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ABSTRACT

The study of the effect of ionization of impurity centres by electric field E_0 on the conductivity of superlattice (SL) has been studied theoretically. It is observed that as the field E_0 increases the current rises reaches a maximum then falls off i.e. show a negative differential conductivity (NDC). Further increase in E_0 leads to an exponential rise of the current. This occur around $E_0 = 3 \times 10^4 \mathrm{V\ cm}^{-1}$. Hence the current density field shows a "N" shape characteristics as against the "n" shape characteristics in the absence of impurity.

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1 Theory

The history for the search for ultra thin materials can be traced quite far back [1-4], however the motivation for their production went up sharply when new types of devices [5,6] where predicted, such as the Bloch oscillator. The advent of new growth techniques like the molecular-beam epitaxy (MBE) in an ultrahigh vacuum [7] and the metal-organic chemical vapour deposition (MOVCD) [8] also gave a boost to the growth of semiconductor superlattice SL. The study of SL has now acquired a multifarious dimensions.

Esaki and Tsu were the first to study the transport properties of a hetero-junction SL in 1970 [5]. They predicted negative differential conductivity (NDC) associated with electron transfer into the negative mass regions of the minizone and Bloch oscillations. Later Esaki et al. [9] observed the (NDC) experimentally using $GaAs - Ga_{1-x}Al_xAs$ superlattice. During the same period Kazarinov and Suris [10] theoretically studied the current voltage $(1 - V)$ characteristic of SL and predicted the existence of peaks corresponding to the resonant tunneling (RT) between ground and excited states of adjacent wells.

The $1 - V$ characteristics of SL has since then been studied extensively by many researchers. Recently the study of $1 - V$ characteristics of SL has again been revisited. Sibille et al. [11,12] studying the $1 - V$ characteristics of SL came out that Bloch electron conduction through the SL miniband is responsible for the NDC over a large range of SL parameters. Their results excluded the mechanism of hopping transport between Wannier-Stark quantized level. Mensah [13] has also studied the $(1 - V)$ charactersitic of SL in the presence of an external electric field $E(t) = E_0 + E_1 \cos \omega t$ and observed particularly that in a quasi-static situation when $\omega\tau \ll 1$ (ω is the frequency of the a.c. field τ-is the electron relaxation time) the current density electric field characteristics shows a negative differential conductivity in the neighbourhood where the constant electric field E_0 is equal to the amplitude of the a.c electric field E_1 and the peak decreases with increasing E_1.

All the studies so far conducted, whether in the range of classically strong field (using Boltzmann transport equation) or in the range of quantizing electric field (where the classical Boltzmann's equation is inapplicable) show the following two important effects: a negative differential conductance and an electron-phonon resonance [14,15]. This is a situation whereby the differential conductivity displays a discontinuity whenever the resonance conditions are reached i.e. $\omega_0 = n\Omega_E$, $n = 1, 2, 3, \dots$ (ω_0 is the frequency of the optical phonon, Ω_E is the separation between adjacent levels of stark ladder $\hbar = 1$).

It is worth noting that all the above mentioned studies were carried out in a SL without impurity centres. It is therefore assumed that the carrier density n_0 in the conduction miniband is independent of the field intensity.

In this paper we will study the effect of ionization of impurity centres by an electric field $E(t) = E_0 \cos \omega t$ (especially the quasi-static case where $\omega\tau \ll 1$) on the current density electric field characteristics of SL. It will be observed that in the presence of impurity centres the current density electric field characteristics is "N" shape unlike the case of absence of impurity centres where it shows "n" shape.

The paper will be organised as follows. Sec. II: Calculation of the ionization probability; Sec. III: Calculation of the electric current density; and finally in Sec. IV: Discussion and Conclusion.

2 Calculation of the Ionization Probability

We shall assume a short-range impurity centre with binding energy $\bar{\varepsilon}$ located at the origin of the coordinate system. The probability of ionization of such an impurity centre will be solved in the semiclassical approach for an electric field $E(t) = E_0 \cos\omega t$ following the approach in [16]. The ionization probability is expressed as

$$W = \exp(-2ImS) \tag{1}$$

where S is the classical action acquired by a particle as a result of below barrier motion. Such a motion occurs in imaginary time [17,18] along the classical path which can be obtained from the equation of motion

$$\frac{dp}{dt} = eE(t) \tag{2}$$

The classical action S is given by

$$S = \int_0^{t_0} L\, dt = \int_0^{t_0} (\varepsilon(t) + \bar{\varepsilon})dt \tag{3}$$

where

$$\varepsilon(t_0) = -\bar{\varepsilon} \tag{4}$$

t_0 is the moment of the onset of tunneling. In calculating S in Eq.(3) we shall assume that the following conditions of the semiclassical treatment are satisfied.

$$\omega \leq \bar{\varepsilon}; \quad \omega \leq 2\Delta; \quad eE_0 d \leq \bar{\varepsilon}; \quad eE_0 d \leq 2\Delta\ .$$

(E_0 and ω are respectively the amplitude and frequency of the external field. 2Δ is the width of the lowest miniband of SL; d is the period of SL; $\hbar = 1$ using energy units). We then orient the field such that it is parallel to the SL axis (oz axis). From Eq.(2) we obtain an expression for p_z using the initial condition $p_z(0) = 0$. This is given by

$$p_z = \frac{-eE_0}{\omega}\sin\omega t \tag{5}$$

hence, substituting Eq.(5) into the usual energy model along the SL axis.

$$\varepsilon(p_z) = \Delta(1 - \cos p_z d) \tag{6}$$

and using the imaginary time $t = it$ we obtain

$$\varepsilon(t) = \Delta[1 - \cosh(\frac{eE_0 d}{\omega}\sinh\omega t)] \tag{7}$$

Similarly from Eq.(4) we obtain

$$|t_0| = \frac{1}{\omega}\text{Arcsinh}\{\frac{\omega}{eE_0 d}\text{Arcsinh}\frac{\sqrt{2\delta+1}}{\delta}\} \tag{8}$$

where $\delta = \frac{\Delta}{\bar{\varepsilon}}$.

Substituting Eq.(7) into Eq.(3) we obtain the action

$$S = i\bar{\varepsilon}[(\delta+1)|t_0| - \delta\int_0^{t_0}\cosh\,(\frac{eE_0 d}{\omega}\sinh\omega t)dt] \tag{9}$$

Eq.(9) cannot be evaluated analytically however it can be evaluated numerically. For a quasi-static case as $\omega \to 0$ S can easily be obtained as.

$$S = \frac{i\bar{\varepsilon}}{eE_0 d}[(\delta+1)\text{Arc}\sinh\frac{\sqrt{2\delta+1}}{\delta} - \sqrt{2\delta+1}] \tag{10}$$

As $\Delta \to \infty$, $d \to 0$ and $\Delta d^2 \to m_0^{-1}$ (parabolic energy spectrum) we obtain

$$S = \frac{2i\sqrt{2m_0\bar{\varepsilon}}}{3eE_0} \tag{11}$$

as expected [20].

The ionization probability can then be written as

$$W = \exp\{-\frac{2\bar{\varepsilon}}{eE_0 d}[(\delta+1)\text{Arc}\sinh\frac{\sqrt{2\delta+1}}{\delta} - \sqrt{2\delta+1}]\} \tag{12}$$

3 Electric Current Density

This problem will also be solved in the quasi-classical condition. This enables us to confine the electrons in the lowest miniband and allow the use of Boltzmann's equation in the τ-approximation.

Moreover, on the right hand side of this kinetic equation we shall include a generation term $G_0(p)$ which represents the ionization of the impurities and recombination term.

The kinetic equation then takes the form [19]

$$\frac{\partial f}{\partial t} + eE_0\frac{\partial f}{\partial p_z} = -\frac{f - f_0}{\tau} + G_0(p) - \frac{f - f_0}{\tau_r} \tag{13}$$

Here f_0 is the equilibrium distribution function, τ_r is the recombinatin time. The solution of Eq.(13) is given by

$$f(p) = \int_0^\infty e^{-t'/\tau_0}dt' G(p - eE_0 t') \tag{14}$$

where

$$G(p) = G_0(p) + \frac{f_0}{\tau_0};\ \tau_0^{-1} = \tau^{-1} + \tau_r^{-1}.$$

With the help of Eq.(14) we define the current density along the SL axis as

$$j_z = e\sum_p V_z(p)f(p) \tag{15}$$

Substituting Eq.(14) into eq.(15) we obtain

$$j_z = e\sum_p V_z(p)\int_0^\infty e^{-t'/\tau_0}dt' G(p - eE_0 t') \tag{16}$$

Taking the energy $\varepsilon(p)$ of the electrons in SL in the lowest miniband as

$$\varepsilon(p) = \frac{p_\perp^2}{2m} + \Delta(1 - \cos p_z d) \tag{17}$$

we get the velocity $V_z(p)$ by differentiating $\varepsilon(p)$ with respect to p_z. This is given by

$$V_z(p) = \Delta d \sin p_z d \tag{18}$$

Inserting Eq.(18) into Eq. (16) and performing the transformation $p \to p - eE_0t'$, the current density assumes the form

$$j_z = ed\Delta \sum_p G(p) \int_0^\infty e^{-t'/\tau_0} \sin(p_z - eE_0t')dt' \tag{19}$$

which further simplifies to

$$j_z = ed\Delta \sum_p \cos p_z d G(p) \int_0^\infty e^{-t'/\tau_0} \sin eE_0t'dt' \tag{20}$$

see [13].

Integrating Eq.(26) for non-degenerate electron gas, we get

$$j_z = ed\Delta \sum_p [G(p) \cos p_z d + \frac{f_0}{\tau_0} \cos p_z d] \cdot$$

$$\cdot \frac{eE_0 d\tau_0^2}{1 + (eE_0 d\tau_0)^2} \tag{21}$$

As indicated [20] in the semiclassical situation we may expect quantum transitions mainly to state $p_z = 0$.

Hence

$$\sum_p G(p) \cos(p_z d) \approx \sum_p G(p) = \frac{NW}{t_0} \tag{22}$$

see [21].

where N is the concentration of the impurity centres; W is the ionization probability obtained in Eq.(12); t_0 is the expression in Eq. (8).

The second term of Eq.(21) is also given by

$$\sum_p f_0(p) \cos(p_z d) = \frac{n_0 I_1(\frac{\Delta}{KT})}{I_0(\frac{\Delta}{KT})} \tag{23}$$

where n_0 is the electron density in the conduction band in the absence of electric field i.e. $E_0 = 0$. $I_k(x)$ is a modified Bessel function.

Inserting Eq.(22) and Eq.(23) into Eq.(21) we finally obtain for the current density

$$j_z = ed\Delta n_0 \frac{I_1(\frac{\Delta}{KT})}{I_0(\frac{\Delta}{KT})} \left[1 + \frac{\tau_0}{n_0} \frac{I_0(\frac{\Delta}{KT})}{I_1(\frac{\Delta}{KT})} \frac{NW}{t_0}\right] \cdot$$

$$\cdot \frac{eE_0 d\tau_0}{1 + (eE_0 d\tau_0)^2} \tag{24}$$

where we define a new electron density n_{eff} and express it as

$$n_{eff} = [n_0 + \frac{\tau_0}{t_0} \frac{I_0(\frac{\Delta}{KT})}{I_1(\frac{\Delta}{KT})} \cdot NW] \tag{25}$$

The current density j_z then takes the form

$$j_z = ed\Delta n_{eff} \frac{I_1(\frac{\Delta}{KT})}{I_0(\frac{\Delta}{KT})} \frac{eE_0 d\tau_0}{1 + (eE_0 d\tau_0)^2} \tag{26}$$

4 Discussion and Conclusion

The field ionization of impurity centres and its effect on the current density of SL has been studied in this paper. The important results obtained have been graphically represented to enable an easy understanding of the phenomenon.

We would like to state that our results are mainly due to electric field ionization without taking into consideration thermal ionization. Such a problem has been studied by Kryuchkov [21] using the steepest-descent (saddle) method.

We present in Fig.(1) the graph of ImS against the electric field E_0 for a given d. It is noted that ImS decreases with increase of the field E_0. However, as we increase the value of d ImS decreases for a given E_0. In Fig. (2) where we presented the graph of ImS against E_0 for a given Δ, we again observed that the ImS decreases with E_0 but this time as Δ increases ImS decreases for a given E_0. From Eq.(12) this implies that the ionization probability increases with the field E_0 and strongly depend on the parameters (Δ, d) of the SL.

In [22] it is noted that for a compositional SL where $d = d_I + d_{II}$ (d_I- is the well width, d_{II} - is the barrier width) Δ depends strongly on the barrier with d_{II}. If the width of the barrier is small e.g. $d_{II} < 50$Å, the miniband have appreciable width for motion along the axis of SL. e.g. $d_{II} = 50$Å, $\Delta = 10$ meV. We, therefore, believe that in fabricating for example modulated doped compositional SL the optimal choice of d, Δ may highly enhance the electron density n and substantially increase the mobility μ as can be seen from Eq.(26).

Another important result of this paper is the nature of the current density as expressed in Eq.(25). It is noted that the current density depends also on the concentration of impurity centres N and the ionization probability W. For a better understanding of the behaviour of the current density as the electric field intensity is increased, we plotted j_z/j_0 against a dimensionless quantity $\frac{eE_0 d\tau}{\hbar}$ (see fig. (3)). We noted a very interesting behaviour different from that of the case when there is no impurity centres. It is noted that, for $N = 0$ i.e. in the absence of impurity centres, Ohm's law is initially obeyed till the current density falls below Ohm's prediction and reaches a maximum. It then falls off with an increasing value of E_0. This phenomenon is called the negative differential conductivity [5,9]. However, in the presence of the impurity centres ($N \neq 0$), on increasing the field intensity E_0, in the classical high field region ionization of the impurity centres predominates and the electrons tunnel into the conducting band. This results in an exponential rise of the current. It is observed that for the parameters $d = 100$Å; $\tau_0 = 10^{-12}$sec, the exponential rise occur when the field intensity $E_0 = 3 \times 10^4 Vcm^{-1}$. It is worthy to note that the nature of the graph is similar to that observed for doped nipi SL by Ploog and Döhler [23].

In conclusion, we have studied the effect of electric field ionization of impurity centres on the current density of SL. We have noted that in the presence of impurity centres the electron density n considerably depends on the field E_0, the miniband Δ, and the SL period d. As a result, the current density j_z/j_0 shows an "N" shape current density field characteristics.

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Figure Captions

Fig. 1. ImS is plotted against E_0 for

•——•; $\Delta = 0.01$ eV, $\bar{\varepsilon} = 0.05$, eV $d = 100\mathring{A}$;

•- - - -•; $\Delta = 0.1$eV, $\bar{\varepsilon} = 0.05$ eV, $d = 125\mathring{A}$;

•— — —•; $\Delta = 0.1$ eV, $\bar{\varepsilon} = 0.05$ eV, $d = 150\mathring{A}$.

Fig. 2. ImS is plotted against E_0 for

•——•; $d = 100\mathring{A}$, $\bar{\varepsilon} = 0.05$eV, $\Delta = 0.15$eV;

•- - - -•; $d = 100\mathring{A}$, $\bar{\varepsilon} = 0.05$ eV, $\Delta = 0.10$eV;

•— — —•; $d = 100\mathring{A}$, $\bar{\varepsilon} = 0.05$eV, $\Delta = 0.05$eV.

Fig. 3. j/j_0 is plotted against $X = \frac{eEd\tau}{\hbar}$ for

•——•; $N = 0$, $T = 77K$, $\frac{I_0}{I_1} = 1.615$;

•- - - -•; $N = 10^{16} cm^{-3}$, $T = 58K \frac{I_0}{I_1} = 1.066$;

•— — —•; $N = 10^{16} cm^{-3}$, $T = 77K$, $I_0/I_1 = 1.615$.

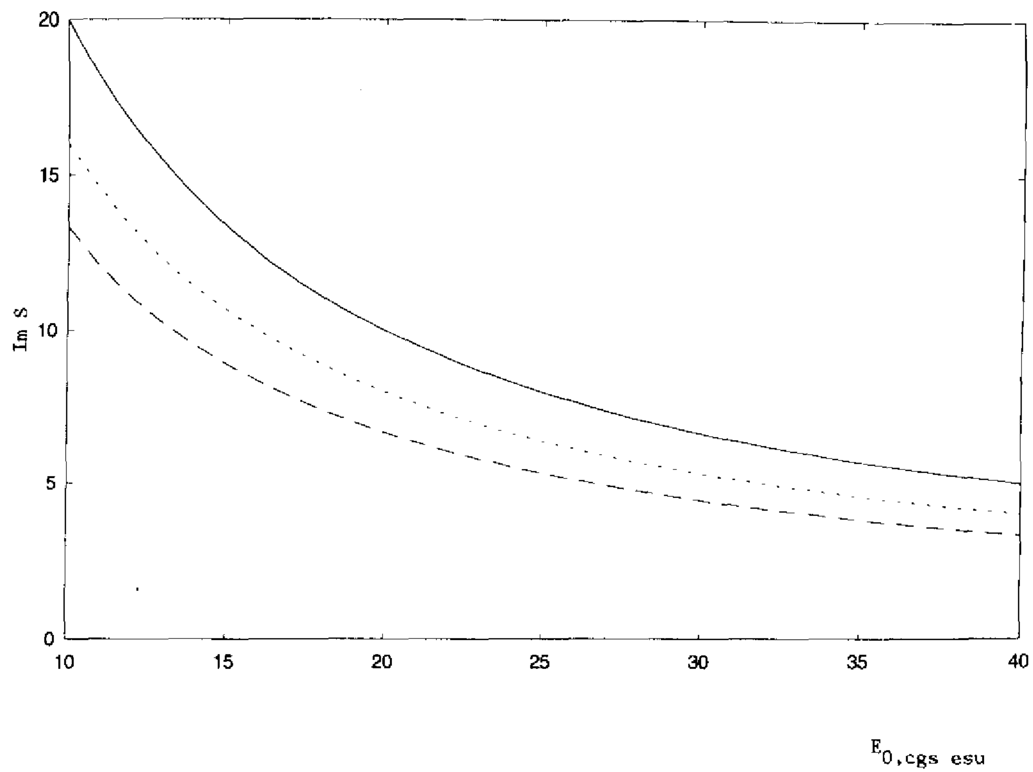

Fig.1

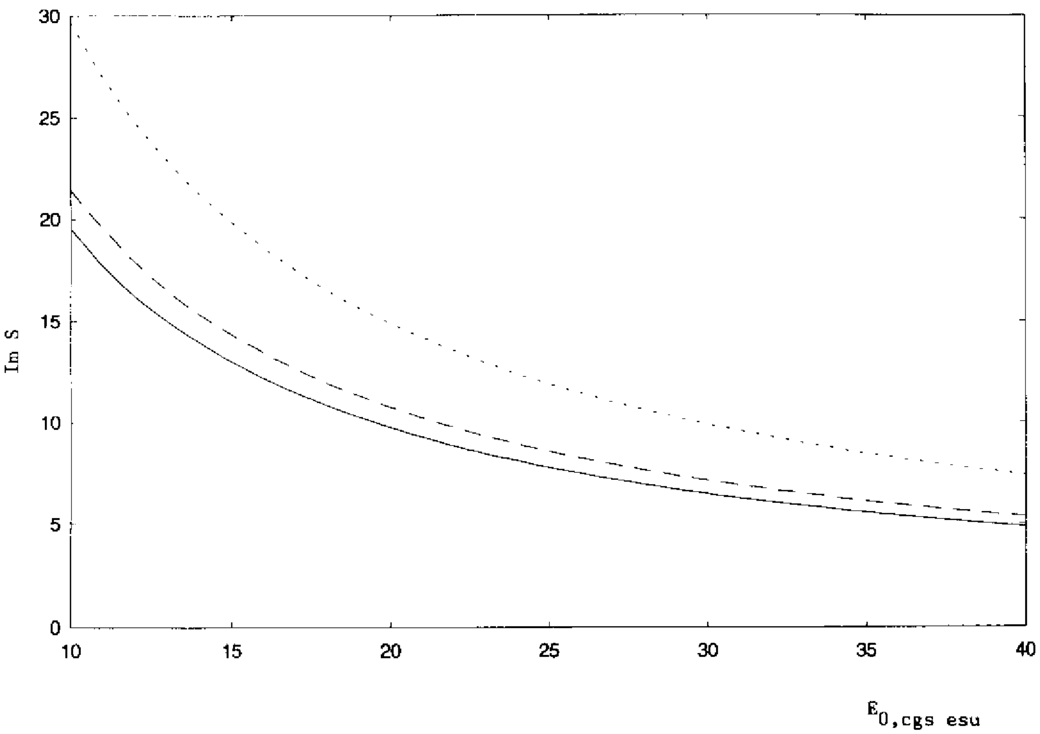

Fig.2

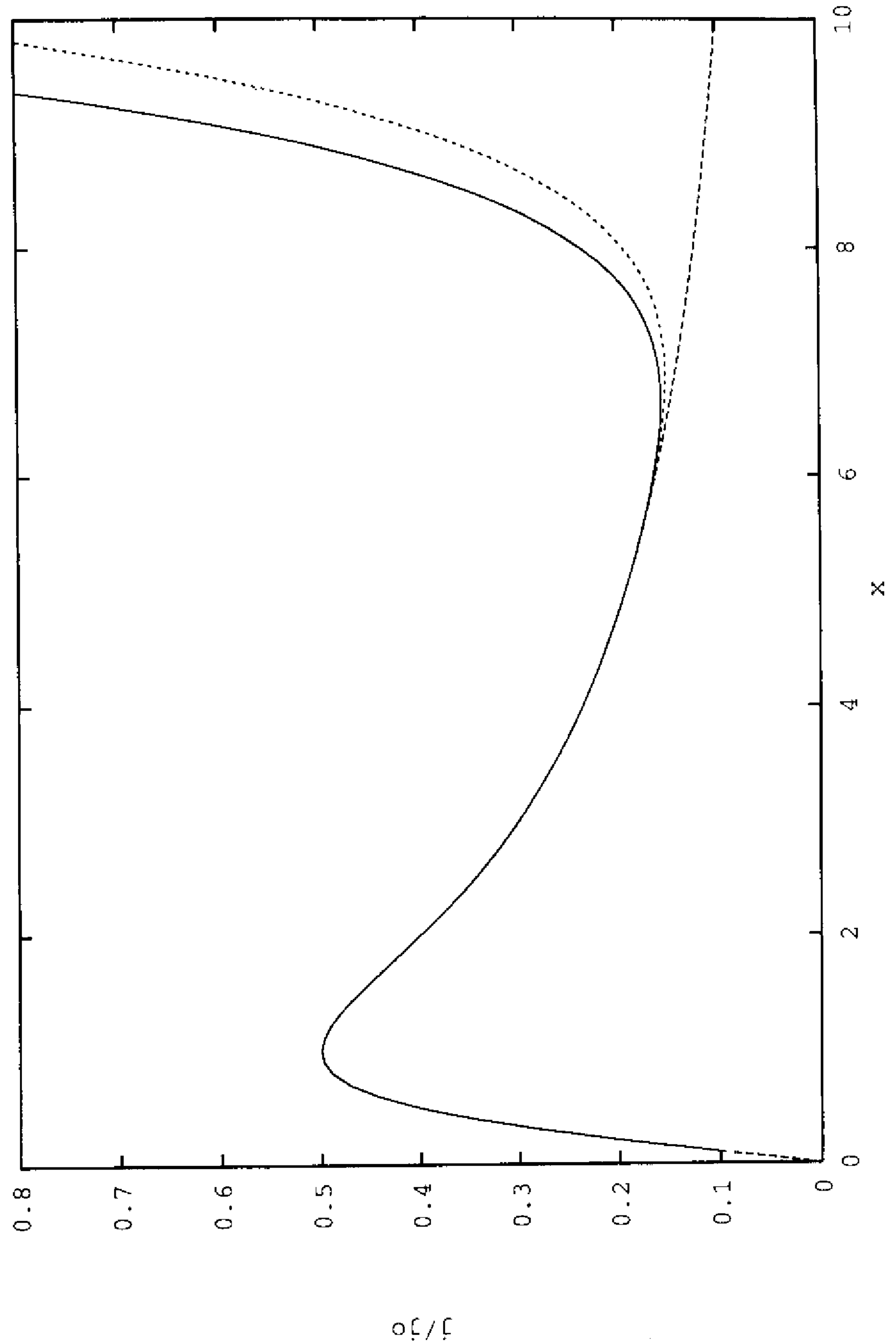

Fig.3