Subsequently, the scores from the two tests were average to create total scores of the participants. Descriptive statistics in a form of means and standard deviations were reported for each Addition and Subtraction test scores.

Analysis of data for research question two

The second research question that guided the study was, "What are the strategies used by in-service mathematics teachers in the estimation of fraction magnitude? To answer this question, strategies used by the participants were checked and their frequencies determined. Hence, the analysis of this particular research question was descriptive in a form of frequencies and percentages.

Digitized by Sam Jonah Library

Analysis of data for research hypothesis one

The first hypothesis that guides the study was, “There is no relationship between the accuracy of in-service teachers’ estimation of fraction magnitude and their years of teaching experience”. To test this hypothesis, data obtained from the fraction magnitude knowledge test, which was calculated using Percentage Absolute Error (PAE) was used. Since there is an inverse relation between level of accuracy and PAE (fraction magnitude scores), the PAE scores were reversed. Thus, each faction magnitude score was subtracted from 100 to get a new magnitude score where high values indicates high accuracy and low values indicated low level of accuracy in the estimations. The reversion was done to ensure that a negative value indicates a negative relation and a positive value depicts a positive relationship.

The Pearson Product-Moment Correlation was performed on the reversed scores. The value of the correlation coefficient (r) was used to determine the nature of the relationship whether low, moderate or strong and the direction whether positive or negative. The weakness of the correlation analysis is that it only shows relationship between variables but cannot determine whether one affects the other. For instance, it cannot determine whether teaching experience influences teachers’ knowledge of fraction magnitude or not.

Analysis of data for research hypothesis two

The second hypothesis of the study states, “There is no significant difference between the level of accuracy of in-service teachers’ estimation of fraction magnitude and the class that they teach”. To test this hypothesis, the

Digitized by Sam Jonah Library

fraction magnitude scores were used. The scores were subjected to normality test. However, it was discovered that the assumption of the normality test was violated. Therefore, the Kruskal Wallis non-parametric test was conducted. This tool was chosen because teachers' knowledge of fraction magnitude was compared across the four categories of mathematics teachers. It was also deemed fit due to the fact that it allows simultaneous comparison of two or more means when the data is not normally distributed. The weakness of this tool is that it does not determine which pairs of categories revealed significant difference among the groups. The Kruskal Wallis test indicated there were differences. Hence a Post-hoc test was further conducted to determine where the differences exist.

Chapter Summary

The main focus of this study was to examine in-service teachers' knowledge of fraction magnitude and to identify whether there is a relationship between their teaching experience and fraction magnitude knowledge. It went further to investigate whether there are significant differences in the fraction magnitude knowledge of mathematics teachers base on the Form/class that they teach. Consequently, the study was situated on the positivists research paradigm.

A Cross-sectional research design was used since the study investigates the level of fraction magnitude estimation accuracy within a specific period. Simple random was employed to select 76 junior high schools in study area. Subsequently, purposive sampling method was used to select 134 in-service mathematics teachers for the study. The study adapted the magnitude knowledge

Digitized by Sam Jonah Library

University of Cape Coast https://ir.ucc.edu.gh/xmlui

instrument from Copur-Genturk (2022) and Siegler et al. (2011). The data was analysed using descriptive as well as inferential statistics.

Digitized by Sam Jonah Library

CHAPTER FOUR

RESULTS AND DISCUSSION

The objective of this work was to investigate the accuracy of in-service teachers' estimation of fraction magnitude, focusing on addition and subtraction questions. Additionally, the study aimed to identify the strategies employed by these teachers in estimating fraction magnitudes. Moreover, the research explored the potential relationship between in-service teachers' accuracy in fraction magnitude estimation and their years of teaching experience. Specifically, it sought to determine whether the accuracy increases or decreases with the duration of teaching mathematics. As the investigation was done at the junior high school level, it went further to examine significant differences in the accuracy of in-service teachers' estimation of fraction magnitude and the classes they teach.

The study used a cross-sectional survey research design. The participants were in-service mathematics teachers who were teaching public junior high schools in the Tamale Metropolis. Simple random and census methods were used to select 134 teachers. Analysis of the data was descriptive and inferential statistics. The findings and their interpretations have been presented based on each research questions and hypotheses. Discussion of the results and the findings were then followed accordingly.

Digitized by Sam Jonah Library

Presentation of Results

It was crucial to thoroughly examine the details of the participants before addressing the research questions and hypotheses guiding this study. This step was essential to determine whether the junior high school mathematics teachers possessed the necessary qualifications and to anticipate the outcomes of the study. Consequently, an investigation was conducted into the educational qualifications of the participants, and the findings are presented in Table 1.

Table 1

Distribution of the Participants Based on Level of Education

Educational Qualification	Frequency	Percentage
Diploma	12	9
Degree	112	83.6
Masters	10	7.4
Total	134	100.0

Source: Field Survey (2023)

Table 1 presents the results of the respondents based on their educational qualifications. It revealed that the majority of the mathematics teachers who participated in the study have received the needed training to teach mathematics at the junior high school level. Specifically, 112 out of the 134 participating teachers, representing 83.5% of the participants, held a Degree. As a result of their education, it is anticipated that these teachers possess the necessary mathematical expertise and a strong grasp of fraction magnitude knowledge.

Digitized by Sam Jonah Library

Research Question One

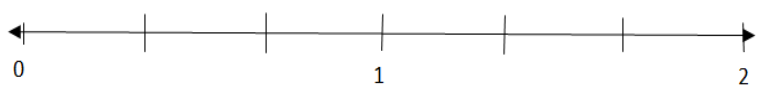
What is the level of accuracy of in-service teachers' estimation of fraction magnitude?

The first research question aimed at exploring the level of accuracy of in-service teachers' estimation of fraction magnitude. To answer this question, the data obtained from the test instrument on fraction magnitude knowledge was used. On the instrument, the participants were required to estimate answers to problems and put dots on the number lines where they think the answer will be located. Accuracy scores were computed from the teachers' estimates using the Percent Absolute Error (PAE) formula.

The instrument comprised three Items for both Addition and Subtraction. The accuracy levels for each individual Addition and Subtraction Items were analysed separately. Initially, the Addition Items were presented, followed by the Subtraction Items. Subsequently, both sets of Items were combined to assess the overall accuracy of participants in both Addition and Subtraction tasks.

Addition Item 1

1. Abu has $\frac{2}{3}$ of a Pizza and his friend has $\frac{1}{4}$ of the Pizza. What point on the number line below represents the amount of pizza they have shared?



The Item aimed to assess teachers' understanding on the common fractions that they teach at the junior high school. They were required to estimate the portion of a Pizza shared by two friends. One of them had $\frac{2}{3}$ and the other has $\frac{1}{4}$ of the Pizza. In estimating this, it is evident that $\frac{2}{3}$ is greater than $\frac{1}{2}$ and $\frac{1}{4}$ is less

Digitized by Sam Jonah Library

than $\frac{1}{2}$, indicating that the correct estimate will be closed to 1. Majority of the participants demonstrated high level of accuracy in this estimation task. Table 2 presents the distribution of the participants' estimation accuracy levels in Addition Item 1.

Table 2

Distribution of the Participants' Levels of Accuracy on Addition Item 1

Accuracy level	Interpretation	Frequency	Percent
0 – 25	High	69	51.5
26 – 50	Average	43	32.1
51 – 75	Low	13	9.7
76 – 100	Very low	9	6.7
Total		134	100.0

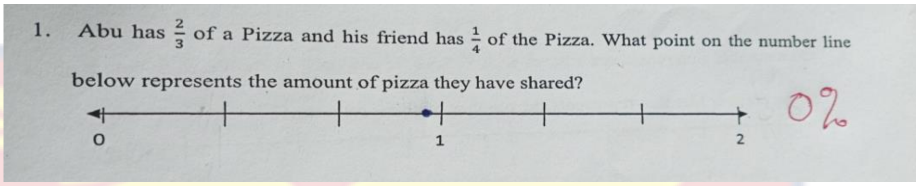
Source: Field Survey (2023)

Table 2 indicates that a majority of the participants, comprising 69 teachers (51.5%), demonstrated a high level of accuracy in their estimations. This implies that their estimates fell within the range of 0 - 25%, indicating proximity to the correct answer. This suggests that many participating teachers possess a strong grasp of fraction magnitudes, especially common fractions. Figure 2 displays a sample of teachers' solutions to Addition Item 1.

Digitized by Sam Jonah Library

Figure 2

Sample Response to the Addition Item 1



Source: Field Survey (2023)

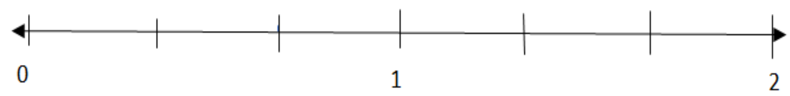
The participants' estimate to the Addition Item 1 is indicated by the dot on the number line as shown in Figure 2. The Item required the sum of $\frac{2}{3}$ and $\frac{1}{4}$ which will be close to but not exactly 1 on the number line, because $\frac{1}{4}$ is less than $\frac{1}{2}$. Therefore, the correct estimate is approximately $\frac{11}{12}$ on the number line. The participant's estimate is equal to the correct answer on the number line and the accuracy score was 0% as shown in Figure 2.

Even though many teachers' estimation accuracy in this particular Item was quite high, a significant portion, 43(32.1%) of them have their estimates falling within the average range 26% - 50%. This means that there was much distance between these participants' estimates and the correct answer. The average level of accuracy was not expected given the training that they received as shown in Table 1. More so, the fractions involved are common fractions which they have learnt at the basic level, therefore a robust understanding of the magnitudes of these fractions were anticipated from them as mathematics teachers. It suggests the some junior high school mathematics teachers have weak grasp of fraction magnitudes

Digitized by Sam Jonah Library

Addition Item 2

A bakery sold $\frac{3}{4}$ of a cake in the morning and $\frac{1}{2}$ of the cake in the afternoon. Locate a point on the number line which represents the total cake the bakery sold.



The Addition Item 2 was to examine teachers' knowledge on fractions greater than 1. It required the respondents to estimate the quantity of the cake sold by a bakery. In other words, the sum of $\frac{3}{4}$ and $\frac{1}{2}$. Performing this operation will yield a fraction that is slightly greater than 1 because $\frac{3}{4}$ is greater than $\frac{1}{2}$. Therefore, the correct estimate is $1\frac{1}{4}$ on the number line. Table 3 presents the distribution of the accuracy scores of the participants on Addition Item 2.

Table 3

Distribution of the Participants Levels of Accuracy of the Addition Item 2

Accuracy level	Interpretation	Frequency	Percent
0 - 25	High	57	42.5
26 - 50	Average	49	36.6
51 - 75	Low	21	15.7
76 - 100	Very low	7	5.2
Total		134	100.0

Source: Field Survey (2023)

The results displayed in Table 3 showed that 57 teachers, which consist of 42.5% of the total participants achieved high level accuracy in Addition Item 2. This number constitutes the majority of the participants. Their estimates are

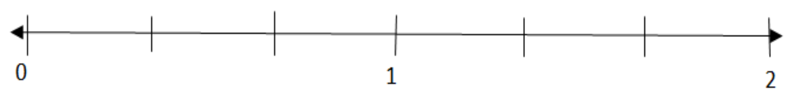
Digitized by Sam Jonah Library

within the range 0% - 25% indicating an excellent knowledge of fraction magnitudes.

However, 21 (15.7%) participants have scores below the average level. In other words, their estimates were far away from the answer on the number line. It means that some participants have challenges with fractions greater than 1. This is worrying because teachers need a good foundation of a concept before they can effectively teach it to others.

Addition Item 3

Estimate $\frac{17}{32} + \frac{39}{63}$ by placing a dot on the number line where you think the sum would be located.



Addition Item 3 required the participants to estimate the sum of two fractions ($\frac{17}{32}$ and $\frac{39}{63}$) . The Item was to test the teachers' conceptual understanding of fraction magnitudes. This is because the magnitude of the fractions involved will be difficult to process through rules and algorithms. Therefore, using unfamiliar fraction will reveal in detail teachers' conceptual knowledge of fractions. Table 4 presents the distribution of the accuracy scores of the participants in Addition Item 3.

Digitized by Sam Jonah Library

Table 4

Distribution of the Participants' Levels of Accuracy on Addition Item 3

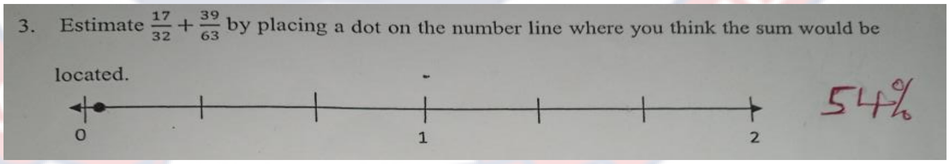
Accuracy level	Interpretation	Frequency	Percent
0 - 25	High	25	18.7
26 - 50	Average	36	26.9
51 - 75	Low	56	41.8
76 - 100	Very low	17	12.6
Total		134	100.0

Source: Field Survey (2023)

Table 4 indicated that majority of the participants' estimation accuracy in Item 3 were below the average. This means that their estimates were far away from the answer on the number line. For instance, 56 participants, representing 41.8% of the participants' accuracy scores were low. In other words, their estimates were within the range 51% - 75%. Figure 3 shows a sample response from the solutions of the teachers to Addition Item 3.

Figure 3

Sample Response to the Addition Item 3



Source: Field Survey (2023)

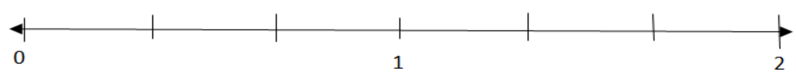
Addition Item 3 as shown in Figure 3 required the participants to estimate the sum of $\frac{17}{32}$ and $\frac{39}{63}$. It is evident that both fractions are slightly greater than 1.

Digitized by Sam Jonah Library

Hence, the correct estimate is approximately $1\frac{1}{7}$ on the number line. However, this particular respondents' estimate is approximately $\frac{1}{16}$ on the number line as indicated by the dot. The computed accuracy score of the teacher was 54%. The teacher was 54% away from the answer on the number. The situation is alarming, looking at the role that teachers' knowledge plays in the learning of students. Teachers are expected to have the knowledge that includes both familiar (common) fractions and unfamiliar fractions. This therefore suggests that the conceptual knowledge of some teachers regarding fractions is low.

Subtraction Item 1

A box contains $\frac{7}{8}$ of a pound of a cereal. If $\frac{3}{4}$ of a pound is taken out. Locate a point on the number line below that represents the quantity of the cereal left in the box.



In Subtraction Item 1, teachers were asked to take $\frac{3}{4}$ out of $\frac{7}{8}$ and the correct estimate is approximately $\frac{1}{8}$ on the number line. Table 5 shows the distribution of the participants' level of accuracies on the Subtraction Item 1.

Table 5

Distribution of the Participants' Levels of Accuracy on the Subtraction Item 1

Accuracy level	Interpretation	Frequency	Percent
0 - 25	High	59	44.0
26 - 50	Average	43	32.1
51 - 75	Low	24	17.9
76 - 100	Very low	8	6.0
Total		134	100.0

Source: Field Survey (2023)

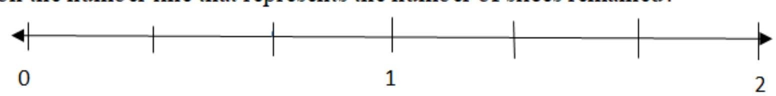
Digitized by Sam Jonah Library

University of Cape Coast https://ir.ucc.edu.gh/xmlui

Table 5 illustrates that a significant majority of teachers, comprising 59 teachers representing 44% of all participants, demonstrated high accuracy levels in their estimations. Their estimates fell within the range of 0% - 25%, indicating a close proximity to the correct answer on the number line. This reflects a strong understanding of subtraction operations involving fractions. However, a portion of participants scored well below the average range. Specifically, 24 individuals, making up 17.9% of the total participants, exhibited lower accuracy levels with scores ranging 51% - 75%. This indicates that there are teachers facing challenges in performing subtraction operations with fractions

Subtraction Item 2

2. On a number line represents a Pizza with 2 slices. If you eat $\frac{1}{4}$ of the Pizza, locate a point on the number line that represents the number of slices remained?



Subtraction Item 2 involves a whole number and a fraction. The participants were required to subtract $\frac{1}{4}$ from 2. Table 6 presents the distribution of the Participants levels of accuracies on subtraction Item 2.

Table 6

Distribution of the Participants' Levels of Accuracy on the Subtraction Item 2

Accuracy level	Interpretation	Frequency	Percent
0 - 25	High	65	48.5
26 - 50	Average	52	38.8
51 - 75	Low	8	6
76 - 100	Very low	9	6.7
Total		134	100.0

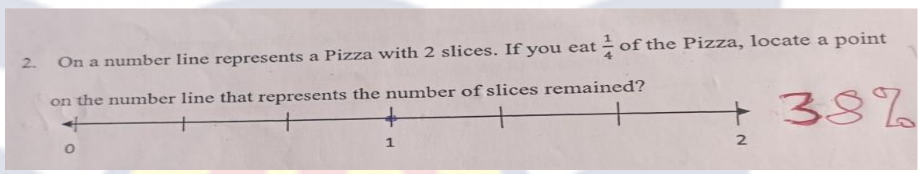
Source: Field Survey (2023)

Digitized by Sam Jonah Library

As indicated in Table 6, many of the participants, 65 consisting of 48% of the total participants demonstrated a high level of accuracy in estimating Subtraction Item 2. However, the significant number, 52(32.8%) of participants who demonstrated average level of accuracy was not anticipated as mathematics teachers. It means that these teachers struggle with fraction problems especially, when subtracting fractions from whole numbers. Figure 4 displays a sample response from the solutions of the teachers to subtraction Item 2.

Figure 4

Excerpt from the Teachers' Response to the Subtraction Item 2



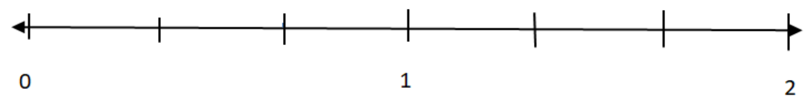
Source: Field Survey (2023)

As indicated in Figure 4, teachers were supposed to take $\frac{1}{4}$ from 2. The correct estimate corresponds to $1\frac{3}{4}$ on the number line. However, the estimate of this teacher corresponds to 1 on the number line as indicated by the dot. This further reflects teachers' challenges processing the magnitude of fractions. A person with a good understanding of fractions would have realised that subtraction $\frac{1}{4}$ from 2 cannot yield 1 because, 1 is exactly half of 2, but $\frac{1}{4}$ is not up to 1.

Digitized by Sam Jonah Library

Subtraction Item 3

Estimate $\frac{43}{69} - \frac{21}{65}$ by placing a dot on the number line where you think the difference would be located.



Subtraction Item 3 was intented to asses the fraction knowledge of the teachers to find out whether their fraction knowledge goes beyond the common fractions like $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$ etc. that are normally found in curriculums materials. A teacher achieving high level of accuracy in this Item will indicate conceptual understanding of fractions. The Item revealed surprising results as majority of the teachers' accuracy scores were far away from the answer. Table 7 presents the distribution of the Participants levels of accuracies on subtraction item 3.

Table 7

Distribution of the Participants' Levels of Accuracy on the Subtraction Item 3

Accuracy level	Interpretation	Frequency	Percent
0 – 25	High	14	10.4
26 – 50	Average	23	17.2
51 – 75	Low	45	33.6
76 - 100	Very low	52	38.8
Total		134	100.0

Source: Field Survey (2023)

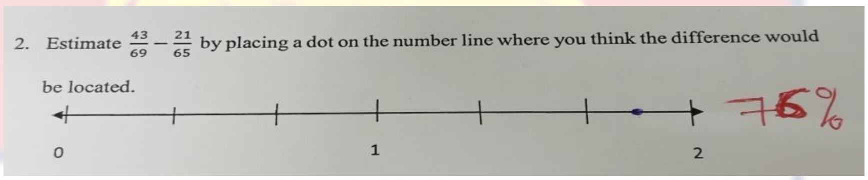
The results displayed in Table 7 revealed that almost all the teachers' accuracy scores in Subtraction Item 3 were below average. For instance, 52

Digitized by Sam Jonah Library

teachers who took part in the test representing 38.8% of the total participants exhibited a very low level of accuracy in their estimations. It means that their estimates were within the range 76% - 100%. Figure 5 shows an excerpt from the solutions of the teachers to Subtraction Item 3.

Figure 5

Excerpt from the Teachers' Response to the Subtraction Item 3



Source: Field Survey (2023)

It can be seen in Figure 5 that the participants estimate corresponds to approximately $1\frac{5}{6}$ as indicated by the dot on the number line. But the Item required the difference between $\frac{43}{69}$ and $\frac{21}{65}$, which is approximately $\frac{1}{3}$. Consequently, this particular teacher was 76% away from the answer. It is sad that this particular teacher could not identify the fact that the bigger fraction $(\frac{43}{69})$ is not up to one, hence, subtracting any number from it cannot yield a number greater than 1. This indicated that some teachers found themselves wanting when encountering problems that cannot be dealt with by the use of procedures (rules and algorithms).

After presenting the participants' levels of accuracies of the individual Items for both Addition and Subtraction, the total scores of the respondents in both groups of Items were determined in order to gain an insight into their overall levels of accuracies. This was done by averaging each participant's estimates for

Digitized by Sam Jonah Library

both addition and subtraction items separately to create a total accuracy score for each participant. For instance, if a participant's estimates for the addition Items 1, 2 and 3 are 49%, 38% and 63% respectively. Then, the Addition accuracy score of the participant is computed as;

(49 + 38 + 63) ÷ 3

= 150 ÷ 3

= 50%

Table 8 presents the distribution of the Addition accuracy scores of the participants.

Table 8

Distribution of the Addition Accuracy Scores of the Participants

Accuracy level	Interpretation	Frequency	Percent
0 – 25	High	53	39.5
26 – 50	Average	47	35.1
51 – 75	Low	21	15.7
76 – 100	Very low	13	9.7
Total		134	100

Source: Field Survey (2023)

Table 8 illustrates that out of the total participants, 53 teachers (39.5%) showcased a high level of accuracy on the Addition Items. Their accuracy rates fell within the range of 0% - 25%. This suggests that a significant portion of teachers possess adequate knowledge of fraction magnitudes.

Digitized by Sam Jonah Library

Yet, there is a concern regarding the accuracy levels of 47 participants (35.1%), which were deemed average. Considering the influence of teachers' knowledge on student outcomes, it was expected that these teachers would demonstrate a higher proficiency in the concept. Moreover, the percentage of participants scoring below average accuracy [15.7% (low accuracy) and 9.7% (very low accuracy)] is alarming. This indicates a significant number of teachers lacking a solid grasp of fractions. Based on these findings, it is evident that some students are likely to encounter challenges with fractions due to their teachers' deficiencies in this area.

The distribution of the participants' levels of accuracies on Subtraction Items is presented in Table 9.

Table 9

Distribution of the Subtraction Accuracy Scores of the Participants

Accuracy level	Interpretation	Frequency	Percent
0 - 25	High	46	34.3
26 - 50	Average	52	38.8
51 - 75	Low	23	17.2
76 - 100	Very low	13	9.7
Total		134	100

Source: Field Survey (2023)

The findings presented in Table 9 revealed that majority (38.8%) of the teachers demonstrated an average level of accuracy in their estimations regarding subtraction, with scores ranging 26% - 50%. This suggests that most teachers

Digitized by Sam Jonah Library

possess a limited grasp of fraction magnitudes, particularly in the context of subtraction. It is expected that all teachers involved ought to have achieved high accuracy because the Items involved fractions that they teach at the junior high school level. However, it is surprising that some teachers scored below the average level (26%). It suggests that teachers have difficulties with fraction just like students.

In order to determine the general performance of participants on both the Addition and Subtraction tasks, the means and standard deviations were computed. The mean score is the average accuracy level of the in-service teachers' estimation of fraction magnitude and standard deviation is the extent to which each respondent's accuracy score differs from the average accuracy level. The means and standard divinations of both addition and Subtraction scores are shown in Table 10.

Table 10

Descriptive Statistics of the Addition and Subtraction Accuracy Scores of Fraction Magnitude

Scores	N	Mean	Std. Deviation
Addition	134	30.72	17.78
Subtraction	134	33.86	18.00

Source: Field Survey (2023)

From Table 10, it can be seen that the mean scores were 30.72% and 33.86% respectively for the addition and Subtraction Items. These values represent the average level of accuracy for all the 134 teachers. This implies that

Digitized by Sam Jonah Library

overall, the teachers' estimate was, on average, 30.37% away from the correct answer on the number line for the Addition Items. On the other hand, their estimate was 33.86% away from the correct answer on the number line for the Subtraction Items. Though the means scores for both the Addition and the Subtraction Items indicated an average level of accuracy, the respondents were more accurate in the estimation of the Addition Items as compared to the Subtraction Items.

Furthermore, the standard deviations were determined to be 17.78 and 18.00 for the Addition and subtraction Items respectively. These values indicate a considerable variability among the teachers' performance, as scores ranged widely from the average. Large standard deviations suggest a wider range of scores, highlighting the diversity in the teachers' understanding of fraction magnitude.

It can be inferred from these descriptive statistics that the teachers' performance in the fraction magnitude test varied significantly. While the mean score of 30.72% and 33.86% for both group of Items indicates an average level of accuracy overall, the standard deviations suggest a substantial dispersion of scores. Some teachers achieved scores well below the mean, indicating a weaker grasp of fraction magnitude, while others attained scores significantly higher, reflecting a strong understanding of the concept.

Research Question Two

What are the strategies used by in-service mathematics teachers in the estimation of fraction magnitude?

Digitized by Sam Jonah Library

The second research question sought to find out the kind of strategies that are used by in-service teachers in the estimation of fraction magnitude. To answer this question, the strategies that were used by the participants in the fraction magnitude test were considered. Strategies used by each participant for the Addition and Subtraction Items were checked and their frequencies and percentages determined. The participants were allowed to choose as many strategies in the lists as they want. However, going through the responses, it was realized that no respondent used more than one strategy. Table 11 shows the distribution of strategies used by the participants in the estimation of the Addition Items.

Table 11
Distribution of the Strategies Used by the Participants in the Estimation of Addition Items

Strategies	Frequency	Percent
Benchmark/Common denominator	68	50.7
Selecting a fraction with similar size/Using the number line	34	25.4
Rounding	21	15.7
Other	11	8.2
Total	134	100.0

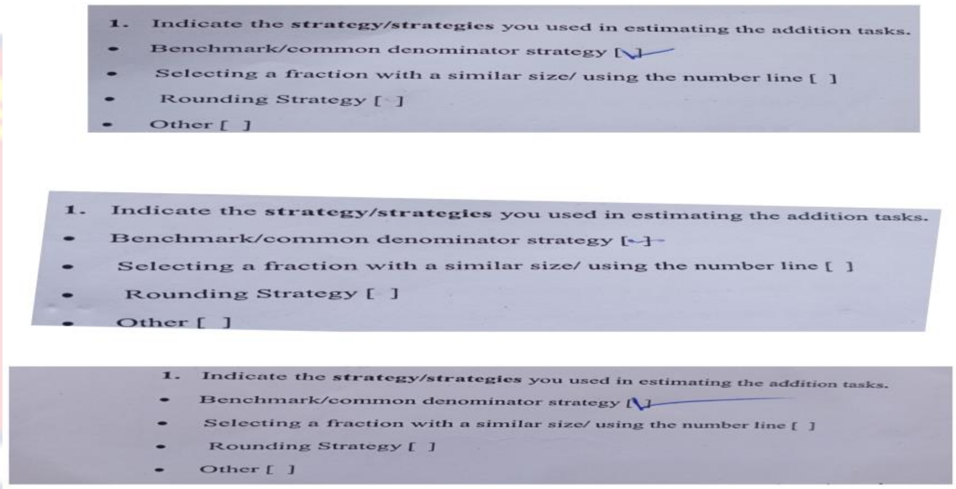
Source: Field Survey (2023)

It can be observed from Table 11 that 68 representing 57.5% of the 134 teachers employed *Benchmark/Common denominator* strategy in the estimation of the Addition Items. Figure 6 presents excerpts from the teachers' responses.

Digitized by Sam Jonah Library

Figure 6

Excerpts from Teachers Who Used Benchmark/Common Denominator Strategy



1. Indicate the **strategy/strategies** you used in estimating the addition tasks.
- Benchmark/common denominator strategy [√]
- Selecting a fraction with a similar size/ using the number line [ ]
- Rounding Strategy [ ]
- Other [ ]

1. Indicate the **strategy/strategies** you used in estimating the addition tasks.
- Benchmark/common denominator strategy [√]
- Selecting a fraction with a similar size/ using the number line [ ]
- Rounding Strategy [ ]
- Other [ ]

1. Indicate the **strategy/strategies** you used in estimating the addition tasks.
- Benchmark/common denominator strategy [√]
- Selecting a fraction with a similar size/ using the number line [ ]
- Rounding Strategy [ ]
- Other [ ]

Source: Field Survey (2023)

The use of the *Benchmark/Common Denominator* strategy as shown in Figure 6 implies that teachers may only be focusing on the denominators of the fractions neglecting that of the numerators. However, both numerator and denominators play a role in determining the magnitude of fractions (Bahr et al, 1983). This means that they were not focusing on the magnitude of the fractions. Based on the majority that relied on this strategy, it can be inferred that generally, junior high school mathematics teachers who took part in the study have partial understanding of fraction magnitudes.

Moreover, the 25.4% of the participants who used correct strategy (*Selecting a fraction with similar size/Using the number line*) that reflected deep understanding of fraction magnitude indicates the presence of teachers with deep understanding of fraction magnitudes. It means that the participants in this category place much emphasis on the magnitude that the fractions represented.

Digitized by Sam Jonah Library

University of Cape Coast https://ir.ucc.edu.gh/xmlui

It was equally important to probe deep into the identified strategies used to determine the effectiveness of each strategy. In other words to determine the level of accuracy achieved with each strategy used. Table 12 gives information of the distribution of accuracies of the identified strategies.

Table 12

Distribution of the Levels of Accuracies of the Strategies Used in the Estimation of the Addition Items

Strategy	Frequency	Levels of Accuracy			
		High	Average	Low	Very low
Benchmark/Common denominator	68	25	32	8	3
Selecting a fraction with similar size/Using the number line	34	24	6	3	1
Rounding	21	2	7	8	4
Other	11	3	2	1	5

Source: Field Survey (2023)

Table 12 presents findings regarding the strategies employed by the participants and their associated magnitudes. Among the 68 participants utilizing the *Benchmark/Common denominator* strategy, 32 (47%), comprising the majority of teachers achieved average accuracy levels, with levels of accuracies ranging 26% - 50%. These results indicate that their estimations were significantly off the mark on the number line, suggesting a limited grasp of fraction magnitude. This underscores the rule-based nature of the *Benchmark/Common denominator* strategy, reflecting only partial understanding of fraction magnitudes.

Digitized by Sam Jonah Library

Conversely, a significant portion of teachers (24 individuals) who opted for strategies such as *selecting fractions with similar sizes or using the number line* demonstrated notably higher accuracy rates, representing 71.6% of all participants. This supports the notion that teachers employing these methods possess a robust understanding of fractions.

Furthermore, the 38% of total participants who rounded either the numerator or the denominator exhibited lower accuracy levels, with estimations ranging from 51% - 75%. This indicates significant deviation from the correct answer, underscoring a lack of comprehension regarding fraction magnitudes. This group constitutes the majority of teachers, further highlighting the limitations of this strategy.

Lastly, teachers categorized under *Other* strategies often utilized faulty methods or resorted to guessing. This classification is supported by the fact that 45% of these teachers displayed very low accuracy in their estimations. It is important to note that while some teachers in this category achieved high accuracy, their success might have been coincidental, possibly due to lucky guesses.

The exploration of strategies employed by respondents in estimating Subtraction Items and their frequencies are detailed in Table 13.

Digitized by Sam Jonah Library

Table 13

Distribution of the Strategies Used by the Participants in the Estimation of Subtraction Items.

Strategies	Frequency	Percent
Benchmark/Common denominator	61	45.5
Selecting a fraction with similar size/Using the number line	32	23.9
Rounding	24	17.9
Other	17	12.7
Total	134	100.0

Source: Field Survey (2023)

A cursory observation of Table 13 suggests that 61 teachers representing 45.5% of a total of 134 participants employed *Benchmark/Common denominator* strategy in the estimation of Subtraction Items. This implies that teachers understanding of fractions and for that matter fraction magnitude is generally based on algorithms in which they used some numbers as benchmarks or find a common denominator to be able to perfume basic fraction operations.

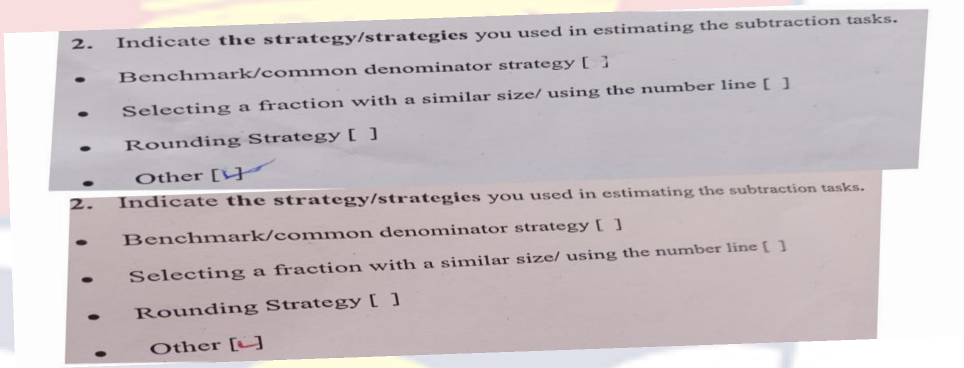
Moreover, it is sad that some teachers fell outside the domains of the conceptual (*Selecting a fraction with similar size/Using the number line*) or the procedural (*Benchmark/Common denominator*) domains and they resorted to strategies that reflects no comprehension of fraction magnitude. For instance, about 12.7% of the teachers were using *Other* strategies in their estimation. This means that they were either using faulty strategies such as directly performing

Digitized by Sam Jonah Library

subtraction with the fractions as it is done in whole numbers. A sample from the responses of the participants is presented in Figure 7.

Figure 7

Sample Responses from the Participants Who Employed the Other Strategies in Estimating the Subtraction Items



2. Indicate the strategy/strategies you used in estimating the subtraction tasks.

- Benchmark/common denominator strategy [ ]
- Selecting a fraction with a similar size/ using the number line [ ]
- Rounding Strategy [ ]
- Other [✓]

2. Indicate the strategy/strategies you used in estimating the subtraction tasks.

- Benchmark/common denominator strategy [ ]
- Selecting a fraction with a similar size/ using the number line [ ]
- Rounding Strategy [ ]
- Other [✓]

Source: Field Survey (2023)

It is evident in Figure 7 that some teachers actually used *Other* strategies in their estimations. This suggests that some junior high school teachers who participated in the study have poor foundation in fractions. Those teachers who even used the correct strategies, most of them did not exhibit high accuracy in their estimation. Investigating in to the distribution of the levels of accuracies achieved with each strategy used suggests that some teachers do not properly understand the strategies they were using, even though they were aligned with the fraction magnitude concept. Table 14 gives information of the distribution of accuracies of the identified strategies.

Digitized by Sam Jonah Library

University of Cape Coast https://ir.ucc.edu.gh/xmlui

Table 14

Distribution of the Levels of Accuracies of the Strategies Used in the Sstimation of the Subtraction Items

Strategies	Frequency	Levels of Accuracy			
		High	Average	Low	Very low
Benchmark/Common denominator	61	22	27	8	4
Selecting a fraction with similar size/Using the number line	32	18	10	3	1
Rounding	24	3	11	8	2
Other	17	3	4	4	6

Source: Field Survey (2023)

It can be observed from Table 14 that among the 61 participants who used the Benchmark/Common denominator strategy, 27 (44%) of them who formed the majority of teachers accuracy levels were average. It means that their estimates were far away from the answer on the number line (26% - 50%). This therefore proved the fact that most of the mathematic teachers do not know how to correctly apply rules involved in the computation of fractions, especially estimating the size of fractions. Conversely, 18 (56%) teachers who *selected a fraction with similar size/Use the number line* were able to achieve a high level of accuracy in their

Digitized by Sam Jonah Library

estimations. This means that the estimates ranged from 0% - 25%, suggesting a good understanding of fractions.

Furthermore, majority of the teachers 11 (45.8%) who used the *Rounding* strategy have their estimates ranging from 26% - 50%, and for that matter their estimates were within the average range. Moreover, is obvious that those teachers who used *Other* strategies were using faulty strategies. This is because the accuracies of 35.3% of the teachers in this group were very low. Their estimates were far away from the correct answer on the number line. This substantiated the fact that they were either guessing or using faulty strategies.

Observations from Figure 14 reveal that out of the 61 participants employing the *Benchmark/Common Denominator* strategy, a majority of 27 teachers (44%) demonstrated average accuracy levels. This indicates that their estimates fell within the range of 26% - 50% on the number line, signaling a significant gap from the correct answer. This finding underscores a prevailing issue among mathematics teachers – a lack of proficiency in applying rules related to fraction computation, particularly in estimating fraction magnitudes.

Conversely, among the 18 teachers (56%) who opted for the strategy of *selecting a fraction with a similar size/using the number line*, a notable achievement was observed. Their estimates reached a high level of accuracy, ranging from 0% - 25%. This suggests a commendable understanding of fractions and the magnitudes they represent.

Furthermore, the analysis of the accuracies of teachers utilizing the *Rounding* strategy reveals that 11 individuals (45.8%) fell within the range of

Digitized by Sam Jonah Library

26% - 50% in terms of estimates, placing them in the average accuracy category. On the other hand, it is evident that teachers employing *Other* strategies exhibited poor accuracy levels. A significant portion, 35.3% of teachers in this group, displayed very low accuracies, with estimates significantly deviating from the correct answer on the number line. This substantiates the notion that these teachers may have been either guessing or utilizing faulty strategies in their estimations.

Research Hypothesis One

The first hypothesis that guided this study was "There is no statistically significant relationship between the level of accuracy of in-service teachers' estimation of fraction magnitude and their years of teaching experience". It investigated the relationship between the teachers teaching experience and the accuracy of their estimation of fraction magnitude. To test this hypothesis, the fraction magnitude scores of the in-service teachers who participated in the study were used. Both the two scores (Addition and Subtraction) were averaged to create a fraction magnitude scores of each participant. For example, if a participants addition and subtraction scores are 34% and 42% respectively, then the fraction magnitude score of the participant is calculated as;

$(34 + 42) \div 2$

$= 76 \div 2$

$= 38\%$

However, since there is an inverse relationship between PAE and the accuracy, a negative relation will mean a positive and the vice-versa. To

Digitized by Sam Jonah Library

University of Cape Coast https://ir.ucc.edu.gh/xmlui

overcome this, the fraction magnitude scores were reverted by subtracting the fraction magnitude score of every respondent from 100. With these new scores, a positive relation was ensured in which a high score indicates a high level of accuracy and low score mean a low level of accuracy. A bivariate Correlation was conducted at 0.05 level of significance and the results presented in Table 15.

Table 15

Results of Bivariate Correlation Between Fraction Magnitude Scores and Years of Teaching Experience

		1	2
Years of teaching experience(1)	Pearson Correlation	1	.661
	Sig. (2-tailed)		.000
	N	134	134
Fraction magnitude score (2)	Pearson Correlation	.661	1
	Sig. (2-tailed)	.000	
	N	134	134

Source: Field Survey (2023)

The results presented in Table 15 indicated a positive, statistically significant and moderate relationship between mathematics teachers knowledge of fraction magnitude and teaching experience revealing a Pearson's correlation coefficient, $r = .661$, $p < .05$. Therefore, the researcher rejected the null hypothesis and concluded that there is a relationship between the level of accuracy of in-service teachers' estimation of fraction magnitude and their years of teaching experience The positive relationship between years of teaching and fraction magnitude estimation accuracy implies that the more, the years of experience of a teacher the likelihood of demonstrating high level of accuracy and the less the

Digitized by Sam Jonah Library

University of Cape Coast https://ir.ucc.edu.gh/xmlui

experience the less the accuracy in the estimation. It can be inferred that teachers who have taught for many years tend to be proficient and knowledgeable as compare to novice teachers. Hence, teachers grow in terms of fraction magnitude knowledge as they teach.

Research Hypothesis Two

The second hypothesis that guided this study was "There is no statistically significant difference between the accuracy of in-service teachers' estimation of fraction magnitude and the classes that they teach". It sought to explore whether there is a significant difference between the fraction magnitude knowledge of in-service mathematics teachers based on the class that they teach. To test this hypothesis, fraction magnitude scores of in-service mathematics teachers were compared across the various classes of junior high school [JHS 1, 2, 3 and Multiple Classes(**Note**: Multiple Class teachers are those teachers who teach more than one class level)]. However, normality test was necessary to find out whether the data (Fraction Magnitude Scores) follows a normal distribution as this will guide in determining the appropriate tool to use in the analysis of the data. In view of this, the Kolmogorov-Smirnov and Shapiro-Wilk tests were conducted. Table 16 presents the results on the normality test of the fraction magnitude scores of the participants.

Digitized by Sam Jonah Library

Table 16

Test of Normality for Fraction Magnitude Accuracy Scores

	Kolmogorov-Smirnov[a]			Shapiro-Wilk		
	Statistic	Df	Sig.	Statistic	Df	Sig.
Fraction magnitude scores	.114	134	.000	.953	134	.000

a. Lilliefors Significance Correction

Source: Field Survey (2023)

A careful observation of Table 16 shows that the normality test for the fraction magnitude accuracy scores is statistically significant (P = .000). It therefore indicates that the fraction magnitude accuracy scores were not normally distributed, necessitating the conduct of the Kruskal Wallis non-parametric test for difference in means among two or more independent samples. Table 17 gives information on the descriptive statistics for the fraction magnitude scores of the participants.

Table 17

Descriptive Statistics for the Fraction Magnitude Scores

	Class level	N	Mean Rank
	JHS1	37	96.86
	JHS2	26	72.23
Fraction Magnitude Scores	JHS3	41	50.46
	Multiple Classes	30	50.47
	Total	134	

Source: Field Survey (2023)

Digitized by Sam Jonah Library

University of Cape Coast https://ir.ucc.edu.gh/xmlui

Mean Ranks displayed in Table 17 were different for each category of teachers. This means that the teachers possess different levels of accuracies fraction magnitude estimation. Multiple Classes and JHS3 teachers demonstrated high level of accuracy in their estimations as compared to their counterparts at JHS1 and JHS2 as reflected in the Mean Ranks (**Note:** *The smaller the value of the PAE, the more accurate it is*). However, the results displayed in Table 17 do not indicate whether the differences in ranks among the different categories of teachers are significant or not. Hence Table 18 presents the Kruskal Wallis Chi-Square test.

Table 18

Kruskal Wallis Chi-Square Test Results of the Four Categories of In-service Mathematics Teachers Based on the Class that They Teach

	Fraction Magnitude Scores
Chi-Square	35.245
Df	3
Asymp. Sig.	.000

a. Kruskal Wallis Test
b. Grouping Variable: Class
Source: Field Survey (2023)

Based on the results presented in Table 18, the researcher rejected the null hypothesis and concluded that there is a statistically significant difference in the fraction magnitude accuracies of among the four groups of teachers, x^2 (3) = 35.245, p = .000, with a mean rank, 96.86 for JHS1 teachers, 72.23 for JHS2 teachers, 50.46 for JHS3 teachers and 50.47 for Multiple Classes teachers.

Digitized by Sam Jonah Library

However, Table 18 does not reveal in detail as to where the difference that exists among the categories lies. This therefore called for the conduct of Post-hoc test to determine which pairs of categories revealed significant difference among the groups at a significance level of 0.05. Therefore, the pairwise comparisons of the differences in the accuracy scores of the four groups of teachers are presented in Table 19.

Table 19

Pairwise Comparisons of Differences in Fraction Magnitude Accuracy Scores of the Teachers

Sample1 - Sample2	Test Statistic	Std. Error	Std. Test Statistic	Sig.	Adj. Sig.
JHS3 - Multiple Classes	-.003	9.325	-.000	1.000	1.000
JHS3 - JHS2	21.767	9.730	2.237	.025	.152
JHS3 - JHS1	46.401	8.801	5.272	.000	.000
Multiple Classes - JHS2	21.764	10.399	2.093	.036	.218
Multiple Classes -JHS1	46.399	9.535	4.866	.000	.000
JHS2 - JHS1	24.634	9.932	4.480	.013	.079

Each row test the null hypothesis that the sample 1 and sample 2 distribution are the same.
Asymptotic significances (2-sided test are displayed. The significance level is .05.
Source: Field Survey (2023)

In the Post-hoc test results presented in Table 19, it is evident that the fraction magnitude estimation accuracy of JHS3 mathematics teachers did not show a statistically significant difference when compared to Multiple Classes teachers (p = 1.000), even though their accuracy was higher. Moreover, the

Digitized by Sam Jonah Library

accuracy of JHS3 teachers was higher and statistically significantly different from JHS2 teachers (p = .025) and JHS1 teachers (p = .000).

Additionally, the fraction magnitude estimation accuracy of Multiple Classes teachers was more accurate and significantly difference from JHS2 teachers (p = .036) and JHS1 teachers (p = .000). Furthermore, the accuracies of JHS2 and JHS1 teachers were found to be significantly different (p = .013).

It can be inferred from the difference in fraction magnitude estimation accuracies exhibited from these groups that the teachers instructing at the higher classes possess a higher level of fraction magnitude knowledge compared to their counterparts in the lower classes. This is evident from the highest level of accuracy demonstrated by the high class (JHS3; accuracy = 50.46, *see Table 17*) and the lowest accuracy exhibited by the lowest class (JHS1; Accurscy = 96.86, *see Table 17*).

Discussion of Results

The findings of the study are discussed according to each research question and hypothesis.

Research Question One

Students' struggle with fractions understanding has been a major concern for mathematics researchers (Siegler et al., 2011, Siegler et al, 2012; Siegler & Pyke, 2013; Siegler & Lortie-Forgues, 2015, Copur-Genturk, 2021, 2022; Ntow, 2022). However, one of the factors that influences the knowledge and performance of students in mathematics is the level of mathematical knowledge possessed by teachers (Depaepe et al., 2015; Copur-Genturk, 2015). Thus, a

Digitized by Sam Jonah Library

teacher with a deep understanding of mathematical concepts is likely to have a greater impact on the students' learning. Conversely, a teacher with a weak knowledge of mathematical concepts will only transmit same to students. As a result, this piece of work aimed to explore the level of accuracy of in-service mathematics teachers' estimation of fraction magnitude.

The results of the study revealed that mathematics teachers' level of accuracy in the estimating of fraction magnitude is average, as reflected in their mean accuracies (30.72 for Addition Items and 33.86 for Subtraction Items, *see Table 10*). It implies that the participants have difficulties processing the magnitude of fractions. This assertion is based on the integrated theory of numerical development which posits that the ability to identify the exact location of a fraction on a number line is a proof of fraction magnitude understanding (Siegler et al., 2011). The average accuracy also means that some junior high school mathematics teachers struggle in comprehending and connecting the symbolic notation of fractions with their magnitudes. It is clear that, errors and misconceptions of students on fractions (Eichhorn, 2018; Odigun 2018; Amuah et al., 2019) are due to teachers' deficiencies in the concept.

The result of the study is similar to many studies. The study of Siegler and Lortie-Forgues (2015) discovered that pre-service teachers have weak knowledge of multiplication and division of fractions. The findings of the study also concur with research by Copur-Genturk (2022) which found superficial understanding of fraction magnitude among Grades 3–7 in-service mathematics teachers in the United States.

Digitized by Sam Jonah Library

University of Cape Coast https://ir.ucc.edu.gh/xmlui

The average accuracy demonstrated by the in-service teachers in this study is an issue of concern because teachers teach students mathematical concepts from the knowledge and experiences that they have. However, nothing more than average can be expected from a student taught by a teacher with an average knowledge of the concept. It is no surprising the Chief Examiners' report at the Basic Education Certificate Examination (BECE) level often mention fractions as one of the areas students perform poorly (WAEC, 2015; 2017). Students at the JHS cannot demonstrate proficiency in factions if those who are training them lack the same.

From the results of this study and that of the previous findings with teachers, it can safely be said that students' difficulties and misconceptions of fractions stem from teachers' weak knowledge of the concept. An exceptional knowledge and mastery of the mathematical content is one of the requirements for effective teaching. However, an average level of accuracy demonstrated by the participants cannot translate to effective teaching.

The difficulties of teachers in rational number conception can partly be attributed to the teacher education curriculums because majority of the teachers who participated (83.6%) were Degree holders. Nonetheless, the average level of accuracy shown in their estimations was not the expected given the educational training that they received.

Moreover, another reason for the weak knowledge of teachers in mathematics as suggested in literature is that many teachers do not patronize teacher professional development programmes. For instance, Osei (2020)

Digitized by Sam Jonah Library

discovered that only 4 out of 18 in-service mathematics teachers of junior high schools who graduated from colleges of education have ever participated in in-service training programmes related to mathematics. Conversely, 2 out of the 15 teachers who passed through distance education programmes have ever attended an in-service training about mathematics. Consequently, their performance in the knowledge of teaching algebra was average.

Furthermore, it is worth noting that both teachers who were found within low and high level of accuracy go through the same training yet exhibited such gross differences in mathematical knowledge. The wide difference in knowledge among the teachers mean that junior high school students who are taking the same courses and are expected to write the same exam (BECE), some are disadvantaged which will ultimately affect their performance in mathematics

Research Question Two

Many studies have identified different kinds of strategies used in solving fraction problems. The kind of procedures and strategies a person employs when solving mathematical problems reveal his/her thinking and level of understanding as posited by the theory of variability. Thus, the ability to determine which method works best and for which problem and the application of different strategies in solving different problems (Siegler, 1995). More so, Mclntosh, De Nardi and Swan as cited in Lemonidis et al. (2018) categorised the strategies into instrumental and conceptual. Whiles the instrumental techniques are the application of rules and algorithms, conceptual, deals with understanding the concepts and their relationships.

Digitized by Sam Jonah Library

The study adapted the fraction magnitude estimation strategies from Siegler et al. (2011) and Copur-Genturk (2022). Whiles some of the strategies reflect conceptual understanding of fraction magnitude (*eg. Using the number line or selection a fraction with a similar size*), others depict instrumental understanding (*Benchmark/Common denominator strategy*). The strategies used by the teachers in estimating the magnitude of fractions depended on the level of their knowledge.

It was discovered that about half of the participants (50.7% for Addition and 45.5% for Subtraction) of the in-service teachers were using the *Benchmark/Common denominator* strategy in their estimation. This is a rule-based strategy in which a teacher selects numbers such as $\frac{1}{2}$, 1 or 2 as benchmarks to guide in the estimation or converting the denominators to same numbers. This means that teachers' understanding of fraction magnitude is based on the application of rules and algorithms. Moreover, it was evident that the whole number bias, which is the application of the whole number thinking to fractions, interfered in the estimation of the participants. Just like students, *whole numbers bias* interferes with fractional thinking of teachers

The teachers who even used strategies that reflect good understanding of fraction magnitude, most of them could not use it properly. For instance, those teachers who used the number line or selected a fraction with similar size in their estimations, out of 34 teachers, 10 teachers were not able to demonstrate high accuracy in the estimation of the Addition Items. (*see Table 12*). Conversely, out

Digitized by Sam Jonah Library

of the 32 teachers who used this strategy to estimate Subtraction Items 14 could not use properly. (*see Table 14*).

The finding of the study is not different from the previous studies. For instance, a study by Toledo et al. (2022) found majority of Brazilian mathematics teachers used strategies that were not in line with an in-depth understanding of fractions. The study revealed majority of the participants were using the flawed *Gap strategy* in comparing of fractions. It also concurred with a study by Copur-Genturk (2022) who discovered that more than half of the Grade-1-7 in-service mathematics teachers employed strategies that were partly in line with the concept of fraction magnitude.

The result is not surprising because the level of accuracies achieved by the participants in both Addition and Subtraction Items as reflected in the means (*see Table 10*). The average accuracy level suggests that the teachers were not using strategies that depict robust understanding of fraction magnitude. This means that the participants are not only deficient in content knowledge but lack the knowledge of the techniques and strategies that will yield correct estimation of fraction magnitudes. As mathematic teachers, in addition to other skills, a mastery of procedural knowledge is necessary for effective teaching.

Hypothesis One

More often than not, employees who have worked for many years in companies or institutions are paid more than those with few years in the work. More so, they are often placed in the managerial and other important positions. This is because it is assumed that workers who have served for many years learn

Digitized by Sam Jonah Library

from experience and hence are more productive in knowledge and smarter than those with few years of work (Yarkwah, 2017).

When it comes to teaching, it is expected that teachers who have taught for many years to be more productive as compared to novice teachers. This is because knowledge is gained through exposure to different categories of learners, learning types, and the development of effective learning techniques. The experience acquired from teaching enables teachers to tailor their approaches to specific pupils ensuring that all students have an equal chance to succeed. Hence, in this context, teachers who have taught for many years are expected to be more accurate in the estimation of fraction magnitudes as compared to their colleagues with less years in teaching.

The study found a positive moderate (r = .661) correlation between the fraction magnitude estimation accuracy and years of teaching experience. The statistically significant relationship means that fraction magnitude knowledge of teachers is likely to grow with respect to their teaching experience. It can be concluded that experience teachers are likely more knowledgeable than inexperience teachers.

The results of this study contradicted several researches (Osei 2020; Corpur-Genturk, 2022). Osei found that basic school teacher's knowledge of algebra does not increase as their years advance in the teaching profession whether they obtain their teaching certificates from regular or distance education. Corpur-Genturk agreed with him when she concluded that teachers' fraction magnitude knowledge does not increase with teaching.

Digitized by Sam Jonah Library

Even though some studies conflicted with the findings of this study, many more studies agree with it. For instance, the study of Laine (cited in Yarkwah, 2017) discovered a positive correlation between teacher experience and student outcomes. It also agrees with what research affirms that, experience is one of the factors that contribute to teacher competency and that teachers with more years of teaching are more proficient compared to teachers with less years of experience (Darling-Harmmond, 2000; Klecker, 2002; Adeyimi, 2008; Harris & Sass, 2011). This suggests that experience matters as far as teaching is concerned

More so, Adeyemi (2008) study in Nigeria found a significant impact of teachers experience on students' performance gains. The study's conclusion was based on the performance of students in senior secondary certificate (SSC) examinations, who were tutored by well experience teachers. A recent study by Copur-Genturk and Li (2023) concluded that even though teachers with deep understanding of mathematical concepts grows faster as compared to teachers with less robust understanding, both increase in mathematical knowledge. Therefore, students will profit more from well experienced teachers as compared to inexperience ones.

The increment in knowledge may be attributed to the teachers interacting with students and curriculum materials in the teaching process. More so, Ghana Education Service (GES) and other stakeholders of education occasionally organize professional development programs for in-service teachers. Hence, they stand a better chance of achieving a deeper understanding of mathematical content as compared to their peers with few or no years of teaching experience.

Digitized by Sam Jonah Library

Hypothesis Two

It is not only the training and educational courses that teachers take in the tertiary institutions and the professional training programmes that influence the level of their mathematical knowledge, but the kind of students and the curriculum that teachers encounter also have an impact on their mathematical knowledge (Hill, 2010; Copur-Genturk 2022).

The study discovered significant differences in fraction magnitude estimation accuracy among junior high school teachers. This implies that in-service mathematics teachers' fraction magnitude knowledge differs according to the class that they teach. In other words, mathematics teachers at lower classes (JHS1and JHS2) had low level of fraction magnitude knowledge as compared to their peers at the high classes (JHS3 and Multiple classes), (*see Table 19*).

The findings of the study disagree with that of Copur-Genturk (2021). She found that the accuracy of teachers' explanations to mathematical concepts was not related to the grade level that teachers taught. The difference in the results may be due the limited variation of grades that the study of Copur-Genturk focused on. Other earlier studies with much variation of classes found no different from the results of this study. For instance, Wilkin (2008) investigation found that upper elementary teachers have more mathematical content knowledge and positive attitude towards mathematics as compared to their colleague teachers at the lower level. Moreover, Hill (2010) also discovered that teachers at the low grades exhibited weak mathematical knowledge as compared to teachers at the high grades.

Digitized by Sam Jonah Library

The studies that focused on rational numbers revealed differences in knowledge among different teachers teaching different grades. A more recent was the study of Copur-Genturk (2022) who engaged grade 1-7 US mathematics teachers. She found that teachers' scores on the fraction magnitude scale were related to the grade that they taught mathematics. She concluded that teachers' mathematical knowledge is associated with the grade that they teach. Moreover, a study Depaepe et al. (2015) discovered that prospective secondary teacher performed better than the prospective elementary teachers. This suggests that the kind of training teachers receive in the teacher training institutions could be a contributory factor to their differences in mathematical knowledge.

The differences in mathematical knowledge among the teachers may also be attributed to the hierarchical nature of mathematics curriculum. Thus, the content and the teaching methodologies differ among different class levels. Teachers at the higher classes (eg. JHS3) teach broad and complex concepts as compared to their peers at the lower classes (JHS1). In the mathematics curriculum in Ghana for instance, in JHS1 (Grade 7) teachers teach learners how to order and simplify fractions to simplest forms. In JHS2 (Grade 8) learners move further to performing simple fraction arithmetic and in JHS3 (Grade 9) teachers guide learners in the application of the fractions to solve practical real-life problems (MoE, 2019). Teachers in the process of teaching revisit concepts learnt at the lower levels thereby broadening their knowledge.

Digitized by Sam Jonah Library

CHAPTER FIVE

SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

This research was premised on the assumption that teachers' knowledge of the subject matter has a positive impact on students' academic performance. In view of this, the level of accuracy of in-service mathematics teachers' estimation of fraction magnitude was assessed to determine the level of fraction magnitude knowledge that they have. The strategies they use in the estimation of fraction magnitude were also explored to find out its alignment with fraction magnitude concept. Furthermore, the study investigated whether teachers' accuracy of fraction magnitude estimation increases as they teach mathematics. It went further to explore if there is a significant difference between the level of accuracy of in-service mathematics teachers' fraction magnitude estimation and the classes that they teach.

The instrument for the study was adapted from Siegler et al. (2011) and Copur-Genturk (2022) and then modified to suit the Ghanaian context. It contained Addition and Subtraction problems that measured the fraction knowledge of the participants. There were number lines which ranged from 0-2 and the participants were required to estimate and put their answers on the number lines at where the answer will be located.

The study used a cross-sectional descriptive survey research design. It employed simple random sampling and census techniques to select 134 mathematics teachers. Participants were teaching mathematics at the junior high schools in the Tamale Metropolis at the time of the study.

Digitized by Sam Jonah Library

Descriptive and inferential statistics were used to analyze the data. Precisely, the accuracy of in-service teachers' estimation of fraction magnitude tasks and strategies used were analysed using descriptive statistics such as mean, standard deviation, frequencies and percentages. For inferential statistics, tools such as correlation were used to explore the relationship between in-service teachers' level of accuracy in estimating fraction magnitudes and years of teaching experience. Kruskal Wallis test was used to investigate the differences in fraction magnitude estimation accuracy among junior high school mathematics teachers.

Summary

In-service mathematics teachers' level accuracy in the estimation of fraction magnitude was investigated in this study. The aim was to find out whether the teachers have the requisite fraction magnitude knowledge to teach mathematics at the junior high school levels.

The study revealed that in-service mathematics teachers possessed an average level of accuracy in the estimation fraction magnitude. This was evidenced when their estimates (means) were 30.72% and 33.86% away from the correct answer on the number line for Addition and Subtraction Items respectively. More so, about 50% of the participants were using strategies that were not aligned with the concept of fraction magnitudes.

Furthermore, a statistically significant moderate correlation was found between the accuracy of in-service mathematics teachers' estimation of fraction magnitude and their years of teaching experience. The positive correlation means

Digitized by Sam Jonah Library

that teachers' understanding of fractions is likely to grow with teaching. It was also discovered that fraction magnitude estimation accuracy of in-service mathematics teachers differed significantly based on the class that they teach.

Conclusions

The study investigated the level of accuracy of fraction magnitude estimation by in-service mathematics teachers of public junior high schools. Below are the conclusions drawn from the study;

Junior high school mathematics teachers' level of accuracy in the estimation of fraction magnitude is average. It means that in-service teachers' understanding of fractions is weak. Just like this study, the teachers' weak knowledge of the mathematical content was discovered in many parts of the world such as Malaysia (Leong et al., 2015) and the United States (Copur-Genturk, 2022). The weak knowledge of in-service teachers on fraction magnitude was further corroborated by the strategies they employed in their estimations. Majority of teachers resorted to procedures that are not aligned with fraction magnitude concept (eg. *Benchmark*/*Common Denominator Strategy*).

Moreover, the study discovered that mathematics teachers' level of accuracy in fraction magnitude estimation increases with their years in teaching. Thus, teachers learn and therefore improve on their knowledge from teaching and encounter with mathematics curriculum. The relation was attributed to the professional development training programmes.

Furthermore, the level of fraction magnitude estimation accuracy was associated to the highest class that a teacher teaches mathematics. In other words,

Digitized by Sam Jonah Library

the higher the class that a teacher teaches, the higher the fraction magnitude estimation accuracy and the opposite is true. The difference in fraction magnitude knowledge was attributed to the difference in the scope of the content that the teachers at the highest classes encounter which is more detailed and wide as compared to their peers at the lowest classes.

Recommendations

Based on the findings, the recommendations below have been put forward for educational practice and policies on the in-service mathematics teachers' knowledge of fractions.

1. Teacher training institutions should improve on the courses that focus on the understanding of the content of fractions, especially the magnitude knowledge of fractions.
2. Pedagogical knowledge of fractions in the teacher education curriculums should be improved by teacher training institutions to help teachers gain an in-depth understanding of correct strategies of processing fraction magnitudes.
3. Stakeholders of education such as Ghana Education Service (GES) and other Non-Governmental Organizations (NGO) with interest in education should frequently organise professional training programmes geared towards the development of sense of fraction magnitude.
4. The head teachers should ensure constant shift of teachers from one class to another to allow each teacher an opportunity to encounter with different curriculums in order to gain an in-depth mathematical knowledge

Digitized by Sam Jonah Library

Suggestions for Further Research

Future studies should be done in other districts in other regions to check for fraction magnitude knowledge of in-service mathematics teachers in junior high schools. It should also focus on the relationship between teachers' educational qualification and fraction magnitude knowledge to find out whether teachers' qualification influences the level of fraction magnitude knowledge. Future research should also investigate the relationship between gender and fraction magnitude knowledge.

Digitized by Sam Jonah Library

REFERENCES

Adeyemi, T. O. (2008). Teachers' teaching experience and students' learning outcomes in secondary schools in Ondo State, Nigeria. *Educational Research and Reviews, 3*(6), 204.

Agbozo, K. K. (2020). Pre-service teachers' expressed attitude toward teaching of fractions and some concepts of fraction. *European Journal of Research and Reflection in Educational Sciences, 8*(3).

Amalina, I. K., Fuad, Y., & Agustina, I. D. (2018). Fraction Magnitude Knowledge: Students Strategy in Estimating Fractions. *Journal of Physics: Conference Series*. *1108*(1), 012058.

Amuah, E., Davis, E. K., & Fletcher, J. (2019). An investigation of junior high school students' ideology of fractions in Cape Coast Metropolis of Ghana. *Journal of Science and Mathematics Education*

Australian Association of Mathematics Teachers (2013). *Top drawer teachers: Fractions.* Retrieve fromhttp://topdrawer.aamt.edu.au/Fractions/Big-ideas

Baah-Duodu, S., Cornelius-Ennin, F., Borbye, S., & Amoaddai, S. (2019). Pre-Service primary school teachers' mathematical knowledge for teaching fractions through problem Solving. *International Journal of Advances in Scientific Research and Engineering.* (ijasre), *5*(3).

Beckmann, S. (2014). *Mathematics for elementary teachers with activities*. Boston, MA; Pearson Education.

Digitized by Sam Jonah Library

Bed, R. A. (2017). Factors affecting difficulties in learning mathematics by mathematics learners. *International Journal of Elementary Education*, *6*(2) 8-15.

Behr, M., Lesh, R., Post, T., & Silver, E. (1983). Rational Number concepts, in R. Lesh and M. landau (eds.) *acquisition of mathematics concepts and processes*. New York, Academic press.

Bailey, D., H. et al. (2014).Development of fraction concepts and procedures in U.S.and Chinese children. *Journal of Experimental Child Psychology,129*(2015),68-83

Bernard, J. B., Golbert, A., & Gabina, S. (2020). Assisting JHS form one pupils to add Unlike fraction using Cuisenaire rod and paper folding: A case at chamba M/A Junior high school. *Teacher Education and Curriculum Studies, 5(3), 53.*

Charalambous, Y. C., & Pitta-Pantazi, D. (2007). Drawing on a theoretical model to study students'undersstanding of fractions. *Educational Studies in Mathematics, 64*(3), 293-316

Christou, K. (2019). *Using refutational text to remedy the multiplication makes bigger misconception. Proceedings from Biennial meeting of the European Association for Research on Learning and instruction (EARLI)*, Aachen, Germany.

Digitized by Sam Jonah Library

Cochran, K. F., & Jones, L. (1998). The subject matter knowledge of pre-service science teachers. *International Handbook of Science Education 2*, 707-718

Copur-Gencturk, Y. (2021). Teachers' conceptual understanding of fraction operations: results from a national sample of elementary school teachers. *Educational Studies in Mathematics, 107*(3), 525-545.

Copur-Gencturk, Y. (2015). The effects of changes in mathematical knowledge on teaching: A longitudical study of teacher's knowledge and instruction, *Journal of Research in Mathematics Education. 46*(3), 280-380.

Copur-Gencturk, Y. (2022). Teachers' knowledge of fraction magnitude. *International Journal of Science and Mathematics Education, 20*(5), 1021-1036

Copur-Genturk, Y., & Li, J. (2023). Teaching matters: A longitudinal study of mathematics teachers' knowledge growth. *Teaching and Teacher Education, 121*, 103949

Cramer, K. A., & White, S. (2010). *Learning rational number concepts and skills in elementary classrooms. In teaching and learning mathematics*. *Translating research to elementary classrooms*, p. 15-20.

Creswell, J. W. (2012). *Educational research (4th ed.).* Boston: *Pearson Education Inc*

Creswell, J. W., & Creswell, J. D. (2017). *Research design: qualitative, quantitative, and mixed methods approaches*. Sage publications.

Digitized by Sam Jonah Library

University of Cape Coast https://ir.ucc.edu.gh/xmlui

Common Core State Standards Initiative. (2010). *Mathematics Standards*. Retrieved from https://www.corestandards.org/Math/Content/3/NF/A/1/

Darling-Hammond, L. (2000). Teacher Quality and Student Achievement. *Educational Policy Analysis Archives. 8,* 1-1.

Depaepe, F., Torbeyns, J., Vermeersch, N., Janssens, D., Janssen, R., Kelchtermans, G.,…Van-Dooren, W. (2015). Teachers' content and pedagogical content knowledge on rational numbers: A comparison of prospective elementary and lower secondary school teachers. *Teaching and Teacher Education*, 47, 82-9

DeWolf, M., & Vosniadou, S. (2015). The representation of fraction magnitudes and the whole number bias reconsidered. *Learning and Instruction*, *37*, 39-49.

Doğan-Coşkun, S. (2018). Are pre-service elementary teachers able to pose problems for the Subtraction of fractions?. *Journal of Educational Research, 5*(2), 94-105.

Dyson, N. I., Jordan, N. C., Rodreigues, J., Barbiere, C., Rinne, L. (2019). A fraction sense for sixth graders with or at risk for mathematics difficulties. *Remedial and Special Education, 41*(4), 244-254.

Education Research Group of Australasia (2013). *Mathematics Education Research Group of Australasia (MERGA).* p. 277-284

Digitized by Sam Jonah Library

Eichhorn, M.S. (2018). When the fractional cookie begins to crumble: Conceptual understanding of fractions in the fifth grade. *International Journal of Research in Education and Science (IJRES)*, *4*(1), 39-54.

Entsie, G. (2021). *Pre-Service mathematics teachers' knowledge for teaching senior high school algebra* [Mphil. thesis, University of Cape Coast]. Retrieved from https:// ir.ucc.edu.gh/xmlui/handle/123456789/7434

Etikan, I., Musa, S. A., & Alkassim, R. S. (2016). Comparison of convenience sampling and purposive sampling. *American Journal of Theoretical and Applied Statistics*, *5*(1), 1-4.

Fitzsimmons, C. J., Thompson, C.A., & Sidney, P.G. (2020). Do adults treat equivalent fractions equally?. Adults' Strategies and errors during fraction reasoning. *Journal of Experimental Psychology: Learning, Memory and Cognition*, *46*(11), 2049

Fozio, L. K., Dewolf, M., & Siegler, R. S. (2016). Strategy use and strategy choice in fraction magntitude comparison. *Journal of Experimental Psychology: Learning, Memory and Cognition*, *42*(1), 1.

Gabriel, F. (2016). Understanding magnitudes to understand fractions. *Australian Primary Mathematics Classroom*, *21*(2), 36-40.

Digitized by Sam Jonah Library

University of Cape Coast https://ir.ucc.edu.gh/xmlui

Getenet, S., & Callingham, R. (2017). *Teaching fractions for understanding: addressing interrelated concepts. In Proceedings of the 40th annual conference of the Mathematics Education Research Group of Australasia*. Melbourne, University of southern Queensland.

Ghana Statistical Service (2014). *District analytic report, Tamale Metropolis*. Retrieved from https://www2.statsghana.gov.gh/docfiles/2010_District_Report/Northern/Tamal 2520Metropolitan

Harris, D. N., & Sass, T. R. (2011). Teacher training, teacher quality and student achievement. *Journal of Public Economics*, *95*(7), 798-812.

Hackenberg, A. J., & Lee, M.Y. (2015). Relationship between students fractional knowledge and equational writing. *Journal for Research in Mathematics Education*, *46*(2), 196-243.

Hill, H. C. (2010). The nature and predictors of elementary teachers' mathematical knowledge for teaching. *Journal for Research in Mathematics Education, 41*(5), 513–545.

Hill, H. C., Sleep, L., Lewis, J. M., & Ball, D. L. (2007). Assessing teachers mathematical knowledge. Second handbook of research on mathematics teaching and learning: *A Project of the National Council of Teachers of Mathematics*, 112

Digitized by Sam Jonah Library

Isaboke, H. G. (2018). *Teacher-preparedness in integrating information and communication technology in instruction in lower primary schools in Nyamira county, Kenya* [Mphil. thesis, Kenyata University]. Retrieve from https://ir-library.ku.ac.ke/bitstream/handle/123456789/18840

Karp, K. S., Bush, S. B., & Dougherty, B. J. (2014). 13 rules that expire. *Teaching Children Mathematics, 21*(1), 18-25.

Karp, K. S., Bush, S. B., & Dougherty, B. J. (2015). 12 math rules that expire in the middle grades. *Mathematics Teaching in the Middle School*, *21*(4), 208-215.

Klecker, B. M. (2002*). The relationship between teachers' years-of-teaching experience and Students' Mathematics Achievement. Paper presented at the annual meeting of the Mid-South Educational Research Association,* Chattanooga, TN

Kor, L. K., Teoh, S. H., Mohamed, S. S. E. B., & Singh, P. (2018). Learning to make sense of fractions: Some insights from the Malaysian primary 4 pupils. International Electronic *Journal of Mathematics Education*, *14*(1), 169-182.

Lee, H. J., & Boyadzhiev, I. (2020). Underprepared College Students' Understanding of and Misconceptions with Fractions. *International Electronic Journal of Mathematics Education, 15*(3).

Digitized by Sam Jonah Library

Lemonidis, C., Tsakiridou, H., & Meliopoulou, I. (2018). In-service teachers' content and pedagogical content knowledge in mental calculations with rational numbers. *International Journal of Science and Mathematics Education, 16*(6), 1127–1145.

Levin, K., A. (2006). Study design III: Cross-sectional studies. *Evidence-based Dentistry*, *7*(1), 24-25.

Lopez, L. A. (2020). *Improving teachers' conceptual knowledge of fractions through online subject-specific professional development* [Doctoral dissertation, University of Houston]. Retrieved from https://hdl.handle.net/10657/8051

Lortie-Forgues, H., Tian, J. & Siegler, R. S. (2015). Why is learning fraction and decimal arithmetic so difficult? *Developmental Review, 38*.

Ministry of Education (2019). *Mathematics curriculum for primary schools (Basic 1-3).* Retrieved from https.//cacca.gov.gh/wp-content/uploads/2019/04/maths-lower-primary-bi-b3.pdf on 17/02/2023

Mou, Y., Li, Y., Hoard, M. K., Nugent, L. D., Chu, F. F., Rouder, J. N., & Geary, D. C. (2016). Developmental foundations of children's fraction knowledge. *Cognitive Development. 39*, 141-153.

Mugenda, O. & Mugenda, A. (2003). *Research Methods; Qualitative and Quantitative approaches*. Nairobi: acts press.

Digitized by Sam Jonah Library

Mullis, I. V. S., Martin, M. O., Foy, P., Hooper, M. (2016). *TIMSS 2015 International results in mathematics.* Retrieved from http:/timss2015.org/timss-2015/mathematics/studentsachievement/trends-in-mathematics-achievement/.

Namkung, J., & Fochs, L. (2019). Remediating difficulty for students with mathematics learning disabilities. *Learning Disabilities: A Multidisciplinary Journal*, *24*(2) 36-48

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: Aut

New World Encyclopedia. (2022). *Fraction(mathematics).* Retrieved from https://www.newworldencyclopedia.org/p/index

Nikita, N. (2019). History of common fractions. Retrieved from https://thestrip.ru/en/face/istoriya-obyknovennyh-drobei-istoriya-vozniknoveniya-drobei-doklad/

Ntow, F. (2022). Uncovering Basic Four Learners' Ideas about the Concept of Equal-Sharing in Fractions. *African Journal of Educational Studies in Mathematics and Sciences, 18*(1), 15-29

Odigun, O., T. (2018). *Analysis of senior secondary school students? gender-based errors committed in solving fraction problems in Oyo state, Nigeria*. Retrieved from https://www.man-nigeria.org.ng/abstract?sid=ABA-EDU-2017-2-38

Digitized by Sam Jonah Library

Osei, W. (2020). *Algebra teaching knowledge of basic school mathematics teachers* [Mphil thesis, University of Cape Coast]. Retrieved from https:// ir.ucc.edu.gh/xmlui/handle/123456789/10804

Pacinello, C. (2018). Factors affecting teacher self-efficacy in elementary mathematics instruction. St. John's University (New York*). School of Education and Human Services.*

Pack, J., Gulcer, B., & McCory, R. (2013). Teaching prospective teachers about fractions: historical and pedagogical perspectives. *Educational Studies in Mathematicatics, 82,* 455-479.

Paik, J. H., & Mix, K. S. (2003). Us and Korean children comprehension of fraction names: A reexamination of cross-national difference. *Child Development. 74* (1), 144-154.

Pedersen, P. L., & Bjerre, M. (2021). Two conceptions of fraction equivalence. *Educational Studies in Mathematics, 107*(1), 135-157.

Pienaar, E. (2014). *Learning about and understanding fractions and their role in the high School curriculum* (Mphil thesis, Stellenbosch University) Retrieved from https: //scholar.sun.ac.za/items/baee062b-8a8d.a07f-78f4b233070f

Reinhold, F., Obersteiner, A., Hoch, S., Hofer, S. I., & Reiss, K. (2020). The interplay between the natural number bias and fraction magnitude processing in low-achieving students. *In Frontiers in Education,* Vol. 5, p. 29.

Digitized by Sam Jonah Library

University of Cape Coast https://ir.ucc.edu.gh/xmlui

Ritchie, S. J., & Bates B. C (2013). Enduring links from childhood mathematics and reading achievement to adult socioeconomic status. *Psychological Science, 24*, 1301-1308

Rice, J. K. (2010). *The Impact of Teacher Experience: Examining the evidence and policyimplications*. Retrieved from https://files.eric.ed.gov/fulltext/ED511988.pdf

Sarantakos, S. (2013). *Social reseach (4th ed.).* London: Palgrave Macmillan

Siegler, R, S. (1995). How does change occur: A mocrogeninc study of number conservation. *Cognitive Psychology*, *28*(3), 225-273.

Siegler, R, S. (2016). Magnitude Knowledge: The common core of numerical development. *Developmental Science, 19*(3), 341-361

Siegler, R. S & Pyke, A., A. (2013). Developmental and individual differences in understanding of fractions. *Developmental Psychology*, *40*(10), 1994.

Siegler, R. S., Duncan G. J., Davis-Kean P. E., Duckworh, K., Claessens, A., Engel, M., …&Chen, M.(2012). Early predictors of High Mathemetics. *Psychological Science, 23*(7), 691-697

Siegler, R. S., Fazio, L. K., Bailey, D. H., & Zhou, X. (2013). Fractions: The new frontier for theories of numerical development. *Trends in Cognitive Sciences, 17*(1), 13-19.

Siegler, R., S, & Lortie-Forgues, H. (2015). Conceptual knowledge of fraction arithmetic. *Journal of psychology Eduaction, 107* (3), 909

Digitized by Sam Jonah Library

University of Cape Coast https://ir.ucc.edu.gh/xmlui

Siegler, R.S., Thompson, C.A., & Schneider, M. (2011). An integrated theory of whole number and fractions development. *Cognitive Psychology*, *62*, 273–296.

Siemon, D., Beswick, K., Brady, K., Clark, J., Faragher, R., & Warren, E. (2015). *Teaching mathematics: Foundations to middle years*. Melbourne, Oxford University Press

Singh, A. S., & Masuku, M. B. (2014). Sampling techniques & determination of sample size in applied statistics research: *An overview*. *International Journal of Economics, Commerce and Management*, *2*(11), 1-22.

Tamale Metro Education Directorate (2023). *Annual report*. Tamale, Ghana.

Thompson, C. A., & Sidney, P. G. (2020). Do adults treat equivalent fractions equally as children? Adults' strategies and errors during fraction reasoning. *Journal of Experimental Psychology: Learning, Memory, and Cognition*, *46*(11), 2049.

Tobias, J., M. (2013). Prospective elementary teachers' development of fraction language for defining the whole. *Journal of Mathematics Teacher Education*,*16*(2),85-103

Toledo, R. V. F., Rosenberg-Lee, M., & Abreu-Mendoza, R. A. (2022). Math teachers' implicit and explicit fraction knowledge: a mixed-methods approach. *Teaching Children Mathematics*, *22*(3), 148-157

Tzur, R., & Hunt, J. (2015). Iteration: Unit fraction knowledge and the French fry tasks. *Teaching Children Mathematics*, *22*(3), 148-157.

Digitized by Sam Jonah Library

Ubah, I., j., K. (2021). The impact of three approaches to the teaching of Grad 5 fractions by three experience teachers. *South African Journal of Childhood Education*, *1*(11), 1-2

Ubah, I.J.A., & Bansilal, S., (2018) Pre-service primary Mathematics teachers' understanding of fractions: An action–process–object–schema perspective', *South African Journal of Childhood Education 8*(2), 539

Vamvakoussi, X., Christou, K., & Vosniadou, S. (2018). Bridging psychological and educational research on rational number knowledge. *Journal of Numerical Cognition,4,*84-106.

Van Hoof, J., Engelen, A. S., & Van Dooren, W. (2021). How robust are learners' misconceptions of fraction magnitude? An intervention study comparing the use of refutation and expository text. *Educational Psychology, 41*(5), 524-543.

Watts, T. W., Duncan G. J., Siegler, R. S., Davis-Kean P. E. (2014). What's past is prologe: Relations between early mathematics knowledge and high school achievement. *Educational Researcher, 43*(7), 352-360.

West Africa Examination Council (2015). *BECE chief examiner's report: Mathematics*. Accra, Ghana: WAEC.

West Africa Examination Council (2017). *BECE chief examiner's report: Mathematics*. Accra, Ghana: WAEC.

Whitacre, I., & Nickerson, S. D. (2016). Investigating the improvement of prospective elementary teachers' number sense in reasoning about fraction magnitude. *Journal of Mathematics Teacher Education, 19*(1), 57-77.

Digitized by Sam Jonah Library

University of Cape Coast https://ir.ucc.edu.gh/xmlui

Williams, G. C. (1986). *Primary Mathematics Today*. Hong Kong: Longman Group (FE) Ltd.

Wilkin, J., L. (2008). The relationship between among elementary teachers content knowledge, attitude, beliefs and practices. *Journal of mathematics Teacher Education, 11*, 139-164.

Wu, H. (2001*). How to prepare students for algebra*. American Educator, *25*(2), 10-17.

Yakubu, A. (2013). *Using Cuisenaire rod approach to improve the teaching methods of teacher trainees in Tamale College of Education, Tamale*. [Doctoral dissertation, University of Winneba]. Retrieved from https://studylip,net/doc/25249141/proposal-on-cuisenaire-rod

Yarkwah, C. (2017). *An investigation into senior high school mathematics teachers' knowledge for teaching algebra* [Doctoral dissertation, University of Cape Coast]. Retrieved from https:// ir.ucc.edu.gh/xmlui/handle/123456789/8480

Yeong, J. I., Dougherty, B. J., & Berkaliev, Z. (2015). Units matter. *Teaching Children Mathematics, 22*(3), 170-176.

Digitized by Sam Jonah Library

University of Cape Coast https://ir.ucc.edu.gh/xmlui

APPENDICES

APPENDIX A

UNIVERSITY OF CAPE COAST

COLLEGE OF EDUCATION STUDIES

FACULTY OF EDUCATIONAL FOUNDATIONS

DEPARTMENT OF BASIC EDUCATION

Dear respondent,

My name is Fuseini Abubakari, an M.Phil. Basic Education student of the University of Cape Coast, Ghana. I am conducting a study to investigate the nature of in-service mathematics teachers' knowledge of fraction magnitude. Rest assured that all information collected will be treated with utmost confidentiality and anonymity. I will not gather any personal identifying details, and your responses will be kept confidential. Your participation in this study is highly appreciated as it is essential for the success of this research.

SECTION A: BACKGROUND INFORMATION

Read carefully and select by making a tick (✓) in the box besides the appropriate option

1. Gender

- Male [ ]
- Female [ ]

2. What is your level of education?

- Diploma [ ]
- Degree [ ]

Digitized by Sam Jonah Library

University of Cape Coast https://ir.ucc.edu.gh/xmlui

- Masters [ ]
- Other [ ]

3. How many years have you been teaching mathematics at the junior high school?
   - 5 years and below [ ]
   - 6-10 years [ ]
   - Above 10 years [ ]
4. Indicate the class/classes you are teaching currently
   - Form 1 [ ]
   - Form 2 [ ]
   - Form 3 [ ]

SECTION B: ASSESSMENT QUESTIONS

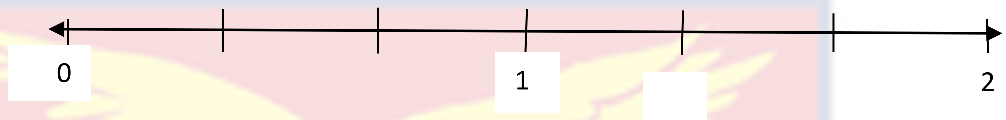
This instrument contains questions on the knowledge of fraction magnitude. It is divided into two parts. Part I and II contain addition and subtraction questions respectively. Each item contains a fraction task, you are expected to estimate and place a dot on the number line below each question where you think the answer would be located. You have 20 minutes to answer these questions. **Please note that the use of calculators is not allowed**.

Digitized by Sam Jonah Library

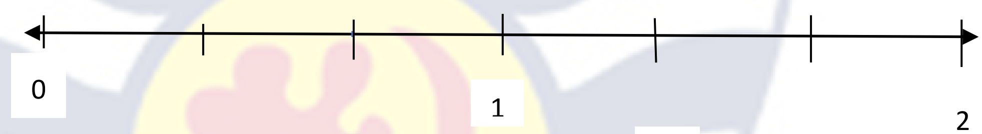
University of Cape Coast https://ir.ucc.edu.gh/xmlui

PART I

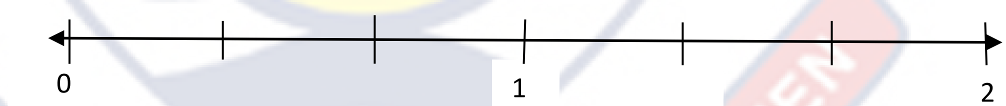
1. Abu has $\frac{2}{3}$ of a Pizza and his friend has $\frac{1}{4}$ of the Pizza. What point on the number line below represents the amount of pizza they have shared?



2. A bakery sold $\frac{3}{4}$ of a cake in the morning and $\frac{1}{2}$ of the cake in the afternoon. Locate a point on the number line below which represents the total cake the bakery sold.



3. Estimate $\frac{17}{32} + \frac{39}{63}$ by placing a dot on the number line where you think the sum would be located.

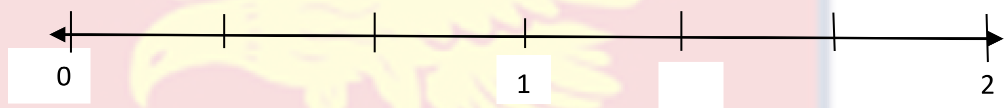


Digitized by Sam Jonah Library

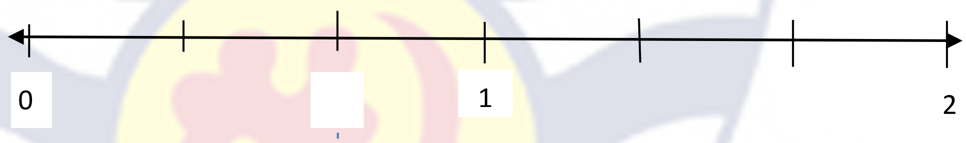
University of Cape Coast https://ir.ucc.edu.gh/xmlui

PART II

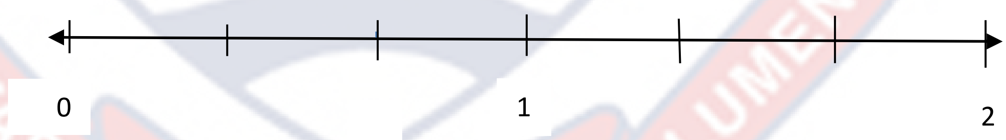
1. A box contains $\frac{7}{8}$ of a pound of a cereal. If $\frac{3}{4}$ of a pound is taken out. Locate a point on the number line below that represents the quantity of the cereal left in the box.



2. On a number line represents a Pizza with 2 slices. If you eat $\frac{1}{4}$ of the Pizza, locate a point on the number line that represents the number of slices remained?.



3. Estimate $\frac{43}{69} - \frac{21}{65}$ by placing a dot on the number line where you think the difference would be located.



Digitized by Sam Jonah Library

University of Cape Coast https://ir.ucc.edu.gh/xmlui

SECTION C: ESTIMATION STRATEGIES

The following are possible strategies in estimating fraction magnitude tasks, please select a strategy/strategies you used in estimating the tasks in **SECTION B** by making a tick (✓) in the box besides the appropriate option.

1. Indicate the **strategy/strategies** you used in estimating the addition tasks**.**

- Benchmark/common denominator strategy [ ]
- Selecting a fraction with a similar size/ using the number line [ ]
- Rounding Strategy [ ]
- Other [ ]

2. Indicate the **strategy/strategies** you used in estimating the subtraction tasks**.**

- Benchmark/common denominator strategy [ ]
- Selecting a fraction with a similar size/using the number line [ ]
- Rounding Strategy [ ]
- Other [ ]

Digitized by Sam Jonah Library

University of Cape Coast https://ir.ucc.edu.gh/xmlui

APPENDIX B

COVER LETTER FOR RESEARCH VISIT

UNIVERSITY OF CAPE COAST
COLLEGE OF EDUCATION STUDIES
FACULTY OF EDUCATIONAL FOUNDATIONS
DEPARTMENT OF BASIC EDUCATION

Telephone: +233 - (0)3321 33379
Cables: University, Cape Coast
Email: basiceducc@gmail.com

UNIVERSITY POST OFFICE
CAPE COAST, GHANA

Our Ref: DBE/32/V.1

17th April, 2023

Your Ref:

Dear Sir/Madam,

LETTER OF INTRODUCTION

The bearer of this letter, Fuseini Abubakar (EF/BEP/21/0009) is an M.Phil.student at the Department of Basic Education, University of Cape Coast.

He is undertaking a study on **"IN-SERVICE TEACHERS' KNOWLEDGE OF FRACTION MANGNITUDE: A CASE OF MATHEMATICS TEACHERS IN TAMALE METHROPOLIS".**

In connection with this, he needs to collect data. The study is academic in purpose and data collected will be treated as confidential.

We would therefore be grateful if you could give him the necessary assistance.

Thank you.

Yours faithfully,

for Prof. Mumuni Thompson
HEAD OF DEPARTMENT

...RTMENT OF BASIC EDU...
...IVERSITY OF CAPE COAST
...APE COAST

Digitized by Sam Jonah Library

University of Cape Coast https://ir.ucc.edu.gh/xmlui

APPENDIX C

LETTER FOR ETHICAL CLEARANCE

Department of Basic Education
Faculty of Educational Foundations
College of Education Studies
University of Cape Coast
Cape Coast.
4th May, 2023.

The chairman
Institutional Review Board
University of Cape Coast
Cape Coast, Ghana.

Dear sir,

APPLICATION FOR ETHICAL CLEARANCE

I am submitting my application for an ethical clearance to enable continue my research. I am a second year student pursuing a Master of Philosophy in Basic Education and conducting a study on "In-service teachers' knowledge of fraction Magnitude: A case of mathematics teachers in Tamale Metropolis.

The study will adopt cross-sectional survey that will involve mathematics teachers teaching at Tamale Metropolis. I am soliciting for an ethical clearance that will enable me access the population.

Attached are the necessary documents.

I am looking forward for your response soon considering the limited time available for the study.

Yours faithfully,

Fuseini Abubakari

Digitized by Sam Jonah Library

APPENDIX D

ETHICAL CLEARANCE LETTER FROM IRB, UCC

UNIVERSITY OF CAPE COAST
COLLEGE OF EDUCATION STUDIES
ETHICAL REVIEW BOARD

UNIVERSITY POST OFFICE
CAPE COAST, GHANA

Our Ref: CES/ERB/ucc/edu/v8-23/19

Your Ref: ....

Date: 1st June, 2023

Chairman, CES-ERB
Prof. J. A. Omotosho
jomotosho@ucc.edu.gh
0243784739

Vice-Chairman, CES-ERB
Prof. K. Edjah
kedjah@ucc.edu.gh
0244742357

Secretary, CES-ERB
Prof. Linda Dzama Forde
lforde@ucc.edu.gh
0244786680

Dear Sir/Madam,

ETHICAL REQUIREMENTS CLEARANCE FOR RESEARCH STUDY

The bearer, Fuseini Abubakar ...., Reg. No. EF/BEP/21/0009 is M.Phil. / Ph.D. student in the Department of Basic Education .... in the College of Education Studies, University of Cape Coast, Cape Coast, Ghana. He / She wishes to undertake a research study on the topic:

In-service teachers' knowledge of fraction magnitude: A case of mathematics teachers in Tamale Metropolis

The Ethical Review Board (ERB) of the College of Education Studies (CES) has assessed his/~~her~~ proposal and confirm that the proposal satisfies the College's ethical requirements for the conduct of the study.

In view of the above, the researcher has been cleared and given approval to commence his/~~her~~ study. The ERB would be grateful if you would give him/~~her~~ the necessary assistance to facilitate the conduct of the said research.

Thank you.
Yours faithfully,

Prof. Linda Dzama Forde
(Secretary, CES-ERB)

NOBIS

Digitized by Sam Jonah Library

University of Cape Coast https://ir.ucc.edu.gh/xmlui

APPENDIX E

APPLICATION FOR RESEARCH PERMIT

Department of Basic Education
Faculty of Educational Foundations
College of Education Studies
University of Cape Coast
Cape Coast.
5th june, 2023

The Metropolitan Director
Ghana Education Service
Tamale.

Dear Sir,

INFORMED CONCENT

I would be very grateful if you could grant me permission to conduct a research in your Metropolis. My name is Fuseini Abubakari, an M.Phil. Basic education student of the University of Cape Coast, Ghana. I am conducting a research on the topic: **In-service teachers' knowledge of fraction magnitude: A case of mathematics teachers in Tamale Metropolis**. The purpose of the study is to investigate the nature of in-service mathematics' teachers understanding of fraction magnitude.

If permission is granted, you will be required to make available the following records;

1. **Number of public junior high schools in the Metropolis.**
2. **Number of Mathematics teachers in the public junior high schools in the Metropolis.**

The researcher is assuring you of confidentiality and anonymity. Therefore, no school or individual shall be identified in the study and at the same time, participants are free to withdraw at any time without consequences.

Attached is the Ethical Clearance letter for your perusal.

I look forward to receiving your response soon considering the timing of my studies.

Thank you in advance for your time and co-operation.

Yours Faithfully,

Fuseini Abubakari
0242708656

Digitized by Sam Jonah Library

University of Cape Coast https://ir.ucc.edu.gh/xmlui

APPENDIX F

LETTER OF PERMIT FROM TAMALE METRO EDUCATION DIRECTORATE

GHANA EDUCATION SERVICE

In case of reply the date and reference number of this letter should be quoted

Our Ref: GES/NR/MEO/TT.17/VOL.2

Your Ref: ....

Email:tmetroedu@gmail.com

REPUBLIC OF GHANA

Metropolitan Education Office
P.O. Box 6,E/R
Tamale, Northern Region
Tel: 0372022090
Date: June 11, 2023

GRANT OF PERMISSION TO COLLECT DATA

Reference to your letter on the above subject matter, I wish to grant you permission to enter some schools in the Tamale Metropolis for the purposes of conducting data.

Mr. Fuseini Abubakari an M.Phil. Basic Education Student, who wants to conduct a research study on a topic: **In-service teachers' knowledge of fraction magnitude: A case of Mathematics teachers** in Tamale Metropolis. He is seeking to collect data on the given topic among Mathematics teachers in the Junior High Schools. Data will be collected on the following;

1. Number of Public Junior High School in the Metropolis
2. Number of Mathematics teachers in the public Junior High School

However, the permission is granted on the assurance that, the student (researcher) will fully comply with safety and protection of the rights and confidentiality of the schools intended to enter.

I am counting on your co-operation.

(NELSON NAAKINIIB KONLAN)
METROPOLITAN DIRECTOR OF EDUCATION
TAMALE

HEADTEACHERS OF
JUNIOR HIGH SCHOOLS CONCERN
TAMALE METROPOLIS
TAMALE

Cc: **Mr. Fuseini Abubakari**
Department of Basic Education
Faculty of Educational Foundations
College of Education Studies
University of Cape-Coast
Cape-Coast

HaRi

METRO. EDUC. OFFICE
TAMALE - NR

Digitized by Sam Jonah Library