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ABSTRACT

Excitation of soliton in superlattice has been investigated theoretically. It is noted that the soliton velocity u and the length L depend on the amplitude E_0 and that an increase in the amplitude causes soliton width L to approach zero and the velocity u to that of light V in homogeneous medium. The characteristic parameters of soliton u, L and E_0 are related by expression $\frac{u}{L\,E_0} = \frac{ed}{2\hbar}$ which is constant depending only on the SL period d. It is observed also that the soliton has both energy $E = 8V^2\left(1 - \frac{u^2}{V^2}\right)^{-1/2}$ and momentum $P = \frac{u}{V^2}\,E$ which makes it behave as relativistic free particle with rest energy $8V^2$. Its interaction with electrons can cause the soliton electric effect in SL.

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I INTRODUCTION

Superlattices are solid state structures in which electrons experience the periodic potential of the lattice and also an additional potential with a period considerably greater than the lattice constant [1]. The study of synthetic superlattices was first put forward by Keldysh in 1962. He suggested that a periodic potential could be produced artificially by a periodic deformation of a sample by the field of a high–power ultrasonic standing wave. Since then studies on superlattices have been intensified and other methods of production suggested. This includes the use of standing light waves, diffraction gratings and thin films with periodically varying thickness [2].

Esaki and Tsu suggested the use of a periodic alternation of different semiconductor films to produce superlattices. In the last few years, with the advent of new growth techniques like the molecular beam epitaxy (MBE) in an ultrahigh vacuum [3] and metal organic chemical vapour deposition (MOCVD) [4] the study of the transport [5–10] optical [11–14], acoustic [15–18], and the thermomagnetic [19–21] properties of this novel material has received a lot of attention.

In this paper, we will study the soliton excitation in SL. It is already hackneyed to refer to the growth of soliton applications in condensed matter physics as "remarkable". What is even more astonishing is that these applications have arisen (or at least become widely appreciated in the physics community) in recent years; for example in the theories of Bloch walls [22] which separate domains in magnetic materials, structural phase transitions [23], liquid ^3He [24], Josephson transmission lines [25], in the theory of the low–temperature conductivity of "one–dimensional" Fröhlich charge–density–wave condensates [26] and most recently in the optical fibre communication [27].

The fundamental solition is spatially localized, highly stable, and has finite energy. It is a localized nonlinear wave that regains asymptotically ($t \to \infty$) its original ($t \to -\infty$) shape and velocity after interaction with a localized disturbance. Due to this property, it moves through a fibre without dispersion and so it is used to transmit information through the process called modulation.

This paper will be presented as follows: Section II, Solitons in a Superlattice; Section III, Discussion and Conclusion.

II SOLITONS IN A SUPERLATTICE

Superlattice semiconductor is strongly anisotropic. This accounts for the splitting of the conduction band into narrow minibands and also the generation and propagation of the soliton wave.

We shall assume that, the characteristic length in which there is a significant change in the electromagnetic field is large compared with the de–Broglie wavelength of electrons or with the SL period.

The electron current density can then be written as

$$j = -e \sum_{p} f(p)\, v\left(p + \frac{e}{c}\, A(r,t)\right) \tag{1}$$

where $A(r,t)$ is the vector potential of the field; $f(p)$ is the distribution function and $v(p)$ is the electron velocity.

The energy $\varepsilon(p)$ of the SL in the lowest miniband is given by

$$\varepsilon(p) = \frac{p_{\perp}^2}{2m} + \Delta\left(1 - \cos\frac{p_z d}{\hbar}\right) \tag{2}$$

Here $p_{\perp}$ is the transverse momentum, p_z is the electron momentum along the SL axis (oz axis), 2Δ is the miniband width, m is the effective mass of electron in the transverse plane and $\hbar$ is the Planck's constant divided by 2π. The electron velocity along oz axis is given by

$$V_z(p)\frac{\partial \varepsilon}{\partial p_z} = \frac{\Delta d}{\hbar}\sin\frac{p_z d}{\hbar} \tag{3}$$

Taking into consideration the fact that the characteristic time for the charge in the field is short compared with the mean free time of electrons, we shall ignore collisions between electrons and the lattice. We shall also consider the the fact that nonlinearity of the high frequency conductivity is mainly due to the nonlinear dependence of the electron velocity on the quasimomentum, then substitute the equilibrium distribution $f_0(p)$ for the $f(p)$ in Eq.(1).

Inserting Eq.(2) and Eq.(3) into Eq.(1) and solving we get

$$j_z = -\frac{e\Delta d}{\hbar}\sin\frac{e}{\hbar c}\, A_z d \sum_{p} f_0(p)\cos\frac{p_z d}{\hbar} \tag{4}$$

Assuming that the electron gas is nondegenerate, we obtain for longitudinal component of the current density after some manipulation the following

$$j_z = -\frac{en\Delta d}{\hbar}\,\frac{I_1\left(\frac{\Delta}{KT}\right)}{I_0\left(\frac{\Delta}{KT}\right)}\sin\left(\frac{e}{\hbar\, c}\, A_z d\right) \tag{5}$$

where n is the conduction electron density, $I_k(x)$ is a modified Bessel function. Substituting Eq.(5) into the vector potential equation

$$\nabla^2 A_z - \frac{1}{V^2}\frac{\partial^2}{\partial t^2} A_z = -\frac{4\pi}{c} j_z \tag{6}$$

where V is the velocity of the electromagnetic wave in the absence of electrons we get

$$\frac{\partial^2 \psi}{\partial t^2} - V^2\nabla^2\psi + \omega_0^2 \sin\psi \tag{7}$$

here

$$\psi = \frac{e}{\hbar\, c} A_z d; \quad \omega_0 = \omega_p^2 \frac{nd^2 \Delta I_1\left(\frac{\Delta}{KT}\right)}{\hbar^2\, I_0\left(\frac{\Delta}{KT}\right)}$$

and ω_p is the Langmuir frequency. Eq.(7) is the usual Sine–Gordon equation. We shall consider a one–dimensional case and seek directly the solution of Eq.(7) for an electric field $E_z = -\frac{1}{c}\frac{\partial A_z}{\partial t}$.

Consider a wave traveling at right angles to the superlattice axis (along the x–axis) at a constant velocity u such that $x - ut = \xi$ and vanishes at $\xi = \pm\infty$.

Eq.(7) becomes

$$\frac{V^2}{\omega_0^2}\left(1-\theta^2\right)\frac{d^2\psi}{d\xi^2} = \sin\psi \tag{8}$$

where $\theta = \frac{u}{V} < 1$.
The solution of (8) is given by

$$\int_{\psi_0}^{\psi(\xi)} \frac{d\psi(\xi)}{\sqrt{E - \cos\psi(\xi)}} = 2^{1/2}\gamma\eta\xi \tag{9}$$

where

$$\eta = \pm 1; \qquad \gamma = \frac{1}{\frac{V}{\omega_0}\sqrt{1-\theta^2}}$$

Evaluating the integral for $E = 1$ for the purpose of this work. We obtain

$$\psi(\xi) = 4\tan^{-1}[\exp(\eta\gamma\xi)] \tag{10}$$

Finally

$$\frac{\partial\psi(\xi)}{\partial t} = \frac{\partial}{\partial t}\left[4\tan^{-1}[\exp(\eta\gamma\xi)]\right] \tag{11}$$

gives the final result as

$$E_z = \frac{2u\eta\gamma\hbar}{ed}\,\text{sech}\,(\eta\gamma\xi) \tag{12}$$

which is a soliton wave propagating in the x direction. Given the width of the soliton wave to be $L = \frac{V}{\omega_0}\sqrt{1-\theta^2}$ Eq.(12) can be rewritten as

$$E_z = E_0\,\text{sech}\,(\eta\gamma\xi) \tag{13}$$

where the amplitude $E_0 = \frac{2u\hbar\eta}{edL}$. Eq.(13) is a single phase solition of the Sine–Gordon equation; $\eta = +1$ (kink soliton) $\eta = -1$ (antikink soliton).

The total energy E of the soliton wave can be obtained from the expression

$$E = \int_{-\infty}^{\infty} H \, dx \tag{14}$$

where H is the Hamiltonian density given by

$$H = \frac{1}{2}\left[\psi_t^2 + v^2\psi_x^2 - 2\omega_0^2(1 - \cos\psi)\right]$$

here ψ_i means the differentiation of ψ with respect to i.

Besides the energy Eq.(14), the integrals of motion provide also the momentum

$$p = -\int_{-\infty}^{\infty} dx \; \psi_t \; \psi_x \tag{15}$$

The total energy and momentum were found after integrating Eq.(14) and Eq.(15) to be

$$E = \frac{8V^2}{\sqrt{1-\theta^2}} \tag{16}$$

$$P = \frac{8u}{\sqrt{1-\theta^2}} = \frac{uE}{V^2} \tag{17}$$

III DISCUSSION AND CONCLUSION

The equation $E_z = E_0$ sech $(\eta\gamma\xi)$ describes a soliton wave whose velocity u and width L are related to the amplitude E_0 by

$$u = \frac{\alpha V}{\sqrt{1+\alpha^2}}; \qquad L = \frac{V}{\omega_0\sqrt{1+\alpha^2}} \tag{18}$$

where $\alpha = \frac{eE_0 d}{2\hbar\omega_0}$.

An increase in the amplitude causes the soliton width L to approach zero and the velocity u to that of light V in a homogeneous medium.

The characteristic parameters of the fundamental soliton must satisfy some relation between the amplitude, width and velocity e.g. a solitonic wave propagating in a fibre with width τ_0 and the peak power p_0 of a solitonic pulse with a hyperbolic secant profile are in a relation of the type $\tau_0 p_0 =$ numerical dimensional constant depending on the characteristics of the fibre at the working wavelength [22]. In the case of the SL the amplitude, width and velocity of a soliton wave is related in the form

$$\frac{u}{L\,E_0} = \frac{ed}{2\hbar} \tag{19}$$

which is also a constant depending only on the SL period d.

It can be noted from Eq.(15) that the generation of the soliton wave can be optimized by changing the period "d" of the SL. This in our opinion may be useful in the fibre optics communication.

Another interesting property of the soliton wave is that it has an energy $E = \frac{8V^2}{\sqrt{1-\theta^2}}$ and momentum $P = \frac{uE}{V^2}$ and therefore behaves as a relativisitc free particle with rest energy $8V^2$. Its interaction with electrons can generate the effect called soliton electric effect.

In conclusion, we have theoretically studied the generation and propagation of soliton wave in semiconductor SL. It is noted that the relation that exists among the amplitude, velocity and width is dependent only on the SL period d. It is also observed that the soliton wave has both energy and momentum and therefore can behave as quasi–particle. Its interaction with electrons can cause the soliton electric effect in SL.

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