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Abstract

The dc acoustoelectric current J^{ac} induced by the interaction of acoustic phonons with the conducting electrons is calculated. The calculation is done in the hypersound regime where $q\ell \gg 1$ (here q is the acoustic phonon number and ℓ is the electron mean free path). The electric field consists of dc field E_0 and a low frequency alternating field E_1. A nonlinear dependence of J^{ac} on E_0 and E_1 is observed. Depending on the direction of the dc field E_0 the acoustoelectric current can either be positive or negative. At $\omega_q = 10^{11}\text{sec}^{-1}$ and in the absence of E_0, J^{ac} is very small and tends to zero at $E_1 = 1.3 \times 10^5 V_m - 1$. Further increase of ω_q from 10^{11}sec^{-1} sees a dramatic shoot "up" of J^{ac}. Lastly the acoustoelectric current in the presence of E_0 is modulated by E_1.

1 Introduction

The acoustoelectric effect is the generation of *dc* electric current (the so-called acoustoelectric current) in a nonbiased device by a coherent acoustic wave or a flux of phonons. The study of acoustoelectric effect in bulk materials has received a lot of attention [1-5]. Recently there has been a growing interest in observing this effect in mesoscopic structures [6-8]. The interaction between surface acoustic waves (SAW) and mobile carriers in quantum wells is an important method to study the dynamic properties of two-dimensional (2D) systems. The SAW can trap carriers and induce acoustic charge transport as has been investigated in a number of systems in view of possible device applications [9]. Also, the SW-method was applied to study the quantum Hall effects [10-12], electron transport through a quantum point contact [13] and lateral nanostructures [14], the fractional quantum Hall effect [15], Fermi surfaces of composite Fermion around a half-filled Landan level [16] and commesurability effects caused by the lateral superlattice induced by a SAW [17]. It has also been noted that the transverse acoustoelectric voltage (TAV) is sensitive to the convex concentration in the semiconductor, thus it has been used to provide a characterization of electric properties of semiconductors [18]. Interface state density [19], junction depth [20] and carrier mobility [21] have been measured with this method.

In this paper we continue the work in [22, 23] where we studied the nonlinear acoustoelectric effect in a semiconductor superlattice. In that paper, the effect of constant electric field on the acoustoelectric current was considered and a nonlinear dependence was observed. Here, we study the presence of constant electric field and low frequency alternating field on the acoustoelectric current. The study of this effect is vital because of the complimentary role it may play in the understanding of the properties of the SL. A similar situation has been considered in a quantum constriction [8] where it was observed that transitions between the propagating and reflecting states can both decrease and increase transmission probability and thus lead both to negative or positive acoustic conductance.

In this work we also noted that the acoustic current is negative or positive depending on the direction of the constant electric field. The low frequency alternating field is modulating the acoustoelectric current.

This paper is organized as follows. In section 2 we outline the theory and conditions necessary to solve the problem. In section 3 we discuss the results and in section 4 we conclude.

2 Theory

Proceeding as in [22, 23], we shall consider the acoustic wave as a hypersound i.e., $q\ell \gg 1$. Under such circumstance the acoustic wave can be interpreted as monochromatic phonons having a δ-function distribution

$$N(k) = \frac{(2\pi)^3}{\hbar\omega_q s}\phi\delta(k-q) \quad \hbar = 1 \tag{1}$$

where k is the phonon wavevector, $\hbar$ is the Planck's constant divided by 2π, ϕ is the sound flux density and ω_q and s are respectively the frequency and group velocity of sound wave, with the wavevector q.

It is assumed that the sound wave and the applied electric field $E(t)$ propagate along the z axis of the SL. The problem is solved in the quasi-classical case, i.e. $2\Delta \gg \tau^{-1}, eE_0d \ll 2\Delta, eE_1d \ll 2\Delta$ (τ is the relaxation time, d is the period of SL, 2Δ is the width of the lowest energy miniband and e is the electron charge). The density of the acoustoelectric current can then be written as [24]

$$j^{ac}\frac{2e}{(2\pi)^3}\int U^{ac}\psi_i(p)d^3p \tag{2}$$

Here $\psi_i(p)$ is the solution of the Boltzmann kinetic equation in the absence of magnetic field; p is the electron momentum and

$$\begin{aligned} U^{ac} &= -\frac{2\pi\phi}{\omega_q s}\left|G_{p-q,p}\right|^2[f(\varepsilon_{p-q}) - f(\varepsilon_p)]\delta(\varepsilon_{p-q} - \varepsilon_p + \omega_q) \\ &\quad + \left|G_{p+q,p}\right|^2[f(\varepsilon_{p+q}) - f(\varepsilon_p)]\delta(\varepsilon_{p+q} - \varepsilon_p - \omega_q) \end{aligned} \tag{3}$$

where $G(p,q)$ is the matrix element of the electron-phonon interaction, $f(\varepsilon_p)$ is the distribution function and $\varepsilon(p)$ is the energy of electron. Introducing a new term $p' = p - q$ and applying the principles of detailed balance, i.e.

$$\left|G_{p'p}\right|^2 = \left|G_{pp'}\right|^2 \tag{4}$$

we express eq.(2) as

$$j^{ac} = -\frac{e\phi}{2\pi^2 s\omega_q}\int |G(p,q)|^2[f(\varepsilon_{p+q}) - f(\varepsilon_p)][\psi_i(p+q) - \psi_i(p)]\cdot\delta(\varepsilon_{p+q} - \varepsilon_p - \omega_q)d^3p \tag{5}$$

where the vector $\psi_i(p)$ is expressed in [25] as the mean free path $\ell_i(p)$.

Thus the acoustocurrent in eq.(5) in the direction of the SL axis becomes

$$j_z^{ac} = -\frac{e\phi}{2\pi^2 s\omega_q}\int |G(p,q)|^2[f(\varepsilon_{p+q}) - f(\varepsilon_p)][\ell_z(p+q) - \ell_z(p)]\cdot\delta(\varepsilon_{p+q} - \varepsilon_p - \omega_q)d^3p \tag{6}$$

For $qd \ll 1$, $G(p_z q)$ is given as

$$|G(p_z,q)|^2 = \frac{\Lambda^2 q^2}{2\sigma\omega_q} \tag{7}$$

here Λ is the deformation potential constant, and σ is the density of the SL. As indicated in [22] in the τ approximation and further, when τ is taken to be constant, ℓ_z is given as

$$\ell_z = \tau v_z \tag{8}$$

where

$$v_z = \frac{\partial\varepsilon}{\partial p_z} \tag{9}$$

The most convincing arguement in favour of this condition is in [26], where it is established experimentally that the relaxation τ is a constant in $GaAs/AlAs$ SL above 40K and is temperature independent.

Inserting eqs.(7) and (8) into eq.(6), we obtain the acoustoelectric current as:

$$j_z^{ac} = -\frac{e\phi|\Lambda|^2 q^2 \tau}{4\pi^2 s\omega_q^2}\int [f(\varepsilon_{p+q}) - f(\varepsilon_p)][v_z(p+q) - v_z(p)]\cdot(\varepsilon_{p+q} - \varepsilon_p - \omega_q)d^3p \qquad (10)$$

Applying this general expression for J_z^{ac} in SL we quote the result of [23] as:

$$\begin{aligned} j_z^{ac} &= -\frac{e\phi|\Lambda|^2 q^2 n d\tau\theta(1-b^2)}{\sigma s\omega_q}\int_0^\infty \frac{dt'}{\tau}\exp(-t'/\tau)\cdot \\ &\cdot\left\{\sinh\left(\frac{\omega_q}{2T}\cos(eEdt')\right)\sinh\left(\frac{\Delta}{T}\cos\left(\frac{qd}{2}\right)\cos(eEdt')\right)\sqrt{1-b^2}\right. \\ &-\frac{\Delta}{T}\sqrt{1-b^2}\sin(eEdt')\sin\left(\frac{qd}{2}\right)\cosh\left(\frac{w_q}{2T}\cos(eEdt')\right)\cdot \\ &\left.\cdot\cosh\left(\frac{\Delta}{T}\sqrt{1-b^2}\cos\left(\frac{qd}{2}\right)\cos(eEdt')\right)\right\} \end{aligned} \qquad (11)$$

3 Results and discussion

We considered an external electric field of the form $E_0 + E_1 \sin\omega t$ where the *ac* field is very weak low frequency regime, i.e., $\omega\tau \ll 1$. Under such conditions we can replace the constant electric field E in eq.(11) with $E_0 + E_1\sin\omega t$ and average the result over time. As can be seen, this cannot be solved analytically hence we used numerical methods. For solutions when $E_1 = 0$ see [23]. We studied the behaviour of the acoustoelectric current when the amplitude of the slow varying *ac* field E_1 is kept constant. The behaviour is shown in Fig.1. It is noted that for $\omega_q = 10^{11}\text{sec}^{-1}$, $\Delta = 1.6\times10^{-20}J$, $\cos(\frac{qd}{2}) = 0.8, \omega\tau = 10^{-1}$ and $T = 300\text{K}$. The acoustoelectric current rises, reaches a maximum, then falls off in a manner similar to that observed during a negative differential conductivity when E_0 is negative. On the other hand, when E_0 is positive the current decreases to a minimum and then rises. This can be attributed to the Bragg reflection at the band edge. At this frequency i.e., $\omega_q = 10^{11}\text{sec}^{-1}$ the graph is symmetrical about the origin. As ω_q increases the symmetry breaks down and at $\omega_q = 10^{13}\text{sec}^{-1}$ we observed that the absolute value of the maximum peak $|J_z^{ac}/J_0^{ac}|_{\max}$ is greater than the absolute minimum value $|J_z^{ac}/J_0^{ac}|_{\min}$ (Fig.2). As can be seen in Fig.2a the ratio of $|j_z^{ac}/j_0^{ac}|_{\max}/|J_z^{ac}/J_0^{ac}|_{\min}$ is about ~ 11 which is quite big. In the presence of *ac* field E_1 we observed that the peak values of j^{ac} decreases as E_1 increases. The E_1 field in this case is behaving as a modulator. A very interesting observation is noticed when $\cos\frac{qd}{2}$ is negative i.e., $|J_z^{ac}/J_0^{ac}|_{\min} > |J_z^{ac}/J_0^{ac}|_{\max}$ in other words there is an inversion. This occurs when $\frac{3\pi}{d} > q > \frac{\pi}{d}$ (see Fig.2b).

We examined herein, the behaviour of the acoustoelectric current J^{ac} when the *dc* field E_0 is kept constant and the amplitude of the *ac* field E_1 is varied. In Fig.3a we observed for $\omega_q = 10^{11}\text{sec}^{-1}$ and $E_0 = 0$, the acoustoelectric current J^{ac} is very small and tends to zero at $\frac{eE_1 d\tau}{\hbar} = 1$. To get the numerical estimate of E_1 for a typical *GaAs/AlGaAs* SL, the following parameters are chosen: $d = 100\text{Å}$; $\tau = 10^{-12}\text{sec}$. For these values we obtained $E_1 \approx 1.3\times10^5 V_{m^{-1}}$. We further observed that as E_0 is increased either in the direction of the acoustic phonons or in the opposite direction J^{ac} increases likewise. With increase in phonon frequency $\omega_q = 10^{13}\text{sec}^{-1}$ J^{ac} increases rapidly even in the absence of E_0 (see Fig.3b).

It is worth noting that at about $E_0 = 2 \times 10^5 V_{m^{-1}}$ and above, E_1 ceases to modulate the acoustoelectric current.

Finally in Fig.4 we present the three dimensional graphs of J_z^{ac}/J_0^{ac} against E_0 and E_1 to give a vivid picture of what is happening.

4 Conclusions

We have studied the acoustoelectric current in SL in the presence of a constant and low frequency alternating field and noted a strong nonlinear dependence of j_z^{ac} on both E_0 and E_1. We observed that in the absence of *dc* field E_0, when $\omega_q = 10^{11}\text{sec}^{-1}$, J_z^{ac} is very small and tends to zero at $E_1 = 1.3 \times 10^5 V_{m^{-1}}$. Further increase in ω_q to 10^{13}sec^{-1} shoots up the acoustoelectric current J_z^{ac} dramatically. When E_0 is introduced J_z^{ac} increases with the increase of E_0. Lastly we noted that E_1 modulates the acoustoelectric current in the presence of E_0 and at field value of $E_0 = 2 \times 10^5 V_{m^{-1}}$ this modulation ceases.

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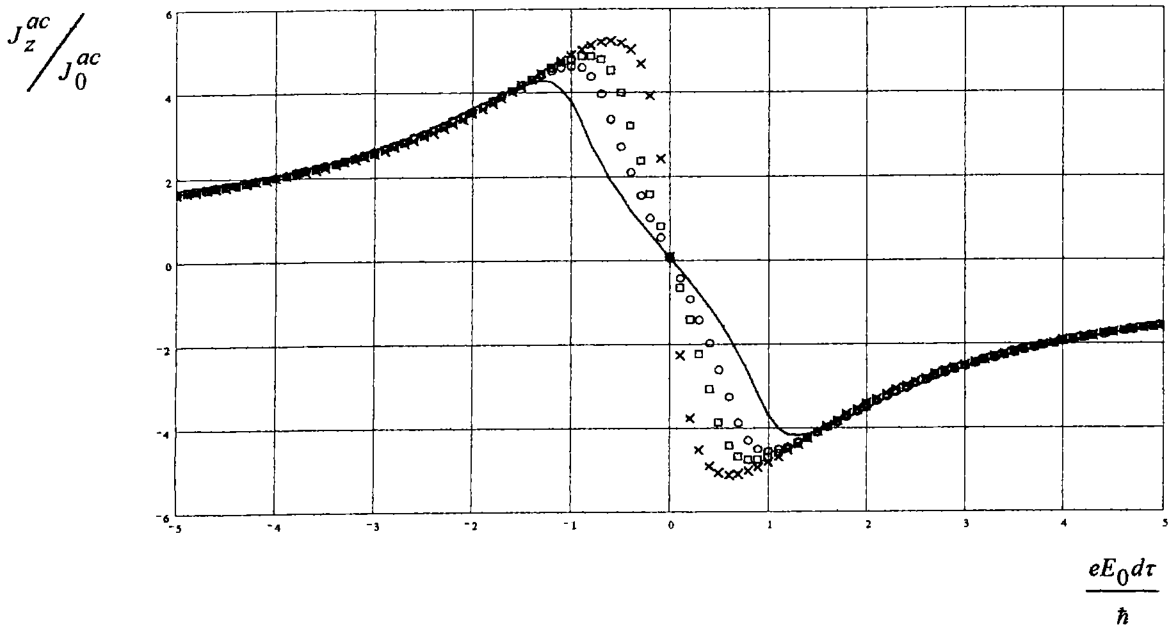
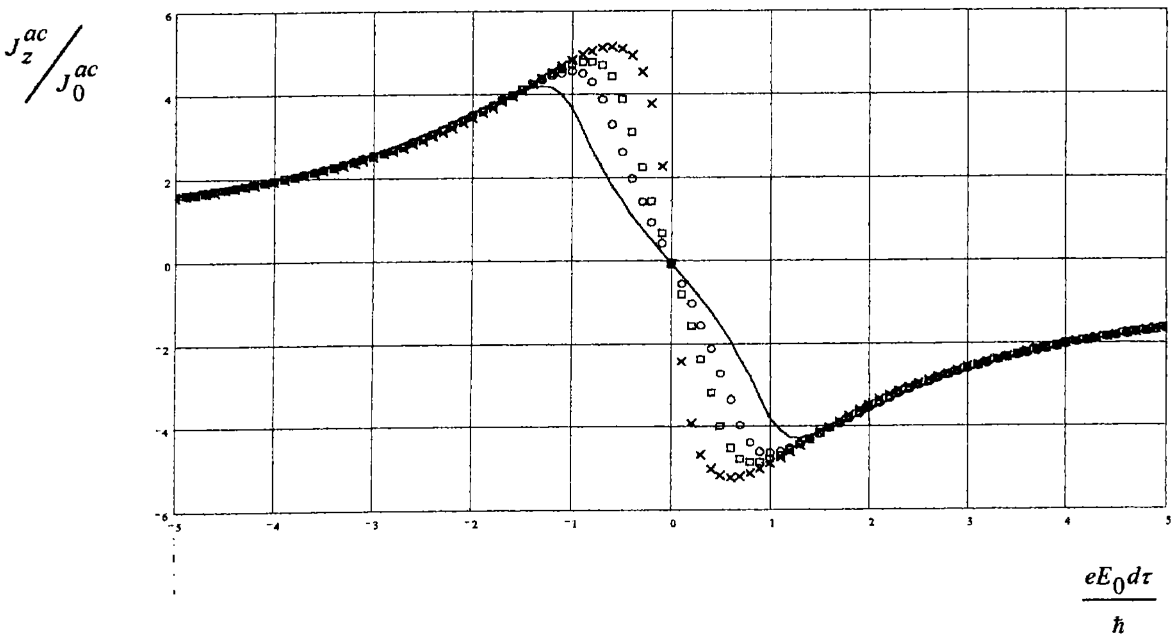


Dependence of J_z^{ac} / J_0^{ac} on $\frac{eE_0 d\tau}{\hbar}$ for ××× $\frac{eE_1 d\tau}{\hbar} = 0$; □□□ $\frac{eE_1 d\tau}{\hbar} = 0.5$; ooo $\frac{eE_1 d\tau}{\hbar} = 0.7$; — $\frac{eE_1 d\tau}{\hbar} = 1$;

For $\omega_q = 10^{11}$; $\Delta = 1.6 \times 10^{-20}$ J ; $\cos\left(\frac{qd}{2}\right) = 0.8$; T = 300K

a



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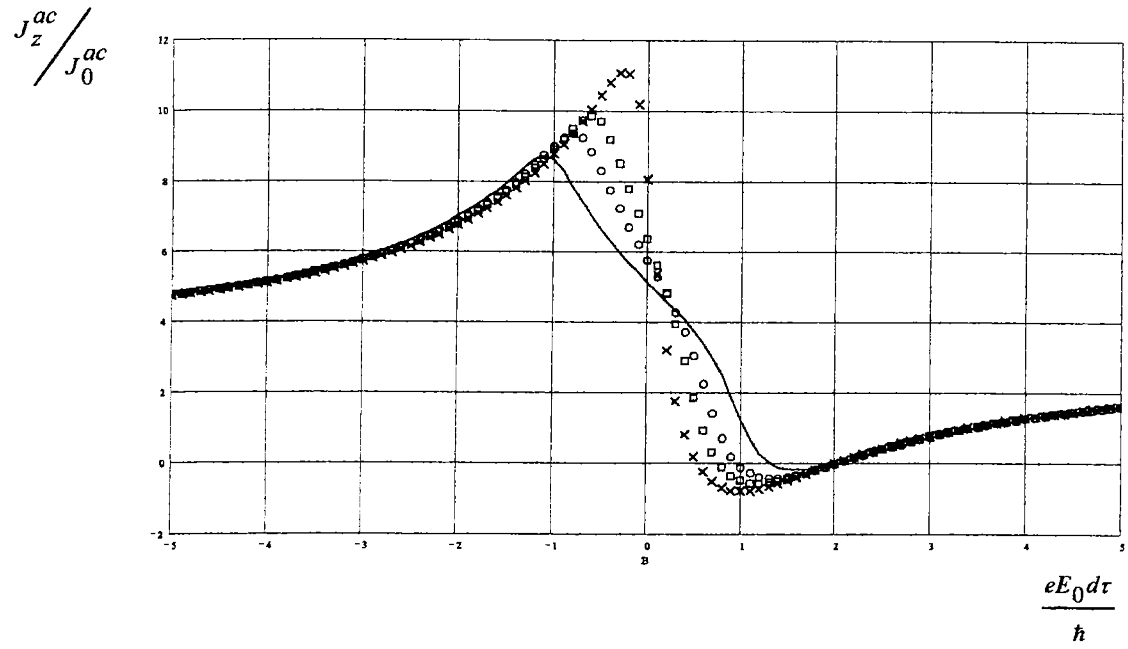
b

Fig 1



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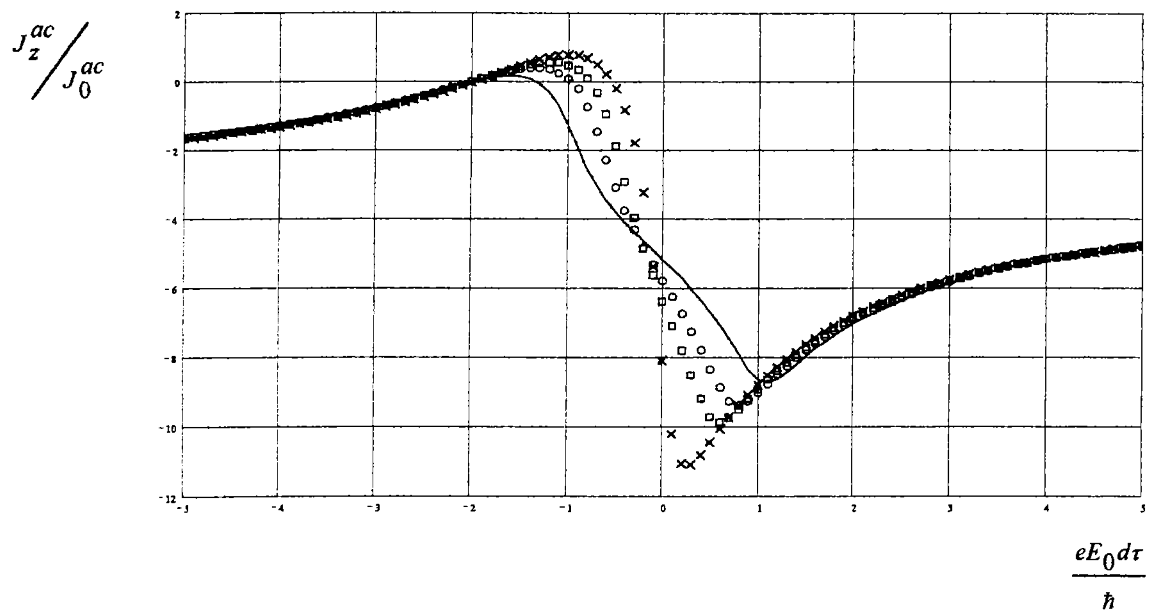

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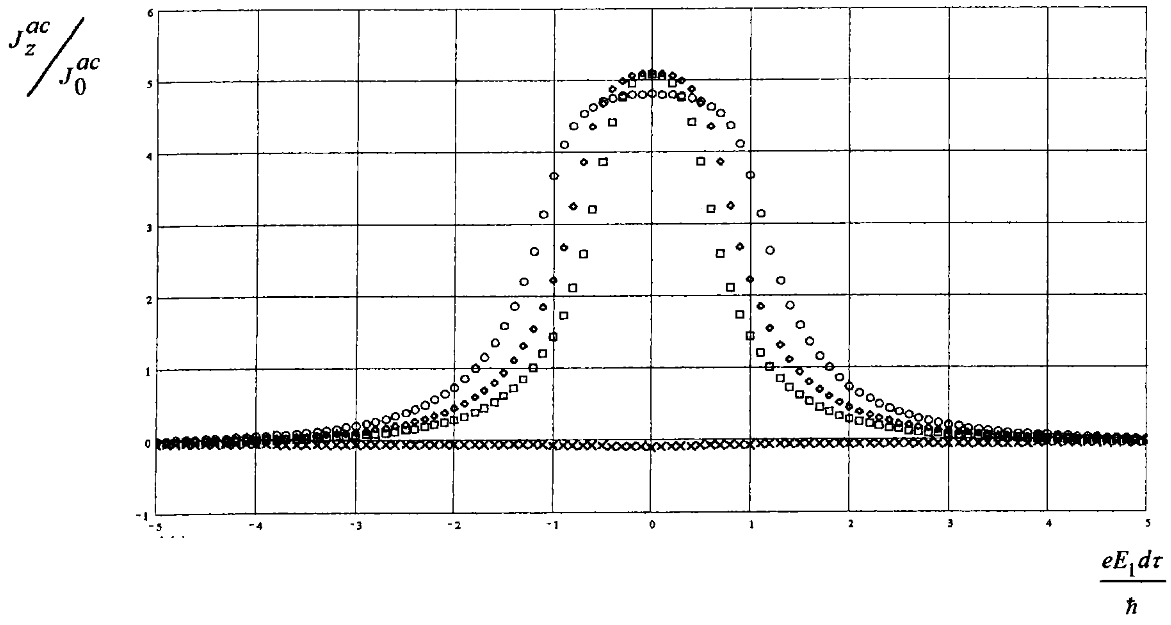
b

Fig 2



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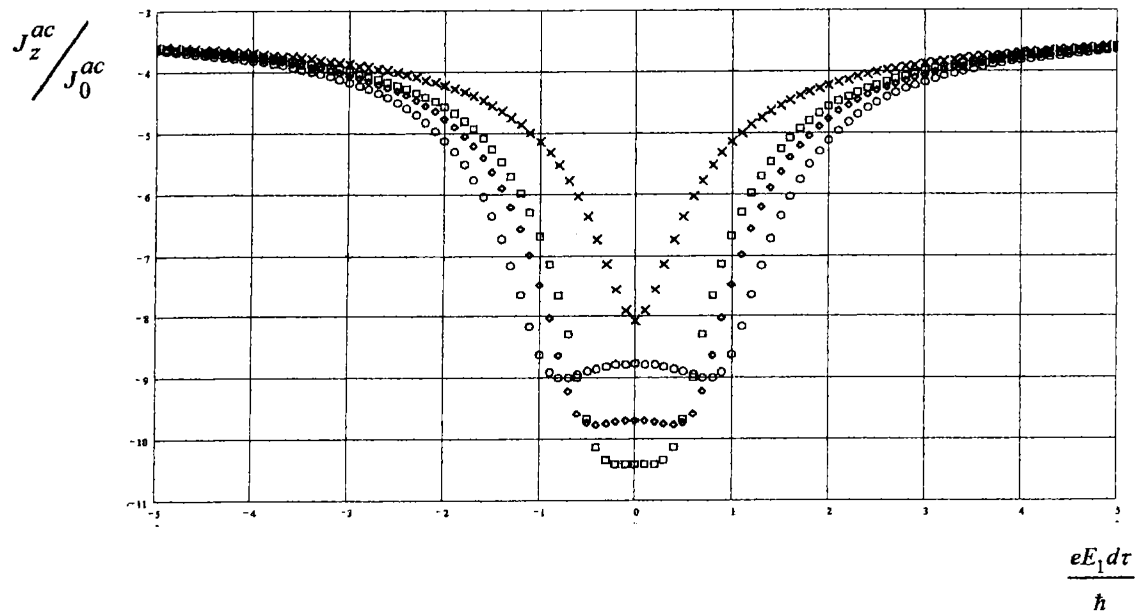

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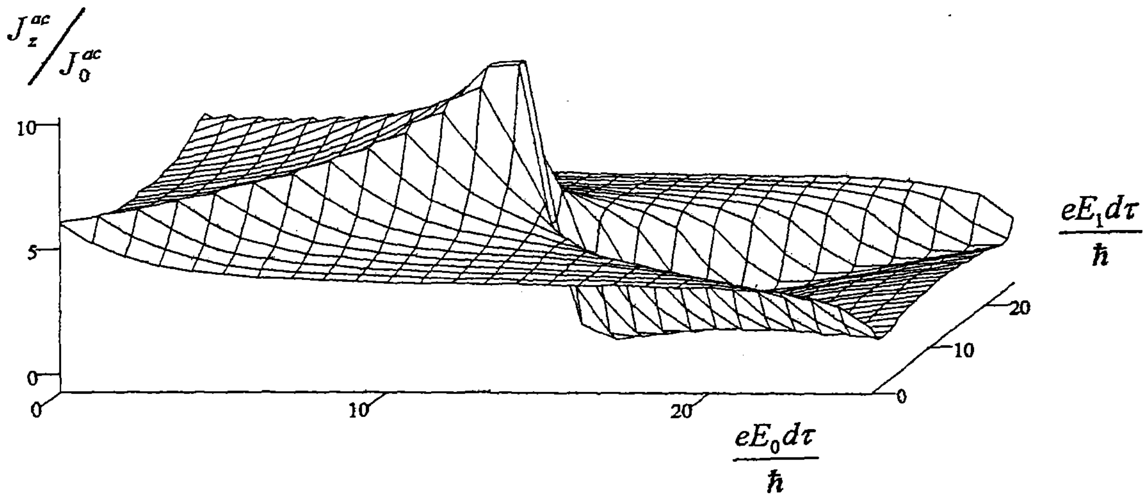
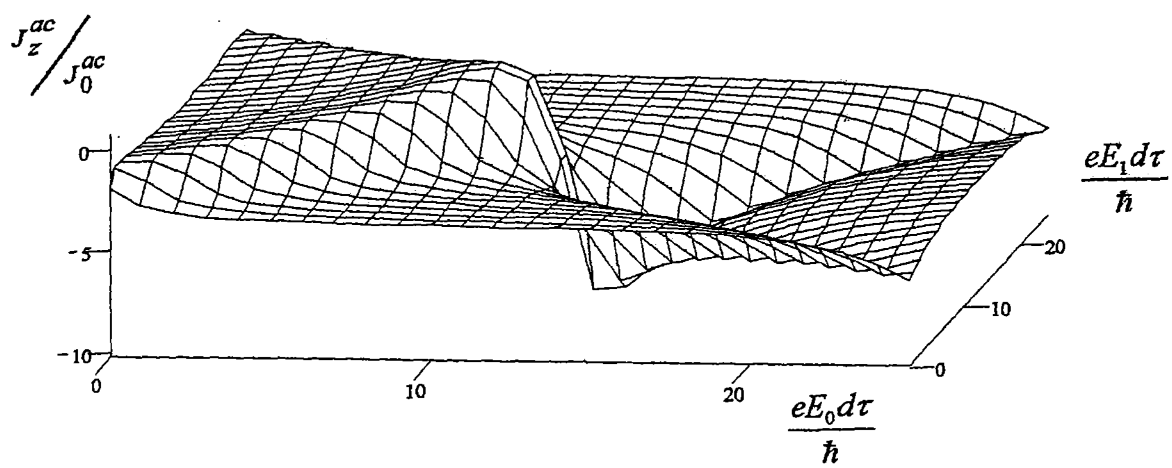
For $\omega_q = 10^{13}$; $\Delta = 1.6 \times 10^{-20}$J ; $\cos\left(\frac{qd}{2}\right) = -0.8$; T = 300K

b

Fig 3



Dependence of J_z^{ac} / J_0^{ac} on $\frac{eE_0 d\tau}{\hbar}$ and $\frac{eE_1 d\tau}{\hbar}$ for $\cos(qd/2) = 0.8$, $\omega_q = 10^{13} s^{-1}$, $\Delta = 1.6 \times 10^{-20} J$, $T = 300K$



Dependence of J_z^{ac} / J_0^{ac} on $\frac{eE_0 d\tau}{\hbar}$ and $\frac{eE_1 d\tau}{\hbar}$ for $\cos(qd/2) = -0.8$, $\omega_q = 10^{13} s^{-1}$, $\Delta = 1.6 \times 10^{-20} J$, $T = 300K$

Fig 4