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USING COOPERATIVE TEACHING AND LEARNING APPROACH TO IMPROVE SENIOR HIGH SCHOOL STUDENTS' LEARNING OF TRIGONOMETRY

BY

RICHARD KYERE ASOMAH

Thesis submitted to the Department of Mathematics and ICT Education of the Faculty of Science and Technology, College of Education Studies, University of Cape Coast, in partial fulfillment of the requirements for the award of Doctor of Philosophy Degree in Mathematics Education

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DECLARATION

Candidate's Declaration

I hereby declare that this thesis is the result of my own original research and that no part of it has been presented for another degree in this university or elsewhere.

Candidate's Signature: ..... Date: 11th May 2022

Name: Richard Kyere Asomah

Supervisors' Declaration

We hereby declare that the preparation and presentation of the thesis were supervised in accordance with the guidelines on supervision of thesis laid down by the University of Cape Coast.

Principal Supervisor's Signature: ..... Date: 12-05-2022

Name: Prof. Douglas Darko Agyei

Co-supervisor's Signature: ..... Date: 11-05-22

Name: Dr. Forster D. Ntow

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ABSTRACT

The study focused on improving the teaching and learning of trigonometry with cooperative teaching and learning approach (CTLA) to promote student-centred approach to learning in Senior High School (SHS) classroom context in Ghana. The research approach for the study was design-based research (DBR), using qualitative and quantitative methods. The DBR consisted of three stages; prototype, implementation and evaluation, and systematic reflection and documentation. At the prototype stage, two exemplary materials with the features of CTLA were designed by the researcher and validated by two experts in mathematics education. Based on the exemplary materials, professional development training was organised for eight in-service mathematics teachers (IMTs) who were sampled on purpose to develop and enact CTLA-lessons. Questionnaires, observations, semi-structured interviews, focus group discussions, student tests and designed lesson documents were the data sources employed in this study. The data were analysed using descriptive statistics, data reduction technique, paired sample t-test, and document analysis respectively. The results of the study revealed that the IMTs improved their pedagogical strategies as a result of enacting lessons anchored on the CTLA. Further, there was significant improvement in the students' achievement test after their participation in the CTLA-lessons. In addition, design guidelines were delineated to serve as a guide in the future developments of IMTs experiences in using CTLA as a pedagogical approach in the Ghanaian classroom context. Implications of the study for policy, practice and further research have been discussed.

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KEYWORDS

Cooperative teaching and learning approach

Design-based research

In-service mathematics teachers

Senior High School Students

Trigonometry

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DEDICATION

To my lovely mum, Madam Monica Adoma Nkansah

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CHAPTER ONE

INTRODUCTION

This chapter brings to the fore the statements in respect of this study's problem, the purpose and the background of the research study. In addition, the research questions, significance of the study, delimitation, and limitations of the study are presented. The chapter ends with processes involved in putting the entire study together.

Background to the Study

Trigonometry has varied applications in life and in its specific sense in mathematics entails geometric, graphical, and algebraic reasoning. In respect of this, its teaching and learning has been conducted since its inception in a piecemeal fashion by prioritizing computation and procedures as a means of instruction in modern times (Thompson, 2008). As a result of the pedagogical complexities associated with the instruction, variety of strategies are adopted by students to make meaning of trigonometric concept and also find possible solutions to the questions related to the concept. Strategies such as memorising definitions and theories, manipulating trigonometric expressions, and graphing functions are mainly employed in respect of this subject matter. Consequently, many rich opportunities for reasoning and sense making in the field of trigonometry are lost. It's against this background that quite a number of researchers (Bressoud, 2010; Blackett & Tall, 1991; Breidenbach, Dubinsky, Hawk, & Nichols, 1992; Weber, 2005) have called for more meaningful pedagogical strategy in supporting students on the concept of trigonometry while taking into considerations the student's sense making of the concept at the end of the instructional period. The conceptual difficulty students face in

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understanding trigonometry could be as result of lack of the appropriate instructional approach in the classroom context, the students development of insufficient conceptual understanding after instruction, students inability to provide critical analysis of problems associated with trigonometry and the confirmation of the wrong processes involved in finding solution to trigonometric problems coupled with its attendant explanation of the results obtained. It is in the light of these observations that the teaching and learning of trigonometry if not taught properly, creates a vacuum of doubt and abstractness in the concept of trigonometry. Moreover, the inappropriate approach to the teaching of trigonometry leads students to drift away from their sense of creativity in the trigonometric classroom context and as a result, memorisation of facts takes precedence over conceptual understanding of the concept. Hence, students are bound to develop a disjointed form of understanding associated with functions in trigonometry (Brown, 2005). For this reason, several studies in literature aimed at teachers' use of the appropriate instructional approach in teaching trigonometry have been conducted (Barnes, 1999; Kendal & Stacey 2003; Miller, 2001; Searl, 2014). It is against this background that, this study takes cognizance of the matrix combination of the ideas obtained from the literature as well as the instruction with which the teacher adapts in achieving realistic teaching in trigonometry. For instance, Weber, (2005); Challenger (2009) have catalogued a number of misunderstandings and misrepresentations on the concept of trigonometry largely from the perspective of the students. In particular, student's inability to relate conceptually, meanings associated with the concept of the three main trigonometric ratios of sine, cosine and tangent of an angle in trigonometry.

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Ross et al. (2011) identified in their study that, there was over reliance on committing to memory as the short cut for learning the three trigonometric ratios sine, cosine and tangent of angles. They pointed out the deficiency in using text books definitions and the mnemonics as an attributable function in the students' inability to understanding the concept of trigonometry. The study further posited that, the teaching and learning in respect of trigonometry is limited to obtaining only the final product (answer) of the question (task) to the disregard of the processes leading to the answer. It is in this regard that, Gillman (1991) asserts that, the development of the three main basic trigonometric ratios is based on some analysis of the concept for a meaningful learning to occur to aid the individual's ability to appreciate the concept of trigonometry.

In furtherance to the problems of the inability on the part of students to learn trigonometry as revealed in literature, Jong and Brinkman (1997) indicates instructional as well as methodological issues as the bane of teachers in ensuring proper understanding of trigonometry. As a result, inappropriate instructional and methodological methods are employed by teachers in the learning environment, thus, putting students in a restricted situation that causes them to misrepresent basic concepts in trigonometry. Hence, the correct conceptual representation which ought to be acquired cognitively in the learning environment in respect of trigonometry is missed. However, the importance of the knowledge acquisition in respect of trigonometry as a course and its consequent application in day-to-day activities cannot be overemphasized (Weber, 2005). As a result, it has become indispensable in the mathematics curriculum of SHS (Ministry of Education [MOE], 2010) in the

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sub-region. It is in this regard that, SHS students are examined on this discipline of study by a governing board of examination councils called West African Examination Council (WAEC). Trigonometry therefore occupies a critical part of the syllabus used by the member countries (sub-region) that constitute the council.

In the WAEC mathematics syllabus, trigonometry is highlighted as an area of concern for countries that are signatory to the council to concentrate. As a result, in Ghana, the planners of the SHS mathematics syllabus incorporate the concept of trigonometry for students to be taught. In particular, the units 2.11 and 3.40 of the second- and third-year mathematics syllabus are dedicated to the concept of trigonometry (MOE, 2010). In this syllabus, there are seven objectives in the SHS mathematics curriculum which stipulates what are to be achieved after instruction, such as the under listed.

> (1) calculate the values of trigonometric ratios of 30°, 45° and 60° (2) find the inverse of trigonometric ratios (3) calculate angles in respect of their elevation and depression in a given right angled triangle (4) calculate angles of elevation and angles of depression specifically for the second-year students (MOE, 2010, pp. 43-53). Again, the third-year outline of the syllabus indicates the following objectives: (1) the application of the three main basic ratios in trigonometry to calculate distances and heights (2) find trigonometric functions and identify maximum and minimum values (3) draw the graphs of trigonometric functions and use them to solve trigonometric equations (MOE, 2010, p. 53).

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The necessity in the student's conceptualisation of trigonometry is implied in the second- and third-year statement of the general objective. This purposed to enable learners find practical solution to problems in real life situations.

To this end, seven general objectives, to be specific, the units 2.11 and 3.40 of the second and third year aspects of the mathematics syllabus are structured with eight content sections sequenced as: tangent, sine and cosine of acute angles; the three main ratios in trigonometry in respect of 30°, 45° and 60°; the use of calculators to read sine, cosine and tangent of angles between 0° and 360°; inverse of trigonometric ratios; angles in respect of their elevation and depression in a given right angled triangle; application of the concept of ratios of trigonometry; graphs of trigonometric equations (MOE, 2010, pp. 43-53).

Trigonometry as a course of study, takes an unparalleled place in mathematics curriculum in different learning institutions in many countries although various nomenclatures attributed to it may differ from one country to the other (Delice & Roper, 2006). It is in continuance to this vital role the concept of trigonometry plays in schools of study that, this study views the students understanding of trigonometry as crucial as remarked in literature "learning trigonometry involves understanding: the definition of triangle; right-angles; recognising them in different orientations; what angle means and how it is measured; typical units for measuring lines; what ratio means; similarity of triangles; how ratio is written as a fraction; how to manipulate a multiplicative relationship; what sin (etc.) means as a symbolic function and so on." (Ross et al., 2011, p. 5). Thus, the current study sough to overcome these conceptual difficulties as a result of which an instructional approach meant to improve students learning of trigonometry was developed. Moreover,

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the student's inability to comprehend basic concepts in trigonometric and teachers lack of the appropriate instructional strategy to teaching trigonometry is compounded by the SHS mathematics syllabus, which mandates instructions on trigonometry as a course be skipped at the first year of the students' academic cycle at the SHS level (MOE, 2010).

Hence, it can be inferred that, this transition from SHS 1 to SHS 2 requires students to possess a sound grasp of a variety of conceptual ideas in order to appreciate the theoretical and practical complexities in trigonometry. It is in response to these gaps of understanding the concepts of trigonometry that varied pedagogical strategies have been researched to have effect on the improvement of students learning. (For example, Brown, 2005; Delice & Roper, 2006; Kendal & Stacey, 2003; Weber, 2005).

Moreover, Hammond, Austin, Orcutt, and Rosso (2001) acknowledged that, all modern approaches to teaching are linked by constructs. For instance, in order to develop ideas and skills, experience and reflection play a major part. Hence, such instructional approach takes into consideration an effect of cultural tendencies and other construct that have impact of experience in relation to how learners make meaning of the subject matter.

Cooperative teaching and learning approach is part of the recommendations offered by the planners of the Ghanaian Mathematics Syllabus for the senior high schools approach (MOE, 2010). This is because characteristically, cooperative learning is practically oriented and collaboratively and participatory inclined, reflexive in process, and purposefully constructed in bridging the loophole created in literature in relation to theory and practice in a classroom (Trauth-Nare & Buck, 2011).

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The choice of this instructional approach is appropriate since previous studies affirms the difficulties accompanying the methods of instructions and as well as student's appreciation of the concept of trigonometry in a learning environment (Burke & Olley 2008; Ergin, Ünsal, & Tan, 2006; Slavin, 2014; Weiner, 1980).

There was therefore the need to develop a more integrative instructional approach anchored on an effective, culturally inclined and a learner centred model that seeks to improve, consolidate and promote deeper understanding of the concept of trigonometry. Whilst making it easy for students to overcome the difficulty that obstruct their conception of trigonometry through the provision of hands-on activities in such an instructional approach. Hence, an instructional approach premised on a cooperative learning model could lead to the improvement of students learning of trigonometry.

Statement of the Problem

There are conceptual difficulties surrounding the teaching of the concept of trigonometry, pieces of evidence of which have been adduced in literature by many researchers to this effect (Ross, Bruce, & Sibbald, 2011; Pritchard & Simpson, 1999; Watson, 2013; Weber, 2005). Trigonometry was described as "the confluence of a number of streams of mathematical difficulty" (Pritchard & Simpson, 1999 p.1246). The conceptual difficulty of trigonometry from the pedagogical perspective was further asserted that "Deep understanding of trigonometry requires the ability to flip between abstract, visual and concrete representations of mathematical objects. In addition, the subject is confounded by inter-relationship between functions" (Ross et al.,

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2011, p. 121). It could be argued, there are problems with the appropriate pedagogy with which to improve students understanding of trigonometry as inferred from (Pritchard & Simpson, 1999; Ross et al., 2011; Watson, 2013) coupled with the difficulties students encounter with concepts such as angles measured in radians, negative angles and angles greater than 360° which are sometimes unfamiliar to the learners an observation made by Weber (2005). Furthermore, the conceptual difficulty students face in learning trigonometry can be narrowed to pedagogy as Gur (2009) contended that, pedagogically. the mediums of instruction are fraught with teacher-centered when it come to the teaching of trigonometry. The problems associated with the teaching of concepts in trigonometry as identified in literature cannot be different in the Ghanaian context. This is because it continues to be a major weakness in the Chief Examiner's report (WAEC, 2011; 2012; 2013; 2018; 2020) on the performance of students in trigonometry as a unit topic in the teaching syllabus for core mathematics in the second cycled institutions (SHS). The WAEC Chief Examiners' Report on mathematics has incessantly acknowledged the concept of trigonometry as part of difficult topics for most learners. For instance, in 2011, the report decried the poor performance of candidates in answering question on trigonometry. Thus, the student's inability to conceptualise and apply their understanding of the various concepts (topics) under trigonometry to answer questions posed to them at the end of their two-year learning of trigonometry at the SHS level. For example, in 2011, the Chief Examiner's Report delineated the pedagogical problems in teaching trigonometry as a result of which almost all the candidates who attempted the trigonometric question below got it wrong:

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Given that sin x = 0.6 and $0^o \leq x \leq 90^o$, find 1-tan x, leaving your answer in the form $\frac{a}{b}$ where 'a' and 'b' are integers. In (a) candidates were to find 1 - tan x in form $\frac{a}{b}$ where a and b are integers given that Sin x = 0.6 and $0^o \leq x \leq 90^o$.

Similarly, in 2012, the Chief Examiner's Report indicated students' inability to diagrammatically represent the trigonometric questions in right-angled triangle and thereby failed to apply the appropriate trigonometric ratios of sine, cosine and tangent of an angle in solving the question. As such, the question was left unanswered by most candidates and those who attempted it.

(a) If tan x = $\frac{15}{8}$, $0^o \leq x \leq$ find the value of Cos x.

(b) Two vertical poles, 3m and 7m long are on the same straight line with a point P on the ground. The shorter pole is 20 m from P and is between P and the longer pole. The angle of elevation of the top T of the longer pole from the top R of the shorter one is 30^o.

Calculate

(i) |RT|;

(ii) the horizontal distance from P to the longer pole, correct to three significant figures;

(iii) the angle of the elevation of T from P, correct to the nearest degree.

Again, the report indicated that, "The part (b) was challenging and demanded very good knowledge in trigonometry since it involved sketching before the question could be answered correctly. Most of the candidates did not make any attempt to answer it but those who attempted it could not draw any meaningful diagram to help them solve the problem" (Chief Examiner's report WASSCE, 2012, p. 335).

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Moreover, in 2020, the Candidates exhibited a rather grave and intricately a gloomy approach in their attempt to deduce a theoretically inclined trigonometric concept. They struggled to apply trigonometric concept to real life related task.

Two points X, Y and Z meters apart are on the horizontal ground. The angle of elevation of a point P from X and Y are 50°and 70° respectively. Q is a point XY produced such that angle = 90°

(i) Illustrate the information on a Venn diagram

(ii) Calculate, correct to two decimal places, the length

a. $\overline{XY}$

b. $\overline{YQ}$

The performance of the students as was remarked by Chief Examiner's Report indicated that "most candidates could not illustrate the information on the appropriate diagram. Q is a point on XY produced such that an angle YQP of 90° was misunderstood or ignored by some instead Q was located between X and Y resulting in wrong and subsequently incorrect answers" (WASSCE, 2020 p. 332). These reports are suggestive of the conceptual difficulty student's face particularly in responding to examination questions that demand the application of concepts in trigonometry at the SHS level. Blackett and Tall (1991) further suggested that, the teaching of trigonometry is fraught with some difficulty as a result of which some form of misunderstanding exist from the perspective of the students. Hence, students find it difficult to study advance courses in mathematics such as pre-calculus and calculus which have their roots in trigonometry (Weber, 2005).

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In the Ghanaian context, Farouq (2017) explored students' error in learning trigonometry among final year SHS mathematics students. The findings of the study disclosed quite a number of errors in process skill and transformation processes committed by the students regardless of the methodology employed in solving trigonometry problems. Subsequently, the study recommended a pedagogical approach meant to improve students learning on the concepts of trigonometry.

It is in the light of the different pedagogical strategies employed in other studies to improve students learning of trigonometry that yielded different results (Kendal & Stacey, 2003) that, the need for a pedagogical approach to teaching trigonometry devoid of conditions where student memorise the content of the subject matter, aims at the end result (product) instead of the processes leading to that result, but rather, a kind of instructional approach where opportunities are afforded the students to make meaning of the subject matter. (National Council of Teachers of Mathematics [NCTM], 2000; Skemp, 1987). In particular, a type of pedagogy that accommodates student's ability to rationalise why concepts related to mathematics exhibit some properties and provide cogent reasons as to its appropriateness in the procedures used. (Skemp, 1987).

The conduct of gradual research into the mediums of instruction relating to trigonometry using DBR approach has been rare. However, many researchers in this field (Kendal & Stacey 2003; Miller, 2001; Searl, 2014) argue that, the current teaching practices of trigonometric concepts in the learning environments do not enhance the student's learning outcomes in trigonometry.

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It is against this background of pieces of evidence of pedagogical difficulties associated with the teaching of trigonometry elsewhere and in the Ghanaian mathematics classroom context using the DBR as a research approach, that a new methodological outlook that improves students understanding of the concept of trigonometry in the learning environment cannot be overstretched. Hence, the study sought to design pedagogical approach that improves students learning of trigonometry using cooperative learning teaching and learning approach (CTLA). This is because the gains that inure to the benefit of students when instructed with CTLA cannot be over emphasized as opportunities are afforded them to interact freely and achieve collective goals. Moreover, it incorporates interactivity as part of its components, students in the learning environment question issues, share knowledge on concepts, clarify variances in respect of concepts and build new meanings in relation to the subject matter (Mercer, Wegerif, & Dawes,1999; Webb & Mastergeorge, 2003).

In Ghana, however, literatures on the use of CTLA as a pedagogical approach to teaching trigonometry in the SHS classroom context have been rare. However, the studies conducted by Assan-Donkoh, Susuoroka, Baah-Duodu and Puotie (2019); Edekor and Agbornu (2020); Bosson-Amedenu, Teku, Vivian and Osei-Asibey (2021) using the CTLA focussed on the teaching and learning of circle theorem at the SHS and a topic in mathematics at the Junior High School (JHS) and College of Education (CoE) respectively. This adds to the inadequacy in the attempts of reviewing literature from the Ghanaian context on the prospects of CTLA in improving students learning on the concepts of trigonometry in the Ghanaian SHS mathematics classroom

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context. Additionally, none of the studies considered the development of a pedagogy anchored on the design-based research (DBR) approach mostly employed in studies that involved the development of an intervention to teach mathematics and other academic courses in Ghana. Thus, a pedagogical approach anchored on the CTLA through the DBR could be the solution to the conceptual difficulty associated with the teaching and learning of trigonometry

Purpose of the Study

The study generally sought to enhance the pedagogical strategies of IMTs through the effective utilisation of CTLA as an instructional approach to improve students learning on the concept of trigonometry in the classroom context particularly, at the SHS in Ghana. Specifically, the study determined:

1. The effectiveness of the CTLA intervention in developing in-service mathematics teachers' (IMTs) pedagogical strategies to improve teaching and learning of concepts in trigonometry.
2. How the improvements in teaching using pedagogical strategies that are anchored on cooperative teaching and learning approach be understood and or explained.
3. How the improvements in students' learning of concepts in trigonometry using cooperative teaching and learning approach be understood and or explained.

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Research Questions

The following questions were therefore formulated to guide the conduct of the study:

1. How effective is the CTLA intervention in developing IMTs pedagogical strategies to improve teaching and learning of concepts in trigonometry?
2. How can the improvements in teaching using pedagogical strategies that are anchored on cooperative teaching and learning approach be understood and or explained?
3. How can the improvements in students' learning of concepts in trigonometry using cooperative teaching and learning approach be understood and or explained?

Significance of the Study

The study, sought to improve students' conceptions and overcome difficulties mathematics students encounter in learning the concept of trigonometry. This instructional approach will help mathematics educators in general and those who teach trigonometry particularly to understand the nature of the difficulty students come across in learning trigonometry. Hence, creating conditions in the learning environment for students to improve their understanding.

The study will further offer educators, planners of the curriculum, and researchers in the area of mathematics education with rich quantitative and qualitative knowledge about the suitable pedagogical approach for teaching trigonometry. It suggests a new medium of instruction that is premised on a cooperative learning model in learning trigonometry with the ultimate aim of

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removing the bottlenecks that hinder the teaching of trigonometry. This is because literature on the pedagogy that is anchored on the DBR approach in respect of trigonometry is scant and quite inadequate. Again, in reviewing literature only a handful sheds light on the vital findings related to students' levels of understanding in respect of the functions in trigonometry (Brown, 2005; Weber, 2005) even though literature attest to its difficulty as a concept for students (Watson, Jones, & Pratt, 2013). This will help mathematics educators to use suitable teaching strategies for the purposes of improving learners understanding.

In urging continual studies in the area of trigonometry, the findings of the current study will challenge researchers in mathematics education to be abreast of the current pedagogical approach to improve learners understanding of trigonometry. Thus, the procedures in relation to the methodology espoused in the design, the development, and implementation of the instructional approach might be extended by other researchers in other settings working under similar or different context to that of Ghana.

Delimitation

In respect of the area covered, this study sought the services of in-service mathematics teachers in the Central Region of Ghana. The study concentrated on the use of cooperative teaching and learning approach to teach trigonometry. The subject, mathematics as used in the study makes references to the concept of trigonometry at the grade levels of SHS 2 and SHS 3 classes since the teaching and learning of trigonometry exclude SHS 1. The study prioritsed this class since the teaching and learning of trigonometry has been problematic according to literature.

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Limitations

The complementary advantages of quantitative and qualitative evidence were employed to collect data for the study. The main research design used was a case study. The findings and conclusions drawn from this study cannot be absolved from the inherent errors and limitations of the tools used in collecting the data, however the diverse methods were complementary when triangulated; thereby improving confidence in the findings. The case study design allowed for only a limited number of participants (the use of eight IMTs in the development of the intervention) to be used in the current study. Thereby, making generalisation of the findings in this study to a larger population not possible. In particular, a descriptive case study was used with the view to making generalisations and analysis that are crucial for the formulation of design guidelines in this study. Hence, the iterative process, as induced by the DBR, was upheld to ensure that explanations given in the study were "independent of any methodological biases" (Harder, 2012, p. 3). The study was context-bound in that, IMTs considered for the study were selected from only one region of Ghana. This could limit the transfer of the study to another region in Ghana. This placed limitation on the extent to which data, as well as outcomes of the study, could be applied to other areas in the field of study.

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Definition of Terms

Trigonometry: It is branch of mathematics that studies relationships between side lengths and angles of triangles.

In-service mathematics Teachers (IMTs): Teachers with requisite qualification to teach mathematics at the SHS

Cooperative Teaching and Learning Approach (CTLA): It is an instructional intervention anchored on cooperative learning model for the teaching and learning of mathematics.

Organisation of the Study

The study is structured into five main chapters. The overview of the current study, an introduction which highlights on the background to the study, statement of the problem, the purpose of the study, research questions, significance of the study, delimitation, and limitations of the study are detailed in Chapter One. Chapter Two deals with the review of related theoretical, conceptual framework and empirical literature on the instructional use of cooperative learning model into teaching. The methodology employed in the study is presented in Chapter Three. This highlights the research design, research approach (DBR), population, sample and sampling procedure. instrumentation, data collection, and data analysis procedures. Chapter Four presents the analysis of the results and discussions of the findings. The discussions on the findings in relation to the research questions raised in this study are also presented in this chapter. Chapter Five is the final chapter. It offers a summary of the key findings, conclusions, and recommendations.

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CHAPTER TWO

LITERATURE REVIEW

This chapter aims at reviewing literature related to this study. It reviews literature on the study's overarching theory of constructivism, the concept of Trigonometry and cooperative learning. The review also covers concepts on professional development training (PDT) and design-based research as an intervention to support teachers to develop pedagogical competence in teaching trigonometric concepts.

The Difficulties in the Teaching and Learning of Trigonometry

Trigonometry is split into two thus, trigonometry I and II in the core mathematics syllabus and as a major component of the elective mathematics in the SHS mathematics syllabus (MOE, 2010). As such

> Learning trigonometry involves understanding: the definition of triangle; right-angles; recognising them in different orientations; what angle means and how it is measured; typical units for measuring lines; what ratio means; similarity of triangles; how ratio is written as a fraction; how to manipulate a multiplicative relationship; what sin (etc.) means as a symbolic function and so on. (Watson, 2009, p. 5).

Watson (2009) attributes the conceptual difficulty students face in learning trigonometric ratios of sine, cosine and tangent of angles as contingent on the child's pre-existing knowledge of the concept of ratios as sharing. Watson (2009) therefore argues that the gap in transitioning from the student's prior knowledge of ratios to the concept of trigonometry accounts for the student's difficulty. Moreover, the research conducted by Byers (2010) also attributed the inability of students to understand the concept of trigonometry to the

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amount of time teachers spend during the introductory stages of the lesson. As result, students are unable to relate conceptually to the content of the lesson as little time is dedicated to the main part of the lesson. Again, Byers (2010) further reported, that quite a number of classrooms learning environment mediums of instruction follows a pattern, similarly, a number of textbooks also exhibited pedagogical practice in a learning environment in respect of trigonometry. Byers found loopholes in the textbook trails as students graduate from one cycled institution to the other especially from secondary to the college level. In respect of the gaps, Byers observed that, textbooks were fixated on ratios, right triangles, and functions in second cycled schools whereas textbooks used vectors, right triangles, tables, and sinusoidal waves at the college level. Clearly, the inconsistencies identified make the learning difficult for students, particularly when such students have not been exposed to the vector approach and ratios as relationship of lengths or sizes of the right-angled triangle as evidenced in the works of Byers (2010); Watson (2009). As a result, problems abound in the instructional output of trigonometry (Chauke, 2013; De Villiers & Jugmohan, 2012). There is therefore the need for instructional intervention that specifies that successful mathematics teaching is a function of teacher content knowledge, pedagogical knowledge as well as pedagogical content knowledge (Shulman, 1986). In this way, such an intervention with the above embedded traits will narrow the gap of understanding mostly experienced by students in the learning environment.

In reviewing literature therefore, one of the research approaches that guarantees the development of a pedagogical intervention meant to overcome the deficit in the teaching of trigonometry is the concept of design-based

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approach (DBR). This is because in a design-based study, one of the main aims is to develop an intervention. The main critical area to ensure the quality and results of research work is the prioritisation of the design of an intervention in the research project. It is against this background that Brown (1992) asserts that, an intervention that if effective should enable teachers transform their learning environment to be reflective of an average class in terms of students' performance supported by realistic technological and personal support. From the above detailed analogy created, an appropriate pedagogical intervention strategy anchored on the DBR process to improve senior high school mathematics students' conception of trigonometry cannot be overlooked.

Concept of Design-Based Research

A comprehensive definition of design-based research was given as "a systematic but flexible methodology aimed to improve educational practices through iterative analysis, design, development, and implementation, based on collaboration among researchers and practitioners in real-world settings, and leading to contextually-sensitive design principles and theories" (Wang & Hannafin, 2005, p. 6). Again, DBR was defined as "a systematic study of designing, developing, and evaluating educational interventions (programmes, teaching-learning strategies and materials, products, and systems) as solutions for complex problems in educational practice" Plomp (2010, p. 4). These definitions were aimed at advancing one's knowledge about the characteristics of these interventions and the processes to design and develop them. As a result, DBR can therefore be inferred from the narratives as a methodological approach to research.

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Design-based research has been given different names by various researchers. For instance, design-based research is sometimes referred to as Design Studies, Design Experiments, Design Research, Development Research, Developmental Research, Formative Research, and Engineering Research (Andriessen, 2006; Plomp, 2010; Wang & Hannafin, 2005). However, notwithstanding the varied forms of the nomenclatures associated with the DBR, the fundamental principles are synonymous (Wang & Hannafin, 2005).

The DBR process uses quantitative and qualitative as part of its methods (Cohen, Manion, & Morrison, 2007; Creswell, 1994; Wang & Hannafin, 2005) specifically a case study and quasi-experimental designs at diverse stages (Andriessen, 2006; Kelly, 2010; Mejia, 2009; Richey & Klein, 2005) to study the process (Richey & Nelson, 1996) of developing an intervention to improving mathematic students' conception of the concept of trigonometry at the SHS level. DBR has certain key features. The DBR can be classified as developmental. The progressive aspect of the DBR process involves the continuous try out of intervention material with the active participation of IMTs working in collaboration with the researcher. van den Akker (2010) elucidated that the qualitative aspect of an intervention in its practical sense is derived from its content and construct validity.

Literature attest to the various paradigms a particular DBR approach can be situated. In conducting the DBR process, Visscher-Voerman, and Gustafson (2004) list four paradigms that are synonymous to each other. Thus, instrumental, communicative, pragmatic, and artistic. In relation to instrumental paradigm, designers specify their goals and outcomes of the

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product to be designed at the beginning of the process. Thus, there should exist some form of uniformity in the connections among the goals, learning situations and processes as well as the outcomes of the design. Again, in using the instrumental paradigm, one explores and formulates the intended outcomes; this ultimately, translates into tangible goals and objectives that are measurable. In instrumental paradigm, priority is given to the later stages of the processes involved in the choice and design as the medium through which the goals and objectives that are tangible will serve as a blueprint for the product. It employs quite a number of activities that are subjective to evaluations in the design process in relation to other paradigms (Visscher-Voerman & Gustafson, 2004).

For communicative paradigm, in this paradigm agreement is reached between the designers and other stakeholders in order for s designs to be considered as good enough. Consultations among the stakeholders involved in the project marks the initiation of the design unlike other paradigms where there is a review of needs and context analysis (Visscher-Voerman & Gustafson, 2004). Thus, they formulate a pool of ideas for consensus to be reached by the stakeholders through dialogue as to the problem, material constraints, basic pedagogical or psychological premises, and tentative concept of product. Design guidelines are deduced through views of stakeholders on the product, process, and successive versions of the product (Visscher-Voerman & Gustafson, 2004).

Pragmatic paradigm involves processes of designing, try-outs, and revisiting a number of prototypes. In the prototyping stage, a number of try-outs are performed by the end users. In this paradigm, effectiveness of the

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products is obtained through the efficacy of the try-outs by the end users. Again, it is worthy to note that, evaluation that are formative in nature, iterations and a number of revisions of prototypes are characteristic of this paradigm (Visscher-Voerman & Gustafson, 2004).

In the artistic paradigm, the process involved in the design of the product is contingent on the experiences and expertise of the designers. It is therefore implied that the guidelines for the design are as result of the insight of the designer. The designers operate under the assumption that their subjective evaluation is the anchor on which products are judged as good. Thus, reality is constructed by the individuals (Visscher-Voerman & Gustafson, 2004). They further argue that, the selection of any of these paradigms is at the convenience of the designers.

Mingfong, Yam, San, and Ek Ming (2010) assert the following qualities that must be characteristic with an effective intervention. Thus, "frameworks for learning, the affordances of the chosen instructional tools, domain knowledge presentation and contextual limitations" (Mingfong et al., 2010, p. 470). In DBR, emphasis is placed on the time, commitment, and contingencies in the design and development of an intervention. They are consequently documented as part of the research process to afford other researchers the opportunity to achieve a similar result in their context (Mingfong et al., 2010).

A design-based approach is steered through stages as result of which Easterday, Lewis and Gerber (2014) identified the following thus, focus the problem, understand the problem, define goals, conceive the outline of a solution, build the solution, and test the solution as six stages through the

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which the DBR process should be conducted as far as the development of an intervention is concerned in a study. In 2010, Plomp consequently, delineated the preliminary research phase, prototyping phase, and assessment phase as the stages of conducting a DBR. Plomp (2010) outlined the various steps involved in the conduct of the DBR process. Thus, preliminary research phase which entails needs and context analysis, review of related literature, and analysis of curriculum materials as its first phase. The second phase of prototyping which is also constructed fundamentally on formative and evaluation and reflection. These processes are necessary as it ensures the success of the intervention in the likelihood of its failure in the first try out. This warrants some revisions of the intervention in its continual implementation (Antwi, 2013). The third phase is the assessment phase. Summative evaluation activities are undertaken to guarantee qualitative aspect of the intervention. Nieveen et al. (2006) corroborated the assertion of the phases of preliminary, prototyping, and assessment as the requisite steps involved in the conduct of DBR in addition to systematic reflection and documentation stage. The addition of systematic reflection and documentation stage is regarded as a continual activity that takes place throughout the study.

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Figure 1 depicts diagrammatic representation of the three phases of design-based research (Plomp, 2010).

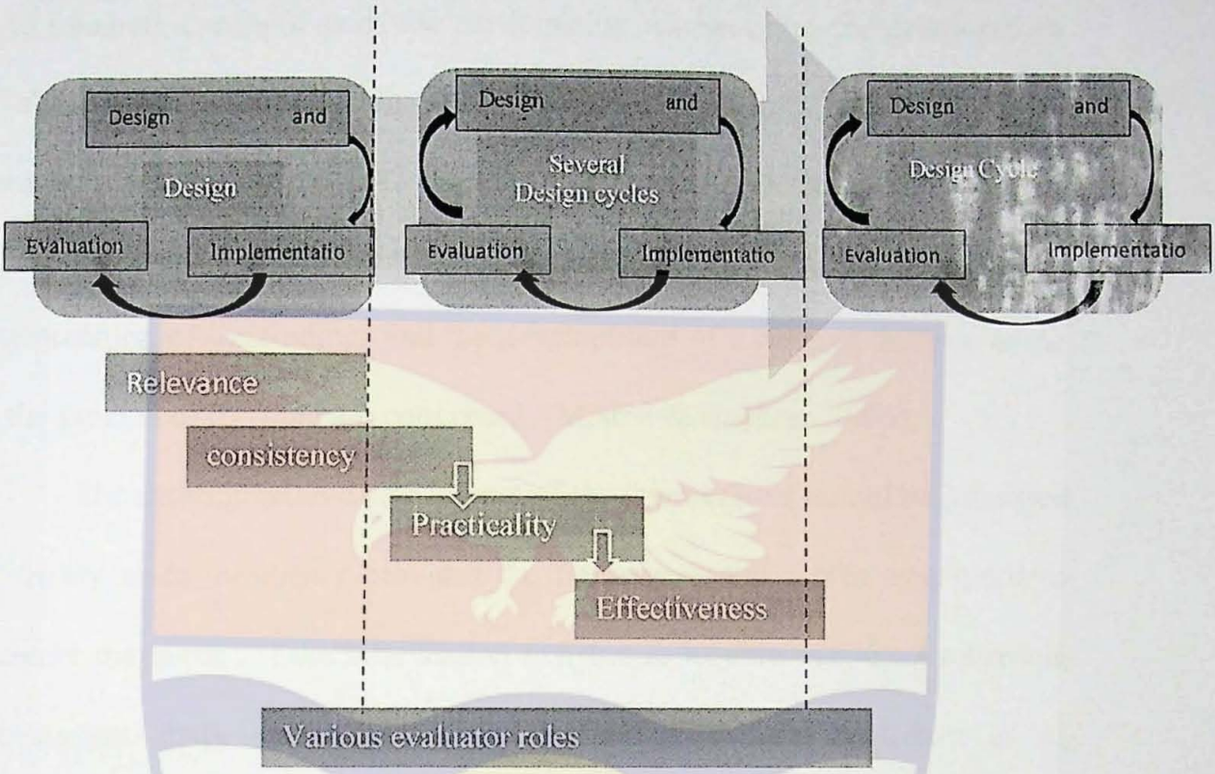


*Figure 1: P*rocesses involved in the three phases of the design-based research. Source: (Plomp, 2010).

The usage of the DBR approach in research is suitable in conditions that necessitate an evaluation of the processes leading to the attainment of the intervention (Antwi, 2013). Again, when it warrants an accurate solution to research questions (Agyei, 2012). Most importantly, when there is the need to come out with challenging educational reforms (Plomp, 2010).

The rationale behind the conduct of DBR, giving the well documented benefits is to develop an instructional approach to teach the concept of trigonometry in part is to bridge the gap in theoretical research and practice (Romme, 2003); make practical and scientific contributions (van den Akker, 2010) to Mathematics education; offer researchers no constraints of the basic research/applied research dichotomy (Keppell et al., 2005); and enhance the professional development of the in-service teachers involved in the study

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(Andriessen, 2006; Plomp, 2010; Romme, 2003) in the area of mathematics education. Again, an equally important reason for adopting DBR for this study is to integrate conceptions of the participating teachers into the development of the instructional approach or products which would be tested in the classroom settings (van den Akker, 2010). Moreover, the rationale for the use of DBR was to provide intervention as result of the application of theory; interpretation of the findings and the development of a class of theories as far as the process of learning are concerned. (Mishra & Koehler, 2006).

The determination of the impact of the intervention should be informed by theory since the theory provides the framework and serves as a guide to measure the impact of the intervention (Mishra & Koehler, 2006). As a result, the current study was informed by the theory of constructivism as its overarching theory. This is because; the study was theorized on an assumption that, the acquisition of knowledge and understanding of trigonometry would be evidenced on the active participation of individual students in the learning environment. This is purposed to make room for the student to make meaning of the subject matter individually and collectively as a group (Prawatt & Floden, 1994). These motives provide a stage for considering theory of constructivism in overcoming the conceptual difficulties students face in learning trigonometry. Hence, the development of an instructional approach informed by the constructivist learning theories in this study through the design-based approach to make the teaching and learning of trigonometry understandable cannot be over-emphasised.

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Theory of Constructivism

Constructivism is a learning theory that involves a learning environment where learners make meaning of the content themselves. It also means that teachers do not directly involve themselves in the learning process but rather guide students to construct their own understanding of the subject matter. In this way learners are the fulcrum around which the teaching and learning process revolves (Bonk & Cunningham, 1998), and teacher's teaching strategies focus on student-centred pedagogical strategies and cognitive learning (Ormrod, 2004).

Constructivism is 'an approach to learning that holds that people actively construct or make their own knowledge and that reality is determined by the experiences of the learner' (Elliot, Kratochwill, Cook, & Travers, 2000, p. 256). Again, Arends (1998) positions the theory of constructivism as a belief in the individual creation of sense making of the subject matter by the student as consequence of experience, and that meaning is influenced by the interaction of prior knowledge and new events.

Constructivism's main idea is that new knowledge is built by learners anchored on their prior learning. Thus Phillips (1995) asserts that, this previous information is what new or revised information of an individual is built from, thus obtaining fresh learning experiences.

There is underlying notion that, the theory of constructivism ensures that, the acquisition of knowledge and understanding is as a result of the active participation of the learner in the learning environment. However, in a passively taught way, the student in the learning environment is viewed as 'an empty vessel' to be occupied with information, whereas it is opposite in the

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constructivist learning environment. Information may be unreceptively established, but understanding cannot take place under such circumstances since it must emanate from the construction of meaningful associations between prior knowledge, new knowledge, and the procedures involved in the learning. As such learning is viewed as the social construction of knowledge and that it is something we do together, in interaction with each other, rather than an abstract concept (Dewey, 1938). The constructivist positions knowledge as existing only within the human brains, and as such it unmatched in the world of reality (Driscoll, 2000). Consequently, the individual picturesque description of his or her world of reality will evolve from their constant development of their innate perception of that world. It must be noted that, the theory of constructivism reinforces a variety of student-centered pedagogical strategies that are directly opposite the conventional system of education, where the sense making of the subject matter is imparted to the learners. Hence, the role of the classroom teacher plays a critical role in facilitation such acquisition of knowledge and understanding. In a typical constructivist classroom learning environment, the teacher serves as the as the coach, guide or a facilitator during instruction as opposed to the instructor role in the conventional learning environment. Again, the teacher creates an enabling environment where students interact with the readily available teaching and learning materials as well cooperative learning-based atmosphere during the teaching and learning process. The teacher ensures that, the students comprehend their previous conceptions, and attends to the activity for the purposes of addressing them and then build on them (Oliver, 2000). Scaffolding is an important feature of ensuring an effective teaching, where

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states knowledge is something that is actively constructed by learners based on their existing cognitive structures. Therefore, learning is relative to their stage of cognitive development" (GSI Teaching and Resource Center, 2015, p. 5)

Thus, pedagogical strategies associated with cognitivist theory helps students in adapting new material of a subject matter to existing information, and empowering them to affect the suitable adjustments to their current knowledgeable framework to make room for such information in the mind of the individual.

Moreover, the social aspect of the theory of constructivism was defined as" learning is a collaborative process, and knowledge develops from individuals' interactions with their culture and society" (Lev Vygotsky, 1978, p. 57). This assertion therefore suggests that, the development of the cultural function of every child seems to occur twice: first, from the child's social level and, later on, at the level of the individual; first, between people (interpsychological) and then inside the child (intrapsychological).

The idea of radical constructivism was advanced by Ernst von Glasersfeld (1974) and avers that, rather than perceiving knowledge through senses, it results from the individual's construction. Students build new knowledge anchored on their existing knowledge. However, radical constructivism stipulates that the knowledge individuals generate communicates nothing about reality to us, but only assist us to adapt to our environment. Thus, knowledge is conceived not exposed. As such, the reality is conceived by the individual is constantly undergoing changes in a space of time to suit ontological reality, even though it can never give a 'true picture' of it (Ernest, 1994, p. 8)

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The tenants of social constructivist theory are achieved through Cooperative, collaborative and participatory forms of teaching and learning. Nonetheless, the current study focusses on cooperative learning as a means of achieving constructivist oriented pedagogical approach to teach trigonometry. According to the Johnson & Johnson model (1998), cooperative learning is an instruction that involves student working in teams to accomplish a common goal, conditioned on the inclusion of the following constructs: Positive interdependence, Individual accountability, Face-to-face promotive interaction, Appropriate use of collaborative skills, Group processing. Again, Slavin and Karweit (1981) averred that cooperative learning can be referenced as pedagogical approach where students conduct their work in small groups or teams purposed to master academic materials and consequently are rewarded for doing well in their groups.

Again, Roger, Olsen, and Kagan (1992) noted to the effect that, the term Cooperative learning is a group learning activity arranged in a manner that allows learning to be reliant on social interactions among students. In this way, group members are held into account and motivated, individually whiles collectively assisting one another in the group in mastering the content of the subject matter. Hence, the concept of cooperative learning cannot be a substitute to a situation where students are merely allocated in group for the purposes of working in activity. Although this methodology is reliably recognized for its efficiency in improving social, motivational and cognitive outcomes of students (Kyndt et al., 2013; Rohrbeck et al., 2003; Slavin, 2014), its application in the learning environment remains a challenge (Sharan, 2010) that needs robust pedagogical strategies (Gillies, 2008). Thus, the study sought

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to improve the instructional approach of teachers in the teaching of trigonometry using the cooperative learning as a medium of instruction.

Again, there is the need for students to appreciate the appropriate use of language to explicate new knowledge and realities which, ultimately affords learners new ways to make meaning of the subject matter in the learning environment (Barnes, 1969; Mercer, 1996). Moreover, when students are engaged in a task in the learning environment, there is the tendency of an increased participation in group discussions, demonstration of more sophisticated level of discourse, fewer interruptions when others speak, and provision of more intellectually valuable contributions (Gillies, 2008; Webb & Farivar, 1999). By encouraging learners to work cooperatively, it leads to the development of the need to consider reaching out to the individual group members to have an ultimate goal of finding solution to the task. Thus, by supporting and motivating one another towards the goal, those who need special assistance are catered for during such interactions in the group (Gillies & Ashman, 1998). Indeed, cooperative learning environments are more relaxed and enjoyable in comparison with traditional classes. This is because, it generates a positive learning environment in the classroom (Roger, Olsen, & Kagan, 1992).

In reviewing the literature therefore, it has been established that a learning environment is said to be cooperative learning-based if and only if such pedagogical technique is anchored on the five main constructs (Johnson & Johnson, 1998). Thus, positive interdependence, face to face promotive interaction, social skills, individual accountability and group processing.

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Positive Interdependence lends credence to the effect that, the student in the learning environment is said to be successful if such students is reliant on the ideas, involvement, and accomplishment of the others in the group. As such, students with sense of positive interdependence exhibit the tendency of reliance on the ideas and contributions of group members and that either they all succeed together or fail (Johnson, Johnson, & Smith, 1991). In this way an opportunity is created in the learning environment where there is dependence on the collective support of the individual group members in the attainment of success in the learning environment. In effect, team members are expected to rely on one another to achieve the set goals of the group. Consequently, a failure on the part of any member in the team leads to an overall reduction in marks scored for the group.

This is one of the pillars which ensure that individual team members in a group are held accountable for their participation in the task assigned to the group and for the purposes of mastery of all of the material to be learned. Individual accountability ensures that students learn together, but perform alone (Johnson et al., 1991). According to them, there are two levels of accountability must be structured into cooperative lessons. This is purposed to strengthen the individual capacities in mastering the subject matter during learning activities in class.

According to Johnson et al. (1991) promotive face-to-face interaction serves as a basic constituent to cooperative learning. It occurs is said to occur in the learning environments where students are afforded the opportunity (time in class) to discuss, ask questions and offer the needed support to each other in the completion of the task assigned to the group. Promotive face to face

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interaction is an important step to establishing cooperative learning in the learning environment. This is because of the provision of feedback via verbal and non-verbal which is a necessity in the success of the group (Johnson et al., 1991).

Under this auspice of group processing, provision is made in the learning environment to afford students, the opportunity to reflect upon the correctness or otherwise of the work as was done by the group. This aids in the determination of the quality of work as executed by the group in respect of the successfulness or otherwise of cooperative learning teams (Johnson et al., 1991). As such, allowing the individual students in the group and the group as a whole some time to reflect on the quality of the task assigned to them will enhance intrapersonal and interpersonal competencies.

Lastly on the tenant of social skills, Johnson et al., (1991) recognises the need for students in the cooperative learning environment to exhibit some competencies for cooperative learning to be effective. They opine that student should be instructed on leadership, decision-making, trust-building, communication, and conflict-management skills. Hence for a successful learning to take place, teachers need to apply the same self-motivated and premeditated teaching of social skills to enhance such competencies in the learning environment (Optiz, 2008).

As a result, of the various scholarly definitions ascribed to the features of the cooperative teaching and learning approach as reviewed above, the study conceptualised these five features for the purposes of developing an intervention that sought to improve the teaching of trigonometry.

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Conceptual Review

To this end, the study reviews the features of cooperative teaching and learning approach-CTLA from the perspectives of the various scholars and as such conceptualises them for the purposes achieving the set objectives in this study.

In deploying positive interdependence, an atmosphere was created to enable the individual students contribute ideas, actively participate and succeed as a group. This was purposed to achieve maximum means of making meaning of the content of the material through the guidance of the facilitator (Johnson et al., 1991). As such, positive interdependence was infused into group activities to ensure that team members have the same objectives and support each other on order to succeed (Johnson et al., 1991). It was in this direction that, members in each grouping elected some of their own colleagues to positions with its accompanying functions. The duties and functions as well as sanctions of these positions were communicated to the entire class before being split into the various groups. The following positions (Team leader, Team recorder and a Team checker) were made elective by each group and the names of such students presented to the facilitator. As such, there was a deliberate effort in the study to elect these positions without the facilitator's involvement in order to inculcate a sense of belonging, responsibility and ownership to the individuals in the group. Again, during the implementation of this feature in the classroom context, instructor's feedback was in the form of a quick follow-up questions to a randomly picked individual member of the group during in-class discussions on the subject matter. This ascertained the level of inclusiveness and the participatory roles individual members in the team

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played towards the resolution of task assigned to a particular group (Johnson et al., 1991). This quick follow-up questions were randomly done to afford members of the group equal chances to respond to questions after group's presentation in class. A randomly picked individual member's correct or incorrect responses to questions, acknowledges the shared responsibility and shared ownership as far as the internal deliberations leading to the completion of the task was concerned. For the purpose of this study, the IMTs use of positive interdependence to teaching was measured in terms of their ability to prepare a lesson in a manner that put students into groups of specific assigned responsibilities on a task.

Apart from, satisfying the condition of inclusivity in completing the task as an indication of the group's collective effort, an individual accountability was employed as a means of awakening non-participating members in the team to sit up and contribute to the internal discussions that ultimately produced the correct solution to the task assigned to the group (Johnson et al., 1991). This was effective since individual team members wanted to avoid being caught unprepared and embarrassed as a result of their non-participation during the group activities. As such, it eliminated if not totally reduced the number of individual team members who ordinarily would have disassociated themselves from the group work and have the task done by the few (Johnson et al., 1991). These activities inexplicably set the tone in demonstrating the power of individual accountability which was a pre-requisite for successful cooperative learning activities throughout the lessons. Moreover, classroom facilitators created a learning environment where activities in the lessons were structured and assigned to elicit responses in two

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levels; first as a group and second at the individual level. In this way, solutions were arrived at the individual level before critiquing each other's work to come up with the final solution reflective of the entire group. This was to enable team members who have superior argument to share ideas with their colleagues so as to reach consensus before a final solution was proffered to the task. This peer review mechanism at the group level were effective since individual members in the group after their participation in the lessons accomplished by themselves tasks which were similar in form and structure assigned them by the instructor. In this way, they learned to do something together so they could do it more easily when they were alone (Johnson et al., 1991). The Individual accountability as a feature of cooperative teaching and learning approach was given a priority in this study, since it was the structural feature needed to daunt and lessen the prospect of free riders or social loafing (Johnson et al., 1991). Since the presence of such individuals inhibits the progress and the individual accountabilities of the group. For the purpose of this study, the in-service mathematics teachers' use of individual accountability to teaching was measured in terms of their ability to prepare a lesson in a manner that ensures students learn together but performs alone.

Further in this study, promotive face to face interaction feature of the CTLA afforded teachers the opportunity to create a platform where students interacted with the readily available teaching and learning materials/resources, tasks and activities in the real classroom context (Johnson et al., 1991). With the group members interactively, providing one another with feedback, challenging reasoning and conclusions, and most importantly, teaching and encouraging one another during the conduct of the lesson enhanced the

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students understanding of the subject matter (Johnson et al., 1991). Hence. reasonable time within the instructional period were allotted for the purposes of creating that enabling environment where individual team members negotiate with the intention of reaching consensus on the correctness or otherwise of a solution to the task. For the purpose of this study, the in-service mathematics teachers' use of face-to-face promotive interaction to teaching was measured in terms of their ability to prepare a lesson in a manner that engages students to share ideas on a subject matter.

In employing the social skill feature of the CTLA in the current study, measures were put in place to inculcate a sense of discipline in the students and ultimately provide good classroom management practices during the teaching and learning process (Johnson et al., 1991). In using this feature, the classroom facilitator positioned students to provide on their own the pre-requisite guidelines (dos and don'ts) needed to facilitate an atmosphere conducive for interactivity on the subject matter during the conduct of the inter and intra group activities in the lesson (Johnson et al., 1991). This ensured proper channel of communication and orderliness in the group. Issues were resolved hierarchically along the chain of these portfolios of elected leadership within the group with the ultimate responsibility resting with the facilitator. For the purpose of this study, the in-service mathematics teachers' use of social skills to teaching was measured in terms of their ability to prepare a lesson in a manner that enable students brainstorm on the Dos and Don'ts (guidelines) prerequisite for successful group discussions/activities

Finally, this study stresses on the processes leading to the resolution of the mathematics problem assigned to the team rather than the final product

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(answer). In this way clarity to the subject matter was sought from two perspectives 1) teacher- where the he/she summarily highlights the core points of the subject matter and offers the explanation in order for the learners to understand and identify the weaknesses in the learners' presentations in the learning environment and 2) students-where the group's solution were presented on the chalkboard for whole class discussions, inputs, alternative superior solution and recommendations if any from the entire class (Johnson et al., 1991). The non-group members (the rest of the students other than those doing the presentation) were encouraged to ask questions in which each of the members of the presenting group was required to be in readiness to respond (Johnson et al., 1991). For the purpose of this study, the IMTs use of group processing to teaching was measured in terms of their ability to prepare a lesson in a manner that promote evaluation of students understanding of the subject matter.

In conclusion, this study was anchored on the social aspect of the theory of constructivism as an approach to teaching. Cooperative learning as an instructional approach (cooperative teaching and learning approach-CTLA) is one of the means through which the theory of constructivism could be achieved. The CTLA has been operationalised in this study as promotive face to face interaction, individual accountability, group processing, social skills and positive interdependence (Johnson & Johnson, 1998). For the purpose of the study, the following operationalisations were made with respect to promotive face to face interaction, individual accountability, group processing, social skills and positive interdependence. Promotive Face-to-face interaction: is characterised by a lesson activity that engages students to share ideas on a

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subject matter. Individual accountability: is characterised by a lesson activity that ensures students learn together but performs alone. Positive interdependence: is characterised by a lesson activity that puts students into groups of specific assigned responsibilities on a task. Social skills: is characterised by a lesson activity that enable students brainstorm on the Dos and Don'ts (guidelines) prerequisite for successful group discussions/activities. Group Processing: is characterised as lesson activity that promote evaluation of students understanding of the subject matter. As result of the detailed description of the features, this study summarily incorporates them from the literature perspectives in an operationalized form as outlined in this study. Thus, in cooperative teaching and learning approach-CTLA, students are placed into groups of assigned responsibilities in respect of the subject matter. They share ideas in a meaningful way in their assigned teams on the concept taught through the activities on the student work sheet, ensuring an active participation of each and every member of the team through the observation of certain agreed dos and don'ts (guidelines) in the team. The result of the group's effort on a task is demonstrated on the chalkboard for the purposes of feedback and consequent modification in respect of errors and misrepresentations committed where necessary.

The responsibility of the teacher in the classroom context in a cooperative environment is that of facilitation. It is important that, mathematics teachers prioritise the holistic development of students during instruction. Therefore, the development of the intellectual capacity of students should not be the overriding goal of mathematics teachers but engaging them in a cooperative learning-based manner to become life-long learners and

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admirers in the knowledge acquisition especially in the area of mathematics is keen. This will ensure that students become independent problems solvers and active team players. literature asserts that cooperative learning advances high-order thinking skills, motivation and improve interpersonal relations as well as the enhancement of motivation and peer relation (Slavin, 1985).

Cooperative teaching and learning approach-CTLA was the pivot on which the lesson was built to provide student centred learning in the classroom context. Hence, this study designed an integrative instructional strategy with the active involvement and contribution of the in-service mathematics teachers and students at the SHS who are the main actors in the implementation of this teaching strategy. Through rigorous analysis of their views and feedback, the administration of research instrument, observation and interviews, the intervention (CTLA) improved the learning outcomes of the students. Thus, a learning experience in this study was said to be cooperative learning-based only when these five features are present. The incorporation of these features in an instructional approach leads to an improvement in learning outcome. For the purposes of this study, an improvement in learning outcome have been conceptualised as one that elucidates its purpose and prioritizes the students in their learning environment and can be thought of as performance-oriented. Its outcome beckons an anticipated level of performance. Thus, a learning outcome in this study can be characterised as an explicit account of what the student is likely to accomplish and the means through which such accomplishment is established.

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Towards the Conceptualization of the Study

Figure 2 presents the conceptual framework for the study. It highlights the study's overarching theory of constructivism specifically, its social aspect of which cooperative learning is a tenant to produce an increased learning outcome in mathematics.

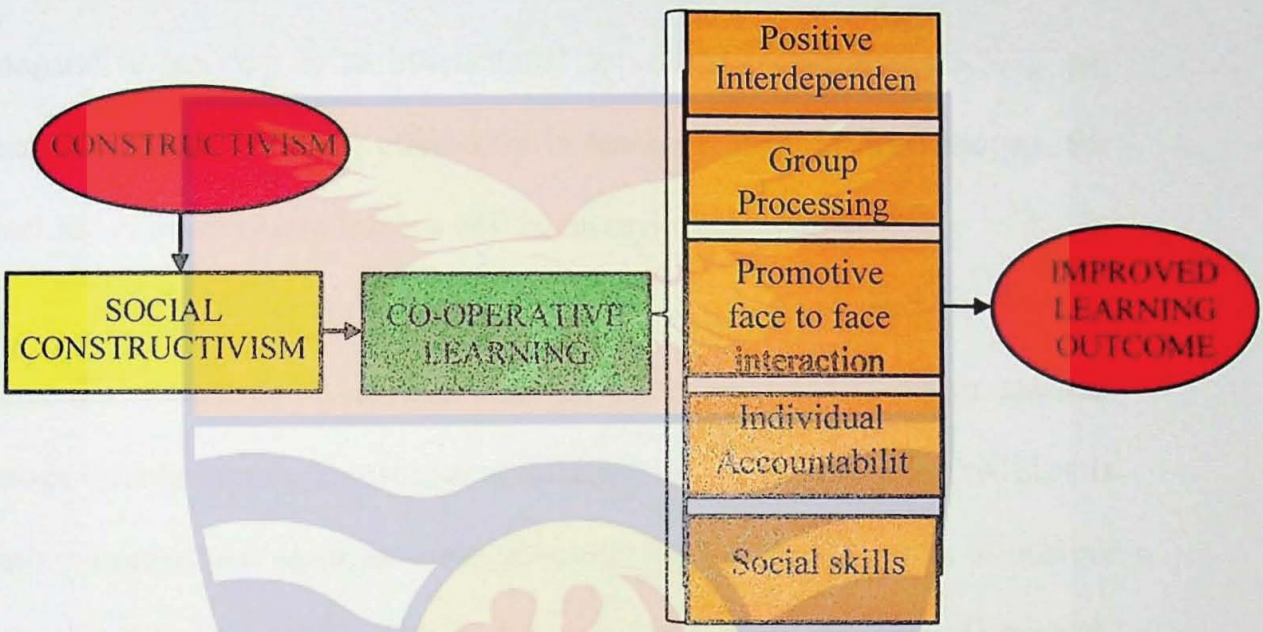


Figure 2: The Conceptual Framework of the Study.
Source: (Author's Own Construct, 2021).

From figure 2, the overarching theory that underpinned the conduct of this study was constructivism. Specifics is made on the social constructivism, which according to Prawatt and Floden (1994) maintain that, the social nature of knowledge, the belief that knowledge is the result of social interaction and language usage, and thus is a shared, rather than an individual experience is what informed the study. As result, a meaningful learning occurs when individuals engage in social activities. Cooperative learning is one of the types of social constructivism which serves as the fulcrum around which this study revolves. Cooperative learning has therefore been operationalised in this study as an instruction that involves student working in teams to accomplish a

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common goal, conditioned on the inclusion of the following features: Positive interdependence, Individual accountability, Promotive face-to-face interaction, social skills and Group processing according to the Johnson et al. (1998) model of cooperative learning. As such, a learning is said to be cooperative learning-based to the extent that the five listed features are present. The framework above illustrates further that, if mathematics teacher's use cooperative learning as an instructional approach (cooperative teaching and learning approach-CTLA) effectively in teaching mathematics concepts, the teaching of mathematics lessons will be more student-centred.

Thus, the study conceptualised that an improvement in learning outcomes would be evidenced from mathematics lessons where teachers design lessons based on cooperative teaching and learning approach-CTLA in such a manner that students make meaning of the materials. It is to achieve this aim that, the current study conceptually discusses these five features of cooperative teaching and learning approach as the anchor on which the intervention was built.

Empirical Evidence

This section highlights the empirical evidence of research findings that are considered relevant to this study. It helps put the current study into perspectives to aid comparison and conclusions of the findings of this study in relation to earlier researches conducted elsewhere. In particular, the review covers topics such as efficacy of the cooperating learning as an instructional approach in the classroom context. In this regard the review was done from the perspectives of both the teachers and students. Finally, the impact of professional development training (PDT) on teacher education was also

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reviewed to ascertain its effect in literature in relation to its initiation in the current study. These were done to aid in the discussion of findings from this study.

Yousef and Younis (2021) conducted their study on the effect of using the cooperative learning method on students 'academic achievement in mathematics for elementary school students. The results of their study depicted an improvement in the learners' academic capabilities in comparison with the conventional method of teaching being used by the regular teachers in the learning environment. Consequently, the study concluded that the use of cooperative learning is superior to the conventional method of teaching.

Akhsanul and Eko (2020) in their study sought to analyse students' self-efficacy reinforcement and their motivation in learning mathematics using the cooperative learning model with the Teams Games Tournament type. The study indicated that a particular type of instructional activity under cooperative learning (Teams Games Tournament) led to the improvement in the learner's abilities by enhancing their understanding of lessons taught in mathematics. This was attributed to the inherent motivation factors of this particular type of the instructional approach employed. As a result, they posited that this particular instructional approach be used in teaching mathematics related courses.

Zakaria, Chin, and Daud (2010) concentrated on Malaysian secondary school students on the effects of cooperative learning on students' mathematics achievement and attitude towards mathematics. The results of the study displayed a rather improved learning on the part of the students in the experimental groups' achievement in mathematics as well as their attitude

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towards studying mathematics. The study also recommended the need for the mastery of the content especially in the areas of mathematics by instructors in their quest to plan and implement cooperative learning well in class.

Shafiuddin (2010) also examined cooperative learning as instructional approach in teaching mathematics. The study was purposed to determine the efficacy of the two pedagogical strategies of cooperative learning and conventional method in learning mathematics at high school level and compare the achievement of the high, average, and low achievers when taught using the cooperative learning method as an instructional approach. The results showed a significant difference between the post-test scores of students in cooperative learning method group and conventional method group. Thus, students perform well when taught through cooperative learning approach. Also, the results communicated that the performance of high, average and low achievers were equal when taught through cooperative learning approach. The study recommended that school curricula should be altered to suit the cooperative teaching and learning approach and as such teachers should be given proper training for the purposes of using cooperative learning as an instructional approach.

Oloyede, Adebowale, and Ojo (2012) conducted a study to determine the efficacy of instructional strategies of competitive, cooperative, and individualistic methods of teaching in the senior high school mathematics classroom context in Nigeria. The findings of their study highlighted a superiority of cooperative pedagogical approach as compared with the rest of the instructions implemented. This is because cooperative teaching and learning strategy was most effective since it facilitated the achievement of

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academic goals and was highly effective at ensuring harmony among students. Consequently, the study recommended for implementation the cooperative teaching and learning strategy as the means through which mathematics could be effectively taught to the understanding and appreciation of the students (Oloyede et al., 2012).

Like Oloyede et al. (2012) and Bukunola and Idowu (2012) also studied the effectiveness of cooperative learning strategies on Nigerian junior secondary students' academic achievement; however, Bukunola and Idowu (2012) concentrated on the basic science. The results revealed that cooperative learning had significant effects on students' post-test academic achievement scores in basic science. Again, the results revealed that the cooperative learning had a significant effect on students' retention of basic science concepts taught. Thus, as student performance improved, their retention also improved. Bukunola and Idowu (2012) recommended that in-service professional development, seminars and workshops should be organised for teachers in order to implement cooperative teaching strategy in schools at all levels.

Whicker et al. (1997) investigated the effects of cooperative learning on students' achievement and attitudes in a secondary mathematics classroom. They averred in their result of the study that, after instructing the students with cooperative learning there were improvement in the students' performance using the pre-test and post-test approach. They therefore contended that, the use of cooperative learning as an instructional approach enhances learners learning outcomes.

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Another study by Javed, Saif, and Kundi (2013) in Pakistan sought to investigate the effectiveness of cooperative teaching and learning approach as an instructional strategy in the teaching of mathematics at secondary school level. The findings showed that, students in the experimental group who studied mathematics through cooperative approach had performed better and obtained greater average than students in control group on the post-tests. Moreover, the effectiveness of the traditional method of teaching and cooperative learning differed significantly in favour of the cooperative learning approach. As a result, the researchers recommended that, cooperative learning approach was the most effective way of teaching mathematics at secondary level since it enables students practically oriented in the learning environment. Javed et al. (2013) insisted that cooperative method is an easy method of teaching and as such should be used often to enhance students sense making of academic materials.

Again, a study by Nurhuda, Lukito, and Masriyah (2018) concentrated on understanding the effectiveness of cooperative teaching and learning approach as a pedagogical strategy on the topic of cuboid and cube volume in mathematics classroom context. From the results cooperative learning as an instructional tool was effective as a result of the appropriateness activities and activeness of the students in the learning environment. Therefore, it was recommended that the cooperative learning as an instructional approach can be used as an alternative to teach various topics in different subjects.

Again, Molla and Muche (2018) focused their study on assessing the impact of cooperative learning approaches on students' academic achievement and laboratory proficiency in biology subject in selected rural schools,

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Ethiopia. Thus, students taught by cooperative learning approach performed better than those who were not as indicated in their results of the study. They contend that, in enhancing students' performance in mathematics related lessons, cooperative learning used as a medium of instruction is critical in achieving such high students' competencies in mathematics.

A study conducted by Ahmadi (2000) assessed the impact of cooperative learning among university students. The researcher investigated the effect cooperative learning has on students' performance, interest, motivation, conceptual understanding, and their attitudes toward mathematics. The results of this study indicated a better superiority of the cooperative learning as an instructional approach in mathematics courses to a more traditional method of instruction. Ahmadi (2000) averred that cooperative learning approach is very dynamic, thus students are more actively involved in the exploration of mathematical concepts and ideas as well as the resolution of specific task.

Finally, in a study conducted by Udu (2018) had findings similar to that of Nurhuda et al. (2018) and Molla and Muche (2018). Udu (2018) aimed at examining the effectiveness of cooperative teaching technique as an instructional approach to enhancing students' academic retention in chemistry. The findings in the study revealed that cooperative promoted better performance and retention than both learning activity package and the lecture method of teaching. This shows the superiority of cooperative learning to promoting students understanding of the subject matter when employed as a pedagogical approach in the classroom context.

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A series of studies (In 'am et al., 2010; Oloyede, et al., 2012; Shafiuddin, 2010; Yousef et al., 2021) on the effectiveness of cooperative learning in improving students' academic performance have been reviewed. It is obvious that all the studies reviewed have some form of coherence, agreement and similarity. This is because all the studies reviewed emphasized the important effect cooperative learning has and its advantage over traditional methods of teaching. Nevertheless, the studies as reviewed (Ahmadi, 2000; Javed et al., 2013; In 'am et al., 2010; Nurhuda et al., 2018; Oloyede, et al., 2012; Shafiuddin, 2010; Yousef et al., 2021; Zakaria et al., 2010) failed to identify the features of the cooperative learning that support content specific pedagogies in their studies. This is because much of the review focused on the effectiveness of CTLA on different subject areas. For instance, on the effectiveness of cooperative learning in teaching English language (Gomleksiz, 2007; Khan & Ahmad, 2013), basic science (Bukunola & Idowu, 2012), biology (Molla & Muche, 2018) and chemistry (Udu, 2018). Tran (2014) also examined the impact of cooperative learning in improving performance in psychology. Although these studies were in different subject areas and cultural context, the findings were consistent.

In the Ghanaian context, Assan-Donkoh et al. (2019) investigated the impact of cooperative learning strategy on mathematics achievements of students. In employing as a medium of instruction to teach the concept of circle theorems, cooperative learning method was used. As a result, they recorded a significant difference in the performances of students taught with and without the cooperative learning. They therefore proposed that cooperative learning be used as a medium of instruction in bridging the

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loophole created among the learners who fall into the categories of high, medium and low achieving capabilities in the learning environment.

Again, Edekor and Agbornu (2020) investigated the effect of cooperative learning strategy on student's performance in mathematics at selected Junior High School in Volta Region of Ghana. In applying cooperative learning as a medium of instruction in the classroom context of their studies, some mathematics lessons were conducted among the junior high school pupils in the municipality. The result of which showed improvement in the students learning abilities in mathematics as opposed to the conventional means of instruction used to enact same lessons. Hence, their recommendation of cooperative learning as the medium of instruction at the basic level of teaching mathematics.

In a more recent study, Bosson-Amedenu et al. (2021) in their analysis of teaching mathematics with the three levels of Jig-Saw, Think-Pair-Share, and Brainstorming, confirmed that when students use Jig-saw learning strategy in Mathematics, their individual academic potentials are enhanced well than when Think-Pair-Share or Brainstorming is used. It was therefore recommended for Jig-saw strategy to be the preferred strategy for learning when mathematics teachers seek to improve deep learning and problem solving among students.

Also, it is significant to note that quite a number of the reviews focused on senior and junior high school students (Like Bukunola & Idowu, 2012; Edekor & Agbornu, 2020; Oloyede et al., 2012), colleges of education (Bosson-Amedenu et al., 2021) while others studied SHS (Assan-Donkoh et al., 2019; Javed et al., 2013; Oloyede et al., 2012; Shafiuddin, 2010) and

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tertiary students (Ahmadi, 2000; Zakaria et al., 2010). Furthermore, majority of the research focused on mathematics, the topics in mathematics studied included algebra and volumes, theorems of circle with little or no attention paid to trigonometry. In relation to context, it is clear that majority of the studies were done in Asian counties such like Malaysia (Zakaria et al., 2010) and Indonesia (In 'am et al., 2010; Yousef et al., 2021) among other countries. There were studies in the African context, for instance in Nigeria (Bukunola & Idowu, 2012; Oloyede et al., 2012; Udu, 2018). Ethiopia (Molla & Muche, 2018) and Ghana (Assan-Donkoh et al., 2019; Edekor & Agbornu, 2020; Bosson-Amedenu et al., 2021). It is apparent that empirical evidence is scant in relation to the design, identification and development of pedagogical approach built on the features of cooperative learning to improve students learning in some content specific areas in mathematics especially in the area of trigonometry. Again, the use of design-based research as a methodologically inclined approach to develop such pedagogies in African contexts particularly from the Ghanaian perspective have been rare if not none existent particularly in designing an intervention anchored on cooperative learning as pedagogical strategy in the classroom context. These denote noteworthy gaps in literature that need to be looked at with empirical research. The study also reviews some research in literature from the perspective of the students on the effectiveness of the cooperative learning as a pedagogical approach in the classroom context.

In a related study Cavanagh (2011) examined students' experiences of active engagement through cooperative learning activities in lectures. The study indicated that cooperative learning used as medium of instruction

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afforded students an opportunity to actively engage the students in a focused manner during learning experiences. As result, the instructional approach positioned the students to enhance their appreciation and understanding of the content of the subject matter assigned to them.

Research conducted by Chen and Chen (2015) also revealed similar findings in relation to the positive perceptions as a result of the implementation of the cooperative learning as an instructional approach in the learning environment. Chen et al. (2015) sought to identify and categorise students' opinion of the use of cooperative learning strategies and the flipped classroom model in an Introductory Statistics course at a university in Taiwan. The findings of the study revealed that all groups identified cooperative learning as an effective instructional strategy in flipped statistics classrooms. Students expressed interest in cooperative learning and saw it as a reliable way of learning. It also identified some impediments, important issues and initial interventions to consider in applying the cooperative learning as a medium of instruction.

Again, Vaughan (2002) examined the effects of cooperative learning on the achievement and attitudes towards mathematics of a group of fifth graders. The students were engaged in 12- week cooperative learning in mathematics. The analysis of pre- and post-test scores revealed positive changes in attitudes and achievement this further underscores the current study's position on SHS students improved scores registered post introduction of the CTLS-based lesson.

It is equally important that other related studies on cooperative learning conducted elsewhere also obtained similar results of favourable learning

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outcomes and improved students' attitudes towards mathematics (Davidson, 1990; Faizah, 1999; Johnson & Johnson, 1994; Ozsoy & Yildiz, 2004; Zakaria et al., 2010). These findings of the studies above, therefore, are congruent with the results of the study as reported by Effandi (2003), Vaughan (2002).

In contrast, Ismail, (2000) found no significant difference in attitudes towards mathematics between experimental and control groups. However, these results could be attributable to the limited duration of three and a half weeks with which the experimental group was subjected to the treatment during the conduct of the study as possible reason for no significant difference between the two groups.

The studies reviewed examined students' perception (Chen et al., 2015) and attitudes (Zakaria et al., 2010; Yousef et al., 2021; In 'am et al., 2010) towards cooperative learning strategies. It is obvious that majority of students have a positive attitude in relation to using cooperative learning and would rather prefer that that their teachers will apply cooperative learning methods in teaching new concepts. The studies reviewed concentrated on management and health science (Phipps et al., 2001), mathematics (Othman et al., 2012), agriculture (Akhtar et al., 2012), English language and English proficiency (Farzaneh & Nejadansari, 2014; Stepanoviene, 2017), and psychology (Reda, 2015). The findings from the studies are mostly convergent although they are from different subject areas and disciplines. Almost all the studies reviewed concentrated on the students' attitudes and perception towards cooperative learning students at the tertiary level and only a few at the second cycled intuitions. It appears that there is a gap in literature in the perception of senior high school students in relation to cooperative learning

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particularly from the Ghanaian perspective, as all the studies reviewed were in relation to achievement test and comparative studies on cooperative learning.

In furtherance to entrenching this instructional approach's efficacy from the perspective of the teachers, the study further reviewed various studies that have also concentrated on the perceptions and attitudes teachers have towards the implementation and use of cooperative learning strategies in the classroom context as a pedagogical approach. This is considered imperative because teachers are regarded by way of the role, they play in the learning environment as facilitators when using cooperative learning as medium of instruction. Hence, their views and attitudes in its implementation in the classroom setting is very valuable in the currents study.

To this end, a study by Jenkins, Antil, Wayne, and Vadasy (2003) which examined how cooperative learning is effective for special education and remedial student. The study further assessed the application of cooperative teaching technique from the perception of teachers. Results from the content analyses of data indicated that teachers were generally positive about cooperative learning's efficacy for students with learning problems, while acknowledging that it worked better for some students than others.

Another study by Ding, Li, Piccolo, and Kulm (2007) studied the effect of intervention in the development of students thinking in mathematics and investigated the perception of teachers towards cooperative teaching and learning. They interviewed six mathematics teachers on their perception of cooperative learning. Participants were drawn from two states in the United States. The results of interviews showed that teachers have a relatively positive view about the effectiveness of cooperative learning. Although the

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number of interview participants was few, it communicates the idea that teachers prefer using cooperative learning to other methods of teaching. They recommend that since teachers have preference for cooperative learning, it needs to be applied more often for the benefit of students.

Koutselini (2008) also applied an intervention study to understand teachers' misconception, understanding and view of cooperative learning. The main aim of the study was to change the understanding and attitude of teachers towards cooperative learning. As part of the specific objectives of the study, the researcher explored participants' attitudes towards cooperative learning, their understanding and why they are hesitant is using it in the classroom. Also, it focused on investigating possible misconceptions of teachers on cooperative learning. A total of ninety-four secondary school teachers in Cyprus were divided into four groups participated in the study. The participants were made up of a total of fifty-eight female and thirty-four male teachers who had two to five-year teaching experience in secondary schools. The intervention initiated in the study succeeded in shifting the secondary school teachers' attitudes and beliefs about cooperative learning from primarily negative to positive. Action research and reflection on experiences and ideas facilitated a dialectic construction of knowledge and a change of attitudes without the threat of imposed theories that cannot be implemented in the actual school environment- a fear that proved to restrict teachers' willingness to try cooperative learning. Along with this fear the lack of experience regarding the differences between cooperative learning and pseudo learning of traditional groups contributed to a number of misconceptions that

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prevent teachers from using cooperative learning as a learning approach with valuable results (Koutselini, 2008).

However, Koutselini (2008) further revealed a negative perception of teachers in relation to applying cooperative learning approach, although this negative perception seemed to get better after intensive training in the application of cooperative teaching and learning approach. An issue that cannot be overlooked is the fact that majority of these studies (Ding et al., 2007; Jenkins et al., 2003; Koutselini, 2008) did not examine the views of teachers in content specific subject. It can therefore be inferred that in overall, there seem to be some inadequacy in terms of empirical evidence from the Ghanaian perspective as evidenced in the review. Particularly, in relation to the design, development and implementation of an intervention anchored on cooperative learning meant to improve students learning on the concept of trigonometry. These constitute significant gaps in the literature that needs to be addressed in order to provide more in-depth information about the use of cooperative learning as a pedagogical approach in the Ghanaian classroom context.

The study conducted by (Johnson, Johnson, & Holubec, 2015; Slavin, 2010; Slavin, 2014) elucidates that, could be employed for the purposes of providing assistance and support to learners who are active cognitively in a direct medium of instruction in formal lessons. The verification of the efficacy in the application of cooperative learning as a medium of instruction is evidenced in research. Cooperative learning is an attributable factor in the fundamental changes in terms of structure from a competitive-individualistic "mass manufacturing" model of organizing to a high-performance team-based

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organizational structure. The activities of cooperative learning could be utilised could be used in all circumstances (i.e., school and school district as well as in the classroom). Moreover, it is the corrective measure for the elimination of individual differences that has inhibited the progress of the students in the learning environment for decades in literature (Slavin, 1991).

Cooperative teaching and learning approach, the implementation of which helps in the exploration of varied abilities of students to enable instructors add to the cognitive base of the students (Johnson, & Johnson, 1998). The competencies of these teachers contribute to the successful implementation of this instructional approach in the classroom context. Hence, the involvement of IMTs in the professional development arrangement (PDA) to effectively implement this pedagogical strategy. This is because authors such as Borko (2004) and Desimone, Garet, Porter, Birman, and Yoon (2002) found out that teacher PDA is an essential mechanism for enhancing teachers' knowledge and instructional practices. In this way, problems that inhibit students' performances psychologically and socially was dealt with through the effective implementation of the PDA and this ultimately increased the wealth of knowledge of the students.

Teacher Professional Development Arrangement (PDA)

The PDA aimed at enhancing teacher's proficiencies in content specific subjects. As such:

> Professional development is about teachers learning, learning how to learn, and transforming their knowledge into practice for the benefit of their students' growth. Teacher professional learning is a complex process, which requires cognitive and emotional involvement of

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> teachers individually and collectively, the capacity and willingness to examine where each one stands in terms of convictions and beliefs and the perusal and enactment of appropriate alternatives for improvement or change (Avalos, 2010, p. 10).

With regard to the focus and main goals of PDA, Guskey (2002) study revealed that PDA brings about changes in teachers' attitudes and beliefs. In recent times, Kazemi and Hubbard (2008) and Opfer and Peder (2011) have stressed the need for PDA since it has the potential to impact many aspects of teachers' professional and personal lives, impacting on teachers' knowledge, competences, and values.

In an extensive survey conducted in the United States with participants (mostly teachers) in respect of other academic disciplines mathematics and science have acknowledged a series of components related to the content and design of PDA programs that teachers tend to value positively. Authors have referred to these features as "features of high-quality PDA" (Bautista, Cañadas, Brizuela, & Schliemann, 2015; Borko, 2004; Desimone, 2009; Darling-Hammond & McLaughlin, 2011; Garet, Porter, Desimone, Birman, & Yoon, 2001). In reference to teachers' self-reported data, PDT activities embedded in these features have positive influence on their acquisition of knowledge and pedagogical practices.

In relation to content of the subject matter, high-quality PDA focuses on the specifics needed for effective implementation of subject matter in the classroom context by the teacher (e.g., mathematics, science, history), thereby equipping the teacher with deeper understanding and appreciation of the subject matter (content knowledge) of pedagogical approach to instruct that

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specific content to students (pedagogical knowledge) and of how students cogitate and make meaning content (knowledge of student thinking) (Carpenter, Fennema, Peterson, Chiang, & Loef, 1989; Penuel, Fishman, Yamaguchi, & Gallagher, 2007).

Chapter Summary

This chapter catalogued the various concepts and theories that grounded the study. It starts with the difficulties in the teaching and learning of trigonometry as a teaching unit in the SHS mathematics curriculum of Ghana. Where literature contextualise its teaching and learning as problematic, with various scholars such as (Farouq, 2017; Gur, 2009; WAEC, 2020) suggesting in literature that the teaching of the concept of trigonometry is fraught with some of difficulty as a result of which there are errors in students understanding. Thus, the pedagogical difficulty that impedes students' conceptual development of the concepts of trigonometry were reviewed.

The study also reviews literature on the concept of design-based research as the research approach to the study. The study adopts the concept of design-based research approach with the view to designing, developing and testing an alternative instructional technique to improve students' understanding of Trigonometry using the cooperative teaching and learning approach-CTLA. It therefore adapts the two stages of prototype phase and assessment phase of Plomp's three phase approach of DBR as the guided approach in the study.

In addition, is the study's overarching theory of constructivism as an approach to teaching. From Phillips (1995) the constructivism theory affords students to be staged at the centre of the learning activity where they can make

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sense of the learning material. Thus, learners are encouraged during instructions to be actively in making meaning of the subject matter in the learning environment.

The study again, reviewed literature on cooperative learning since it is one of the tenants of constructivism. The benefits, its empirical use from the perspectives of the teachers and the students as a methodological approach to teaching were reviewed. In furtherance to this, the study discussed the features of cooperative teaching and learning approach from the perspectives of the various scholars and as such operationalises them for the purposes achieving the set objectives of the study. It ends with a review on the impact of professional development arrangements initiated in similar studies conducted elsewhere.

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CHAPTER THREE

RESEARCH METHODS

This chapter presents the research method that was used in carrying out this study. It specifies the rationale for the methodological approaches used in the research. In particular, the concepts of research design, research paradigm and the professional development structures that the study adapted. The methodology was constructed on design-based research approach. The design-based research approach consisted of prototype stage, implementation and evaluation stage as well as systematic reflection and documentation stage. The study highlights the population, research instruments developed and how reliabilities and validity of the design-based research approach adopted for the study were ensured. The data collection and its analysis procedures under the three stages of the design-based approach were discussed.

Research Approach

The research approach for this study was anchored on the designed based research (DBR). In particular, the DBR serve as receptacles to generate and test theory (Cobb et al., 2003). Hence its use as a methodological approach to establish design principles and theories of a pedagogical strategy aimed at improving the conceptual difficulties students faces in learning trigonometry. DBR is used in many contextualised fields of discipline; however, this study is situated in the context of in-service teacher development in which researcher collaborates with the managers (teachers) of the classroom learning environment to help in the development of a professional community (Lehrer & Schauble, 2000; Stein, Silver, & Smith, 1998). It is against this background that, the study incorporates in-service mathematics teachers at the

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SHS as participants to collaborate the effort of the researcher to develop an intervention purposed to improve the teaching and learning of trigonometry. Thus, making designed based research, highly interventionist in nature (Cobb et al., 2003). The current study, however, adopted two phases of DBR identified by Plomp (2010), as prototyping phase and Assessment phase. Thus, these two phases as identified by Plomp (2010) were modified into a three-staged research approach during the conduct of the study. As a result, these stages were coined in the current study as stage one (prototype stage), stage two (implementation and evaluation stage) and stage three (systematic reflection and documentation stage). Consequently, the preliminary phase of Plomp (2010) was skipped in favour of the last two stages since the current study begun with the proof-of-concept where the problem identification and setting of goals were based on the previous researches and in-depth review of literature (Agyei, 2012; Nihuka, 2011). This seems to suggest that, the preliminary phase of Plomp (2010) was covered through literature review and proof of concept study thus the adaption of the last two phases. In addition, Plomp (2009), "informed by prior researches and review of relevant literature, researchers in collaboration with practitioners can design and develop workable and effective interventions by carefully studying successive version (or prototypes) of interventions in their target contexts, ..." (p. 13). As a result, Plomp (2010) approach of DBR was adopted and modified to include prototype stage, implementation and evaluation stage and systematic reflection and documentation stage in the conduct of the current study. The DBR process that was used for the study is presented in Figure 3 below

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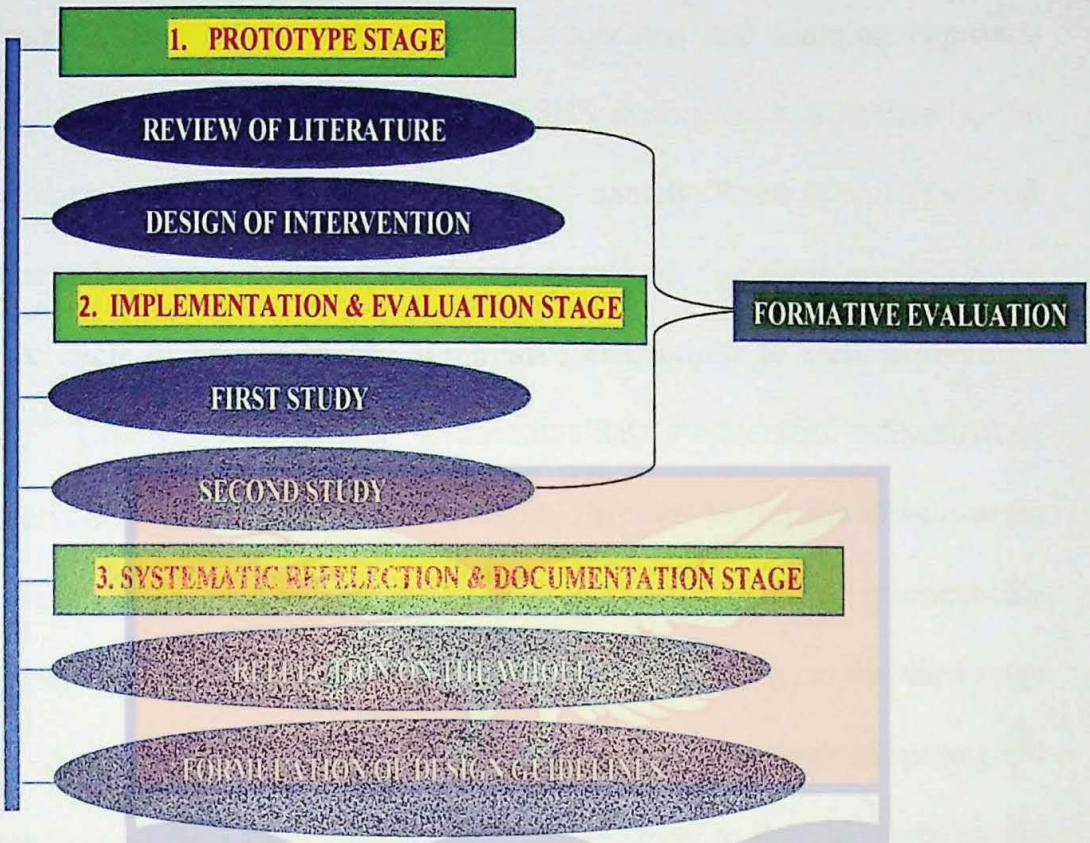


Figure 3: Diagrammatic representation of design-based research process for the study.
Source: (Author's Own Construct, 2021).

Figure 3 shows the diagrammatic representation of the stages involved in the conduct of the study. From the figure 3, the initial process (stage one) started from the prototype stage where through review of related literature, the first two prototypes (exemplary materials- CTLA intervention) were developed by the researcher for expert appraisal, this was purposed to attest to the intervention's relevance and consistency in terms of the scope of the content and the curricular practices in the SHS syllabus in respect of the subject matter (mathematics). The essence of which was to validate the intervention. Thus, initial design guidelines obtained from literature helped in designing the exemplary materials at this stage of the study. The second stage of the study was the implementation and evaluation stage. This stage of the study entails a professional development arrangement (PDA). The rationale

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for the conduct of the PDA was to train and equip eight in-services mathematics teachers to with cooperative teaching and learning approach (CTLA) competencies in the teaching of SHS mathematics. Specifically, the PDA comprised two phases of: 1) first study- namely, Phase 1; and 2) second study-namely, Phase 2. The two phases served as the pivot on which the iterative cycle of formative and summative evaluations as well as redesign revolved. This was done in order to accommodate a systematic refinement of the intervention developed and ascertained. This was to test the effectiveness of the intervention through peer-teaching activities as well as the practicability and utility of the intervention in real SHS classroom. Finally, at the third stage of the study, thus at the systematic reflection and documentation stage. I reflected on all the two stages involved in the study to document the useful design guidelines for possible replication.

Prototype Stage

The first stage (prototype stage) of the study involved three distinct parts of: (1) the review of literature relevant to the study, (2) the design of two prototypes which were first developed by the researcher (exemplary material-CTLA intervention), and (3) the appraisal of the draft prototypes by experts. The researcher was able to come up with the initial design guidelines of the study as a result of the review of related literature. As a result, the researcher developed the first two drafts prototypes as the exemplary material for the study. The draft prototypes were developed based on the theoretical and conceptual framework of the study (features of cooperative teaching and learning approach) (Johnson & Johnson model, 1998) and the design guidelines. The application of the cooperative teaching and learning approach

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(CTLA) was appropriate in that, the CTLA is a corrective measure for the elimination of individual differences, advances thinking skills that are high and improves social relations (See Appendix T-T1&T2). The draft prototypes took into consideration the scope of the content of the Ghanaian mathematics curriculum for SHS. The intervention (draft prototypes) was denoted in the current study as cooperative teaching and learning approach (CTLA) and served as the exemplary material for the participants. For the purposes of enhancing the validity, practice and the quality of the of the two draft prototypes developed by the researcher, they were given to two experts to appraise in the areas of instructional design and mathematics education. These were aimed at improving the quality of the CTLA and inform the construction of the initial design guidelines for the study, the views and suggestions from the experts were relied on. Thus, the appraisal from the two experts was crucial in improving the exemplary material and the initial guidelines of the study. Two lessons that were activity-based on the topics of *the trigonometric ratios of sine, cosine and tangent of angles* and *angles of elevation and depression* from the Ghanaian core mathematics syllabus for SHS were developed at the prototype stage.

Implementation and Evaluation Stage

The rationale for the second stage of the study was to provide a description of the extent to which the intervention that was designed by the in-service mathematics teachers was delivered as intended. Thus, there was iterative stages of formative and summative evaluation (thus, a formative evaluation of two rounds of peer teaching at phase one as well as one round at

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phase two and a summative evaluation at the real classroom context) and re-design of the intervention.

Under this stage, a professional development arrangement (PDA) (see Table 1) was initiated for the in-service mathematics teachers who were involved in the study. The rationale for the conduct of the PDA were partly to train the participants (IMTs) and partly to afford them the opportunity to acquire the competencies needed to use the CTLA in teaching trigonometry at the SHS level. The PDA initiated at the second stage of the study consisted of two phases: 1) Phase 1(namely first study); and 2) Phase 2 (namely second study). These two phases of 1 and 2 were used as the platform to establish an iterative cycle of formative and summative evaluations as well as redesign purposed to refine the intervention gradually in the study. An introductory training workshop served as the first medium of engagement with the IMTs in all of the two phases. There were two aspects of the PDA that was organised for the two cohorts of the IMTs involved in the study. Thus, the first aspect requires the IMTs to participate in an initial training workshop and the second aspect demands that, as a result of the participants' experiences with the exemplary materials, they enact their own CTLA-interventions. Figure 4 gives an account of the activities at each phase of the PDA.

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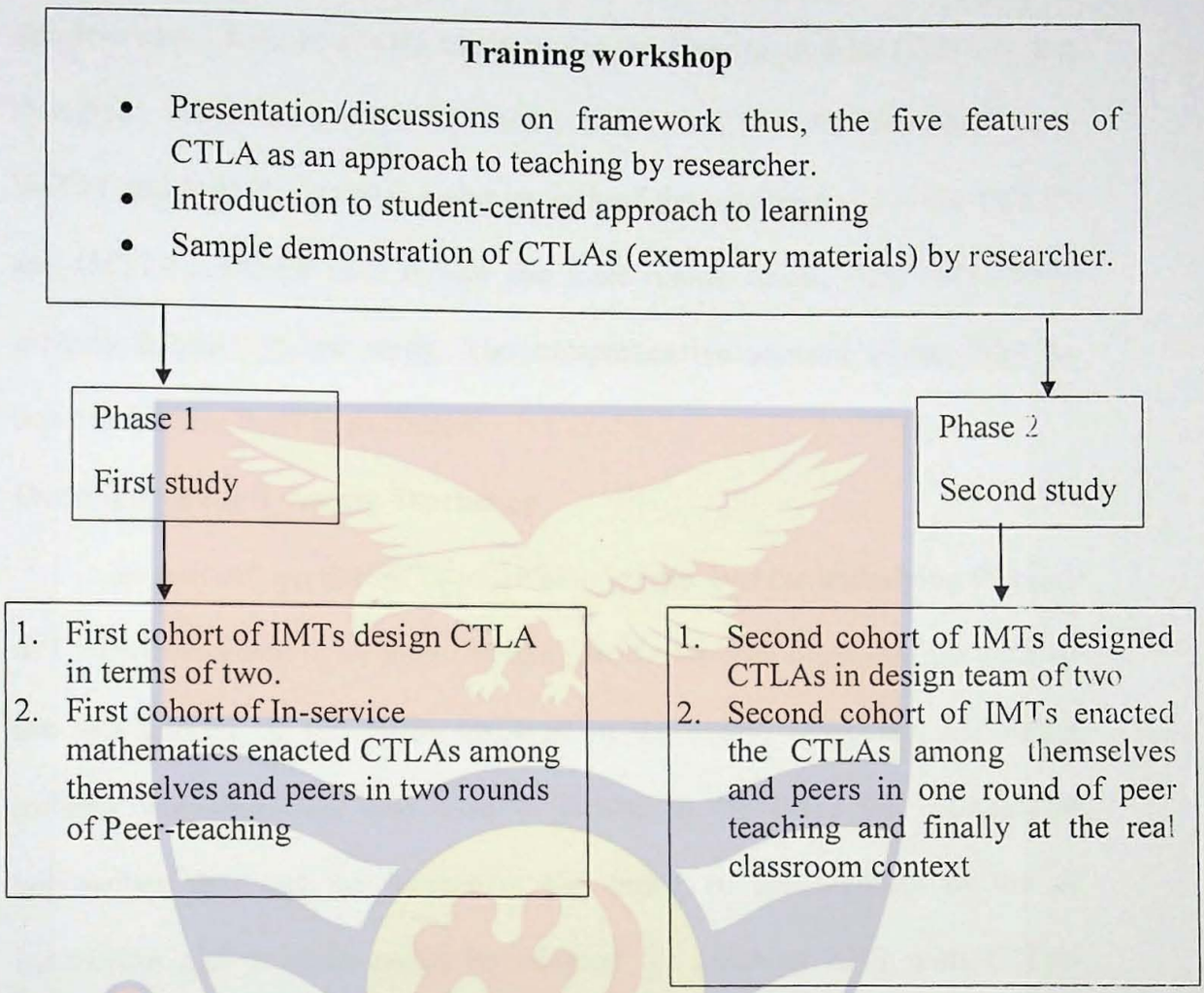


Figure 4: Professional development arrangement adapted for the study.
Source: (Author's Own Construct, 2021).

It is imperative to remark that, the two cohorts of IMTs worked in teams to explore the scope and content of the Ghanaian SHS mathematics syllabus (MOE, 2010) particularly in the area of trigonometry. The participants also worked in teams of two in the design and development of the CTLA intervention. In the first study, the first cohort of four IMTs was split into teams of two: in-service mathematics teachers' team one (IMTT1) and in-service mathematics teachers' team two (IMTT2). The groupings of the IMTs into the two teams were not as a result of any criteria but in the manner the participants sat at the first introductory stage of the PDA. Hence, such arrangements of their first seating arrangements were made permanent

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throughout the entire PDA. Similarly, the last cohort of four IMTs were put into two teams thus, in-service mathematics teachers' team three (IMTT3) and in-service mathematics teachers' team four (IMTT4). In particular, both IMTT1 and IMTT2 had two males in each of the two teams whereas IMTT3 and IMTT4 consisted of a female and three males. In all, eight IMTs were actively engaged in the study. The comprehensive account of the PDA as deployed in the study is presented.

Overview of the Training Workshop

In order to get the IMTs to develop insight into the underlying theories of CTLA-intervention, an initial training workshop was organized for each of the two cohorts of the IMTs involved in the study. Moreover, the initial training workshop was also used to expose to the IMTs the pedagogical approaches that put the learner at the center of the medium of the of instruction and as such could be adopted for teaching SHS with CTLA-intervention in order to make the mathematics learning environment activity-oriented. An overview of the PDA activities for the eight IMTs involved in the current study is presented in Table 1.

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Table 1: Professional Development Training Activities

Week	Facilitator	Topic	Stage
Week 1	Researcher	Overview /rationale of the training	Wk.
Week 1	Researcher	Exposing the IMTs to the conceptual framework of the study	Wk.
Week 2	Researcher	Introduction to Cooperative learning approaches	Wk.
Week 2	Researcher	Introduction of the five features of CTLA	Wk.
Week 3	Researcher	Introduction/demonstration (first teaching) and analysis of the intervention for teaching mathematics	Wk.
Week 4	IMTs	Design of CTLA (intervention) mathematics lessons for selected topics in Trigonometry.	D
Week 5	IMTs	First teaching try-outs by two IMTs among their colleagues/researcher	Wk./e.
Week 5	Researcher/ IMTs	Revision of designed lessons by IMTs based on feedback	Wk./d/e
Week 6	IMTs	Second Teaching try-out by the next two IMTs after possible modifications of the first teaching try-outs to peers/researcher	Wk./e
Week 7	Researcher/ IMTs	Revision of designed lessons by IMTs based on feedback	

Note: Wk. = workshop, d = design, e = execution, IMTs= In-service mathematics teachers.
Source: Fieldwork, 2021.

In preparing the IMTs, presentations and discussions were used as means of interaction to get the participants to appreciate the diverse ways through which these theories underpinned: a) the design of CTLA-interventions with particular emphasis on the concept of trigonometry at the SHS, b) the preparations in relation to pre-lessons with CTLA and c) the pedagogical approach to adopt in deploying CTLA-intervention. Most importantly, the introduction of the five features of the CTLA to the IMTs largely served as the theory that underpinned a teaching strategy that was constructivist oriented. Since, the study's objectives were aimed at exploring IMTs experiences in developing and enacting CTLA-lessons (intervention

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implementation) in a constructivist learning environment. Hence, the theory of constructivism informed this pedagogical approach (CTLA) to teaching. Thus, the IMTs at the PDA were exposed to "*Positive Interdependence, Individual Accountability, Promotive face-to-face Interaction, Social Skills and Group Processing*" features of CTLA as the key ingredients that characterised the pedagogical strategy adopted in this study. This was necessary in developing the CTLA-intervention. Additionally, the introduction and discussion of conceptual framework served as the knowledge needed to practicing an activity-oriented teaching with CTLA.

Thus, discussions in relation to every single feature of the conceptual framework and the relationship between them were extensively exhausted at the initial training workshop. This was aimed to ensure understanding of the features and how they are applied in the learning environment. Again, the workshop facilitated the IMTs appreciation of how the relationships that existed between the features of the CTLA work in other disciplines and ought to inform their teaching in trigonometry. Thus, develop the competencies to advance their pedagogical approach in using an activity based CTLA as a medium of instruction in the mathematics classroom learning environment.

Moreover, in achieving the practical effect of using the CTLA, additional activities were initiated at the workshop stage to get the IMTs to design and develop a comprehensive lesson plans and activity sheets based on the five features of the CTLA. This was aimed at getting the IMTs to familiarise themselves with the operational effect of the five features of the CTLA to achieve maximum lesson presentation in a manner that is student centred and meaningful to the students in the learning environment. As such,

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the participants involvement at this stage of the activities were all geared at ensuring that the IMTs will be able to design on their own a lesson that is activity-oriented using student worksheet sheet and other lesson resources that prioritise the learner as the main focus of the lesson. This was envisaged to get IMTs in connecting ideas from one lesson to another informed by: 1) the selected instructional learning objectives, and 2) the mathematics content as prescribed in the SHS core mathematics syllabus.

Finally, the researcher enacted a sample demonstration of the exemplary intervention (thus, CTLA lessons) in each cohort of IMTs at the initial training workshop. Thus, the two drafts designed at the prototype stage (see Appendix T-T1 and T2). Further, to obtain a practical effect of modeling and demonstrating the application of the features of CTLA as a medium of instruction in the real classroom situation. The researcher taught the two cohorts of four IMTs in two sessions with the exemplary intervention. Thus, the researcher conducted two rounds of teaching try outs with the first and second cohort of IMTs in phases one and two respectively in the current study. The IMTs appreciation of the underpinning principles and conditions for the application of the CTLA were facilitated by the teaching try-outs conducted by the researcher.

Additionally, the teaching try outs conducted by the researcher were purposed to achieve a practical insight on how the relationships that exists in the study's theories and concepts as well as the ideas discussed characterised the development of CTLA- intervention at the initial training workshop. The deployment of CTLA intervention was prefaced on the idea that it has the

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ability to inform IMTs as to the practical implementation of the five features of CTLA to teaching constitutes in the study.

Overview of the Design and Teaching Try-outs of the CTLA-intervention by IMTTs in the PDA

The IMTs developed their version of the CTLA intervention as a result of the experiences with the exemplary materials. They then tried it out with their colleagues serving as students in two rounds. Thus, the intervention was tried out with their colleagues who served as students (teachers teaching SHS mathematics) in two rounds by the first cohort of IMTs in Phase 1, where a member each of the two IMTTs taught.

Similarly, the second cohort of IMTs also had their intervention tried out in two rounds of teaching try-outs in phase two which were initiated first, among their colleagues (i.e., Team members and the student teachers) and secondly, in the real SHS mathematics classroom context. As result of the feedback received from each try out, the CTLA intervention was improved and refined. These iterations were critical in the improvement of the intervention. Thus, the formative evaluation of the design was conducted for all two rounds of peer-teaching at the first study and one round of another peer-teaching try out at the second study as well as the summative evaluation (the introduction of the CTLA-intervention at the SHS classroom) at the last teaching try-out in the second study.

An opportunity afforded the researcher through the feedback obtained from the IMTs development of many detailed designs purposed for enactment of CTLA lessons at the first and second study to improve on the initial design guidelines of the study. The first and second study lasted for fifteen days each.

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Since the studies were scheduled at the convenience of the respective IMTs (in order not to interrupt the flow of their teaching periods), some of the days were postponed to make room for the presence of all the teachers involved in the study. This is because the IMTs were performing their duties at post in the various SHS where the study took place. For example, there were re-scheduled times of appointment with the IMTs since the study coincided with the admission of the first-year SHS students which had just begun and the services of the teachers were needed most of the times. As a result, proceedings in relation to the PDA were delayed with postponement of most of the agreed dates. However, the content deliberated in each of the cohorts in each session of the PDA was the same, although the timelines for the two phases were different.

Reflection and Documentation Stage

Under this last stage, a retrospective analysis was taken from the prototype stage where the intervention was drafted and at the peer teaching try outs activities as well as the introduction of the intervention in the real classroom context with the CTLA-intervention. In a manner that is student-centred with the use of an activity-oriented worksheet (implementation and evaluation stage) of the study. This analysis was purposed to do a thorough review of the last two stages of the development of the intervention and document the final design guidelines for the study.

Thus, the developed competencies of the IMTs in the design of CTLA lessons were instrumental in formulating final design guidelines (theory) that ensures the development of an effective pedagogical strategy (CTLA). Hence, they served as the yardstick for the explanations attributed to the

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improvements in the students' appreciation of the concepts in mathematics especially in the area of trigonometry as well as the improvements in the IMTs pedagogical strategy.

Research Paradigm

The study was grounded in the interpretivist approach (naturalist) of social constructivism as the choice of research paradigm. This study was theorised on the assumption that, the social construction of knowledge is achieved when individuals interact with the readily available teaching and learning materials in their environment and make sense of the environment in which they work and live (Creswell, 2007). The study's epistemological position attributes one's acquisition of knowledge and understanding to the experiences and the meanings they derive from and bring to such experiences. This position informed the current study. Consequently, such meanings would be multiple and varied. Hence, the suggestion that, the IMTs perceptions of the use of CTLA to teach trigonometry at the SHS level and students' experiences with this phenomenon as an attributable factor in this study. Since, there were differences and varied experiences which culminated in multiple realities after the study.

Thus, the current study sought to understand and report same these multiple realities. To accomplish this, qualitative data gathering technique was employed in a cooperative teaching and learning-based context to provide the participants the chance to provide detailed description of their understandings. This enabled the researcher report on the meanings and responses obtained from the in-depth interviews with the participants.

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To this end, the qualitative approach, afforded the IMTs the enabling environment to offer a comprehensive analysis that makes room for a self-retrospection into the manner in which they conceived their methodology of teaching anchored on the cooperative-based teaching and learning approach as pedagogical strategy in teaching mathematics (trigonometry) at the SHS in Ghana.

Research Design

Descriptive case study design was employed since the study sought to explore and understand the meanings IMTs of the participating SHSs derived from their experience of using CTLA in teaching trigonometry. Merriam (1998) asserts that a descriptive case study is one that presents a detailed account of the phenomenon under study and is useful for investigating innovative programmes and practices. The study explored how IMTs of the participating SHS designed, developed and enacted lessons anchored on the CTLA-intervention as an instructional strategy in to improve students learning of concepts in trigonometry. Descriptive case studies are generally descriptive of culture or sub-culture, and also try to find out the key phenomena (Dudovskiy, n. d.). Zainal (2007) account of descriptive case studies as a phenomenon in its natural setting occurring in a data in question is in line with the study's development of an instructional approach in the participants own environment. The aim was to analyze sequences of interpersonal events after a period of time (Dudovskiy, n. d.; Zainal, 2007) to help improve the intervention in overcoming the difficulties in the teaching of trigonometry. Thus, the development of the guidelines for this study was initiated.

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Case study was considered appropriate since this study consolidates the views of students and incorporates valuable inputs from the participating IMTs at the various phases of the study to fine tune the instructional approach so as to yield an increase in learning outcomes. In addition, the main research question for the study was coined in the form of a "how" question. According to Yin (2003; 2009), a case study is appropriate when the study is purposed to give answers to how, why and where questions. Hence the use of case study as the most appropriate research design in helping to explain both the process and outcome of a phenomenon of improving students learning of trigonometry through complete observation, reconstruction and analysis of the cases under investigation (Tellis, 1997). The current study was descriptive in nature since the purpose of the study was to design, develop and enact CTLA lessons as an instructional approach aimed at improving students learning of trigonometry. Moreover, the study sought to achieve this through generation of theories for possible replication by other scholars.

According to Dyer and Wilkins (1991) the employment of case study in research helps researchers to come out with new theoretical relationships and be able to question old ones leading to deeper and better understanding of the phenomenon under study. Further, the study included professional development training as result of which the IMTs from the two different SHS (school X and school Y) were put into two teams for the purposes of developing cooperative teaching and learning-based lessons as an intervention. This is in line with case study with embedded units since the IMTs served as units of analysis in the study. According to (Yin, 2003), the descriptive case study gives the researcher the ability to look at subunits that are located within

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a larger case as it was the case in this study where various teaching try outs under the iterative phases were analysed within a larger case of developing an intervention aimed at improving students understanding of trigonometry. In this regard, the study adopted an Interpretivist approach (naturalist) as the choice of research paradigm owing to the qualitatively inclined nature of the research.

Population

The study engaged two cohorts of four IMTs (thus, professional mathematics teachers who teach mathematics at the SHS) in four out of the ten SHS in the Cape Coast metropolis. Thus, professional mathematic teachers who were credited with Bachelor's education degree in mathematics from recognised tertiary awarding institutions (university level) were used in the study. This is because, they could be said to possess the pre-requisite content knowledge of the subject matter and can bring to bear critical and reflective thinking in their analysis of educational issues in general as well as curricular and pedagogical issues in the study.

This, in a way, can help to improve the teaching of mathematics at the SHS in Ghana. Finally, teachers who met the criteria were known as in-service mathematics teachers (IMTs) thus, those identified as currently teaching core mathematics and elective mathematics at the SHS two and three were given a priority to participate in the study. The gender distribution of the IMTs in the study is shown in Table 2.

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Table 2: Gender Distribution of IMTs Involved in the Study

Gender	N	%
Male	7	87.5
Female	1	12.5
Total	8	100.0

Source: Fieldwork, 2021.

From Table 2, it is evident that majority of the IMTs involved in this study were males with only one of IMTs being a female. The age of IMTs was thought to be relevant to the study therefore the researcher sought to ascertain the mean age of the IMTs. The average age of IMTs in this study was 37 years (SD = 6.33). The minimum age was observed to be 27 years while the maximum age was 42 years. It was found that the IMTs in the study had an average teaching experience of approximately 10 years (SD = 3.85). This goes to show the extent to which IMTs involved in the study had ample experience in teaching mathematics at the SHS level. The SHS based distribution of IMT is displayed shown in Table 3.

Table 3: SHS-based Distribution of IMTs Involved in the Study

School	N	%
School X	4	50
School Y	4	50
Total	8	100.0

Source: Fieldwork, 2021.

Mathematics students in SHS 2 were also involved in the study at the implementation and evaluation stage where the interventions were implemented in a real classroom context to assess the effectiveness and utility

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of the interventions. The gender distribution of the students in the study is shown in Table 4.

Table 4: Gender Distribution of Respondents (Students)

Gender	Frequency	Percentage (%)
Male	45	81.8
Female	10	18.2
Total	55	100.0

Source: Fieldwork, 2021.

Table shows that more than half of the respondents were males, representing 81.8% of the total sample size. The average age of students in this study was 17 years (SD = 1.50). The minimum age was observed to be 15 years while the maximum age was 24 years. From the test of frequencies and percentages, all 55 (100%) of the respondents were SHS 2 students. This is so because the teaching and learning of trigonometry starts from the second year at the SHS (MOE, 2010). Hence, the focus of the study on the second year SHS students. The two SHS, thus, School A and School B are presented in Table 5.

Table 5: Student's Frequency Distribution Based on School Type (School A&B)

School	N	%
School A	29	52.7
School B	26	47.3
Total	55	100.0

Source: Fieldwork, 2021.

I used the IMTs because I believed that their involvement in the study could enhance their ability to use CTLA and augment their pedagogical strategies in teaching mathematics in senior high schools upon completion of the study.

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Sampling Procedure

Convenience sampling technique was employed to select four out of the ten SHSs in the Cape Coast Metropolis in the Central Region of Ghana for the conduct of this study. In particular, the convenience sampling was used in the selection of two SHSs (senior high schools X and Y) for the first study and the last two selected SHS (senior high schools A and B) for the second study at the implementation and evaluation stage of the study. This was because the convenience sampling technique allowed the researcher to select SHS whose infrastructural as well as human resources (both administrative authorities, IMTs and students at the selected SHS) was at the disposal of the researcher for an uninterrupted conduct of the study. This decision is line with Rose, Spinks and Canhoto's (2015) whose opinion was synonymous to the one shared in the current study. Thus, a vital ingredient in a case study design is the researcher's capacity in accessing his/her case easily. Hence, the employment of convenience sampling technique in this study was vital in the acquisition of the rich evidence which led to an improvement in interpreting the phenomenon studied. Again, eight and six professional mathematics teachers from two senior high schools X (N=8) and Y (N=6) respectively were selected by convenience to serve as students (coined in the study as student teachers) during the peer-teaching try-outs stages at the implementation and evaluation stage of the study.

Whereas, purposive sampling technique was used in the selection of the eight IMTs (teachers who were taken through the PDA for the purposes teaching with the intervention) from SHS X (N=4) and Y (N=4) as well as

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fifty-five SHS mathematics students (from SHS A (N=29) and B (N=26)) as participants in this study. In particular, the first four (4) IMTs were purposively selected out of the seventeen professional mathematics at SHS X whiles, the last cohort four IMTs were selected out of fifteen professional mathematics at SHS Y. Additionally, the fifty-five SHS mathematics students were also selected on purpose. This is because the teaching and learning of trigonometry starts from SHS two (MOE, 2012). Hence, IMTs in such classes (SHS 2 and 3) were of interest during the selection process. In addition, purposive sampling technique was considered in the current study since it afforded the researcher the opportunity to tap into the rich information at the disposal of the IMTs. This was a necessity in responding to the research questions (Fraenkel & Wallen, 2006). Again, this technique was employed as result of the "researcher's experience and knowledge "of the IMTs as participants in the study (Kothari, 2004). Thus, the researcher took into considerations certain traits of accessibility, commitment and seriousness as the main criteria for selecting the participants (IMTs). In Ghana, in-service mathematics teachers teach both elective mathematics (elective subject) and core mathematics (general core subject) or any of the two depending on the modalities at the school. Hence, the selection on purpose of the in-service mathematics teachers teaching at SHS 2 was meaningful in the study. This is because the conceptual understanding of trigonometry begins at the second year.

Data Collection Instruments

The instrument (i.e., IMTs Semi-structured Interview Guide) was adapted from Agyei (2012) and administered to the IMTs. The items were

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structured along the following headings: Personal information, Planning and Preparation prior to the lesson, during the lesson, lesson activities, and post-review of the lesson from the perspectives of the IMTs at the end of instructional period during the conduct of the study (see Appendix A). The instrument was administered to all the eight IMTs who taught lessons with the CTLA-intervention. In particular, after each round of the peer-teaching try out and at the real classroom implementation of the CTLA lessons at the implementation and evaluation stage of the study. This was to enable the researcher know the extent to which the IMTs improved in their knowledge and skills of the features of the CTLA in the design, development and enactment of the CTLA lessons.

The SHS Mathematics Students Semi-Structured Interview Guide (see Appendix B) was developed by the researcher and for the purposes of appraisal were given to the supervisors who are experts in mathematics education at the Department of Mathematics and ICT Education, University of Cape Coast. The inputs received were incorporated in the design of the instrument. The essence of which was to improve on the validity of the interview guide. The instrument was administered to the SHS students after participating in the CTLA lessons enacted by the IMTs at SHS A and B. This afforded the researcher an opportunity to know the extent of the SHS students experience and views on the new methodology of teaching (CTLA-intervention) they were introduced to in the classroom context.

The IMTs Pre- and Post-CTLA Self-Assessment Questionnaire was adopted Agyei (2012) and administered to the first and last cohort of the four IMTs prior to and post introduction of the PDA (see Appendix C). This

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instrument was used to track the changes in the IMTs development of knowledge of the five features of the CTLA. This helped in the determination of the IMTs development of knowledge in relation to the CTLA-intervention's utility and practicability in the learning environment.

The SHS Mathematics Students Perceived Effectiveness of the Intervention Questionnaire of the CTLA-lessons designed by the IMTs (see Appendix D) was also measured from the perspectives of SHS students who participated in the lessons. Thus, the instrument (an adaptation from Agyei, 2012) was used to measure the perceived effectiveness of the lessons in the areas of the five features underpinning CTLA-intervention. This questionnaire comprised 20 items grouped under the subscales of positive interdependence (P.I), promotive face to face interaction (F.F), individual accountability (I.A), group processing (G.P) and social skills (S.S). The Cronbach's alpha reliability estimates for all the subscales: I. A (3 items, $\alpha = .62$), P.I (5 items, $\alpha = .80$), F.F (4 items, $\alpha = .75$), G.P (3 items, $\alpha = .53$) and S.S (5 items, $\alpha = .53$) were considered to be reliable; According to Hinton, McMurray and Brownlow (2014) Cronbach's alpha reliability estimates of 0.5-0.75 is moderately reliable for the conduct of a study. Hence, Cronbach's alpha reliability estimates of 0.53 for subscales of group processing and social skills with was considered to be reliable in the conduct if this study.

The SHS Mathematics Students Experience with Intervention Questionnaire (see Appendix E) used in this study, prioritised the student's experiences with the intervention. Hence, the adaption of item from Agyei (2012); Koh (2013) in designing the instrument. The essence of adapting these items were to accomplish two things; (1) to know the extent to which the

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intervention used as the medium of instruction improved and contributed to the student's appreciation of the mathematical concepts taught especially in the areas of trigonometry. (2) to ascertain the extent to which the intervention was perceived to be cooperative based by the students. This questionnaire also comprised 24 items which were grouped under three (3) sub-scales about students' experiences with the cooperative teaching and learning approach-CTLA lessons. The sub-scales were: 1) *Interest* (3 items, α = .83)-this was aimed at finding out if the lessons engaged the learners' attention; 2) *Comprehension* (6 items α = .81)- this was used to determine how clear and understanding the lessons were, and 7) *Delivery* (9 items α = .88 -this was used to determine whether the content was well explained during the use of the CTLA as the medium of instruction. With a reliability estimate of Cronbach alpha ranging from 0.81 to 0.88 the items have relatively high internal consistency and as such were deemed reliable.

The discussions that took place at the focal groups interactions also served as a source of data collection during the teaching try-outs stages of the study. Thus, the peer-teaching activities at the first and second studies of the implementation and evaluation stage of this study were guided by the IMTs Semi-structured Interview Guide (see Appendix F). Discussions were held after each session of the teaching try-outs. These sessions were meant to solicit ideas from the IMTs for the purposes of improving the intervention. Thus, to check the appropriateness of the CTLA-intervention and the extent to which it promoted the teaching of trigonometry. In this way, the weaknesses associated with the intervention were identified and minimised while making

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efforts to maximise its strengths. These comments and suggestions were used as useful feedback which augmented the improvement of the intervention.

Observations were employed to collect data particularly in the course of the implementation of the intervention. The cooperative learning observation protocol (CLOP) was used in the observation of the CTLA lessons and the teacher's instructional strategy (see Appendix G). Hence, its adaptation and consequent modification in the current study in recording and measuring the occurrences of the features of the CTLA-intervention. The instrument has inter-rater reliability agreement of $\kappa = 0.67$. Thus, this is "substantial agreement," which is only second to "almost perfect agreement." (Landis & Koch, 1997, p. 23)

Finally, an achievement test in the form of pre-test and post-tests designed by the IMTs were administered to the students at SHS A and SHS B in their real classroom learning environment. The pre-test and post-test were used to explore the students' conceptual understanding of Trigonometry. The items were constructed based on the learning objectives of trigonometry outlined in the mathematics curriculum for senior high school (MOE, 2010). The items were reviewed by the researcher before its administration for the purposes of ensuring that, the post-test items were similar to those in the pre-test, the structure of the items was different. Thus, providing uniformity in the difficulty levels of the test items constructed by the IMTs. As result, a simple random sampling technique was employed to reach out to 10 students from school (i.e., SHS A and SHS B) to be interviewed after being instructed with the CTLA intervention.

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The instruments used in the collection of data for the purposes of this study were used to address research questions one (RQ1), research question two (RQ2), and research question three (RQ3) as required.

Table 6 presents a summary of the data collection instruments discussed above and their links to addressing RQ1, RQ2 and RQ3.

Table 6: Summary of Data Collection Instruments for Research Questions (RQ)- RQ1, RQ2 and RQ3

Research question	Instruments
RQ 1	IMTs Pre- and Post-CTLA Self-Assessment Questionnaire (For Pre and Post Instructional Assessment)
	IMTs Enacted Lessons (e.g., lesson plan documents and activity sheets)
	Cooperative Learning Observation Protocol (CLOP) (For During Instructional Assessment)
RQ2	SHS Mathematics Students Perceived Effectiveness of the Intervention Questionnaire (For Post Assessment)
	IMTs Semi-structured interview Guide (For Pre-During and Post Instructional Assessment)
	Cooperative Learning Observation Protocol (CLOP) (For During Instructional Assessment)
	IMTs Focus Group Discussion Guide (For Post Instructional Assessment)
RQ3	SHS Mathematics Students Pre- and Post-tests (Achievement test)
	SHS Mathematics Students Semi-Structured Interview Guide (For Post Instructional Assessment)
	SHS Mathematics Students Experience with Intervention Questionnaire (For Post Instructional Assessment)

Source: Fieldwork, 2021.

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Data Collection Procedure

Data collected in this study lasted for five weeks for Phase 1 (i.e., from the 22nd of February, 2021 to the 29th of March 2021) and six weeks for Phase 2 (from the 12th of April, 2021 to the 25th of May, 2021). Data were collected from the diverse phases as delineated in the research approach. In particular, data were collected from the design stage of the intervention as result of the review of the related literature, professional development training and implementation of the intervention via the peer-teaching try-outs initiated at the implementation and evaluation stage (and where the intervention was used as a medium of instruction in the real class room learning environment) after which a retrospective analysis was taken at the systematic reflection and documentation stage The research instruments employed in the current study such as interviews, participatory observations, and document analyses were premised on the assertions made by Andriessen (2006).

Hence, in this study, data were collected through interview, observation, focused group discussion and questionnaires. An interview guide was designed by the researcher after which it was given to the study's supervisors for their input before its administration. The inputs received were incorporated to improve the validity of the instrument designated as Appendix A (see Appendix A and B). The IMTs and SHS interview guides were administered to the eight IMTs and ten SHS mathematics students from SHS A and B at the end of the teaching try outs stages and post introduction of CTLA-lessons respectively in the study.

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In order to augment the data received from the interview. an observation guide, cooperative learning observation protocol (CLOP) was adapted in the study. The CLOP used as an observation tool throughout the second stage (phases one and two) of the study (see Appendices G). This instrument (CLOP) evaluates the overall effectiveness of CTLA used by teachers as they engage students in cooperative tasks in the classroom. For the purpose of this study, the instrument was adapted to evaluate students' engagement in the features of CTLA for effective group and individual student learning of trigonometry. It was used to take a comprehensive note from the field thus; the CTLA features were rated and recorded in the classroom by the researcher. In the column next to the CTLA features, is the rating of the observed indicators that inform the features of the CTLA in a given time interval as, 3- high, 2-medium, or 1-low and 0-non-existent. In the note's column, observational characteristics such as evidence of group discourse, task activity and/or objectives were also recorded. Hence, the use of this tool in rating (high, medium, low, or not existent) the various indicators that informed the presence or absence of the features of cooperative learning (Johnson et al., 2007) at the introduction of the intervention in a real classroom and the peer teaching try out stages of the study.

In addition to the data received at the implementation and evaluation stage, the IMTs intervention such as lesson plans, teaching and learning resources as well as student's worksheet were collected for the purposes of authenticating the information from the other participants in the study. Questionnaires, reviewed from literature, were adopted and modified to suit the Ghanaian socio-cultural context of the study. This minimised ambiguities

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and ensured clarity of expressions in eliciting appropriate responses to the sets of statements on the questionnaires. This enabled the researcher to collect data that measured attitudes of the in-service mathematics teachers and perceptions of the SHS mathematics students about such sets of statements on the questionnaire (Wilkinson & Birmingham, 2003).

Data Processing and Analysis

The study adopted qualitative and quantitative methods. The aim was to draw on the strengths of qualitative and quantitative data gathering techniques to analyse the data received at the diverse phases (prototype stage and implementation and evaluation stage) of the study. This augmented the explanations the study sought to generate in finding a possible solution or new understandings to the conceptual difficulty students face in learning Trigonometry. Hence, making these methods attractive because of the multiple point of integration (Teddle & Tashakkori, 2006).

The study employed the Statistical Package for Social Sciences (SPSS) as a tool in analysing the quantitative data in the research. In analyzing the data, first, scale construction was done to determine the sub-scales (and their reliabilities) on the data collected using the effectiveness of intervention questionnaire and Students' Experiences with the intervention questionnaire. Furthermore, descriptive statistics specifically, means and standard deviations were employed to provide summary of the results. The pre and post-tests adapted in the study were also analysed using descriptive statistics.

The achievement test employed in the study was in the form of pre-test and repeated as post-test in the areas of trigonometry taught. Essay type of questions were set from application three ratios of sine, cosine and tangent of

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an angle in SHS A and compound angles in SHS B. As such, a total mark of 5 was awarded to the items for which marks above 2.5 and below 2.5 were interpreted as favourable (positive) score and unfavourable (negative) score respectively in SHS A, whiles a total mark of 10 was scored with the score of above and below 5 interpreted as favourable (positive) score and unfavourable (negative) score respectively in SHS B. The paired sample *t*-test was used in determining the mean difference between the two sets of observations in each of the SHS. The use of this tool of analysis was employed since observations in the pre-test and post-test conducted in SHS A and B were independent of one another with insignificant outliers in terms of the scores and in addition were found to be approximately normally distributed. Hence the use of paired sample *t*-test to determine the improvement (if any) in the students' pre-test and post-test scores in two SHS A and B who participated in the CTLA lessons.

In observing the lesson, cooperative learning observation protocol (CLOP) was used. In using the CLOP to observe the CTLA lesson, the various indicators that informed the features of the CTLA were rated as high (H), medium (M), low (L) and Non-existent (N). These indicators were coded as 3-H, 2-M, 1-L and 0-N. Subsequently, an average mean of 1.5 was sued to describe a particular feature of the CTLA as above average, while a score below 1.5 indicated below average. The observed data on the CLOP was also in the form of a text, these texts which accounted for the observed features of the CTLA were tabulated and analysed in terms of its frequency of occurrence in the teaching and learning process.

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The data reduction technique was employed in analysing the data received from the IMT semi-structured interview guide, SHS mathematics students semi-structured interview guide and the IMTs focus group discussion guide after being transcribed from audio to text (Miles & Huberman 1994). The themes were identified in relation to the conceptual framework of the study. To this end, IMTs perspectives as themed from interviews and focus group discussions were grouped in line with the research questions. Again, the SHS students' perspectives as themed from the interview were also analysed along the students' experiences with the CTLA intervention. In order to appreciate and understand the processes of development in the conduct of the study, the views of the IMTs and SHS students which were contradictory to the research questions were equally treated as separate information vital to the understanding of the current study. For the purpose of refining, coding and recoding, as well as the identification of opinions contradictory to the study, the groupings of such views from the IMTs were explored. As a result, interpretations to the last groupings of the themes were made. In order to explain the: 1) the pre-and post-tests responses from the students and 2) data accumulated from the IMTs CTLA-lessons (i.e., activity sheets and lesson plans) with particular attention on the CTLA lesson activities developed by the IMTs, document analysis was employed. Tables and graphs were used to present quantitative evidence while themes, verbatim quotations, non-verbal expressions, pictures as well as excerpts from the qualitative data sources were used where necessary to present evidence.

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Validity and Reliability of the Interview Guide

In ensuring the content validity of the questions on the interview guide, the first draft of the guide was given to two experts in Mathematics and ICT education department. The quality of the items in the interview guide was improved upon by the recommendations from the experts. Again, the IMTs were afforded credible opportunity to add where necessary pieces of information they deem relevant to the already given information, this was purposed to ensure internal validity of IMTs interviews granted in the study. Owing to the qualitative nature of the current study, trustworthiness of the study was ensured based on criteria of" credibility", "transferability", "dependability" and "confirmability" (Lincoln & Guba, 1985).

Credibility

In facilitating a deeper understanding of the processes involved in the development of the CTLA intervention, the evidences from the qualitative and quantitative at different stages of the study were analysed for such inference. It is against this background that, data sources at diverse stages and analysis of procedures in the current study were adopted to establish triangulation (Agyei, 2012). Moreover, in ensuring honesty on the part of the IMTs, options were given to the participants be part or otherwise during the selection process. In this way, only those IMTs who were willing participated in the study.

Again, I established a very good relationship with the students and the IMTs involved in the study. This was purposed to make them feel comfortable and appreciated for being part of the study. The IMTs were also encouraged to be honest and impartial during the PDA. Additionally, the researcher adopted an approach of rephrasing the questions posed during the interview segment

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aimed at erasing statements that were inconsistent and unreflective of the views of the IMTs and the SHS mathematics students. The dialogues that were transcribed from audio to text were given to respondents to cross-check for the purposes of confirming if their responses to the questions asked were reflective of their views during the interview sessions, and that, they were the true representations as captured by the researcher.

Transferability

Fundamental to the assertions made on transferability by Lincoln and Guba (1985) is to afford readers a comprehensive overview of the phenomenon under study. This is to allow readers judge for themselves the possibility replicating this study in another context. Although, the current study was time and context-bound, the generalizations of its findings cannot be extended beyond its scope, however, adequate descriptions have been made in relation to the study's settings, the methods applied, and findings so as to afford other researchers the details of this study for the purposes of contextualising similar studies in their settings.

Dependability

A criterion sought in the current study to achieve dependability was sourced from the multiple data collections from the diverse stages in the DBR process adopted for the study (triangulation of data). Thus, there were repeated observed data from in-service mathematics teachers' CTLA lessons and the techniques of coding and recoding employed in the data analysis were all instrumental in realisng the dependability criterion. Moreover, the weaknesses associated with the techniques in collecting multiple sources of data were strengthened. Again, verifications in relation to the consistency of the

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methodology and procedures applied during the conduct the study were checked to be an acceptable process of inquiry by the two supervisors assigned to the study.

In order to achieve a standardised document from this research, the reports were thoroughly reviewed through the entire study. The feedbacks received were used to improve and shape the current document (reports) into the required standard. Again, for the purposes of facilitating further research work, a detailed description of the processes within the study was outlined. Finally, in validating and ensuring the utility of the CTLA intervention. the appraisal from the 2 experts in the areas of instructional design and mathematics education contributed in the validation of the exemplary interventions. Thus, their views and suggestions were incorporated to improve the quality of the CTLA interventions.

Confirmability

In qualitative research, Shenton explains confirmability as a researchers' "comparable concern to objectivity" Shenton (2004, p. 72). Moreover, Guba (1981) also avers that, confirmability is to ensure an unbiased interpretation of the data from the analytical perspectives of the researcher. Thus, in this study, confirmability was guaranteed through the triangulation of the data in order to reduce researcher bias; and as such credit is given where necessary, ideas from other researchers whose works shaped the decisions and the methodology applied in the current study are duly acknowledged and that the findings of the study are reflections of the responses (thus, devoid of my personal impressions) of IMTs and the SHS students involved in the study.

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Ethical Considerations

In observing ethical issues in this study, the study prioritised some precautionary measures in the collection of the data. Amongst them is the clearance sought from the Institutional Review Board, University of Cape Coast (see Appendix K). The views initiated by the board were incorporated in the study which helped in shaping the study. Permission was obtained from the Head of Department of Mathematics and ICT Education, UCC. Consequently, a letter authorising this researcher to embark on the data collection exercises was issued. As a result, the researcher selected the IMTs who participated in the study. In furtherance to this, consent forms were given to the participants with the notion of deciding or otherwise as far as the participation in this study is concerned (see Appendix L). The participants were given the opportunity to be part of the study or opt out after the researcher had briefed them about the rationale of the current study. As such none of the participants was pressured or intimidated to respond to the issues raised in the study. In addition, the participants were immune from the open display of their names, phone numbers or anything that might connect the completed form to the responding IMTs.

Summary of the Chapter

The discussions and explanations in connection with the methodology applied during the conduct of the study have been detailed in this section of the study. The philosophical basis that anchored this study was interpretivism paradigm; as a result, the current study was to a large extent qualitative. In relation to interpretivism posture, inter-subjective approach was the driving force of this study. Thus, ideas that are interpretivist inclined were regarded as

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vital for the development of knowledge and skills with particular attention on the appreciation of the processes delineated. Additionally, the ideas of the DBR approach adapted as well as case study design which is descriptive in nature were used to track for the purposes of understanding the processes of development of the IMTs as they incorporate CTLA as a meaningful and student-centred pedagogical tool for teaching mathematics



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CHAPTER FOUR

RESULTS AND DISCUSSION

An outline for the presentation of the results of the study is discussed in this chapter. It delineates the presentations in the form of tables and themes in response to the research questions in the study. Further, the findings are discussed.

The main rationale for the conduct of this study was to enhance the pedagogical strategies of IMTs through the effective utilisation of CTLA as an instructional approach to improve students learning on the concept of trigonometry in the classroom context particularly, at the SHS in Ghana. This is purposed to make the teaching and learning of trigonometry student-centred and meaningful. For the purpose of the study, CTLA involves student working in teams to accomplish a common goal, conditioned on the inclusion of the following features: Positive Interdependence, Individual Accountability, Promotive face-to-face Interaction, Social Skills, Group Processing (Johnson & Johnson, 1998).

Research Question One (RQ1)

The research question one (RQ1) was "how effective is the CTLA intervention in developing IMTs pedagogical strategies to improve teaching and learning of concepts in mathematics?" The question sought to establish the effectiveness of the CTLA intervention in developing IMTs pedagogical strategies to improve teaching and learning of concepts in mathematics.

The effectiveness of the CTLA-based intervention was conceptualised to mean the extent to which the five features of the intervention (i.e., Positive interdependence, Individual accountability, promotive Face-to-face

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interaction, social skills and Group processing) were reflected in the lessons taught. The following definitions of the features have been used:

1. *Promotive face-to-face interaction* was characterised by a lesson activity that engaged students to share ideas on a subject matter.
2. *Individual Group accountability* was characterised by a lesson activity that ensured students learn together but performs alone (independently).
3. *Positive interdependence* was characterised by a lesson activity that put students into groups of specific assigned responsibilities on a task.
4. *Social skill* was characterised by a lesson activity that enabled students to brainstorm on the dos and don'ts (guidelines) prerequisite for successful group discussions/activities, and
5. *Group Processing* characterised a lesson activity that promote evaluation of students understanding of the subject matter.

The research question was measured and reported from two perspectives: 1) IMTs developed pedagogical strategies as obtained from their reported lessons at the implementation and evaluation stage of the study and 2) Students' perceptions about the effectiveness of CTLA as experienced in classroom context of the study. The two perspectives are further discussed in detail in the next section.

In-service Mathematics Teachers Developed Pedagogical Strategies

To respond to how the two cohorts of four in-service mathematics teachers developed their pedagogical strategies on the eight lessons they

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taught at both the peer teaching and the real classroom situation, data was collected from the IMTs reported lessons, observed data and semi-structured interview to verify whether they were reflective on the features of CTLA. The IMTs lessons (lesson plans and activity sheets) were analysed through document analysis. In using the cooperative learning observation protocol (CLOP) to observe the CTLA lessons, the researcher took notes of the occurrences of the features of the CTLA during the teaching and learning process on the field. Additionally, the indicators that informed the occurrences of the features of CTLA were rated as high (H), medium (M), low (L) and Non-existent (N) during the enactment of the CTLA lessons. Possible scores to the indicators on the CLOP were on a scale ranging from high (3) to non-existent (0). Thus, a score of 1.5 (and above) was used to describe the feature of the CTLA as above average and below average (when the score is below or above 1.5). The field notes from the CLOP together with the text that was transcribed from the audios of IMTs interview were analysed by the identification of themes and patterns in relation to the research question one of this study.

The result showed that the feature of positive interdependence of CTLA-based teaching was realised in the interventions designed and enacted by the IMTs. The evidence gathered from the observed data in the course of the IMTs' implementation of the lesson showed that they were able to engage their students in groups with specifically assigned responsibilities on the task during the teaching and learning process. Table 7 shows evidence of the extent to which the IMTs lessons were perceived to be positive independent.

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Table 7: Rating of Positive Interdependence in IMTs Taught Lessons

Lesson	Team	Feature Observed			Average	
		The lesson includes a wide range of activities where students are				
		Put into groups	Elected into positions	Assigned specific task	Mean	SD
SCHOOL (X & Y)						
Angles of elevation	T1T1	3	3	2	2.67	0.34
Angles of depression	T1T2	2	2	3	2.33	0.41
Trigonometric ratios	T2T1	2	2	1	2.00	0.62
Trigonometric identities	T2T2	2	1	3	2.00	059
Compound angles	T3T1	0	1	1	0.67	0.31
Trigonometric identities	T3T2	3	3	3	3.00	0.72
SCHOOL (A & B)						
Compound Angles	T4T1	3	2	3	2.67	0.49
Application of trigonometric ratios (angles of elevation)	T4T2	3	3	2	2.67	0.58

High-3, Medium-2, Low-1, Non-existent-0, SD-standard deviation
Source: Fieldwork, 2021.

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From Table 7, an overall mean ($\boldsymbol{M} = 2.25$, $\boldsymbol{SD} = 0.51$) for positive interdependence was registered at the peer teaching phases in schools X and Y as well as real classroom situation at schools A and B at the implementation and evaluation stage depicting an above average score for the indicators that informed this feature during the teaching and learning process. Thus, IMTs use of this feature was rated high during the lessons on angles of elevation, and trigonometric identities which depicted a complete observation of the indicators on the CLOP. For instance, at school A, the IMT created the platform for students to elect themselves into the positions of a leader, recorder and time keeper after the students had been put into groups of five as displayed on the CLOP. Figure 5 shows excerpts of positive interdependence on the CLOP.

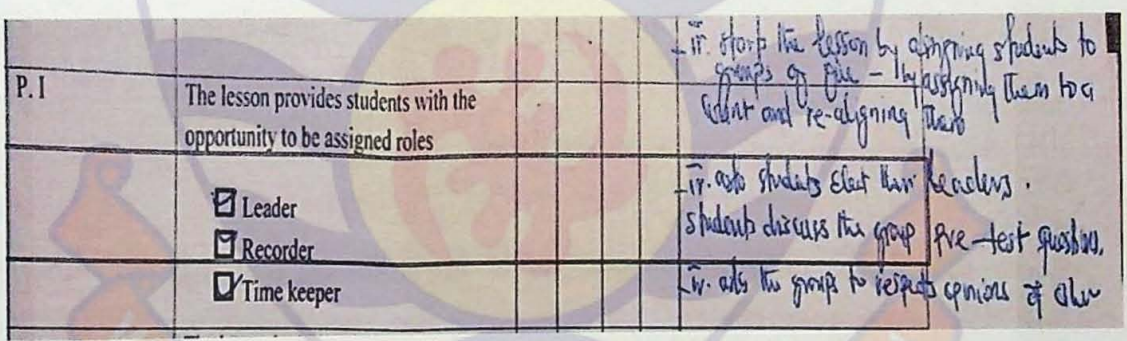


Figure 5: Excerpt of positive interdependence on the CLOP.
Source: Fieldwork, 2021.

However, a "medium" and "non-existent" implementation of some of the indicators of this feature was observed during the lessons on angles of depression, trigonometric identities and compound angles. From the above analysis on the frequency of occurrence of positive interdependence in the lessons taught, it was observed that all the four teams of IMTs planned their lessons to be reflective of this particular feature of positive interdependence in such a way that students were put into groups of specifically assigned

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responsibilities on a task according to the total number of students in the classroom. For instance, in teaching a lesson on *compound angles* at school B, an assignment of students into groups of four (as a result of fewer number of students in the class -26) were observed. This translated into six groups of four students with two out of the six groups having a total number of five students each during the conduct of the lesson on the *compound angles*. Again, at school A, it was observed at the introductory part of the lessons on *angles of elevation* that the students had been re-aligned into groups of five after the IMT shared the lesson's instructional objectives with the students. There were five groups of five students each with the sixth group recording four students in that group to make up for the students' numerical strength of 29 in the classroom. Consequently, in each of the lessons observed, the students elected themselves to the roles of a leader, the group recorder and time keeper. This division of labour in the group was observed to have kept the respective members of the group focused on the task as was assigned to the group by the IMTs. I also observed that adequate time was allowed to enable the group members think through the questions and tasks before the teacher called on the group leader to respond to the question on behalf of the group. This ensured an evenly distribution of task within group members. For example, on a particular task on *angles of elevation* during the lesson at school A, students were observed to be engaged in a series of activities in the group. While, the inverse functions of the trigonometric ratios as well as the ratios of the sides of adjacent, hypotenuse and opposite to the angles were computed with a calculator by a student in the group, another student in the same group was observed representing the concept of angle of elevation in a right-angle

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triangle form. I further observed that another group member was assigned to record the final correct solution after consensus on the alternate ideas shared in the group had been reached and finally the leader of the group was tasked to explain to the entire class the basis of their solution to the angle of elevation when the group was called to present their work on the chalkboard by the IMT. As such, the individual task led to the mastery of the content of the concept in applying trigonometric ratios to find angles of elevation. In this way, difficult task which could not be answered individually was solved and eloquently articulated during the real classroom teaching experience after the students were put into groups. Figure 5 provides evidence of students working in groups on task.



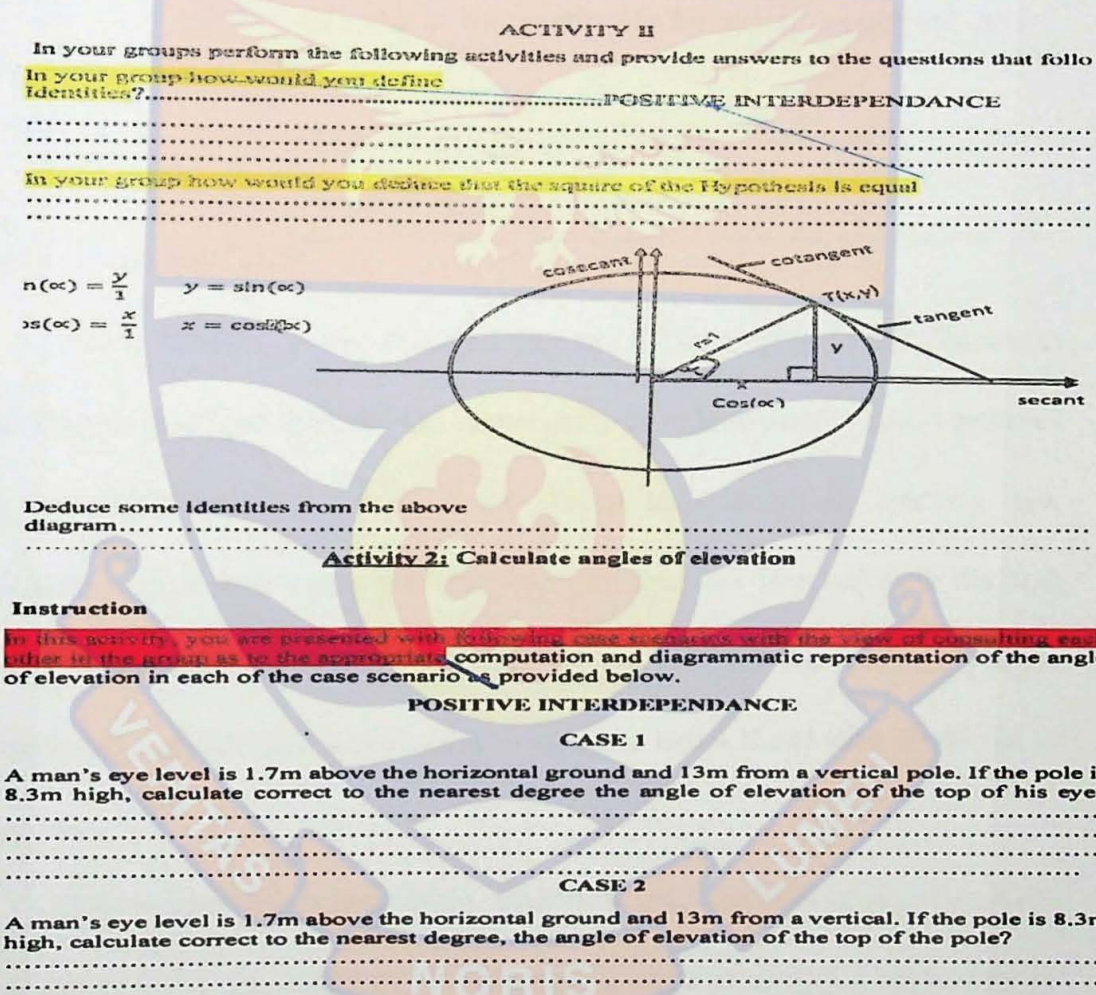
Figure 6: Pictures of students working in groups on a task.
Source: Fieldwork, 2021.

Analysis of the activity sheets (component of the lesson plan document) also revealed that the lessons promoted the placement of students into groups of assigned responsibilities in the teaching and learning of *trigonometric identities* and *angles of elevation* respectively in SHS X and SHS A where the intervention was introduced. For instance, the activity sheet component of the lesson plan on *trigonometric identities* and *angles of*

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elevation were designed purposefully to be reflective on positive interdependence feature of the CTLA. This was to afford students in the groups the opportunity to brainstorm on the topics introduced by the IMTs with the view to finding solution to the task. Figure 7 shows the potions of the activity sheet that promoted positive interdependence in the lesson of *trigonometric identities* and *angles of elevation* respectively.



ACTIVITY II

In your groups perform the following activities and provide answers to the questions that follo

In your group how would you define identities?.....POSITIVE INTERDEPENDANCE

.....

.....

.....

In your group how would you deduce that the square of the Hypothesis is equal

.....

.....

$\sin(\alpha) = \frac{y}{1}$ $y = \sin(\alpha)$

$\cos(\alpha) = \frac{x}{1}$ $x = \cos(\alpha)$

Deduce some identities from the above diagram.....

.....

Activity 2: Calculate angles of elevation

Instruction

In this activity, you are presented with following case scenarios with the view of consulting each other in the group as to the appropriate computation and diagrammatic representation of the angle of elevation in each of the case scenario as provided below.

POSITIVE INTERDEPENDANCE

CASE 1

A man's eye level is 1.7m above the horizontal ground and 13m from a vertical pole. If the pole is 8.3m high, calculate correct to the nearest degree the angle of elevation of the top of his eye?

.....

.....

.....

.....

CASE 2

A man's eye level is 1.7m above the horizontal ground and 13m from a vertical. If the pole is 8.3m high, calculate correct to the nearest degree, the angle of elevation of the top of the pole?

.....

.....

Figure 7: Excerpt of activity sheets showing positive interdependence in the lessons of trigonometric identities and angles of elevation.
Source: Fieldwork, 2021

Similar observations were made from the interview data. The IMTs responses in respect of the question: "Describe how the placement of students into groups facilitated the lesson in teaching mathematics" confirmed this. The results are reported in Table 8.

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Table 8: Excerpt of Positive Interdependence on the CLOP

IMTs	Responses
T1T1	This made the students realise the need to engage one another and learn new ways of solving questions from their friends through explanation.
T2T2	Grouping the students brought about the sharing of ideas and knowledge acquisition to promote effective learning among students.
T3T1	Those assigned roles were focused on the roles assigned them, this made the work of the group organised and smooth.
T4T1	They learned from each other. The brilliant ones helped the weak ones in each group.

Source: Fieldwork, 2021

From Table 8, it was revealed that the IMTs employed this feature in their lessons to afford students the opportunity to independently make sense of the new materials as a result of the shared experiences in learning new approaches from their colleagues through explanations in relation to the task assigned to the group. As a result, this feature enabled the IMTs to get the students who understood a particular concept to assist those who could not in the group.

The results from observation, interview, and analysis of activity sheets all attest to IMTs placement of students into groups of assigned responsibility during the conduct of lessons in the study. Thus, an application of positive interdependence in the study created a platform that allowed students to bring to bear their collective prior knowledge on the subject matter and consequently, positioned teachers to provide opportunities to students to offer complementary ideas on the task assigned to the group in order to make sense of the subject matter. This is because assigning students to groups in the

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teaching and learning process promotes students understanding of the subject matter taught.

In the case of the promotive face-to-face interaction feature of CTLA-intervention, the design and implementation of all the two cohort of four CTLA lessons for mathematics showed that the IMTs were able to ensure in their lessons that students engaged themselves in a divergent knowledge expression on the concepts taught. Thus, students shared ideas by constructing their own meaning on various concepts in mathematics during the teaching and learning process in the study. For instance, in enacting lessons on *angles of depression* in school Y the IMT was observed to have engaged students in the groups to develop the correct diagrammatic representation of the of case scenarios on angles of depression presented to groups in a form of task.

As a result of the ideas shared, multiple representations in the form of right-angle triangle were deduced as the representation of angle of depression from the case scenarios and different trigonometric ratios were applied in finding solution to the task. Thus, the students were observed in their various groups reasoning, challenging, piecing their individual thoughts on the processes (calculations) leading to the correct solution to the task. Table 9 shows evidence of the extent to which the IMTs CTLA-lessons were perceived to be reflective of promotive face-to-face interaction feature in the conduct of the lessons.

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Table 9: Rating of Promotive Face to Face Interaction in IMTs Taught Lessons

Lesson	Team	Feature Observed			Average	
		The lesson provides students with the opportunity to				
		Share ideas on the task	Interact with student's worksheet	Stimulate each other to draw out new ideas	Mean	SD
SCHOOL (X & Y)						
Angles of elevation	T1T1	2	2	1	1.67	0.45
Angles of depression	T1T2	2	2	2	2.00	0.52
Trigonometric ratios	T2T1	3	3	2	2.67	0.62
Trigonometric identities	T2T2	2	3	3	2.67	071
Compound angles	T3T1	1	0	1	0.67	0.49
Trigonometric identities	T3T2	3	3	2	2.67	0.84
SCHOOL (A & B)						
Compound Angles	T4T1	3	2	3	2.67	0.62
Application of trigonometric ratios (angles of elevation)	T4T2	3	3	3	2.67	074

High-3, Medium-2, Low-1, Non-existent-0, SD-standard deviation
Source: Fieldwork, 2021.

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From Table 9, an overall mean ($\boldsymbol{M} = 2.13$, $\boldsymbol{SD} = 0.62$) for promotive face to face interaction was registered at the peer teaching in schools X and Y as well as real classroom situation at schools A and B during the implementation and evaluation stage. Thus, IMTs use of this feature was rated high during the lessons on trigonometric identities, trigonometric ratios and compound angles which depicted a complete observation of the items on the rubric. Figure 8 shows excerpts of this high rated observation of promotive face to face interaction as noted on the CLOP.

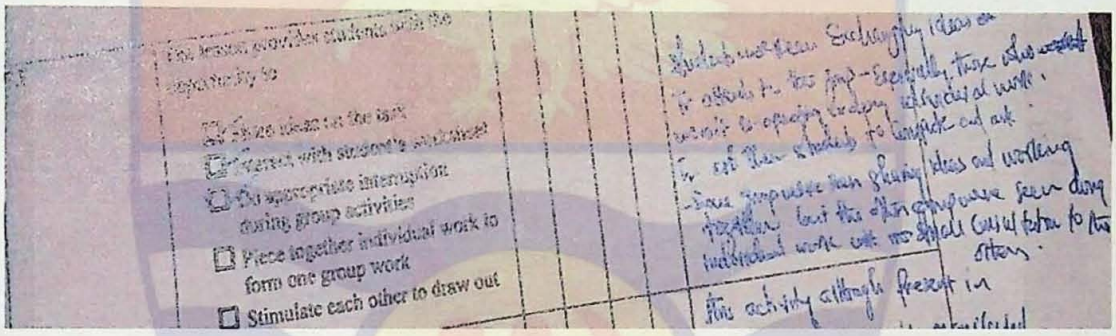


Figure 8: Excerpt of promotive face to face interaction on the CLOP.
Source: Fieldwork, 2021.

Nevertheless, a" medium" and "non-existent" observation of some of the indicators of this feature was realised during the lessons of angles of elevation and depression as well as compound angles. For instance, at the peer teaching stage at school X in the lessons on angles of elevation and depression the IMTs did not fully engage the students on the need for consensus building on the concept taught after the students had been put onto groups during the conduct of the lesson. An analysis from the above indicates that, the lessons enacted by the IMTs were reflective on the promotive face to face interaction feature of the CTLA. Thus, teachers observed in the various lessons provided sufficient time and opportunities for students to share ideas on the various concepts taught. For instance, it was observed at school X during the teaching

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and learning process on the concept of trigonometric identities that the IMT through the application of this feature engaged students to use the concept of Pythagoras theorem to deduce the trigonometric identity of $Sin^2x + cos^2x = 1$. Figure 8 shows images of students from school A and school B sharing ideas on *angle of depression* and *trigonometric identities* together in groups.

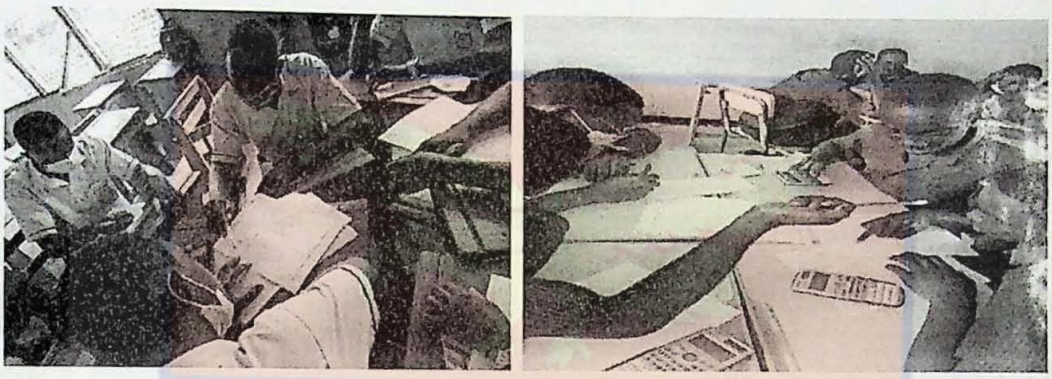


Figure 9: Pictures of students sharing ideas in teams on task.

Source: Fieldwork, 2021.

Similarly, from the interview conducted for the IMTs, it was revealed by the respective IMTs that the CTLA-lesson promoted face-to-face promotive interaction feature during the teaching and learning process among the students. This opinion was elicited in response to the question: "In what way did the lesson activities encourage learners to share ideas by constructing their understanding of the concept taught?"

The collective decision was that the CTLA-lesson provided opportunity for students to share ideas on the concepts taught. Some specific responses gathered from the IMTs have been summarised as;

T1T1: *The students in the group were able to correct each other's errors and it helped each of them to improve.*

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T2T2: *Learner's cooperation and discussion cleared some mathematical misconceptions formed by their colleagues during group activities.*

T3T2: *Students with misconceptions about certain aspects of the topic got support from their colleagues and upon consensus building came out with the best.*

T4T1: *It was made easier for one to acknowledge his strengths and weaknesses since one's fault could be easily identified and corrected by his fellow counterpart in a group.*

In the same way, an analysis of the activity sheets (component of the lesson plan) used by the IMTs in their lessons show clearly that the CTLA interventions for teaching mathematics projected more of the promotive face-to-face interaction feature in comparison with other features of cooperative teaching and learning-based activities in a lesson. This is because the teachers designed most of the activities in such a way that show students were motivated to brainstorm on the concepts before responding to task on the activity sheet. Students working in groups in the classroom were afforded the opportunity in the lesson to share ideas on a task. A further analysis of the activity sheet anchored on the promotive face to face interaction feature of the CTLA created conducive atmosphere in the learning environment where students challenged each other's opinion, interacted with the readily available teaching and learning materials as well as their colleagues during the teaching and learning process. This motivated students to accept each other's strengths and weaknesses in the learning environment and collectively assisted each other to improve on their weaknesses as team members. Figure 9 shows some

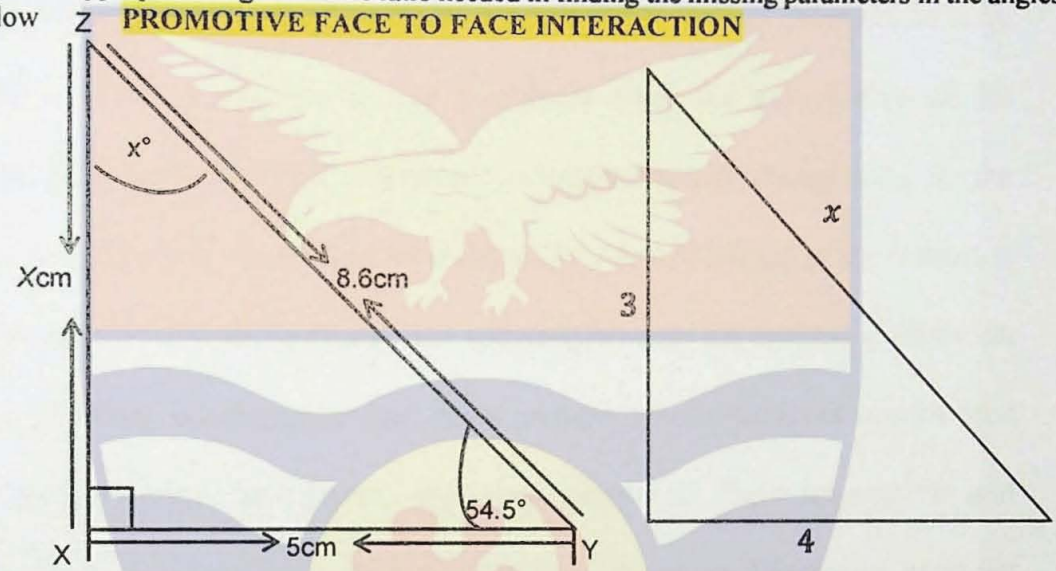
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excerpts of the IMTs designed CTLA-interventions that portrayed activity-sheet through face-to-face promotive interaction efforts among students in the learning environment in all the four lessons taught.

CORE MATHEMATICS (SHS 2)
THE ANGLES OF ELEVATION
Student's Activity worksheet sheet

Activity 1: **Determining the appropriate trigonometric ratio in a right-angle triangle.**
Instruction
In this activity, you are presented with the diagrams to enable you ***share ideas*** **in your groups as to the appropriate trigonometric ratio needed in finding the missing parameters in the angles below**



.....
.....

Figure 10: Excerpt of activity sheets showing promotive face to face interaction in lesson.
Source: Fieldwork, 2021.

Based on the evidence from observation, interview, and analysis of activity sheets as a component of the lesson plan point to a projected feature of the promotive face-to-face interaction in a manner that allowed divergent views of the students to be shared for the purposes of reaching consensus on the appropriate solution to the task in the lessons taught. This is because, promotive face to face interaction provides the platform for teachers to situate the learner as the focus of the activities that are purposed to enhance understanding of the subject matter in lessons.

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The Social skills feature of the cooperative leaning-based intervention focused on how the in-service mathematics teachers designed CTLA-interventions to connect the lesson to a conducive learning environment. Thus, a learning environment where doing mathematics was not intimidating, contrary ideas were respected and mathematics classroom teachers facilitated concept development under instruction. For instance, during the teaching and learning on the concepts of *angles of elevation*, the IMT was observed to have fostered a whole discussion on the guidelines from the perspective of the students that facilitated and promoted successful group discussions in the lesson. As a result of the student adherence to these guidelines in the lesson, it was observed during the teaching and learning process on *angles of elevation* that an enabling environment had been created where students contributed freely to the group discussions and such provoked their innovative and creative skills in leaning the concept of angle of elevation. Moreover, as result of the application of social skills feature in the classroom context of school A. It was observed that students were able to abstract, generalize and apply the three trigonometric ratios sine, cosine and tangent to find solution to the angles on elevation in the case scenarios (activity-sheet bearing questions on angles of elevation) provided in class. This implies that lessons that are social-skills-oriented equip students to respect the views of their colleague in the group, share ideas in a harmonious environment and apply the concepts learnt in the classroom to solve real-life problems. Table 10 shows evidence of the extent to which the In-service mathematics teachers' lessons were observed to be reflective of social skills feature during the teaching and learning process.

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Table 10: Rating of Social Skills Feature in the Lessons Taught by IMTs

Lesson	Team	Feature Observed				Average	
		The lesson includes a wide range of activities where students					
		Manage Conflict during group activities	Respect the views of the other students in the group	Harmonize the dos and don'ts in class	Ensure orderliness in the group discussion	Mean	SD
SCHOOL (X & Y)							
Angles of elevation	T1T1	3	2	3	3	2.75	0.54
Angles of depression	T1T2	2	3	3	3	2.75	0.61
Trigonometric ratios	T2T1	2	2	2	2	2.00	0.42
Trigonometric identities	T2T2	2	3	3	3	2.75	0.54
Compound angles							
Trigonometric identities	T3T1	1	1	1	2	1.67	0.48
SCHOOL (A & B)	T3T2	3	3	3	2	2.75	0.59
Compound Angles							
Application of trigonometric	T4T1	3	2	3	3	2.75	0.64
ratios (angles of elevation)	T4T2	3	3	2	3	2.75	0.73

High-3, Medium-2, Low-1, Non-existent-0, SD-standard deviation

Source: Fieldwork, 2021.

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From Table 10, an overall mean ($\boldsymbol{M} = 2.52$, $\boldsymbol{SD} = 0.57$) for social skill was registered at the peer teaching phases in schools X and Y as well as real classroom situation at schools A and B at the implementation and evaluation stage depicting an above average score for the indicators that informed this feature during the teaching and learning process. Thus, IMTs use of social skills feature was rated high during the lessons on angles of elevation and depression, trigonometric ratios and identities which depicted a complete observation of the indicators on the CLOP. For instance, at the real classroom implementation of this feature at schools A and B, the IMTs fostered whole discussions on the guideline's pre-requisite for the conduct of the lessons especially during the intra group and inter-group discussions on the concepts taught. Figure 11 shows excerpts of this high rated observation of social skills as noted on the CLOP.

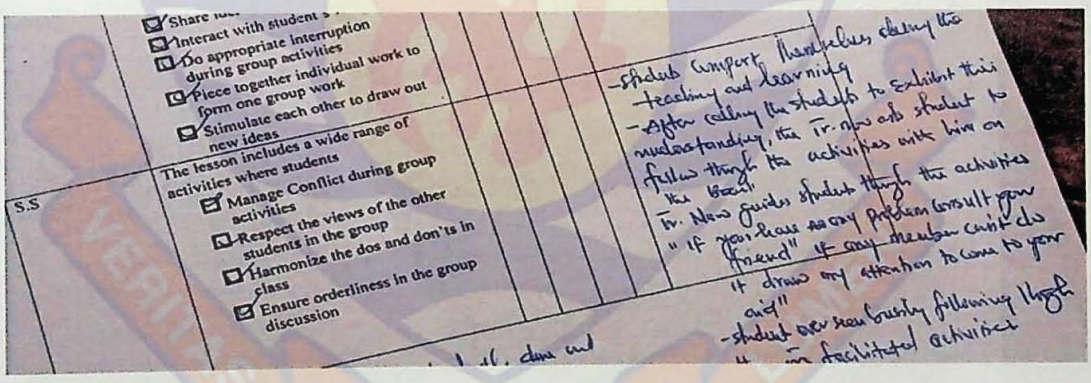


Figure 11: Excerpt of social skills on the CLOP.
Source: Fieldwork, 2021.

Nonetheless, "medium" and "non-existent" implementation levels of some of the indicators of this feature were observed during the lessons on trigonometric ratios and compound angles respectively at schools X and Y. For example, at school X, the application of the social skills feature was implied in the learning environment. Thus, the IMT cautioned the students to

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respect the contributions of their colleagues and report any act of insubordination in the group to him.

From Table 10, it was observed that the IMTs planned their respective lessons to be reflective of this feature (thus, with an overall mean of 2.52). As such students were observed to be focused on the task assigned to them by the IMTs. Thus, as a result of the application of positive interdependence (thus, placement of students into groups of assigned responsibilities) in the lessons of angles of elevation in the two senior high schools. The IMTs were able to ensure successful inter and intra relationships between the groups. For instance, in the absence of the teacher, the students were observed busily interacting fruitfully among themselves in their various groups towards the resolution of the task assigned to them by the teacher. This feature therefore ensured minimal or no interference in the group activities by the teacher. The images in Figure 12 were excerpts of the learning environment even in the absence of the direct supervision from the class teacher.



Figure 12: Excerpts of the learning environment in the absence of the direct supervision from the class teacher.
Source: Fieldwork, 2021.

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From Figure 12, the images are suggestive of a serene atmosphere that was created with no minimal interference from the teacher in the classroom as students were busily attending to the task throughout the entire lesson. The students understanding of the concepts involved in the topics were enhanced through their adherence to the guidelines fostered in the class. This made it possible for the IMTs to push the students into mathematical abstraction, generalisation and application of concepts which otherwise would have been lost in the traditional learning environment.

Moreover, an analysis of the lesson plan document also shows evidence of the social skills of the lesson taught by the IMTs. The IMTs designed some of the tasks on the activity sheets to imply the need for a sound learning environment in order for the students to reflect, think through and conjecture to aid in the conceptualisation of the topics under instruction. Figure 13 below shows excerpt of the lesson plan document that projected social skills feature of the CTLA in the lessons taught by the IMTs.

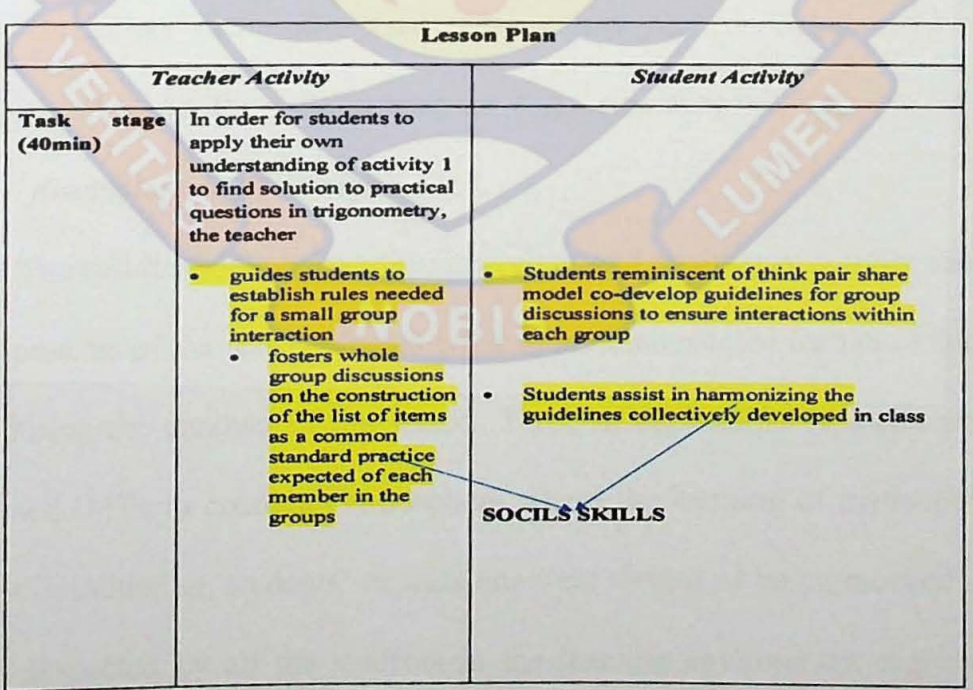


Figure 13: Excerpt of the lesson plan that shows social skills
Source: Fieldwork, 2021.

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Likewise, the IMTs responses in respect of the question: "Describe how the guidelines in the lesson facilitated the teaching of mathematics" have been reported in the study. The collective decision was that the lessons provided an enabling environment pre-requisite for successful group discussions and sound learning practices. Some specific responses gathered from the IMTs have been summarised as;

T1T1: *It ensured that each student in the group stayed focus and participated fully because the class was under control.*

T2T2: *The students were put on their toes and as such were disciplined, this facilitated teaching and learning of the angles of depression I taught in class*

T3T2: *It actually guides student's conscience about what to do and what not to do in the activity process in order to attain the set goals in the lesson.*

T4T1: *It created an environment where students spoke their minds on issues being discussed without any fear or being mocked by their colleagues as I go round monitoring discussions in the groups.*

The results from observation, interview, and analysis of activity sheets as components of the lesson plan all point to an implemented feature of social skills during the conduct of the lesson. Thus, an application of social skills positioned IMTs to create an atmosphere where the learning of mathematics was not intimidating, students' expressions were devoid of being mocked and instead respected by all the students in the learning environment especially during inter and intra group discussions in the lesson. The social skills feature

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of the CTLA created a sense of discipline in the teaching and learning process and yielded good classroom management practices during the conduct of the lesson.

Group processing was conceptualised in the study as teaching and learning process where teachers act purposefully to evaluate students understanding of the subject matter. The lessons enacted in this study were reflective of the group processing feature of the CTLA. For instance, during the enactment of a lesson on *compound angle* in school B, groups of students upon completion of their assigned task were called to the chalkboard to explain the processes leading to their solution to the task. Consequently, multiple solutions to the task on the compound angles from different groups were displayed in the classroom on the *compound angle*. This afforded students the opportunity to choose from the multiplicity of the solutions (thus, the use of sine (A+B), tan (A+B), and cos (A+B)) in resolving the task as displayed on the chalkboard from various presentations made in class by the groups. For instance, in school A, the IMT instructed lesson on the concept of *angles of elevation*. I observed that the students employed multiple techniques to find the angle of elevation during the teaching and learning process as a way of providing constructive feedback to colleagues in the groups when the opportunity was afforded them to present their solution on the chalkboard. Table 11 shows evidence of the extent to which the In-service mathematics teachers' lessons were perceived to be reflective of group processing feature.

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Table 11: Frequency of Occurrence of Group Processing in IMTs Lesson Taught

Lesson	Team	Feature Observed				Average	
		The lesson provides students with the opportunity to					
		Critique each group's solution	Do group presentation on the Chalkboard	Make inputs into other's work	Do summary of the lesson's key highlights by the IMT	Mean	SD
SCHOOL (X & Y)							
Angles of elevation	T1T1	1	0	0	2	0.75	0.51
Angles of depression	T1T2	2	2	1	3	2.00	0.47
Trigonometric ratios	T2T1	2	2	2	2	2.00	0.61
Trigonometric identities	T2T2	1	3	0	3	2.00	0.45
Compound angles	T3T1	1	0	2	0	1.00	0.31
Trigonometric identities	T3T2	3	3	3	2	2.75	0.58
SCHOOL (A & B)							
Compound Angles	T4T1	3	1	2	3	2.25	0.64
Application of trigonometric ratios (angles of elevation)	T4T2	3	3	2	3	2.75	0.71

High-3, Medium-2, Low-1, Non-existent-0, SD-standard deviation

Source: Fieldwork, 2021.

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From Table, an overall mean ($\boldsymbol{M} = 2.00$, $\boldsymbol{SD} = 0.54$) was registered for group processing which depicts a fairly utilisation of this feature during the teaching and learning process. The IMTs use of group processing feature was rated high during the lessons on angles of elevation and trigonometric identities which depicted a complete observation of the indicators on the CLOP. For instance, at the real classroom implementation of this feature at school A and X, the IMTs evaluated students understanding of the concepts taught. Figure 14 shows excerpts of this high rated observation of group processing feature as noted on the CLOP.

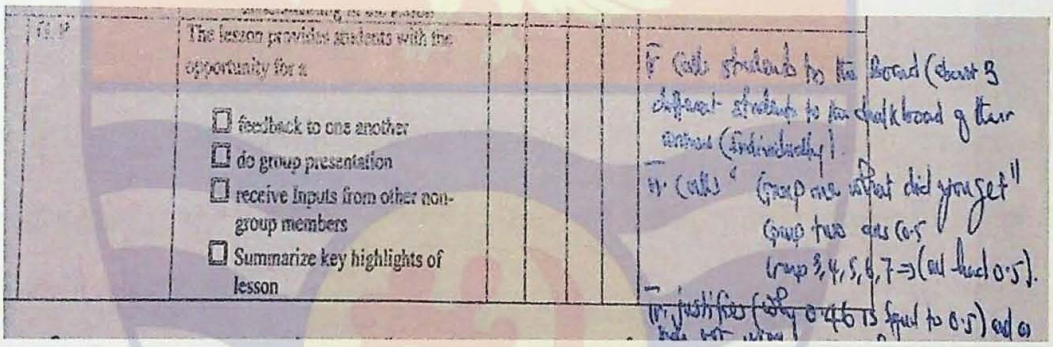


Figure 14: Excerpt of group processing feature on the CLOP.
Source: Fieldwork, 2021.

Nonetheless, a "medium" and "non-existent" observation of some of the indicators of IMTs evaluation of the students understanding of lessons on angles of depression, trigonometric ratios and identities were recorded on the CLOP mostly from schools X, Y and B. Again, on this feature of group processing Figure 15 shows pictures of the group's work on the chalkboard, monitoring and evaluation exercises of students during the implementation of the CTLA-lessons.

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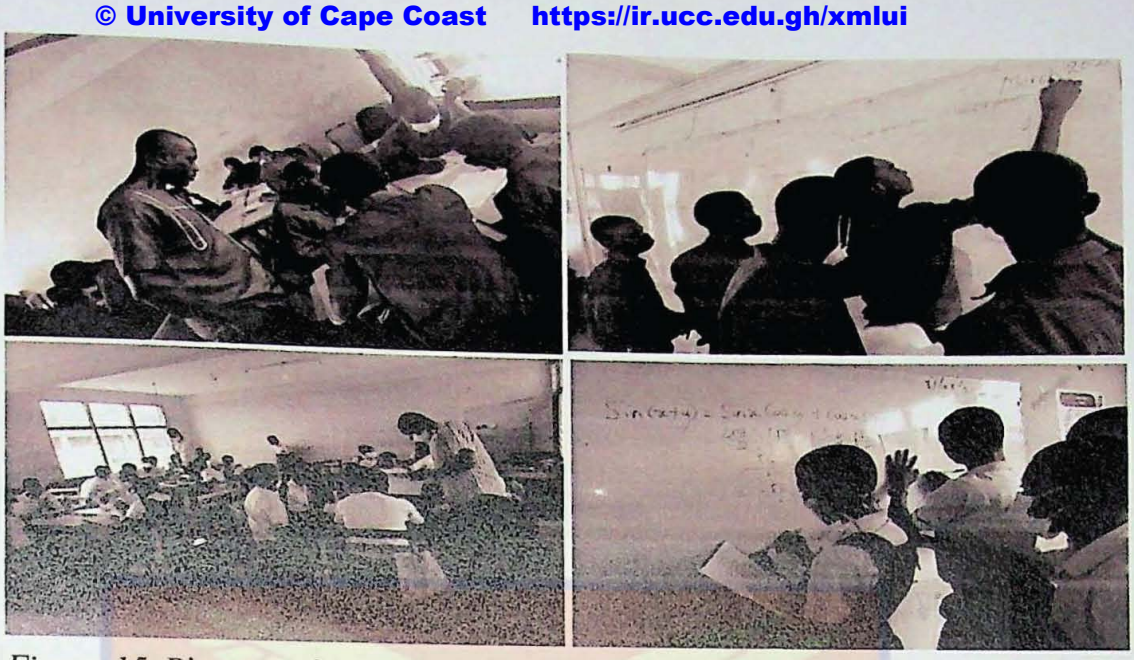


Figure 15: Pictures of group presentation by students on the chalkboard
Source: Fieldwork, 2021.

An analysis of the IMTs prepared lesson also projected the feature of group processing in its construction. This was evidenced during the implementation of the CTLA lessons in the real classroom learning experience. Figure 16 shows the feature of group processing as used in the IMTs lesson plan.

Lesson Plan		
	Teacher Activity	***Student Activity***
Conclusion stage (15 min) **GROUP PEOCESSING**	In order to provide clarity and correct errors with the aim of helping students to grasp concept taught, the teacher • calls to the board, the group's recorder to present solutions on behalf of the group • fosters whole group discussions on the various presentations on the solutions to the problem • solicits alternate methods/inputs of finding solution to the questions • Finally, sums up the whole lesson by highlighting the key concepts discussed as Teacher then brings closure to the lesson by • inviting questions from students based on the set instructional objectives of the lesson giving assignment sheets to students	• Group leaders lead discussions on the solutions to the problems after the group's recorder presentation of the solution on the board • students provide constructive feedback their peers on the solutions presented on the board • students provide alternate solutions to the solutions presented on the board • Students listen and take notes of the core points as the teacher provides clarity to some questions. • Students provide feedback responses to teacher's questions and then take assignment sheets to be solved as homework

Figure 16: Excerpt of the IMTs lesson plan projecting group processing feature
Source: Fieldwork, 2021.

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In support of the group processing observation, the interview revealed that the intervention promoted group processing. These comments were obtained in response to the question: In what way did the lesson activities promote group processing? The responses elicited from the IMTs have been summarised as;

T2T1: *It helped to clarify and summarise the individual contributions together.*

T3T2: *It helped raise the confidence level of the students since they got the solution right.*

T4T 2: *This activity helped the teacher to get feedback, know what to develop in the minds of the students and clear any misconception.*

T4T1: *It made the students to be part and parcel of the teaching and learning process.*

From the observations, interviews, and analysis of the lesson plan shows that the lessons enacted by the IMTs were reflective of the group processing feature of the cooperative teaching and learning environment. This is because, an application of the group processing feature provided teachers with defined processes to facilitate the evaluation of students understanding of the subject matter. Thus, teachers were positioned to afford students the platform to think through the quality of their group work during the conduct of the lessons.

Finally, the features of CTLA-intervention that was implemented during the teaching and learning process also had evidence of individual accountability. This is because at school X, the IMT engaged their peers (who

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served as students in the classroom) on the concept of *angles of depression* at the individual level thus, these peers were engaged at the personal level to provide diagrammatic representation of the case scenario (questions bearing angles of depression on the students work sheets) on angles of depression before finally asking the individual students in the groups to put their separate work output together as the group's solution to the task. In school B, the students were observed providing solutions to questions on *compound angles,* a subject matter they had difficulty answering prior to the introduction of the CTLA-intervention. For instance, the students had no clue to the concept of compound angles in respect of the three trigonometric ratios of sine, cosine tangent of angles SHS A and SHS B. Consequently, after the students were engaged at the individual level and received support from team members in the group by way of clarifications and discussions on the concept of *compound angles* in the group, the individual students were observed to have provided accurate solutions to the questions. Table 12 shows evidence of the extent to which the IMTs lessons were observed to be reflective on the feature of individual accountability in the learning environment.

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Table 12: Rating of Individual Accountability in Lessons Taught by IMTs

Lesson	Team	Feature Observed				Average	
		The lesson includes a wide range of activities where students					
		Participate in the lesson	Contribute ideas in the lesson	Engage themselves in the lesson	Piece together individual work to form one group work	Mean	SD
SCHOOL (X & Y)							
Angles of elevation	T1T1	1	2	1	1	1.25	0.42
Angles of depression	T1T2	1	0	1	1	0.75	0.53
Trigonometric ratios	T2T1	1	2	2	1	1.50	0.48
Trigonometric identities	T2T2	2	1	1	1	1.25	0.51
Compound angles	T3T1	1	1	0	0	0.50	0.45
Trigonometric identities	T3T2	3	2	3	2	2.50	0.63
SCHOOL (A & B)							
Compound Angles	T4T1	1	2	3	2	2.00	0.52
Application of trigonometric ratios (angles of elevation)	T4T2	3	3	2	3	2.75	0.71

High-3, Medium-2, Low-1, Non-existent-0, SD-standard deviation

Source: Fieldwork, 2021.

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From the Table, an overall mean $(\boldsymbol{M} = 1.56, \boldsymbol{SD} = 0.53)$for individual accountability was observed at the peer teaching phases in schools X and Y as well as real classroom situation at schools A and B at the implementation and evaluation stage depicting an above average score for the indicators that informed this feature during the teaching and learning process. Thus, IMTs use of this feature was rated high during the lessons on the application of trigonometric ratios (angle of elevation). For instance, at the real classroom implementation of this feature at school A, the IMT created an avenue for students to share ideas at the group and individual levels, actively participating in the teaching and learning process and contributing ideas in their respective groups on the concept taught. These observations depicted a complete observation of the feature of individual accountability on the CLOP. However, an observation of "medium" and "non-existent" implementation of this feature was observed during the lessons of angles of elevation and depression, trigonometric identities as well as compound angles which depicted some low rated and non-existent indicators on the CLOP. For example, at the peer teaching implementation of this feature at schools B, X and Y, the IMT partially implemented the indicators on the CLOP. Figure 17: shows excerpts of individual accountability on the CLOP.

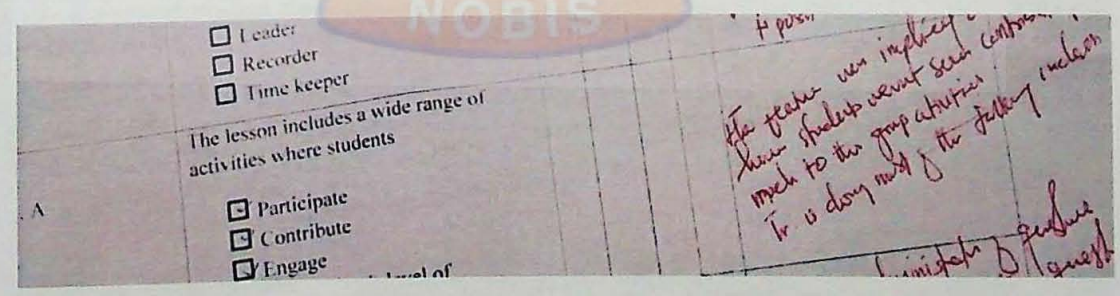


Figure 17: Excerpt of individual accountability on the CLOP.
Source: Fieldwork, 2021.

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Analysis of the observation data during the teaching and learning process showed that all the four teams of IMTs organised their lessons in a manner that engaged students in lessons independently on tasks in the construction of knowledge. It was also observed that the IMTs prioritised this feature by engaging individualistic group members to find solution to a task before piecing the respective individual solutions together as the group's solution. This form of engagement (individual accountability) in the lessons provided the opportunity for the teachers to evaluate each student's learning ability. As such, students' weaknesses were identified by the IMTs. Consequently, such weak students identified were attended to by the respective teachers in the class. The presence of individual accountability (especially, where students work on task individually) during the lesson also afforded the IMTs the opportunity to assess each student's performance on activities rather than groups' performance. Figure 18 below provides evidence of students working individually on task.

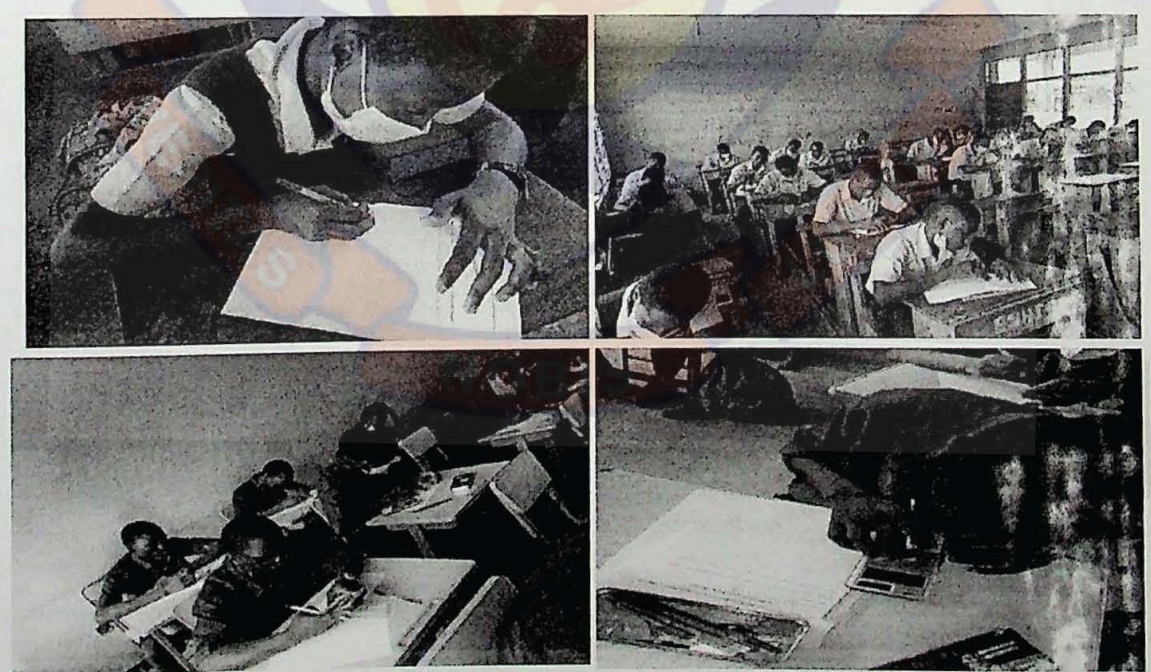


Figure 18: Pictures of students working on a task during the implementation of the individual accountability feature.
Source: Fieldwork, 2021.

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The IMTs were interviewed for the purposes of eliciting responses as to whether their lessons engaged students individually in the lesson, the teachers' responses have been quoted in Table 13. The responses were in respect of the question: "Describe how you engaged the learners individually in teaching mathematics".

Table 13: IMTs Responses to the Question of How They Engaged Students Individually in the Lessons

IMTs	IMTs responses (quote responses)
T1T1	*The student shared ideas in the group and the most convincing concept or explanation were adopted by all members in the group.*
T2T2	*They shared ideas in the group and it helped them to avoid mistakes and it did boost their confidence to approach mathematical problems.*
T3T1	*Through group activities student learnt from each other and they were also guided by contributions made from sharing of strategies and methods in solving questions.*
T4T1	*Student learnt in groups, new ideas were conjured as well as new methods and when students were asked questions independently, they used the methods they got from these groups to improve the ones they knew already.*

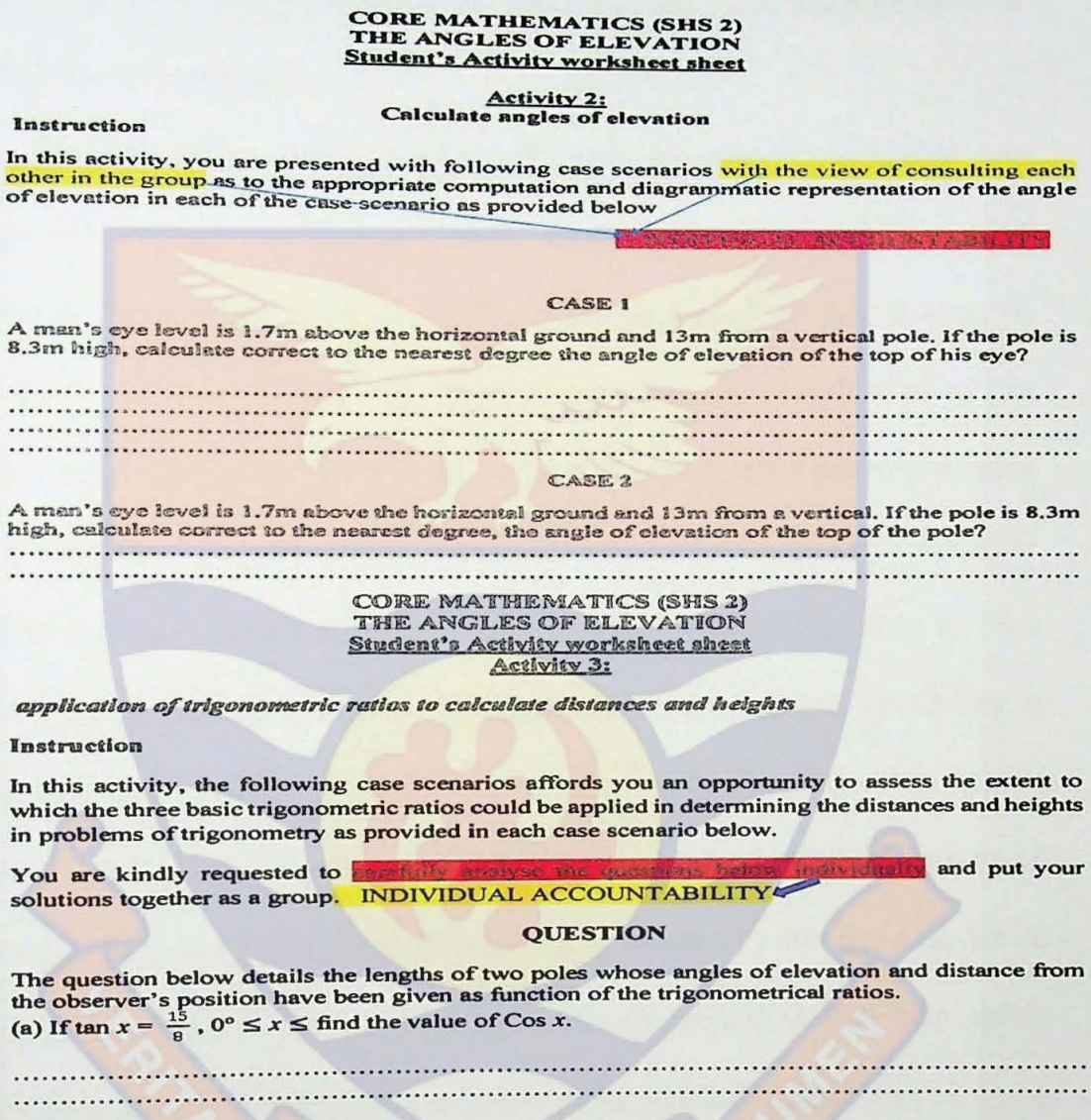
Source: Fieldwork, 2021.

Analysis of the activity sheets (component of the lesson plan) also revealed that the CTLA- intervention promoted the individual accountability of students in the teaching and learning of mathematics. Even though majority of activities were done in groups, there were some activities that required the

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students to work individually in producing the answers to the questions on the activity. Figure 19 shows the portions of the activity sheet that promoted individual accountability of students in the lesson.



CORE MATHEMATICS (SHS 2)
THE ANGLES OF ELEVATION
Student's Activity worksheet sheet

Activity 2:
Calculate angles of elevation

Instruction

In this activity, you are presented with following case scenarios with the view of consulting each other in the group as to the appropriate computation and diagrammatic representation of the angle of elevation in each of the case scenario as provided below

CASE 1

A man's eye level is 1.7m above the horizontal ground and 13m from a vertical pole. If the pole is 8.3m high, calculate correct to the nearest degree the angle of elevation of the top of his eye?

.....
.....
.....
.....

CASE 2

A man's eye level is 1.7m above the horizontal ground and 13m from a vertical. If the pole is 8.3m high, calculate correct to the nearest degree, the angle of elevation of the top of the pole?

.....
.....

CORE MATHEMATICS (SHS 2)
THE ANGLES OF ELEVATION
Student's Activity worksheet sheet
Activity 3:

application of trigonometric ratios to calculate distances and heights

Instruction

In this activity, the following case scenarios affords you an opportunity to assess the extent to which the three basic trigonometric ratios could be applied in determining the distances and heights in problems of trigonometry as provided in each case scenario below.

You are kindly requested to carefully analyse the questions below individually and put your solutions together as a group. INDIVIDUAL ACCOUNTABILITY

QUESTION

The question below details the lengths of two poles whose angles of elevation and distance from the observer's position have been given as function of the trigonometrical ratios.

(a) If $\tan x = \frac{15}{8}$, $0° \leq x \leq$ find the value of Cos x.

.....
.....

Figure 19: Excerpt of activity sheets showing Individual Accountability of students in the lesson on angles elevation
Source: Fieldwork, 2021.

The results from the observation, interview and analysis of activity sheets are suggestive of the fact that IMTs engaged students individually in their lessons by way of ensuring mastery of the content of the subject matter in the lesson. Thus, the results seem to suggest that an implementation of Individual accountability gives room to teachers to position students as active participants in the group activities and provides them with the requisite

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knowledge to discourage and lower the likelihood of "free riders" or "social loafing" (students who would sit idle and allow the few brilliant students to perform the task) in group activities. Hence, its application created an enabling environment where group members accomplished the same kinds of tasks by themselves in the group.

In conclusion evidence adduced from observation, interview and component analysis of the lesson plans suggest that, all the eight enacted lessons on the concepts of trigonometry were reflective of the five features of the CTLA. Thus, the study has revealed that, with the application of the positive interdependence in the lesson, questions which could not be answered by the individual students were responded to correctly when the students were put into groups. Again, with the employment of promotive face to face interaction, it was realized that, multitudes of methodologies to finding a solution to a task within the groups were realised in the learning environment. Most importantly, a minimal or no interference was observed in the lessons of the IMTs as a result of conducive atmosphere created by the dos and don'ts (guidelines) during the conduct of the lessons (social skills). The group processing served as the vehicle through which students taught and received feedback from colleagues by way of the groups' interactions and presentations as well the teacher's support in understanding the subject matter. Finally, an opportunity afforded IMTs to evaluate each student's ability (individual accountability) in the learning environment during the conduct of their lessons led to the weaknesses on the part of students that were identified to be addressed. Hence, it can be concluded that the lessons taught in the study by

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the IMTs were effective as they projected the five features of the CTLA at the different phases of the implementation and evaluation stage.

Students' Perceptions About the Effectiveness of the CTLA-Lessons

The effectiveness of the CTLA intervention for mathematics lessons was also measured quantitatively by administering a 5-point Likert scale questionnaire (SHS mathematics student perceived effectiveness questionnaire) to 55 students at two SHS (School A and B) during real classroom implementation of the lessons designed by the IMTs. This questionnaire comprised 20 items grouped under the five subscales (see Table 14) of the CTLA. The students were to indicate the extent to which they agree or disagree to the items after they had been exposed to the CTLA lesson. Possible responses to the items on the five-subscale questionnaire were on a scale ranging from strongly disagree (1) to strongly agree (5). Thus, for the subscales, a score below and above 3 depicts favourable and unfavourable opinions of the respondents. Table 14 shows the results Students' Perceived Effectiveness of the CTLA intervention for mathematics lessons based on school-type.

Table 14: Students' Perceived Effectiveness of the CTLA Lessons Based on School-type (N=55)

CTLA Subscale	School-type	n	Mean	SD
Positive Interdependence	School A	29	4.23	0.52
	School B	26	3.56	0.94
Face-to-Face Interaction	School A	29	4.31	0.55
	School B	26	4.19	0.78
Social Skills	School A	29	4.23	0.37
	School B	26	3.96	0.74
Group Processing	School A	29	4.33	0.46
	School B	26	3.98	0.76
Individual Accountability	School A	29	4.28	0.49
	School B	26	3.87	0.97

Source: Fieldwork, 2021.

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From Table 14, it can be inferred that positive interdependence ($\boldsymbol{M} = 4.23,\ \boldsymbol{SD} = 0.52$) and ($\boldsymbol{M} = 3.56\ \boldsymbol{SD} = 0.94$) and social skills ($\boldsymbol{M} = 4.23,\ \boldsymbol{SD} = 0.37$) and ($\boldsymbol{M} = 3.96,\ \boldsymbol{SD} = 0.74$) were the subscales that recorded the lowest means with differences in standard deviation for schools A and B respectively. This means, it appears the IMTs had difficulty placing their respective students (class sizes of 29 and 26 for schools A and B respectively) into groups of proportionate sizes and most importantly organizing the learning environment with strict adherence to the dictates of the guidelines as was set out by the students for the purposes of creating a conducive learning environment during the teaching and learning process from the perspective of the students in the classroom. Group processing recorded the highest means and standard deviations of ($\boldsymbol{M} = 4.33,\ \boldsymbol{SD} = 0.64$) and ($\boldsymbol{M} = 3.98,\ \boldsymbol{SD} = 0.76$) for schools A and B respectively, depicting the IMTs effective evaluation of students' understanding of the concepts of trigonometric ratios and compound angles taught in the real classroom situation in schools A and B respectively.

However, in general, an overall mean of ($\boldsymbol{M} = 4.28,\ \boldsymbol{SD} = 0.43$) and ($\boldsymbol{M} = 3.91,\ \boldsymbol{SD} = 0.84$) for schools A and B respectively were recorded indicating that the respondents agreed that the CTLA-intervention was effective although the standard deviation suggests that the responses were not clustered around the mean value. The overall picture as presented in the above analysis indicates that the CTLA lessons designed by the IMTs were effective (i.e., projected all the five features of CTLA). This seems to suggest that the CTLA -intervention for trigonometric lessons was effective in developing

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IMTs pedagogical strategies to improve teaching and learning of concepts in trigonometry.

In concluding, it can be observed that the analyses of the two perspectives measured as presented in the above by both the qualitative and quantitative pieces of evidence indicate that the CTLA-based lessons designed by the in-service mathematics teachers were effective and consequently projected all the five features of CTLA as conceptualised in the study. The results seem to suggest that by developing and enacting the CTLA-mathematics lessons the IMTs teachers developed their pedagogical strategies to improve their teaching of concepts in the mathematics they taught.

Research Question Two

Research question two was "how can the improvements in teaching using pedagogical strategies that are anchored on cooperative teaching and learning approach be understood and or explained?". Thus, the question sought to explain how the improvements in teaching using CTLA as a medium of instruction in the learning environment could be understood and or explained.

Based on the data obtained from the study, the improvements in teaching using CTLA as a pedagogical strategy to teach mathematics can be understood and or explained from two main perspectives; 1) IMTs developed knowledge and skills in enacting cooperative teaching and learning approach lessons (thus, lessons reflective of the five features of cooperative teaching and learning approach) at the implementation and evaluation stage and 2) IMTs improved content knowledge in the subject matter. The two perspectives are further explained below.

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In-service Mathematics Teachers' Developed Knowledge and Skills in Enacting CTLA Lessons.

In responding to research question two, the IMTs developed knowledge and skills in enacting CTLA lessons, data was measured quantitatively by administering a 5-point Likert scale questionnaire (IMTs Pre- and Post-CTLA Self-Assessment Questionnaire) prior to and post introduction of the PDT to the two cohorts of four IMTs who were involved in phases one and two at the implementation and evaluation stage as mentioned in chapter 3 of the study. Thus, data was collected from IMTs at the two senior high schools (school X and Y) in Cape Coast. This questionnaire comprised 20 items grouped under the five subscales (see Table 14) of the CTLA. IMTs were to indicate the extent to which they agree or disagree to the items after they had been introduced to the enactment of CTLA lesson. Possible responses to the items on the five-subscale questionnaire were on a scale ranging from strongly disagree (1) to strongly agree (5). Thus, for the subscales, a score below and above 3 depicts favourable and unfavourable opinions of the respondents.

The purpose of the CTLA self-assessment questionnaire (pre and post) was to track the IMTs knowledge of the features of the CTLA intervention before and after the PDT to ascertain the extent of their developed knowledge and skills in enacting CTLA lesson. The results of the Pre and Post CTLA self-assessment questionnaire of eight IMTs are presented. Thus, the results of first batch of four IMTs form School X and last batch of four IMTs from School Y are presented in Tables 15 and 16 respectively.

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Table 15: IMTs Pre and Post PDT on CTLA in the First Study

CTLA Subscale	Pre test		Post test		
	Mean	SD	Mean	SD	Gains
PI	2.13	.29	5.00	.00	2.87
FF	2.58	.32	4.63	.24	2.05
SS	2.54	.37	4.88	.16	2.34
GP	2.35	.33	4.44	.17	2.05
IA	2.74	.32	4.14	.17	1.40

PI- Positive Interdependence, FF- Promotive Face to Face Interaction, SS- Social Skills, GP- Group Processing, IA- Individual Accountability
Source: Fieldwork, 2021.

Generally, the results depicted an improvement in all the CTLA subscales (positive interdependence, promotive face to face interaction, social skills, group processing and Individual accountability). This is because the mean scores for the IMTs' CTLA self-assessment before the professional development training are lower than the mean scores after the professional development training. For example, for the first cohort of IMTs, it can be observed from Table 15 that positive interdependence (P.I) was rated as the highest with mean gain of (P.I, Gain = 2.87) followed by social skills (S.S, Gain = 2.34) face to face interaction (F.F, Gain = 2.05), group processing and Individual accountability recording equal gains (G.P, Gain = 2.05) and (I.A, Gain = 1.40) respectively. Table 16 presents the mean scores for pre and post CTLA self-assessment for the last cohort of IMTs who were involved in second study at the implementation and evaluation stage.

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Table 16: IMTs Pre and Post PDT on CTLA in Second Study

CTLA Subscale	Pre test		Post test		
	Mean	SD	Mean	SD	Gains
PI	2.34	.29	4.92	.67	2.58
FF	2.48	.32	4.73	.55	2.25
SS	2.35	.37	4.85	.72	2.50
GP	2.33	.33	4.15	.54	1.82
IA	2.31	.32	3.81	.67	1.50

PI- Positive Interdependence, FF- Promotive Face to Face Interaction, SS- Social Skills, GP- Group Processing, IA- Individual Accountability
Source: Fieldwork, 2021.

Similarly, analysis from Table 16 reiterates an improvement in all the CTLA self-assessment for the second cohort of IMTs. For example, before the PDT, the IMTs indicated that they had less or no knowledge in relation to any form of activity where during the teaching and learning process deliberate effort ought to be made for the group's mastery of the subject matter to reflect on the individual student mastery of that subject matter. Again, the IMTs indicated that they had inadequate knowledge in relation to any form of activity during lessons where students project their work (solution to a task) on the chalkboard purposed to elicit alternate ideas (different approach to the resolution of the question) from other students in the learning environment. The lowest recorded mean values of Individual Accountability-I.A ($\boldsymbol{M} = 2.31$, $\boldsymbol{SD} = 0.32$) and GP- Group Processing ($\boldsymbol{M} = 2.33$, $\boldsymbol{SD} = 0.33$) affirm these assertions. However, the Post-PDT revealed that IMTs seem to have improved not only on the subscales of I.A ($\boldsymbol{M} = 3.8,1\ \boldsymbol{SD} = 0.67$) and G.P ($\boldsymbol{M} = 4.15$, $\boldsymbol{SD} = 0.54$) with gains of 1.50 and 1.82 respectively over their pre-PDT but in all domains relating to CTLA. For instance, FF- Promotive Face to Face Interaction ($\boldsymbol{M} = 4.73$, $\boldsymbol{SD} = 0.55$) an increment of 2.25 means gain from

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the Pre-PDT mean with a higher standard deviation. Also, P.I and S.S recorded mean values of 4.92 (SD = 0.67) and 4.85, (SD = 0.71) respectively.

It is obvious from Table 15 and Table 16 that, both first and second cohorts of IMTs had little knowledge about the use of CTLA in teaching mathematics prior to the professional development training conducted in the study. However, the results seem suggestive of IMTs being knowledgeable about the grouping of students with some assigned roles (P.I) and organising the learning environment to be accommodative of the views of other student during the teaching and learning process (S.S). This is not surprising since it lends credence to the misconceived definition attributed to the use of CTLA as just the organisation of students into groups with some assigned tasks to the detriment of the other features which corroborate to ensure a cooperative-based learning environment (with all of the five features at play in the classroom). The responses given by the IMTs (first cohort and second cohort) suggest that as a result of their developed knowledge and skills in designing and enacting CTLA-lessons, they gained deeper understanding and appreciation of CTLA as an instructional approach in the learning environment, identified weaknesses in the CTLA intervention for mathematics lessons (as observed under focus group discussions) and in addition, became more informed about the features of the CTLA to adopt in order to maximise learners understanding and conceptualisation of mathematical concept.

Further, the IMTs were interviewed and their lessons observed to provide the qualitative data to explain improvements in teaching using CTLA. Thus, data was collected from the IMTs reported lessons, observed data, focus group discussions and interview. The IMTs lessons (lesson plans and activity

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sheets) were analysed through document analysis. Also, data from the interview and focus group discussions were analysed by the identification of themes as well as patterns in relation to the research question two.

For instance, the lesson plan designed by team 4 (T4T1 and T4T2) and team 3 (T3T1 and T3T2) for teaching of the topics: Compound Angles and three ratios of trigonometry respectively showed evidence of IMTs development of their competencies in teaching SHS trigonometry with CTLA through a series of teaching try outs with an improved lesson plan document. This is because the first lesson plan document designed by team 4 (T4T1 and T4T2) and team 3 (T3T1 and T3T2) depicted an incomplete reflection on all the features of the CTLA (thus, there was an omission of the features of face-to-face promotive interaction and individual accountability). Figure 20 shows excerpt of illustrations of IMTs developed lesson plan on compound angles team 4

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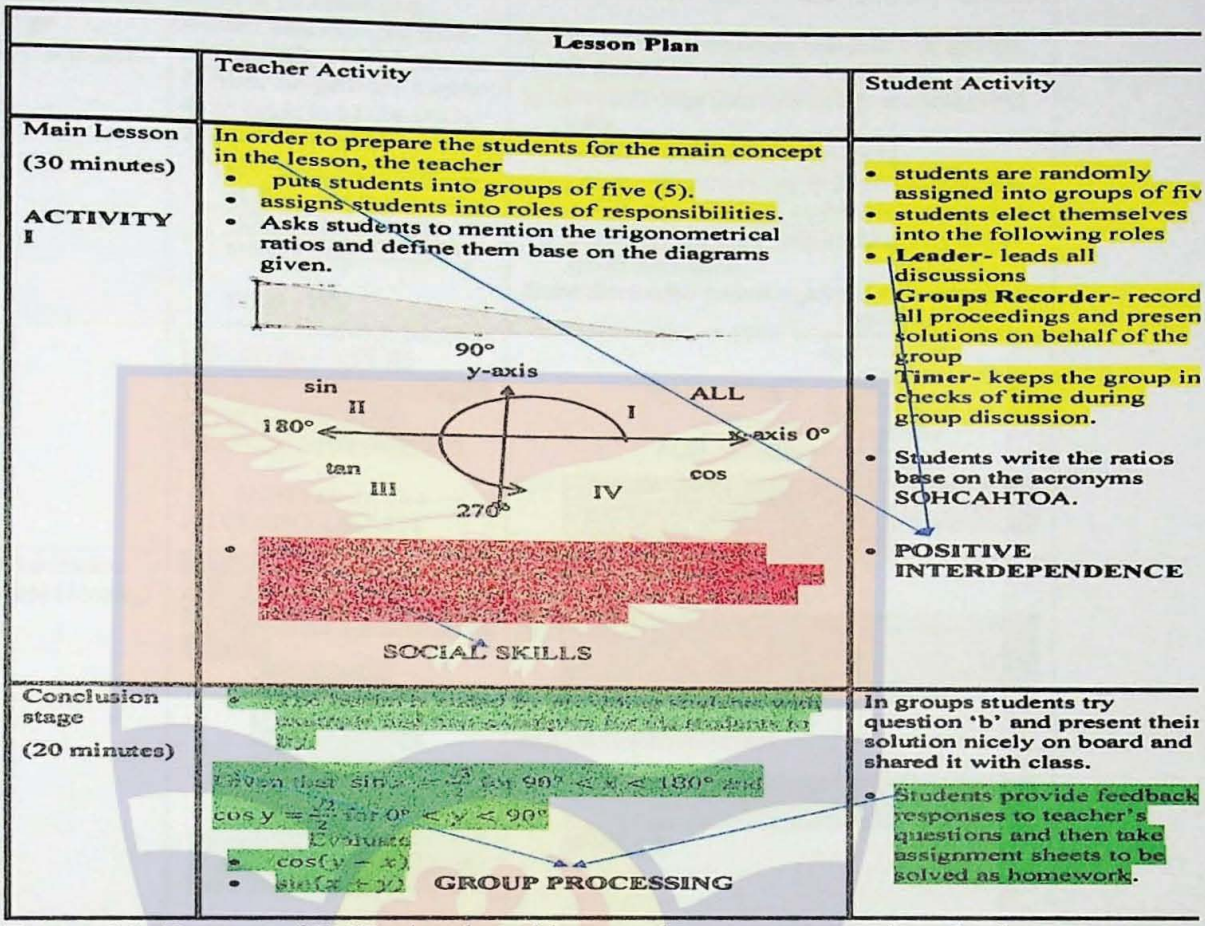


Figure 20: Excerpt of IMTs developed lesson plan on compound angles by team 4.

Source: Fieldwork, 2021.

Similarly, the design of the first lesson plan on the concept of trigonometric ratios by team 3 showed evidence of absence of face-to-face promotive interaction and social skills features of the CTLA although they were implied during the teaching and learning process. Figure 21 shows excerpts of the first lesson designed by team 3.

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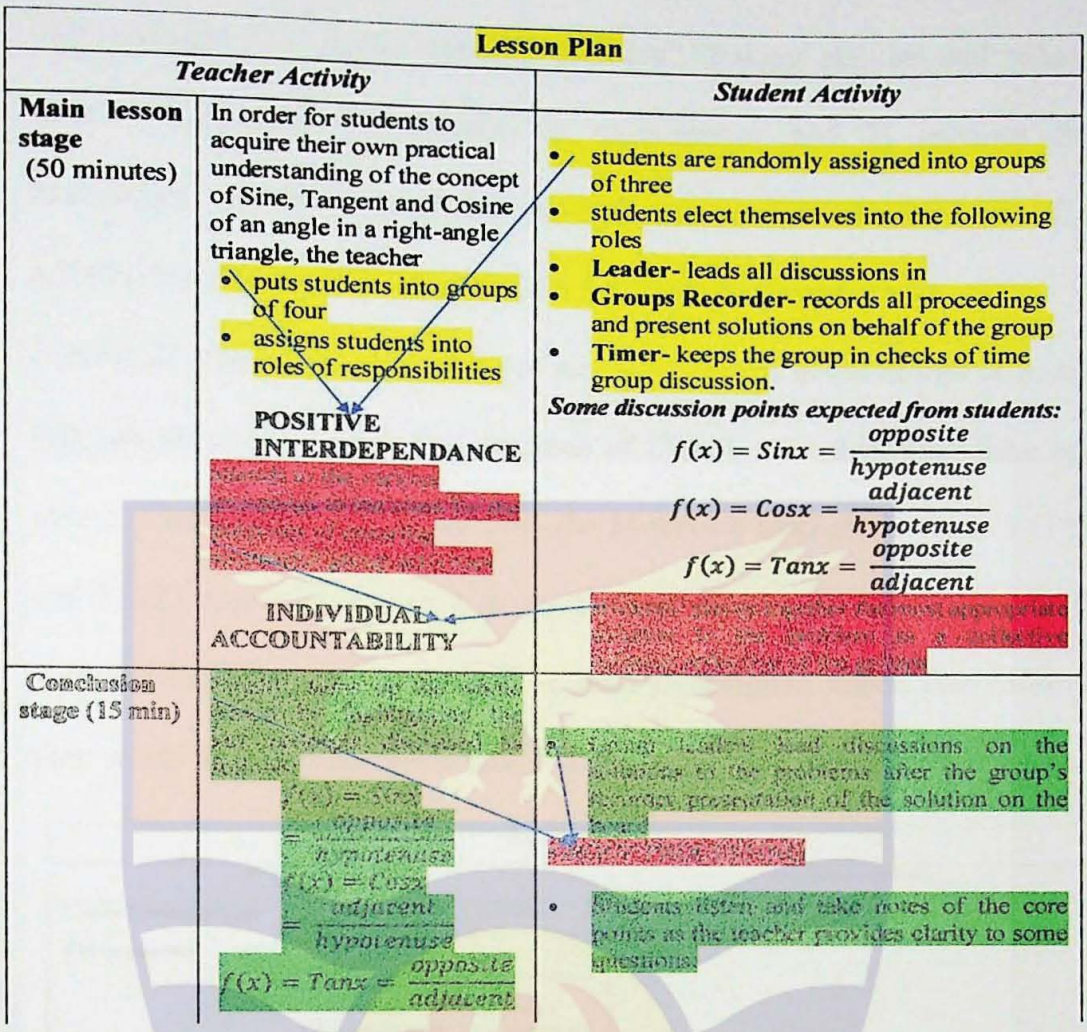


Figure 21: Excerpt of IMTs developed lesson plan on trigonometric ratios by team 3.
Source: Fieldwork, 2021.

However, after a series of teaching try outs, focal group discussions and cooperation among team members, teams 3 and 4 improved in their competencies in the design of lessons reflective on all the features of the cooperative teaching and learning approach on the concepts of trigonometric ratios and compound angles respectively. In particular, the IMTs in team 4 (T4T1 and T4T2) and team 3 (T3T1 and T3T2) used their knowledge of the affordances of the features of CTLA to 1) guide learners in the acquisition of knowledge through the use of the students worksheet of activities to conceptualise compound angles and the three trigonometric ratios of sine, cosine and tangent of angles taught at the real classroom context at the two

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SHS (schools A & B) 2) stimulate learners' thinking abilities and enhance their understanding of concepts in mathematics and 3) enhance their pedagogical approach in student-centred manner (i.e. making use of an activity-oriented based lesson reflective on the features of CTLA).

Figures 21 and 22 show evidence of an improvement in the design of lessons that are reflective on the five features of CTLA over their first developed lesson. Thus, an illustration of how the in-service teachers in team 3 (T3T1 and T3T2) improved in their lesson plan document to be reflective on all the five features of the CTLA as seen in their re-designed lesson plan informed their teaching of trigonometric ratios in the senior high school (schools A).

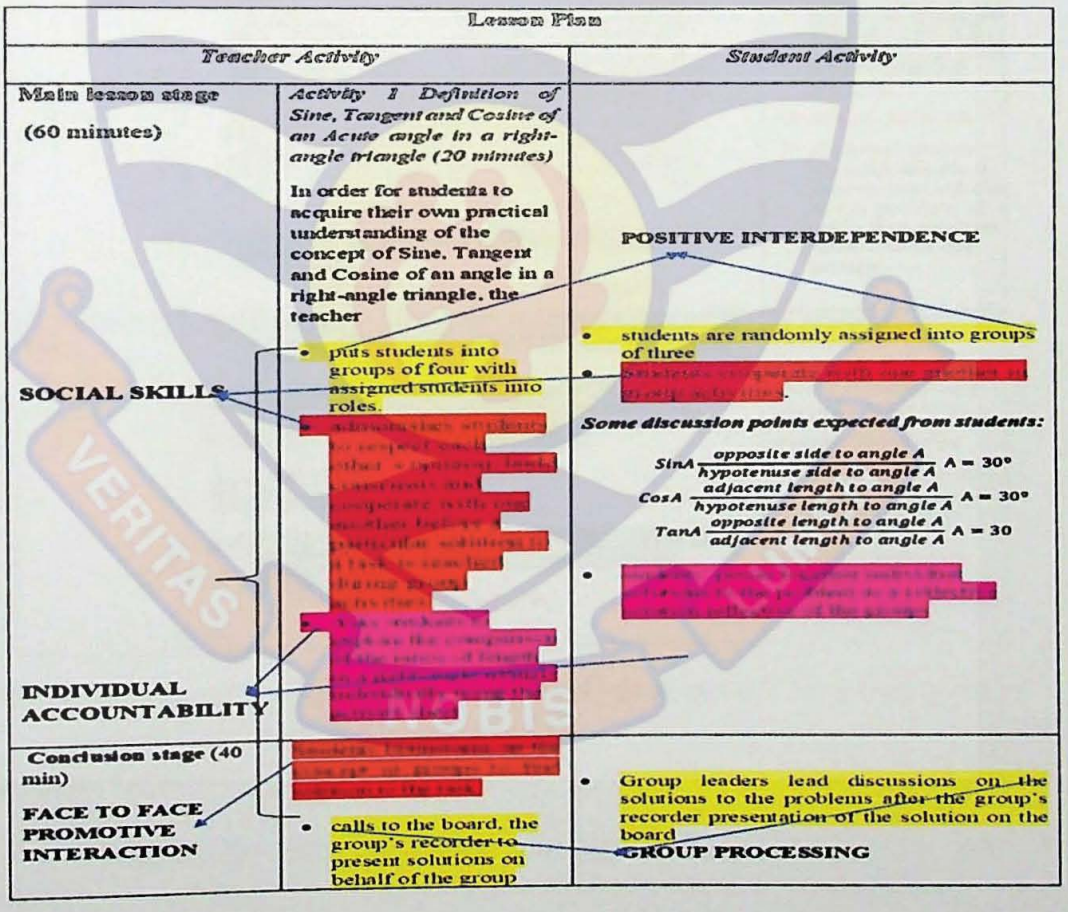


Figure 22: Excerpt of the IMTs (in Team 3) developed CTLA lesson on trigonometric ratios.
Source: Fieldwork, 2021.

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Similarly, the IMTs in team 4 (T4T1 and T4T2) made use of their improved lesson plan document reflective on the all the features of the CLTA in a way that enhanced their teaching strategies as evidenced in Figure 22. Figure 23 below illustrates how the IMTs in team 4 (T4T1 and T4T2) developed CTLA as seen in their lesson plan informed their teaching of compound angles in the senior high school (schools B).

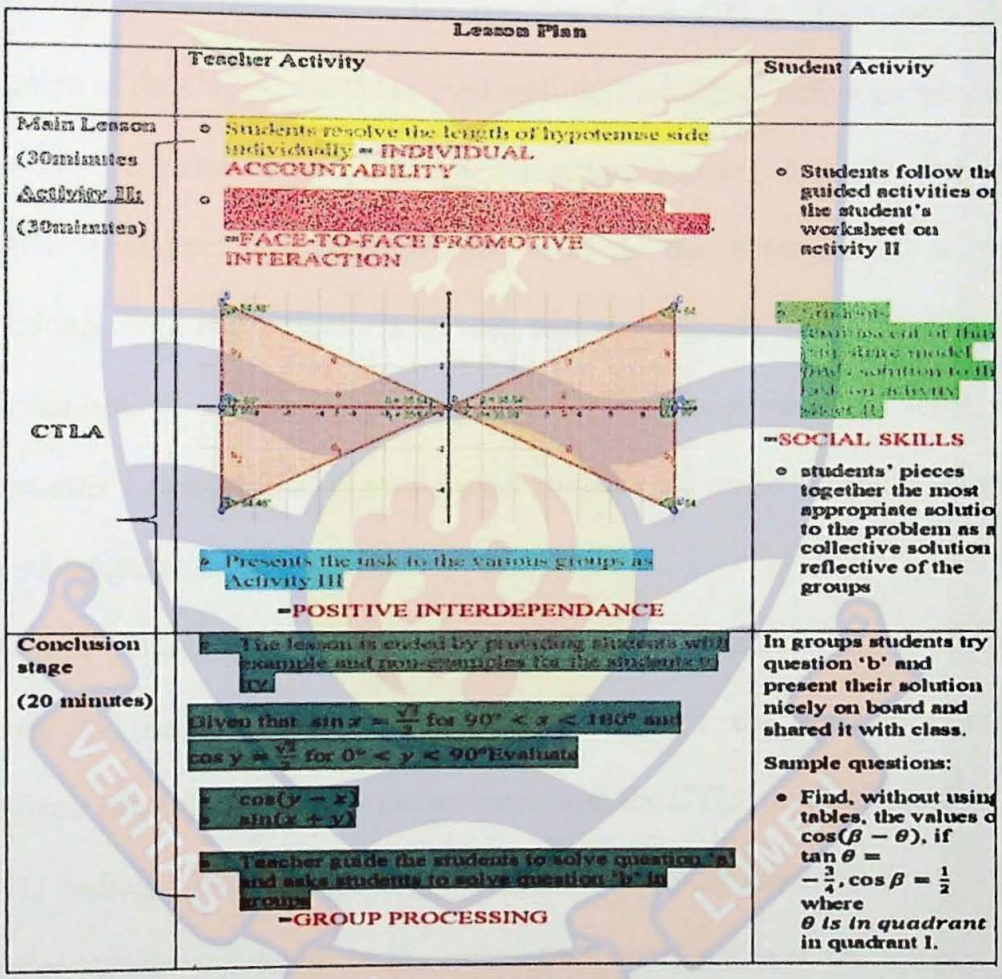


Figure 23: Excerpt of the IMTs (in Team 4) developed CTLA lesson on compound angles.
Source: Fieldwork, 2021.

The IMTs also confirmed during an interview session after the enactment of the CTLA-based interventions that there had been considerable improvement in the teaching and learning process because of their enhanced appreciation of the CTLA. According to the IMTs, the incorporation of the features of CTLA in teaching and learning of mathematics with particular

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reference to the concept of trigonometry in SHS improved their pedagogical approach to the teaching of mathematics as whole. This is because concepts in trigonometry, for example, the three ratios of sine, cosine and tangent of acute angles in a right-angle triangle, the compound angles, the angles of elevation and depression as well as the trigonometric identities which they had always taught in abstraction in a typical traditional classroom context was taught with ease through the affordances of the features of the CTLA. Thus, with the application of the CTLA, the IMTs used well-defined sets of activities relevant to the subject matter, readily available teaching learning material and the student's worksheet that was tailored towards the achievement of the instructional learning objectives of the lesson to guide students in their conceptualisation of the subject matter. This approach of teaching mathematics helped the IMTs to guide students to make meaning of the concepts taught in the study.

The following are some of the responses given by the IMTs in response to the question: Was the integration of the activity-oriented worksheets helpful in teaching mathematics with the CTLA in this lesson?

T1T1: *Indeed, I have always taught my lessons without any form of activity especially in the area of trigonometry over the years of teaching at the SHS level, but the inclusion of activity-oriented worksheets in teaching angles of elevation ensured an orderly and systematic presentation of the concept during the teaching and learning process, so it was helpful.*

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T1T2: *Personally, I think it was helpful and ought to be encouraged this is because the materials helped to mobilize all the knowledge base needed to guide the teaching and learning process, an experience which facilitated concept formation in class I'm, therefore, of the view that, the use of the activity sheets should be replicated in other topics.*

T2T1: *The inclusion of activity-oriented worksheet provoked the students' interest in the concept taught and motivated me to develop a good understanding of some topics in trigonometry which I have always taught in abstraction.*

T2T2: *As a result of its activity-oriented nature, students understood the concept taught, they made sense of the concept themselves with little or no input from me (teacher). I only guided them as compared to my usual method of feeding them with formulae of sine cosine and the rest.*

T3T1: *Yes, it helped me to engage the students and sustain their attention throughout the lesson. Hence, its use in mathematics related courses (trigonometry) should be encouraged*

The focus group discussion also reiterated that the IMTs improved in the development of knowledge and skills in enacting CTLA-lessons. For example, in one of the focus group discussions, the group demonstrated that the PDA helped them to develop competencies and skills (CTLA) needed to design CTLA-lessons for mathematics lessons. This implies an improvement

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in the competencies of the IMTs in the design of CTLA lessons. The quotes represent the specific views of some of the in-service mathematics teachers in the focus group discussion. The responses were in respect of the question: "How did the initial training workshop (a component of the PDA) influence your preparation to teach concepts in trigonometry?"

T1T1: *I never knew all along my lessons were premised on the wrong notion of organising cooperative learning, well after the workshop (PDT) I was able to design an activity-based worksheet in an area like trigonometry to teach, it has made my lesson very rich.*

T1T2: *It's an eye opener to me since the idea of getting students to project their group work on the board so as to elicit alternate ideas from other students was completely missing when using group related work in my class but after the PDT, the use of this approach in particular led to the understanding of the students. Something I hardly do in my usual lessons.*

T2T1: *I had a little or no knowledge on the integration of CTLA in teaching. However, after the PDT it prompted me to re-strategise my teaching plans in order to make it more participatory from the perspective of the side of the students since during the teaching and learning process I only served as a guide instead of my usual role I used to play in the class.*

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T2T2: *The training has broadened my knowledge about the use of CTLA in teaching mathematics; this enabled me to get students to construct their own understanding of the concepts taught. Hence, my lessons were student centred, a situation in the class opposite to what I'm used to, thus feeding the students with the formulas for them to memorise.*

As a result of the improvement in the CTLA, the in-service mathematics teachers were given the opportunity to scrutinise and offer constructive criticism on the teaching approach of their colleagues (Peers). These discussions were held after each of the peer-teaching try outs at the first study of the implementation and evaluation stage for the purposes of fine tuning the intervention in the next peer-teaching try out of the study. In this way, opportunities were afforded the IMTs as to what could be done to improve the intervention. Some of the responses to the question: "What can be done to improve the CTLA- intervention designed and enacted by your colleagues?" The following suggestions were given to team 2 (T2T1 and T2T2) after their first lesson on the topic of: Identities of trigonometric function.

T1T1: *Well, I'm of the opinion that, in as much as the lesson captured the attention and achieved the set objectives of the lesson, the time allotted to each group to explore with the aim of making sense of the concept was very minimal. I think much could have been achieved if the time for each group had been extended.*

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T1T2: *I think a critical look should be taken at how the various roles assigned to the students in the group could be enforced. This will shift the attention of the students to be more focused as there is division of labour where each plays his or her role for optimal benefit of the entire group.*

Similarly, the IMTs whose lessons had been critiqued during the implementation of their lessons were also afforded an opportunity to provide feedback to their colleagues after they had enacted their lesson. Thus, team two (T1T1 and T1T2) taught their lesson on the topic of: angles of elevation and depression.

T2T1: *In my own opinion, the lesson was student-centred approach which was full of activities by the students themselves. Again, it was an effective and efficient way to teach the concept of angles of elevation and depression owing to its conceptual difficulty in nature.*

T2T2: *Their lesson was on point, in that, it did encourage full class participation in the lesson, the concept of angles of elevation and depression was to a large extent in my view understood. Finally, it's a good method of teaching since it does encourage peer-teaching among the students as well.*

The analysis as obtained from the IMTs pre and post self-assessment quantitative questionnaire, interview, focus group discussions and lesson plan document attest to IMTs improvement in the knowledge and skills in the enactment of CTLA lessons (thus, lessons reflective of the five features of cooperative teaching and learning approach) in the study

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In-service Mathematics Teachers' Improved Content Knowledge in their Subject Matter

In tracking the in-service mathematics teachers' improved knowledge in the content of the subject matter, the implementation and evaluation stage of the study served as the gateway to respond to IMTs improvement in the content knowledge of the concepts taught. Thus, at the beginning of the workshop, it was observed that some of the IMTs had difficulties in explaining and responding to certain questions on some aspects of the concepts in trigonometry. For instance, I observed that some of the IMTs were relying on the mnemonics (memorised versions) such as the "SOHCAHTOA" and the trigonometric identities of "$Sinx^2 + Cox^2 = 1$" in teaching whiles others could not provide the diagrammatic representation of concepts of angles of elevation and depression in the form of right-angled triangle to enable them apply the appropriate trigonometric ratios (sine, cosine and tangent of angles) to find solution to the questions asked during the researcher's sample demonstration of the CTLA lesson to the IMTs.

However, after a series of teaching and learning processes with the CTLA, the IMTs exuded confidence and mastery of the same concepts (angles of elevation and depression as well as the trigonometric identities) they had difficulty with prior to the PDA. Consequently, the interview data gathered were also supportive of this observation made. For example, some of the IMTs disclosed in an interview that they have personally benefited a lot as a result of their involvement in the study. Thus, it has helped them to gain a better appreciation and understanding of some concepts in mathematics especially in the area of trigonometry which they had difficulty prior to the study. The

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following were some of the IMTs remarks after the implementation of the CTLA-lesson. These responses were in respect of the question: "What general comment can you make about using CTLA-intervention in teaching trigonometry in SHS?"

T1T1: *I have been using mnemonics to teach trigonometry but the use of the student worksheet of activities has added another dimension of understanding in getting the students to appreciate the processes involved in trig ratios as the relationships of the lengths of the right-angled triangle.*

T2T1: *I had difficulty in teaching some concepts in trigonometry especially using diagram to distinguish between the upward and downward movement of the eye in locating objects using the eyes level, but I think I was able to overcome such anxiety. My lesson today was the best as compared to my classroom style of teaching.*

Analysis of lesson plan documents (activity-oriented student worksheet) also corroborated this fact that the IMTs improved their knowledge of the content in the subject matter. This assertion is made on the basis of two main criteria that were used to measure the IMTs knowledge of the content in the subject matter. The first criterion was the quality of the information provided in the lesson plan documents and the second criterion was the quality of delivery (i.e., clarity in explanation of concepts). Figure 24 shows excerpt of the activity sheet that was used to engage students in their various groups to make meaning of the concept of the three trigonometric ratios of sine, cosine and tangent of angles in trigonometry during the implementation of the

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CTLA- lesson designed by team 3 (T3T1 and T3T2) for the teaching of the topic: ratios of sine, cosine and tangent of angles.

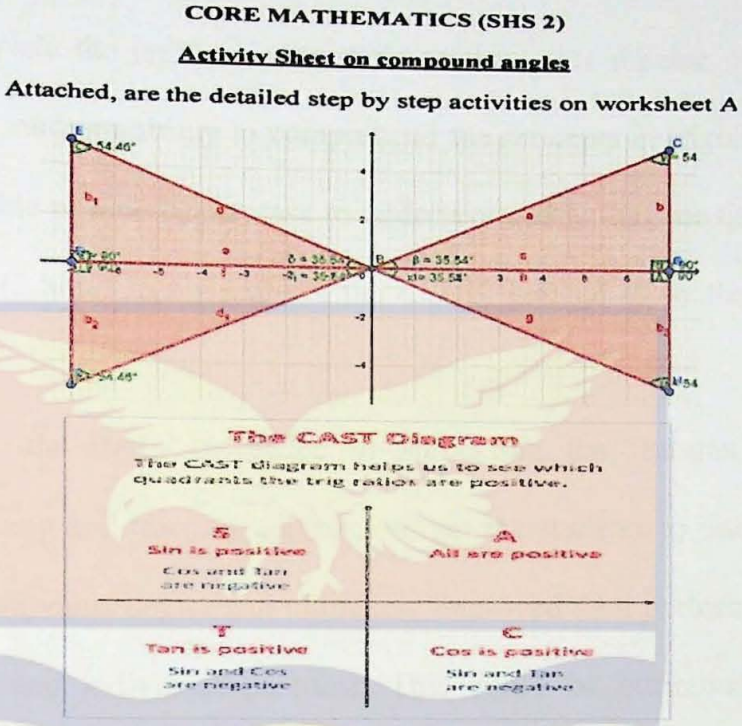


Figure 23: Excerpt of the activity sheet used in the lesson of ratios of sine, cosine and tangent of angles by team 3.
Source: Fieldwork, 2021.

Likewise, Team 4 (T4T1 and T4T2) used the activity-oriented worksheet in Figure 24 to engage students to construct their own understanding of the concept of compound angles in relation to sine, cosine and the tangent of angles. For example, the IMTs employed the affordances of the features of CTLA to teach compound angles at school B. In the remarks of the Head of Department of Mathematics at school B, when he was informed of the topic the students were going to instructed on (compound angles). He remarked.

> *"eeeiii (an exclamation of shock) are you going to teach trigonometry? These students haven't even completed the SHS one course in mathematics although they are in SHS2"*

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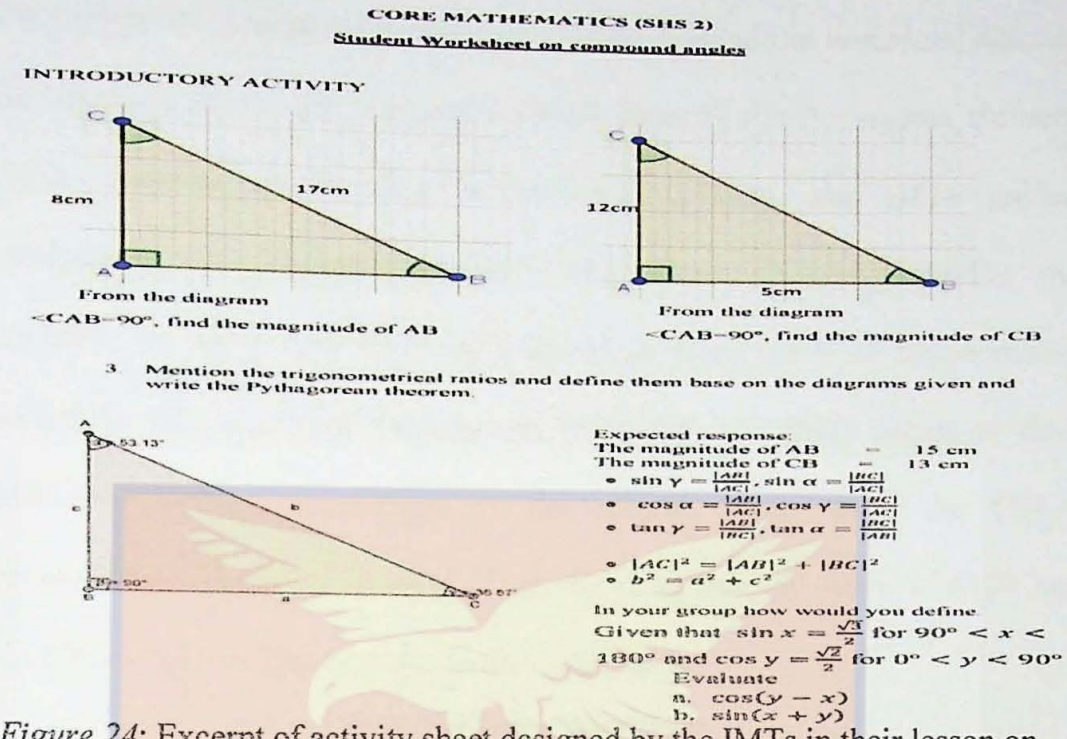
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The above remarks show the extent to which the students in school B were perceived to be incapable of appreciating any concept in trigonometry. Moreover, the Head of Mathematics Department was emphatic in their inability to complete the previous semester's mathematics scheme of work. Hence found the students ability to comprehend the concepts in trigonometry virtually impossible or a herculean task to undertake taking into consideration the limited time (2 hours as observed of the CLOP) granted us by the school authorities.

However, the IMTs succeeded in harnessing the features of the cooperative teaching and learning approach to get the students to understand the concept of *compound angles* and ultimately improved their performance at both the group and individual post-test. This could be attributed to an improved content knowledge of the concept of *compound angle* by the IMTs. The two cohorts of IMTs did not only provide quality information in their lesson plan documents but also provided good explanations of concepts in the subject matter as a result of their improved content knowledge. The IMTs were able to use appropriate annotated diagrams to support the exploratory activities on the student's worksheet of the concepts taught. The clarity in their expression and their ability to handle students' questions in the learning environment as observed on the CLOP all proved that their content knowledge improved with the use of the CTLA as a pedagogical strategy for trigonometric lessons in mathematics. Figure 25 shows excerpts of the detailed activities on the student's worksheet designed by the IMTs in the lesson on *compound angles*.

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CORE MATHEMATICS (SHS 2)

Student Worksheet on compound angles

INTRODUCTORY ACTIVITY

8cm 17cm

From the diagram

<CAB=90°, find the magnitude of AB

12cm 5cm

From the diagram

<CAB=90°, find the magnitude of CB

3. Mention the trigonometrical ratios and define them base on the diagrams given and write the Pythagorean theorem.

Expected response:

The magnitude of AB = 15 cm

The magnitude of CB = 13 cm

- $\sin\gamma = \frac{|AB|}{|AC|}, \sin\alpha = \frac{|BC|}{|AC|}$
- $\cos\alpha = \frac{|AB|}{|AC|}, \cos\gamma = \frac{|BC|}{|AC|}$
- $\tan\gamma = \frac{|AB|}{|BC|}, \tan\alpha = \frac{|BC|}{|AB|}$
- $|AC|^2 = |AB|^2 + |BC|^2$
- $b^2 = a^2 + c^2$

In your group how would you define

Given that $\sin x = \frac{\sqrt{3}}{2}$ for $90° < x <$ $180°$ and $\cos y = \frac{\sqrt{2}}{2}$ for $0° < y < 90°$

Evaluate

a. $\cos(y - x)$

b. $\sin(x + y)$

Figure 24: Excerpt of activity sheet designed by the IMTs in their lesson on compound angles.

Source: Fieldwork, 2021.

Again, with particular reference to team 1 (T1T1 and T1T2) who instructed their peers on the teaching of topic: trigonometric identities, the IMTs used an activity in the form of right-angled triangle to explain how trigonometric identities were derived using the unit circle method. He explained to the students that with a pair of compasses and a rule, the students were to construct a unit circle of a centimeter (radius) using the origin as the center and pick any arbitrary point say $T(x, y)$in the first quadrant. From their knowledge of tangent of an angle as the ratio of the opposite side to the adjacent side, they were asked to find tan α from the diagram as shown on the students' worksheet (see Figure 26). In addition, the IMTs tasked the students to find the relation that connects all three sides of the right-angled triangle (applying the Pythagoras theorem). Finally, a student each from the different groups was asked to write down the Pythagoras theorem (Students write the Pythagoras theorem mathematically as: $x^2 + y^2 = 1$). The IMTs then guided

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the students to substitute the expression for the vertical and horizontal sides of the diagram above into the mathematical form of the Pythagoras theorem (thus, students obtained$Sinx^2 + Cox^2 = 1$). Finally, the IMTs guided students to write equivalent forms of the relation above by dividing the equation by $\sin^2 x$, then with $\cos^2 x$ to obtain other forms of trigonometric identities. The clarity of explanations offered by the IMTs attests to their improved content knowledge of the subject matter with the CTLA intervention. Figure 26: Excerpt of activity sheet used by Team 1 (T1T1 and T1T2) for the teaching of trigonometric identities.

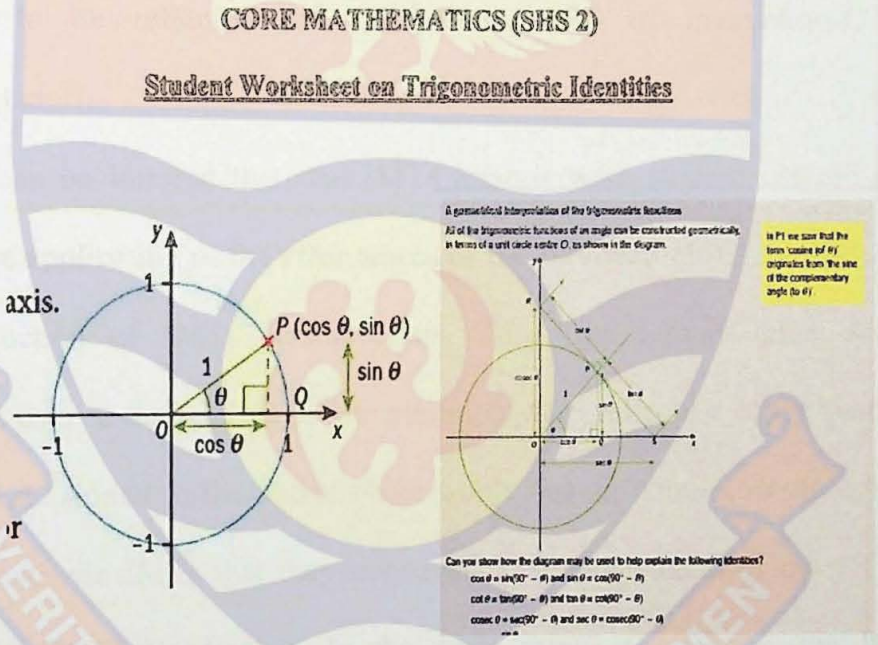


Figure 25: Excerpt of activity sheet used by Team 1 (T1T1 and T1T2) for the lesson on trigonometric identities.
Source: Fieldwork, 2021.

Even though there was a balance (equilibrium) in the assimilation and accommodation processes in the minds of the in-service mathematics teachers developed CTLA, they encountered some challenges with the use of CTLA-intervention for mathematics lessons especially in the area of trigonometry. Some of the challenges they faced during the initial stages of the peer-teaching

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try-outs were the difficulty in integrating the new method of teaching into their existing (old method of teaching) one. For instance, after introducing to them to the new method of teaching (CTLA) an IMT (T3T1) was observed in one of the peer-teaching sessions using his usual old method of teaching where he did all the teaching without a single involvement of the student-teachers (during the one of the peer-teaching activities-where other teachers served as students in the class at the implantation and evaluation stage of the study) in any form of class activity, grouping etc. (thus, the said IMT could not assimilate) however most of the teachers corrected this by broadening their horizon (schemas) to be reflective of the new method of instruction-CTLA (accommodation).

It can be inferred that, the IMTs lessons were student centred as a result of the application of the affordances of the five features of the CTLA in the construction of IMTs lesson plans. Thus, the lessons plan and it consequent uses in the real classroom situation were reflective of the features of the CTLA. Added to the above observation was an admission through an interview with the IMTs that they improved in the enactment of the CTLA-lessons post PDT introduction. In the focal group discussions, the IMTs mentioned that the PDA helped them to develop the pre-requisite competencies and skills needed to design the CTLA intervention. Lastly on the issue of an improved content knowledge of the subject matter, it was observed that, some of the IMTs who prior to the PDT had misconceived notions on some aspects of the concept trigonometry (angles of elevation and depression) performed creditably post PDT in teaching with the CTLA.

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The overall effect in answering research question two (2) was that, the IMTs showed improvement in the post self-assessment on the use of the CLTA as a pedagogical approach in the classroom context as was measured quantitatively in the study. This assertion was qualitatively supported through the IMTs interviews, lesson plan document analysis and focal group discussions. Thus, the IMTs developed the competencies in the affordances of the five features of CTLA to enact CTLA-based lessons in the classroom context. As a result of which, an improved content knowledge of the IMTs prior to and post introduction of the PDT was observed in the study.

Research Question Three

The research question three was "how can the improvements in students' learning of concepts in mathematics using CTLA be understood and or explained?" Thus, the question sought to establish how the improvements in students' learning of concepts in mathematics using cooperative teaching and learning approach could be understood and or explained.

The improvements in students' learning of concepts in mathematics were explained from two perspectives; (1) the positive perceived students' experiences with the CTLA lessons and (2) enhanced students' learning outcomes as a result of the exposure of SHS students to the intervention (CTLA) during the implementation and evaluation stage. The two perspectives are explained in details in the next section.

The Perceived Positive Students' Experiences with the CTLA-Lessons

The students' dispositions towards the CTLA-lessons were determined by the use of students' experiences questionnaire that was administered to 55 students in the two SHS (School A and School B) immediately after each of

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the lessons. Possible responses to the items on the five-subscale questionnaire were on a scale ranging from strongly disagree (1) to strongly agree (5). Thus, for the subscales, a score below and above 3 depicts favourable and unfavourable opinions of the respondents. The results of the school-based analysis of the students in school A and School B are shown in Table 18.

Table 17: Students' Experiences with CTLA -Intervention Based on School-type

Experience Subscales	School-type	N	Mean	SD
Delivery	School A	29	4.12	0.47
	School B	26	3.91	0.58
Interest	School A	29	4.34	0.58
	School B	26	3.77	1.00
Comprehension	School A	29	4.16	0.47
	School B	26	3.69	0.83

Source: Fieldwork, 2021.

The three subscales from the analysis indicated that generally, the students from both schools expressed positive dispositions towards the CTLA lessons. With students in school A rating their experiences with the intervention highly in respect of the interest ($\boldsymbol{M} = 4.34$, $\boldsymbol{SD} = 0.58$) as against School B($\boldsymbol{M} = 3.77$, $\boldsymbol{SD} = 1.00$). School B recorded the subscale of delivery ($\boldsymbol{M} = 3.91$, $\boldsymbol{SD} = 0.58$) as the highest rated experience with the intervention as against school A($\boldsymbol{M} = 4.12$, $\boldsymbol{SD} = 0.47$). The comprehension subscale also recorded means of ($\boldsymbol{M} = 4.16$, $\boldsymbol{SD} = 0.47$) and ($\boldsymbol{M} = 3.69$, $\boldsymbol{SD} = 0.83$)for school A and school B respectively.

Although there were perceived differences in the dispositions of students' attitude towards the CTLA-intervention in the two senior high schools involved in the study, the IMTs pedagogical strategy improved the students learning of concepts in trigonometry taught, as evidenced on the

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above average mean scores of ($\boldsymbol{M} = 4.20$, $\boldsymbol{SD} = 0.50$) and ($\boldsymbol{M} = 3.79$, $\boldsymbol{SD} = 0.80$) registered on all the subscales (see Table 17) post introduction of the intervention in schools A and B respectively at the real classroom situation. It can further be observed that, the students experience with the intervention was favourable irrespective of the school type it was deployed. Thus, the IMTs use of the intervention improved students understanding and ultimately yielded enhanced learning outcomes as a consequence of which the students in the two schools registered higher mean scores on the subscales of interests, comprehension and delivery in the study.

Similarly, the information gathered through the interview seems suggestive of the student's improvement in their learning outcomes in trigonometry as a direct consequence of their perceived positive experiences with the CTLA- intervention. This assertion is based upon the fact that all five students who were interviewed immediately after the implementation of the two lessons designed by the IMTs expressed positive experiences with the intervention. Below are some of the sentiments expressed by the students.

Student 1: *This new method of teaching is very different but effective and more engaging way of teaching mathematics. In fact, I really admired the way and manner in which the lessons were delivered. {Delivery} Now I understand the topic and can teach my friends in class {Comprehension}.*

Student 2: *It is very effective and should be adopted for the next generation this is because the activity sheet made the lesson more practical and understandable to our group. we*

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were able to solve our question as a group {Delivery, Comprehension}

Student 3: *It is a very good method of teaching because when I saw the questions, I thought we won't be able to do it but after the groupings in class we shared ideas on the topic and was able to do that same question I had no idea about, so it should be encouraged. There was good teacher and student relationship and this helped me to open up when I didn't understand anything during the group activities. {Delivery}*

Student 4: *It was quite interesting and not boring. The teacher gave us more examples on the topic for us to try on the activity worksheet. This new method of teaching has brought many ideas and concepts on the same topic I couldn't understand, but I'm now able to answer questions and I think it should be introduced in other schools well. {Interest, Delivery, Comprehension}.*

Student 5: *My overall impression of this new method of teaching is a good one. This is because; this new method helps to promote both group work and individual work when it comes to studying. The teacher motivated us to solve the task at our personal level before putting our answers together as a group. I was able to answer the questions I found as difficult in the textbook. {Delivery, Comprehension}*

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These reports reiterate the students' positive experiences (See Table 17) with the CTLA lessons and serve aa a possible reason for the improvements in students' learning of mathematics concepts that were taught.

Enhanced Students' Learning Outcomes in Trigonometry

As part of research question 3, the improvement in students' learning using the CTLA was further understood and explained from the perspective of the students improved learning outcomes. Data collected particularly during the implementation and evaluation stage of the study in which the second cohort of IMTs taught CTLA lessons in two SHS in Cape Coast were used in addressing this question. During the process, the two groups of IMTs: Team 3 (T3T1 and T3T2) and team 4 (T4T1 and T4T2) conducted pre-and post-tests consisting of the same items on the topics: angle of elevation (application of trigonometric ratios) and compound angles respectively before and immediately after implementing each of their lessons. The test required students to detail the processes involved in arriving at the answers to the questions (thus, show working- proof of the steps involved in the resolution of the question). The test was scored out of five in school A and out of ten in school B. Tables 18 and 19 show the results of the paired sample *t*-test that was used to determine if the mean differences in the pre-test and post-test of the scores obtained by the students in prior to and post introduction of the CTLA in the respective lessons were significant in both schools.

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Table18: Paired Sample T-test for SHS A

			Paired differences			
	M	*SD*	*Md*	*T*	df	*p*
School A Post-test scores	4.38	.677	3.03	24.014	28	.000
School A Pre-test scores	1.34	.897				

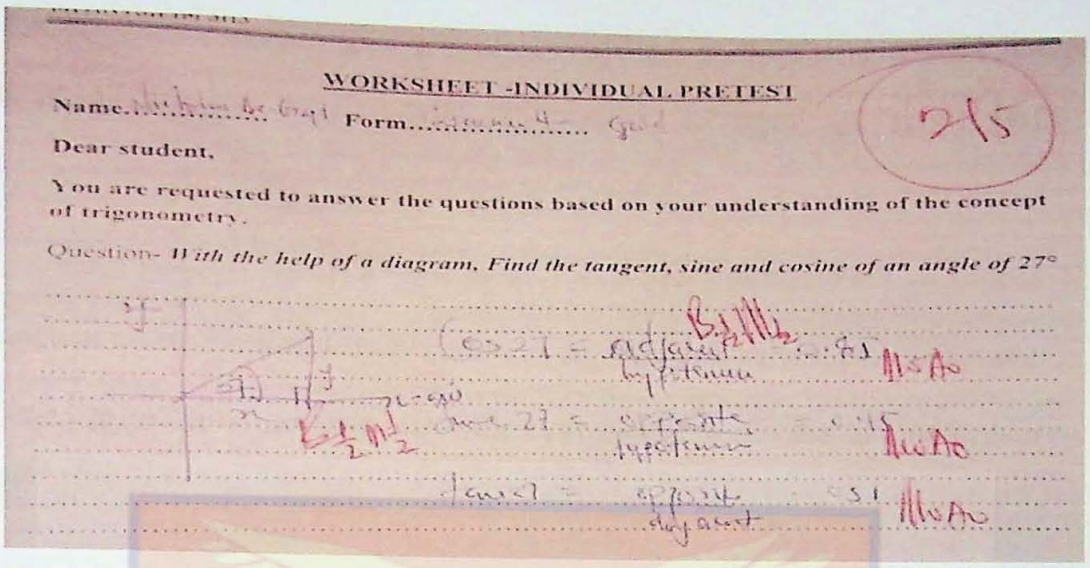
Source: Fieldwork, 2021.

From table 18, there was a significant mean difference in the pre and post test scores of the students (M = 5.54, SD = .761; t (25) = 37.13, p = .000). In addition, eta squared was calculated to determine the effect size statistic of the CTLA-intervention. Thus, an eta squared statistic of 0.95 was obtained from the analysis which depicts a 'large' effect size in the differences in the student's pre-test and post-test scores obtained before and after the introduction of the CTLA lessons. This shows the students' improved performance in the post-test after the introduction of the CTLA intervention in the real classroom situation in SHS A.

Again, the evidence of the improvement in the performance of students in school A before and after the introduction to the intervention is shown in Figures 27A to 27B. Thus, Figures 27A to 27B show excerpts of the performances of individual student's pre-test and post-test administered in the study. It can be observed that the students showed improvement in their test scores after the introduction of the intervention. This evidence follows through all the questions as the student provided clarity to the various steps (show working-proof of the method employed in answering the question) in solving the questions and demonstrating conceptual understanding through the construction of appropriate diagrams in their respective solutions.

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WORKSHEET -INDIVIDUAL PRETEST

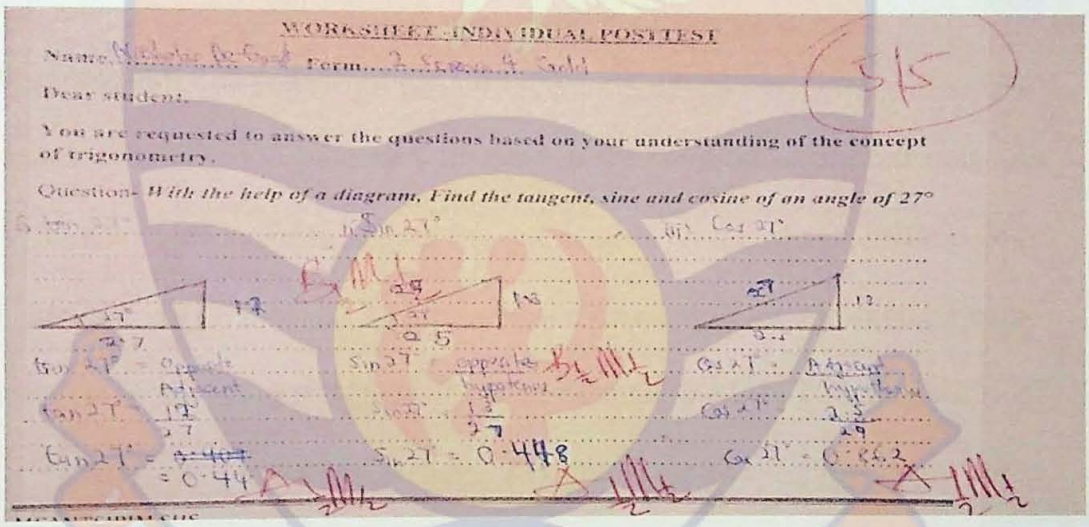
Name..... Form.....

Dear student,

You are requested to answer the questions based on your understanding of the concept of trigonometry.

Question- *With the help of a diagram, Find the tangent, sine and cosine of an angle of 27°*

Figure 26A: Excerpt of student 1 individual post-test score.
Source: Fieldwork, 2021.


WORKSHEET -INDIVIDUAL POSTTEST

Name..... Form.....

Dear student,

You are requested to answer the questions based on your understanding of the concept of trigonometry.

Question- *With the help of a diagram, Find the tangent, sine and cosine of an angle of 27°*

Figure 27B: Excerpt of Student 1 individual pre-test score.
Source: Fieldwork, 2021.

For example, from Figure 27A and 27B, student 1 pre-test score increased from 2 out of 5 to 5 out of 5 on the individual pre-test and post-test respectively. As observed from the excerpts, the student 1 could not account for the processes leading to the answer that was asked on the three trigonometric ratios of sine, cosine and the tangent of the angle given during the pre-test. In particular, the question (*with the help of the diagram find the sine, cosine and the tangent of an angle of 27 degree*) could not be established

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by way of drawing the proper right-angle triangle diagram to illustrate the concept of question coupled with his inability to apply the appropriate trigonometric ratio to solve the question in the pre-test.

However, in the post-test, after the IMTs through their enhanced competencies in enacting CTLA lesson (thus, grouped the students with some assigned roles, provided them with the opportunity to discuss at length their ideas on *sine, cosine and the tangent of an angle* through the provision of related activities on the students work sheet individually and collectively as a group as well as the provision of platform for the groups to project their solution on the chalkboard for the purposes soliciting alternate ideas and views from non-group members during the teaching and processes) the students were able to account for the processes that led to the solution of pre-test item. As such, student 1 was able to answer the questions with the correct methodology and the appropriate diagram to represent the concept of the question he could not answer in the pre-test. Hence, it can be asserted that, the effective utility of the CTLA in the lesson might have accounted for the improvement in the student 1 individual pre-test score.

Similar findings were observed in the results in SHS B. Thus, students' performance improved post introduction of the CTLA intervention by the IMTs. Table 19 shows the results of the paired sample t-test of the scores obtained prior to and post introduction of the CTLA lessons to students in school B.

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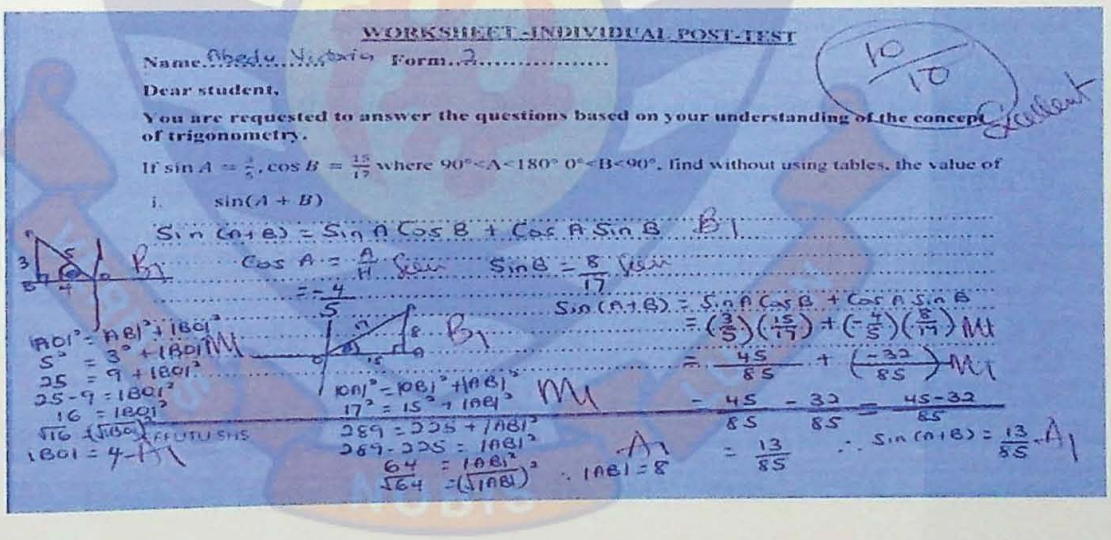
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Table 19: Paired Sample T-test for SHS B

			Paired differences			
	M	*SD*	*Md*	*T*	df	*p*
School B Post-test scores	7.92	1.093	5.54	37.131	25	.000
School B Pre-test scores	2.38	1.169				

Source: Fieldwork, 2021.

The Table shows the paired sample t-test with a significant (mean difference = 5.538, 95% *CI:* 5.231 to 5.846) mean difference. A very large eta squared (eta squared = 0.98) was obtained from the analysis which depicts that, there was significant difference in the student's pre-test and post-test scores obtained prior to and post introduction of the CTLA lessons. Thus, students' performance improved after they were introduced to the CTLA lesson on the concepts taught. This assertion was confirmed by analysis of how students answered the questions as demonstrated by Figure 28.


WORKSHEET -INDIVIDUAL POST-TEST

Name..... Form..2.....

Dear student,

You are requested to answer the questions based on your understanding of the concept of trigonometry.

If $\sin A = \frac{3}{5}, \cos B = \frac{15}{17}$ where $90° < A < 180°$ $0° < B < 90°$, find without using tables, the value of

i. $\sin(A + B)$

$\sin(A+B) = \sin A \cos B + \cos A \sin B$

$\cos A = \frac{A}{H}$ $\sin B = \frac{8}{17}$

$= -\frac{4}{5}$

$\sin(A+B) = \sin A \cos B + \cos A \sin B$

$= \left(\frac{3}{5}\right)\left(\frac{15}{17}\right) + \left(-\frac{4}{5}\right)\left(\frac{8}{17}\right)$

$= \frac{45}{85} + \left(\frac{-32}{85}\right)$

$= \frac{45}{85} - \frac{32}{85} = \frac{45-32}{85}$

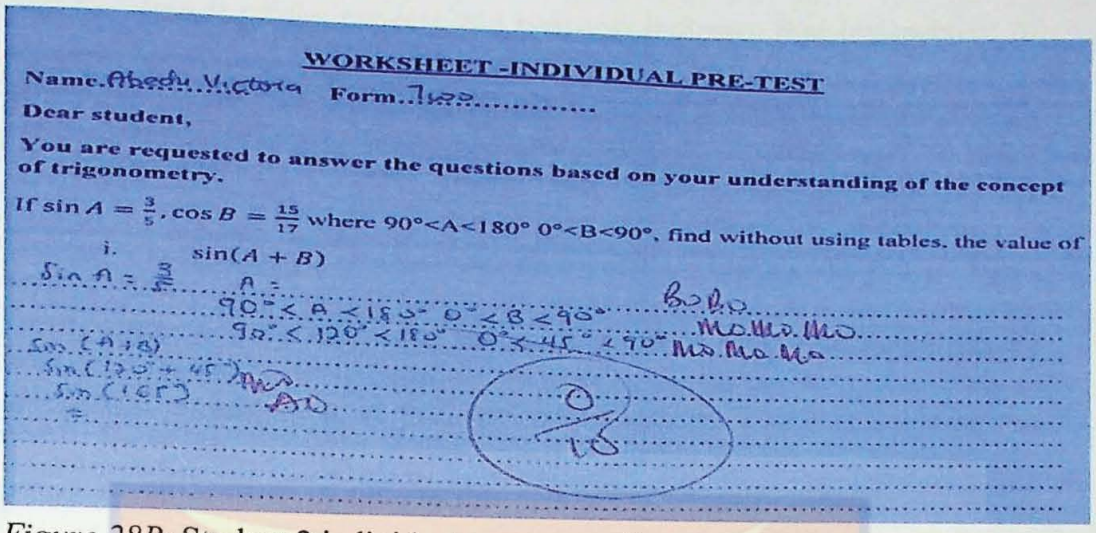
$= \frac{13}{85}$ $\therefore \sin(A+B) = \frac{13}{85}$

10/10 Excellent

Figure 28A: Student 2 individual post-test performance.
Source: Fieldwork, 2021.

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WORKSHEET -INDIVIDUAL PRE-TEST

Name. Abedu Victoria Form. Two

Dear student,

You are requested to answer the questions based on your understanding of the concept of trigonometry.

If $\sin A = \frac{3}{5}, \cos B = \frac{15}{17}$ where $90° < A < 180°$ $0° < B < 90°$, find without using tables, the value of

i. $\sin(A + B)$

$\sin A = \frac{3}{5}$ A =

$90° < A < 180°$ $0° < B < 90°$

$90° < 120° < 180°$ $0° < 45° < 90°$

$\sin(A+B)$

$\sin(120° + 45°)$

$\sin(165°)$

No

No

0

10

Figure 28B: Student 2 individual pre-test performance.

Source: Fieldwork, 2021.

It can be observed from Figures 27A and 27B that, student 2 pre-test score increased from 0 out of 10 to 10 out of 10 form the pre-test to the post-test. As can be inferred from the Excerpts, student 2 could not answer the question (thus, *if* $\sin A = \frac{3}{5}, \cos B = \frac{15}{17}$ *where* $90° < A < 180°$ $0° < B < 90°$, *find without using tables, the value of* $\sin(A + B)$) that was asked on the concept of compound angle in the pre-test. Thus, student 2 failed to provide any form of diagrammatic representation of the concepts of sine and cosine needed to find solution to the question. Again, he/she could not recall the appropriate sum of angles for the trigonometric ratios although such references were provided prior to the pre-test.

However, in the post-test, after the IMTs enacted CTLA-lesson on *compound angles* in the real classroom situation, student 2 was able to answer the question with the score of 10 out of 10. Thus, student 2 filled the gap by detailing the systematic approach as demanded by the question (see Figure 28B). Hence, the use of CTLA in the lesson might have accounted for the increase in the student 2 pre-test score. The analysis from the paired sample t-

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test and excerpts from the pre-test and post-test indicates that irrespective of the school the CTLA was deployed, the performance of the students improved.

Qualitative data gathered during the implementation of the CTLA lessons were also suggestive of enhanced students learning outcomes. Thus, responses from the interview data indicated improved learning outcomes by students after participating in CTLA-enacted lessons in the real classroom situation. In furtherance to the confirmation of the results obtained from the pre-test and post-test conducted at the school, some of the students I interviewed attributed their improved learning outcomes to the implementation of the CTLA-enacted lessons. For instance, in relation to the topic: the application of three trigonometric ratios of sine, cosine and the tangent of angles, the students selected using the simple random sampling technique at SHS A indicated that, they had a better appreciation of the concept taught as a result of the new pedagogical approach (CTLA) they were introduced to in class. Some of the responses obtained from the students in relation to the question: "How will you assess today's lesson?" were captured as:

Student 1: *Today's lesson was a success; this is because I have always struggled to understand anything in trigonometry, but I understood the topic easily due to the method of teaching introduced. This method of teaching allowed me to think deeply and improved my understanding and appreciation of the trigonometric ratios taught.*

Student 2: *Very successful because when they came, I wasn't getting what they were doing but at the end, I got to understand and even know about the relevance of the*

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trigonometric ratios. You see trigonometry is one difficult topic I don't know anything about it but today Haa!!! Ha! (a sigh of relief of happiness) I have understood.

Student 3: *It was a success. I was able to solve most questions I had difficulty with especially in "trig ratios". In fact, sir, I have not come across any of their (IMTs) process/techniques of solving trigonometry question and their technique was much easier than that I see in the textbook. All their questions on the trigonometric ratios were solved using their method of teaching and all the solutions were correct. This was really a good experience if I am to compare it with my core mathematics teacher way of teaching us in class.*

Student 4: *Success: my reason of choosing success is that; the teacher made us know the various ways of solving questions under trigonometry through the group presentations we had in class, this enabled me in particular to understand the lesson. Finally, I am able to solve questions under this trig ratio I could not do before.*

Student 5: *It was successful. This is because before the lesson I had no knowledge about trigonometry so from my perspective I thought it was difficult but at the end of it all, by God's Grace am able to do trigonometry. so, it was successful.*

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Student6: *To be honest, before today's lesson, I had almost little knowledge about trigonometry although we have had lessons on the topic but during the course of the lesson, I have learnt much enough so yes, I was able to answer the teacher's questions with the understanding I got from my group members, the lesson was a success.*

Similar observations were recorded during the interview session with the students who were selected at random (thus, using simple random sampling technique) after the IMTs instruction on Compound Angles in SHS B. The responses obtained from some of the students in relation to the question: How will you assess today's lesson? Recorded as:

Student 1: *Today's lesson was a good one. How ideas were shared in the group made it less difficult for our group to find solution to task assigned to us by the teacher, so now I can say it was easy because I can do it alone. I like the manner of teaching even if nothing at all by the discussions we had on the topic you could get something inside (head) if there is nothing in your mind.*

Student 2: *Today's lesson was very great. It went successfully because I think from the individual contributions during the group presentation, some of the students who don't speak in class were seen contributing to the group activities in my group on the compound angles identities and its applications which is under trigonometry and I understood it very well too since I got my question correct.*

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Student 3: *Today's lesson has broadened my mind about certain aspects of the identities in compound angles under trigonometry in elective mathematics and partly physics. This is because my views were improved upon as a result of the group activities on the task. So, I can say Today's lesson was very successful to me since I'm able to answer questions I found it difficult to solve.*

The sentiments expressed by students above concerning the CTLA-enacted lessons corroborate the student-centred assertive nature of the intervention. As such, students learning outcomes during the implementation of the CTLA-lesson improved as a result of the IMTs exposure of the students to the cooperative teaching and learning approach as the medium of instruction in the learning environment.

Thus, based on the evidence gathered from the student's pre-test and post-test and interview data it can be asserted that, the enhanced learning outcome in the performance of the students in the two SHS was as a result of the implementation of the CTLA-intervention as a pedagogical approach in the real classroom situation.

Discussion of Results

This section highlights and discusses the main key findings as emerged based on the three research questions that guided the study. This is purposed to evaluate whether the CTLA intervention introduced in the study was effective in promoting SHS students understanding of the subject matter (trigonometry), improved teachers' pedagogical strategies as an instructional approach in the context of the SHS classroom and finally whether the intervention enhanced

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learning outcomes of the SHS students especially in the area of trigonometry. Thus, the key findings are delineated in themes in relation to the research questions as well as the related literature reviewed in Chapter 2 of this study.

The Effectiveness of the CTLA Intervention in Developing IMTs Pedagogical Strategies to Teaching.

The research question one sought to establish the efficacy of the CTLA intervention in developing IMTs pedagogical strategies to improve teaching and learning of concepts in mathematics. For the purpose of this study, effectiveness of the cooperative teaching and learning-approach was conceptualised to mean the extent to which the five features of CTLA (Positive interdependence, Individual accountability, promotive Face-to-face interaction, social skills and Group processing) were reflected in the design and implementation of the lessons. Both qualitative and quantitative data were collected in order to determine the effectiveness of the CTLA-intervention in developing IMTs pedagogical approach to improve the teaching and learning of concepts in mathematics. Generally, findings emerged from the results of the study revealed that the CTLA-lessons designed and implemented in the study were effective in developing the IMTs pedagogical strategies to improve the teaching and learning of mathematics. This is because the CTLA-lessons promoted student-centred teaching and learning process in a constructivist manner which yielded improved students' learning outcomes. Both qualitative and quantitative results from the perspective of the in-service mathematics teachers and the 55 students from the two senior high schools showed that all the five features of CTLA-intervention were fully applied during the teaching and learning process and hence, effective as conceptualised in the study.

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The qualitative results (e.g., Figures 4-18 and Table 7-14) showed that the IMTs used the CTLA-lesson to 1) group their learners with some assigned roles in each group in learning the subject matter (Positive Interdependence); 2) engage learners to share ideas in groups in order to access divergent knowledge expressions for the purposes of conceptualising the mathematical concepts taught in the lesson (promotive face to face interaction); 3) organise the mathematics classroom learning environment in such a way that, students' personal experiences and views were respected in order to contribute to the solution of the assigned task (social skills); 4) measure students' performances on activities through monitoring, evaluation and feedback (thus, students project their solution to the assigned task on the chalkboard for the purposes soliciting for alternate methods of solving that same task-(group processing); 5) put together alternate ideas (solutions) from individual students for the purposes of reaching consensus on a solution reflective of the group's individual effort (individual accountability).

Similarly, the quantitative results confirmed that the CTLA-lessons promoted student-centred teaching (social skills, group processing, individual accountability, face to face promotive interaction and Positive Interdependence) in teaching SHS mathematics (see Table 15 and Table 16). It is clear from Table 15 and Table 16 that, the overall mean values recorded for the five features of CTLA for school A ($\boldsymbol{M} = 4.28$, $\boldsymbol{SD} = 0.43$) and school B($\boldsymbol{M} = 3.91$, $\boldsymbol{SD} = 0.84$)indicated student-centred teaching. This implies that the CTLA-lessons designed and implemented in the study was effective in developing IMTs pedagogical strategies to improve the teaching of trigonometry in SHS in Ghana. These findings from the current study are

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consistent with the findings of studies conducted in the Ghanaian context by Assan-Donkoh et al., 2019; Edekor, & Agbornu, 2020; Bosson-Amedenu et al., 2021 who sought to conduct a comparative study on cooperative learning in relation to the traditional method of teaching.

Again, the findings of the study are unswerving to that of Mehta and Kulshrestha (2014) who mainly sought to develop and implement instructional material anchored on cooperative learning approach as well as the effect of cooperative teaching and learning technique on the social and cooperative skills of students. The results of the study indicated that, they developed positive interdependence, face-to-face interaction skills, and feeling of individual accountability as compared by Mann Whitney U test as a result of which the study concluded that, there was an effective utility of the cooperative teaching technique on the topics taught. Thus, as result of cooperative teaching technique, the students developed the features of cooperative skills which is consistent with the 55 senior high school students' development of the features of CTLA post introduction of the CTLA lessons in their real classroom learning environment in the current study (see Table 7 and 15-overall mean values recorded for the five features of CTLA for school A ($\boldsymbol{M} = 4.28,\ \boldsymbol{SD} = 0.43$) and school B ($\boldsymbol{M} = 3.91,\ \boldsymbol{SD} = 0.84$).

Again, the studies (Johnson, Johnson, & Holubec, 2015; Johnson, Johnson & Smith 1991; Johnson et al., 1998) posited that a lesson designed and anchored on the five main features of cooperative learning has the prerequisite potential to turn such an environment into one where the individuals respect (social skills), rely (positive interdependence), share (promotive face to face interaction), account to each other (individual

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accountability) in an atmosphere promising of students' accomplishment of task and learning (group processing). Again, they contended that a learning environment is said to be cooperative based if and only if such pedagogical technique is anchored on the five main constructs (Johnson et al., 1998). Thus, positive interdependence, face to face promotive interaction, social skills, individual accountability and group processing. These findings are consistent with the current study as all the features described above were found to be relevant in the Ghanaian SHS mathematics classroom learning environment. Thus, IMTs pedagogical strategies developed in the current were observed to be reflective of the five features of the CTLA during the teaching and learning process as well as in their lesson plans (see IMTs document analysis from figures 4-18) used in the lesson. Hence in the current study, lessons that were deemed effective reflected on the five main features of the CTLA as was found in the studies of (Johnson et al., 1998).

However, the current study, is in contrast with the study conducted by Lopata, Miller and Miller (2003) on self-reported relative use of cooperative learning among exemplar teachers. The findings from their study indicated a significant disparity between overall reported actual and preferred use, as well as a significant discrepancy between actual and preferred use on each of the four features of cooperative learning. It was further noted in their study that, the use of individual accountability accounted for a greater development of the staff in terms of the use of the features. In the current study, individual accountability accounted for the least development of the IMTs competencies in the use of the five features of the CTLA initiated at the implementation and evaluation stage of the study. This was observed to be the least applied during

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the conduct of the lessons and was subsequently perceived as the lowest on the mean rating by the IMTs from the perspective of the students (Individual Accountability-$\boldsymbol{M} = 2.3,\ \boldsymbol{SD} = 0.32$).

Based on the findings discussed above, the study concludes that the CTLA-intervention that was designed and implemented in this study was effective in developing IMTs pedagogical strategies to improve the teaching of concepts in mathematics.

Improvements in the CTLA as Pedagogical Strategy in Teaching Mathematics

The research question two of the study sought to understand the improvements in the CTLA as an instructional strategy in the real classroom context (SHS classroom) of the study. In answering this research question, two perspectives were used to explain the improvements in teaching using CTLA as pedagogical strategy in teaching mathematics. The improvements in teaching were explained from the perspective of; 1) IMTs developed knowledge and skills in enacting CTLA-lessons and 2) IMTs improved content knowledge in the subject matter. In overall, results from both qualitative and quantitative data showed that, the improvements in the in-service mathematics teachers' knowledge and skills were reflective of their developed utility of the features of CTLA as an instructional approach in the classroom context (see- Figures 21-23 and Tables 15-16) of the study.

There were two cohorts of IMTs for the first study and second study who were taken through PDA arrangement to acquaint themselves with the pre-requisite knowledge and skills needed to advance their pedagogical strategy with the CTLA, especially in the area of trigonometry in mathematics.

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The results obtained from the analysis of their lesson plan documents, interviews and focus group discussion data indicated that the IMTs developed their knowledge of CTLA during the PDA conducted in the study. This is because prior to the PDA, the IMTs' knowledge of CTLA was measured and the results revealed that most of them had inadequate knowledge and misconceived notions about the application of CTLA in the classroom context. For instance, majority of the IMTs were of the opinion that grouping students (positive interdependence) in an organized environment (social skills) and assigning students to task in order to get them to find solution to that task (promotive face to face interaction) constituted the use of cooperative teaching and learning approach in the classroom context.

However, the lesson plan documents designed by the IMTs and implemented during the conduct of the study, for instance, disclosed that IMTs did not only develop in relation to CTLA as a unique body of knowledge but also sought to bring to bear a matrix combination of the five features of CTLA as instructional strategy in the classroom context during their enactment of the CTLA lessons (see Figure 21-23). This finding clearly shows that, the IMTs views about the nature of the CTLA (Johnson & Johnson, 1998) are fundamental in the appreciation of the improvement in the pedagogical approach (CTLA) employed by the IMTs in the context of this study. This ostensibly led to a positive change in the in-service mathematics teachers' views on teaching mathematics with cooperative teaching and learning approach.

The results from the study (see Table 15 and Table 16) further proved that there were improvements in the IMTs knowledge and skills for using

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CTLA as pedagogical strategy in teaching the subject matter. The overall results for both Phases show an improvement in all the features of CTLA with largest areas of change occurring in CTLA-related sub-scales: Positive Interdependence (gain=2.87), Face to Face promotive interaction (gain = 2.25), Individual Accountability (gain = 2.05), and Group Processing (gain = 2.05). This indicates IMTs development in their knowledge and skills in the features of Positive Interdependence and Face to Face promotive interaction related sub-scales much better as they might have perceived them as familiar knowledge. The results from the interview data also confirmed that, the PDA was critical in the IMTs acquisition of knowledge and skills needed to design and enact CTLA-lessons.

The outcome of the current study aligns with similar studies whose employment of PDA helped in enhancing teachers incorporation of new pedagogical strategies in education (Ho, Watkins & Kelly, 2001; Hew & Brush, 2007). Professional development training and its components played a significant role in the current study. One of the reasons for the teachers' developed CTLA was the nature of the effective PDA initiated in the study. The PDA for this study was in three stages: 1) an introductory workshop for the IMTs 2) design of CTLA- lessons by the IMTs in teams of two and 3) implementation of the lessons designed by the IMTs (see Table 2). During the implementation, there was an iteration of teaching try-outs by the IMTs in peer teaching sessions.

With regard to the focus and main goals of PDA, authors such as Borko (2004) and Desimone, Garet, Porter, Birman and Yoon, (2002) found out that teacher's PDT is an indispensable tool to enhance teachers'

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knowledge and practices in instruction. To this effect the study employed the PDA as the main tool in shaping the IMTs knowledge on the features of the CTLA. As a result, changes became evident in the pedagogical strategies of the IMTs during the implementation and evaluation stage of the study. This affirms the study conducted by Guskey (2002) where he asserts PDA brings about changes in teachers' attitudes and beliefs. In recent times, Kazemi and Hubbard (2008); Opfer and Peder (2011) have stressed the need for PDA as a result of its potency to affect positively areas of the teacher's competences, knowledge, values as well as their personal and professional lives.

Notwithstanding the positive change in the IMTs developed knowledge in the features of what constitute a CTLA; they encountered some challenges with the use of CTLA intervention for trigonometric lessons. Some of the challenges they faced during the initial stages of the peer-teaching try-outs were the unwillingness to let go of the old pedagogical strategy on the part of some of the IMTs in order to make room for this new method of teaching. For instance, one of the IMTs (T3T1 to be specific) was observed combining the old method of teaching with the new one during the teaching try-outs phases at the implementation and evaluation of the study. This is consistent with the studies of Blatchford, Kutnick, Baines, and Galton (2003) that delineated the problems that teachers face in the introduction of CTLA as pedagogical tool in the classroom context. They postulated on the conditions that necessitate the application of intervention such as the CTLA in the teaching and learning environment.

The IMTs inability to teach from the learner perspective was a challenge encountered during the observation of the CTLA lessons. Thus, post

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PDA interview with the IMTs indicated that, they were deeply rooted in the teacher-centred pedagogical approach in teaching the subject matter. The IMTs viewed the methodology of teaching as teacher centred. This finding seems to align with Agyei's (2012) who asserted that, the training of prospective teachers in Ghana is preoccupied with teacher-centred pedagogical approach.

According to Westbrook et al (2013), since there has been a shift from teacher dominated pedagogies as result of curricular reforms to instructions that learners are central to the teaching process, there is the need for teachers to use constructivist pedagogical strategies to enhance teaching and learning. It is, however, important to mention that with the series of teaching try-outs, the IMTs were able to improve on these challenges mentioned above and successfully incorporated CTLA as a pedagogical strategy to enhance their CTLA lessons in the study. Moreover, the IMTs exhibited an enhanced content knowledge of the subject matter as a result of this improvement in IMTs knowledge and skills in enacting CTLA-lessons.

Improvements in Students' Learning Outcomes Using CTLA as Instructional Strategy to Teach Trigonometry

The research question three sought to explain the improvements in students' learning using CTLA particularly in the area of trigonometry. The discussions in connection with this research question were presented from two perspectives as reported in the results section as; enhanced students' learning outcomes and perceived students' experiences with the CTLA-based lessons. Enhanced students' learning of concepts in trigonometry with CTLA-intervention can be explained based on the improvements that occurred in

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students' learning outcomes as depicted in Figure 27 and Figure 28. Analysis of the results (27A, 27B and 28A, 28B) at the individual pre-test and post-test scores of the students showed that, the CTLA-intervention enhanced students' learning of concepts in the subject matter. This improvement in students' learning outcomes seem to have resulted from the use of student's worksheet of activities, the student centred approach to learning and the IMTs knowledge of the features of the CTLA in the teaching of the topics such as: the three trigonometric ratios of sine, cosine and tangent of angle as well as the compound angles, the trigonometric identities and angles of elevation and depression. The results of this study are consistent with the studies (Ahmadi, 2000; In 'am et al., 2010; Oloyede et al., 2012; Shafiuddin, 2010; Yousef et al., 2021) who assessed the impact of cooperative teaching and learning strategy in teaching mathematics students. The study affirmed the effect cooperative learning has on students' performance, interest, motivation, conceptual understanding, and their attitudes toward mathematics. The results established that, students under the tuition of cooperative disposition towards mathematics improved as well as their performance as was the case in the current study where there were improvements in the performance of students in the two schools the IMTs taught (see Tables 18-19). They indicated a better superiority of the CTLA as an instructional approach in mathematics courses to a more traditional method of instruction. This observation was also implied in the current study although the current study did not compare the CTLA intervention directly with the conventional method of teaching. The results align with this study because the views of the students on the new teaching method (CTLA) agree with sentiments as expressed in the study of Ahmadi

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(2000) (see qualitative analysis of students' responses to the researcher interview). Again, in the said studies of (Ahmadi, 2000; In 'am et al., 2010; Oloyede, et al., 2012; Shafiuddin, 2010; Yousef et al., 2021), they averred that, CTLA is very dynamic, thus students are more actively involved in the exploration of mathematical concepts and ideas as well as the resolution of specific task. This was also observed in the current study as the introduction of the activity-oriented worksheet made the students active participants in constructing their own understanding of the subject matter.

In a related study, Zakaria, Chin, and Daud (2010) also researched on the effects of cooperative learning on students' mathematics achievement and attitude towards mathematics. However, Zakaria et al. (2010) concentrated on Malaysian secondary school students. The results displayed a rather improved learning outcome on the part of the students in the experimental groups' achievement in mathematics as well as their attitude towards studying mathematics. This further confirms the improved learning outcomes of the students (see Figures 27A, 27B, 28A, 278B) post introduction of the CTLA intervention in the two SHS involved in the study.

Again, Shafiuddin (2010) also examined CTLA in learning mathematics. The rationale behind the conduct of their study was to assess the efficacy of the cooperative learning employed as a pedagogical approach in the classroom context. Thus, comparing the two pedagogical approaches of conventional method of teaching and cooperative learning. Subsequently, it was revealed that on the pre-test and post-test analysis that, the achievement levels amongst the high, average, and low learners were higher when cooperative learning was used as the medium of instruction as opposed to the

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conventional method of instruction. The current study's student's performance on the pre-test and post-test corroborates this study as IMTs after exposing students to the CTLA-lessons registered much improved mathematics achievement in their post-test in comparison with their pre-test scores.

Moreover, Whicker et al. (1997) findings in their study further affirm similar observations in the current study. In their study, the effects of cooperative learning on students' achievement and attitudes in a secondary mathematics classroom were found to be favourable. They found that students who experienced CTLA lessons performed better on the test scores than students in the comparison group and as such, affirms the current study's pretest and post-test scores where students improved on their pre-test scores as result of the introduction of the CTLA-intervention.

Again, Vaughan (2002) observed the effects of cooperative learning on the students' performance and attitudes in relation to mathematics. The analysis of pre- and post-test scores showed favourable changes in the students' attitudes and performance. This further affirms the current study's position on SHS students' disposition towards the intervention introduced and consequently the improved scores registered post introduction of the CTLA-lesson. The students in an interview confirmed that, the use of worksheet of activities, the student centred approach to learning and the IMTs knowledge of the features of the CTLA helped them to advance their knowledge acquisition of the subject matter (trigonometry). During the implementation of the CTLA-lessons, it was observed that the IMTs were able to use worksheet of activities, the student centred approach to learning and the knowledge of the features of the CTLA in a well-coordinated manner to aid the students in their

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conceptualisation of the topics in trigonometry such as the angles of elevation, the trigonometric ratios of sine, cosine and tangent of angles, the compound angles as well as the identities in trigonometry in a real classroom context which would have been taught in abstraction in a traditional mathematics classroom context.

The findings above seem to imply that the CTLA- intervention provoked students' understanding of concepts in mathematics which indicates the extent to which the CTLA-lessons improved the students understanding of the trigonometric concepts taught. This explanation is consistent with the other studies advanced in this section of the discussion in view of the gains that inured to the benefit of SHS students when they were instructed with cooperative teaching and learning approach. This is because opportunities were afforded them to interact freely and achieve collective goals in the learning environment. Moreover, the CTLA incorporated activity oriented-based worksheet as part of its components which enables students in the learning environment to question issues, share knowledge on concepts, clarify variances in respect of concepts, and build new understandings on the concepts taught (Mercer, Wegerif & Dawes, 1999; Webb & George, 2003).

Another perspective from which the enhanced students' learning of concepts in mathematics with CTLA- intervention could be explained is from the point of view of students perceived positive experiences with the CTLA-lessons. The results from Table 18 showed that the students seem to express positive depositions after their encounter with the CTLA-lessons for mathematics lessons. For example, the students indicated that the CTLA-lessons were generally interesting and enjoyable with a lot of activities which

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helped them to be effective participants in the lesson. This observation in the current study is consistent with the assertion that cooperative learning classes are more relaxed and enjoyable than traditional classes, because there is a positive environment in the classroom (Roger, Olsen & Kagan, 1992; Slavin, 2014). The students further pointed out that the use of students' worksheet of activities in deducing the concepts themselves in mathematics augmented the effort of the IMTs to make the content of the lesson clear and as such facilitated their understanding of subject matter. (See interview with some elected SHS students after the implementation of the CTLA lessons).

In a further study of Akhtar, Perveen, Kiran, Rashid and Satti (2012), they studied students' attitude towards cooperative learning in the classroom. The results contended that cooperative learning makes learning interesting, provides fun, done in satisfactory situation and enhances their ability to socialise. This finding further aligns with the current study's students' experiences towards the implementation of CTLA in the real SHS classroom context. This is because, the views of the students expressed after the introduction of the intervention were favourable on all the subscales measured in the current study. The results in the current study show that all the three components (Delivery, Comprehension and Interest) for both schools recorded high means with differences in their standard deviations to indicate the extent to which the students' experiences and attitude toward the implementation of the CTLA intervention in the classroom context were deemed favourable and positive. Interest recorded the highest mean value($\boldsymbol{M} = 4.34$, $\boldsymbol{SD} = 0.58$), Delivery ($\boldsymbol{M} = 4.12$, $\boldsymbol{SD} = 0.47$) and Comprehension ($\boldsymbol{M} = 4.16$, $\boldsymbol{SD} = 0.47$) for school A while for School B, Delivery ($\boldsymbol{M} = 3.91$, $\boldsymbol{SD} = 0.58$)

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recorded the highest mean, Interest ($\boldsymbol{M} = 3.77$, $\boldsymbol{SD} = 1.00$) and Comprehension($\boldsymbol{M} = 3.69$, $\boldsymbol{SD} = 0.83$). The findings of this study are also consistent with studies of (Assan-Donkoh et al., 2019; Bosson-Amedenu et al., 2021; Davidson, 1990; Edekor & Agbornu, 2020; Effandi, 2003; Faizah, 1999; Johnson & Johnson 1994; Ozsoy & Yildiz, 2004; Vaughan, 2002; Whicker et al., 1997; Zakaria et al., 2010). However, the current study contrasts with the findings by Ismail (2000). Ismail found no significant disparity in attitudes of the students towards mathematics between experimental and control groups in his study. The limited time (three and a half weeks) for the conduct of this study might have influenced the researcher's observation of "no significant difference between the two groups" in the study.

Even in other disciplines, the effectiveness of cooperative learning has proven to be effective like Oloyede et al. (2012), Bukunola and Idowu (2012) also studied the effectiveness of cooperative learning strategies on Nigerian junior secondary students' academic achievement; however, Bukunola and Idowu (2012) concentrated on the basic science. The sample for this study was the total number of students in the intact classes used in the selected co-educational Junior Secondary Schools. The results revealed that cooperative learning had significant effects on students' post-test academic achievement scores in basic science. Again, the results revealed that the cooperative learning had a significant effect on students' retention of basic science concepts taught. Thus, as student performance improved, their retention also improved.

In general, the results indicated that student perceptions of the cooperative teaching and learning approach were favourable. Hence, the

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improvement in students' learning of concepts in trigonometry using CTLA as a pedagogical strategy in the classroom context.

Summary of the Chapter

In this chapter, three research questions were formulated for the purposes of developing in-service mathematics teachers' knowledge and skills to incorporate CTLA to teach SHS mathematics in a manner that is student-centred. The results from the qualitative and quantitative evidence indicates that the exemplary materials designed for the conduct of study were characteristic of student-centred teaching with the incorporation of the five main features of and thus, signify the type of CTLA-intervention that fits the realities in the Ghanaian SHS mathematics classroom. The study further discloses the improvements in the knowledge and skills on the part of IMTs in construction lessons that were reflective of the five features of the CTLA. Thus, the CTLA-intervention developed in the study was found to be effective as conceptualised in the study. Additionally, the improvements in teaching using CTLA- pedagogical strategies can be explained from the IMTs developed knowledge and skills in enacting CTLA-lessons.

In conclusion, the study revealed that the improvements in students' learning of concepts in trigonometry can be explained and or understood from their enhanced learning outcomes and their perceived positive experiences with CTLA- lessons. Hence, the findings in this study affirm empirically the results of most studies conducted on CTLA elsewhere and in the Ghanaian context as discussed above.

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CHAPTER FIVE

SUMMARY, CONCLUSION AND RECOMMENDATIONS

This chapter presents the summary of the key findings of the study to provide conclusions based on which recommendations for practice, policy and further research are made. In addition, the processes involved in validating the final design guidelines using the DBR approach are presented.

Summary

The rationale for the conduct of this study was to enhance the pedagogical strategies of IMT through the effective utility of CTLA as an instructional strategy to improve students learning on the concept of trigonometry in the classroom context particularly, at SHS in Ghana. This is purposed to make the teaching and learning of mathematics student-centred and meaningful.

In order to achieve this objective, purposive sampling technique was employed to select two cohorts of four in-service mathematics teachers and convenience sampling techniques was also used in selecting four Senior High Schools involved in the study. A sample of eight in-service mathematics teachers and 55 students from two senior high schools (where CTLA lessons were introduced in the real classroom learning environment) were selected for this study. Three research questions guided the study. The research questions that guided this study were; "how effective is the CTLA intervention in developing IMTs pedagogical strategies to improve teaching and learning of concepts in mathematics?", "how can the improvements in teaching using pedagogical strategies that are anchored on cooperative teaching and learning approach be understood and or explained?" and "how can the improvements in

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students' learning of concepts in mathematics using cooperative teaching and learning approach be understood and or explained?".

The research method employed was descriptive case study design (Merriam, 1998). This means both quantitative and qualitative data techniques were employed to explore and understand the meanings IMTs derived from their experiences of using CTLA to teach trigonometry. Data was collected at the various stages of the study as result of the design-based approach (DBR) adapted in the study. Thus, at the prototype as well as the implementation and evaluation stages of the study.

In particular data was collected through different instruments. Thus, questionnaires for both IMTs and SHS mathematics students, observation protocol for observing the teaching and learning process with the CTLA, semi-structured interviews conducted for both SHS mathematics students and IMTs, focal group discussions conducted for the IMTs and the administration of pre-test and post-test for students at SHS A and SHS B. The DBR process was employed as the research approach for the conduct of this study. In addition, the descriptive case study was used as the research design in the current study.

The quantitative research instruments (thus, students perceived effectiveness of the intervention questionnaires, IMTs CTLA pre and post self-assessment questionnaire and students' experiences with the intervention questionnaire) in the study were analysed using descriptive statistics specifically, means and standard deviations as well as paired sample t-test. Again, scale construction was employed to analyse the students' experiences with the intervention. The achievement tests (i.e., pre-test and post-test) were made up of one (1) open ended mathematics question (essay-type) for each of

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the lessons taught under trigonometry. The question (test) was awarded a total mark of 5 to which marks above or below 2.5 was interpreted as favourable (positive) score and unfavourable (negative) score respectively in SHS A, whiles a total mark of 10 was scored with the mean score above and below 5 interpreted as favourable (positive) and unfavourable (negative) scores respectively in SHS B. In addition, a sample pair t-test was used to determine the significance in means of the students' pre-test and post-test scores obtained in the test in the two SHS. The observed data on the CLOP was in the form of field notes (text) and indicators that informed the features of CTLA. To this end, the indicators were rated and coded as a result of which the outcome was analysed using descriptive statistics (means and standard deviation). Additionally, the data obtained from the interview and the focus group discussions were transcribed from audio to text. Data reduction technique (Miles & Huberman 1994) was used in anlysing the observed data, interview, field notes (text) recorded in the CLOP and focus group discussions.

Key Findings

In view of the analysis of the results for the three (3) research questions formulated in this study, the following are the summary of the key findings that emerged. The findings are presented in the order of the research questions i.e., from research question 1 to 3 respectively.

The first research question sought to establish the effectiveness of the CTLA intervention in developing IMTs pedagogical strategies to improve teaching and learning of concepts in mathematics.

The following were observed:

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1. the study revealed that the CTLA-lessons was effective as conceptualised in the study in developing the in-service mathematics teachers' pedagogical strategies to improve teaching and learning of concepts in mathematics.

    a. IMTs lessons developed and enacted, projected evidences of high Positive interdependence, Individual accountability, Promotive Face-to-face interaction, Social skills and Group processing as reflective of the five features of a CTLA-lesson; Individual Accountability however, was the least reported by the teachers.

    b. Students' who participated in CTLA-lessons reported high perceptions about its' effectiveness. They ranked all the five features they observed about the CTLA-lesson: Positive interdependence, Individual accountability, Promotive Face-to-face interaction, Social skills and Group processing high.

Second research question of the study sought to explain how the improvements in teaching using pedagogical strategies that are anchored on cooperative teaching and learning approach can be understood and or explained. The improvements in teaching using CTLA-lessons can be understood and explained as a result of the:

1. IMTs developed knowledge and skills in enacting cooperative teaching and learning approach lessons as were reflected in both the qualitative and quantitative data collected. The mean gains of scores (before and after designing and enacting their CTLA- lessons) for all the five features were significant.

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2. IMTs improved content knowledge in the subject matter. The study revealed an improved content knowledge of the IMTs post introduction of the PDT. As such, same concepts the IMTs found difficulties in explaining before the study were explained better in a way that improved their teaching with the CTLA-lessons.

Finally, the research question three of this study sought to establish how the improvement in students' learning of concepts in mathematics using cooperative teaching and learning approach could be understood and or explained.

The following were observed:

1. that, the enhanced learning outcomes in the performances of the students as reported in the study was one possible reason that explained the improvements in students' learning using the cooperative teaching and learning approach;
2. the study further revealed improved and significant performance in students' pre-post-tests scores after participating in the CTLA intervention in all the two senior high schools involved in the study.
3. that the students' overall positive dispositions towards the CTLA-lessons which were reflected in high means scores on the subscales - interest, comprehension and delivery, depicted favourable experiences with the intervention is another reason that explained the improvements in students' learning using the cooperative teaching and learning approach;

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Conclusion

This section reflects on the processes involved in the conduct of the study and subsequently highlights the implications of the findings as discussed on practice, policy and suggested research on senior high school mathematics education. As evidenced in the study, the DBR approach of the study which consisted of the design, development, and the teaching try-outs of the intervention served an effective approach for equipping IMTs knowledge and skills in the development and enactment lessons in a manner that promoted the features of CTLA. In addition, the iterative nature of the DBR approach was essential as it allowed for systematic development and improvement of the IMTs skills in designing the CTLA-intervention. This was considered necessary as it also afforded the researcher the opportunity to gain vital information into how the IMTs developed their knowledge and skills for the incorporation of the CTLA-interventions and also into their teaching practices.

The three stages employed in the current study was crucial in facilitating the development of CTLA- intervention that was effective. This was crucial in the provision of a realistic solution to the teacher-centred pedagogical approach currently in use for the teaching of mathematics. For example, the *prototype stage* provided opportunity for an innovative review of the literature. This was vital in the identification of the initial design guidelines of the study. The stage also afforded experts the opportunity to ascertain the practicability as well as the improvement in the validity of the exemplary materials which were developed by the researcher; hence, the prototype stage in the study was considered as key for the production of a

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quality CTLA- intervention with features that best fit the realities in the Ghanaian classroom learning environment.

Thus, the DBR approach has been instrumental in formulating the design guidelines that could inform theory. In this way, future development of IMTs competencies for the incorporation of CTLA- lesson could be replicated.

The *implementation and evaluation stage* of the DBR approach adapted for the study served as the pivot around the entire research revolves. The iteration of the features of the CTLA was emphasised as result of the PDA conducted at this stage of the study. The formative evaluation and redesign as completed in the professional development training arrangements through the iterative cycle afforded the researcher the platform to provide comprehensive description of the intended implementation of the designed intervention. As such, the peer-teaching try-outs and the feedbacks that were given after each try-out became essential to ascertain the effectiveness of the CTLA-intervention.

The final stage of the DBR process provided a reflective account of the entire research. Thus, the reflection and documentation of the whole research processes involved in the study enabled theory formulation (i.e., the refined and final design guideline) in the design of an effective CTLA- lesson. Hence, it is vital to note that, the last stage of the current study was fundamental in the explanation of how the improvements in both the teaching of trigonometry with CTLA and the students' appreciation of trigonometric concepts happened during the processes of implementing the CTLA-lessons.

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Recommendations

The recommendations are made to provide useful information to informed policy, improved practice and for further research. Based on the findings of the study, recommendations are made to respond to the various actions to be taken by key stakeholders in mathematics education. The study, therefore, recommends the following for practice, policy and further research.

Recommendation for Practice

Based on the results of the implementation of the CTLA-lesson teaching technique through the three stages of the DBR framework, the design principles for developing SHS Mathematics teachers' competencies to teach the concept of trigonometry were formulated. The study has proposed design guidelines to guide practice and training of in-service and pre-service teachers in enacting and implementing CTLA- lessons. The following are the design guidelines that emerged from the study:

1. It is very critical to develop initial CTLA-based material in the form of exemplary lesson in any design and professional development training for in-service and pre-service teachers. The exemplary material provides practical samples for participants of the study to gain an in-depth understanding of what a CTLA- intervention constitutes.
2. It is important to adopt an activity-based lesson as a means of maximising the affordances of the features of the CTLA in teaching the subject matter. They are essential for the development of CTLA - oriented competencies.
3. Placement of in-service and pre-service teachers in teams of assigned responsibility in the design, development and enactment of the CTLA-

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based intervention is a means of ensuring cooperation among the participants in the study to work together, learn from each other and ultimately improve the intervention as weaknesses identified are shared and improved upon.

4. The conduct of an initial training workshop is a vital tool for the deployment of CTLA-oriented teaching and learning competencies for the participants of the study. This approach of professional development training arrangement is theory-induced and content-focused. Hence, it is considered essential for preparing both in-service and pre-service teachers to effectively incorporate CTLA into their pedagogy.
5. Scaffolds and student-centred experiences such as hands-on activities in relation to the subject matter, deliberations in respect of CTLA-oriented instructional strategies, and teaching try-outs with peers should be incorporated in both pre-service and in-service teacher's professional development training.
6. Adoption of constructivist pedagogical approaches is key to a successful implementation process with CTLA- intervention. The constructivist pedagogies provide opportunities for more student-centred and cooperative learning-based environment among students in the implementation process.
7. Provision of enabling conditions such as grouping of students with some assigned roles (positive interdependence), engagement of students at the individual and group level to master the subject matter (individual accountability), the creation of platforms for sharing of

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ideas (promotive face to face interaction), organising the learning environment to be accommodative of students' views (social skills) and evaluating the students understanding of the subject matter (group processing) on the subject matter are key to the effective implementation of a CTLA- lesson.

The study further indicates that students' disposition toward the CTLA was positive on all subscales (interest, comprehension and delivery) depicting a favourable experience of the intervention. As a result of which an enhanced learning outcome in the performances of the students were reported irrespective of the type of the two SHS involved in the study. In order to replicate this success in other mathematics classroom context, the study therefore recommends that

a. SHS could consider prioritising professional development training to sharpen SHS teacher's competencies in using the CTLA in subject specific content pedagogies.

b. SHS mathematics teachers are admonished to make conscious efforts to adapt the CTLA in order to increase the proficiency of SHS students in the area of mathematics.

In addition, the study has identified from both the quantitative and qualitative analysis from the perspectives of the IMTs prior to and post introduction of the PDA that the features of positive interdependence (P.I), social skills (S.S) and promotive face to face interactions (F.F) was effectively used by the IMTs. However, the features of individual accountability (I.A) and group processing (G.P) were minimally applied, creating a gap in the teaching and learning process with the CTLA. As such, there is the need for curriculum

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planners, in-service training providers, teacher educators to be well informed and updated on these features of individual accountability and group processing in addition to the positive interdependence (P.I), social skills (S.S) and promotive face to face interactions (F.F). The study therefore recommends that:

a. In-service training providers especially at the SHS level should lay emphasis on the features of individual accountability (I.A) and group processing (G.P) in addition to rest of the features of CTLA to equip mathematics teachers to realise to the fullest the potentials of CTLA as pedagogical approach in the classroom context.

Recommendation for Policy

a. Teacher education institutions are urged to consider training prospective teachers on how to enact mathematics lessons using CTLA.

Suggestion for Further Research

The study focused on the features of CTLA-intervention that can be designed to fit the realities in the SHS classroom in Ghana in a manner that makes the teaching of mathematics cooperative learning-based. Even though the study focused on these features of CTLA-based intervention to improve the teaching of trigonometry in Ghana especially at the SHS level, the application of CTLA in the classroom context was operationalised to cover the five features of cooperative learning (Johnson & Johnson, 1998) in the study. In view of this, I recommend that further studies be conducted to find out the

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impact of other features of the cooperative teaching and learning approach as advanced in literature to teach mathematics.

Again, the study also showed that the mathematics students' disposition towards the CTLA differed on some of the subscales measured in SHS A and SHS B during the introduction of the CTLA-intervention to them in the real classroom context. Based on the results, only three features of promotive face to face interaction, social skills and positive interdependence appear to be employed regularly whiles the remaining two individual accountability and group processing were least experienced during the teaching and learning process with the CTLA by the IMTs. The rationale for the IMTs preference for the application of only three out of five features of the CTLA in a lesson could not be established since it was beyond the scope of this study. Some reasons might have accounted for this. Hence a further study meant to understand these reasons could unlock the realisation of the full potential of all five features of CTLA when applied simultaneously in teaching and learning process in the classroom context.

Finally, the study was limited to the development of IMTs competencies for the incorporation of CTLA intervention into the teaching and learning of mathematics at the SHS level. Thus, it would be worthy to note of a replication of this study in other subject disciplines such as biology, physics and chemistry as well as art related subjects at the SHS level; since the deployment of CTLA as a pedagogical strategy is not limited to mathematics.

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REFERENCES

Abdallah, M. S., & Wegerif, R. B. (2014). *Design-based research (DBR) in educational enquiry and technological studies: A version for PhD students targeting the integration of new technologies and literacies into educational contexts*. ERIC Clearinghouse.

Agyei, D. D. (2012). *Preparation of pre-service teachers in Ghana to integrate information and communication technology in teaching mathematics.* Unpublished doctoral dissertation, University of Twente, Enschede, The Netherlands.

Ahmadi, M. H. (2000). The impact of cooperative learning in teaching mathematics. *Primus, 10*(3), 225-240.

Akhtar, K., Perveen, Q., Kiran, S., Rashid, M., & Satti, A. K. (2012). A study of student's attitudes towards cooperative learning. *International Journal of Humanities and Social Science*, *2*(11), 141-147.

Andriessen, D. G. (2006). *Combining design-based research and action research to test management solutions.* Paper presented at the 7th World Congress Action Learning, Action Research and Process Management, Groningen.

Antwi, V. (2013). *Interactive teaching of mechanics in a Ghanaian university context*. Amersfoort: Drukkerij Wilco.

Assan-Donkoh I., Susuoroka, G., Baah D., Baah-Duodu, S., & Puotie, Z. (2019). Cooperative learning strategy in teaching and learning of circle theorem in mathematics: A case of Nana Brentu senior high school in Aowin Municipality in Ghana. *Open Science Journal of Mathematics and Application*. *7*(1), 1-13.

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© University of Cape Coast https://ir.ucc.edu.gh/xmlui

Avalos, B. (2010). Teacher professional development in teaching and teacher education over ten years. *Teacher and Teacher Education, 27*(2), 10–20.

Bautista, A., Cañadas, M. C., Brizuela, B. M., & Schliemann, A. D. (2015). Examining how teachers use graphs to teach mathematics during a professional development program. *Journal of Education and Training Studies*, *3*(2), 91-106.

Blackett, N., & Tall, D. (1991) Gender and the versatile learning of trigonometry using computer software. In F. Furinghetti (Ed.), *Proceedings of the 15th Conference of the International Group for the Psychology of Mathematics Education* (pp. 144-151), Assisi, Italy.

Blatchford, P., Kutnick, P., Baines, E., & Galton, M. (2003). Toward a social pedagogy of classroom group work. *International Journal of Educational Research,* 39(1), 153-72.

Bonk, C. J., & Cunningham, D. J. (1998). Searching for learner-cantered, constructivist and sociocultural components of collaborative educational learning tools. In Bonk, C. J. & King, K. S. (Eds.), *Electronic Collaborators: Learner-cantered technologies for literacy, apprenticeship, and discourse* (pp. 25–50), New Jersey, USA.

Borko, H. (2004). Professional development and teacher learning: Mapping the terrain. *Educational Researcher*, *33*(8), 3-15.

Bosson-Amedenu S., Teku E., Vivian N., & Osei-Asibey E. (2021). A study of cooperative strategies that enhance individual mathematical achievement among college of education students in Ghana. *Teacher Education and Curriculum Studies*. *6*(1), 28-32.

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© University of Cape Coast https://ir.ucc.edu.gh/xmlui

Challenger, M. (2009). *From triangles to a concept: a phenomenographic study of A-level students' development of the concept of trigonometry*, Unpublished doctoral dissertation, University of Warwick.

Chauke, W. (2013). *A report on analysis of grade 12 students' performance in mathematics paper II examination of 2012*. Gauteng Department of Education.

Chen, L., Chen, T. L., & Chen, N. S. (2015). Students' perspectives of using cooperative learning in a flipped statistics classroom. *Australasian Journal of Educational Technology*, *31*(6), 98-118.

Cobb, P., Confrey, J., DiSessa, A., Lehrer, R., & Schauble, L. (2003). Design experiments in educational research. *Educational Researcher*, *32*(1), 11-23.

Cohen, L., Manion, L., & Morrison, K. (2007). *Research methods in education* (6th ed.). London: Routledge.

Creswell, J. W. (1994). *Research design: Qualitative and quantitative approach.* California: SAGE Publications, Inc.

Creswell, J. W. (2007). *Advanced mixed methods research designs*. Thousand Oaks, CA: Sage Publications.

Darling-Hammond, L., & McLaughlin, M. W. (2011). Policies that support professional development in an era of reform. *Phi Delta Kappan*, *76*(8), 597–604.

Davidson, N. (1990). *Cooperative learning in mathematics: A handbook for teachers*. CA: Addison-Wesley Publishing Company.

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© University of Cape Coast https://ir.ucc.edu.gh/xmlui

De Villiers, M., & Jugmohan, J. (2012). Learners' conceptualisation of the sine functions during an introductory activity using sketchpad at grade 10 level. Educ. *Matem. Pesq., São Paulo*, *14*(1), 9-30.

Delice, A., & Roper, T. (2006). Implications of a comparative study for mathematics education in the English education system. *International Journal of the IMA*, *25*(2), 64-72.

Desimone, L. M. (2009). Improving impact studies of teachers' professional development: Toward better conceptualizations and measures. *Educational Researcher, 38*(3), 181–199.

Desimone, L. M., Porter, A. C., Garet, M. S., Yoon, K. S., & Birman, B. F. (2002). Effects of professional development on teachers' instruction: Results from a three-year longitudinal study. *Educational Evaluation and Policy Analysis*, *24*(2), 81-112.

Dewey, J. (1938) *Experience and Education*. New York: Collier Books.

Ding, M., Li, X., Piccolo, D., & Kulm, G. (2007). Teacher interventions in cooperative-learning mathematics classes. *The Journal of Educational Research*, *100*(3), 162-175.

Dyer Jr, W. G., & Wilkins, A. L. (1991). Better stories, not better constructs, to generate better theory: A rejoinder to Eisenhardt. *Academy of Management Review*, *16*(3), 613-619.

Easterday, M. W., Lewis, D. R., & Gerber, E. M. (2014). Design-based research process: *Problems, phases, and applications*. Proceedings of International Conference of the Learning Sciences (pp. 317-324). Boulder, Colorado.

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Edekor L., K., & Agbornu S. (2020) Cooperative learning strategy and students' performance in mathematics in junior high school in Hohoe municipality, Ghana. *American Journal of Educational Research*, 8(9), 693-697.

Effandi, Z. (2003). *The effects of cooperative learning on students in a matriculation mathematics class*. Ph.D. thesis. Universiti Kebangsaan Malaysia, Bangi.

Elliot, S. N., Kratochwill, T.R., Cook, J., & Travers, J. (2000). *Educational psychology: Effective teaching, effective learning* (3rd Ed.). Boston, MA: McGraw-Hill College.

Ergn, İ., Ünsal, Y., Tan, M., & Mustafa, T. A. N. (2006). 5E modeli'nin öğrencilerin akademik başarisina ve tutum düzeylerine etkisi:"yatay atiş hareketi" örneği. *Ahi Evran Üniversitesi Kırşehir Eğitim Fakültesi Dergisi*, *7*(2), 1-15.

Faizah, M. G. (1999). *Kesan pembelajaran koperatif menggunakan alat ujian pencapaian dalam matematik.* Projek Penyelidikan Sarjana Pendidikan. Universiti Kebangsaan Malaysia.

Farouq, S. M. (2017). Ghanaian senior high school students' error in learning of trigonometry. *International Journal of Environmental & Science Education*, *12*(8), 1709-1717.

Farzaneh, N., & Nejadansari, D. (2014). Students' attitude towards using cooperative learning for teaching reading comprehension. *Theory and Practice in Language Studies, 4*(2), 287-292.

Fraenkel, J. R., & Wallen, N. E. (2006). *How to design and evaluate research in education*. (6th ed.). New York: NY: McGraw-Hill.

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Garet, M. S., Porter, A. C., Desimone, L., Birman, B. F., & Yoon, K. S. (2001). What makes professional development effective? Results from a national sample of teachers. *American Educational Research*, *38*(4), 915-945.

Gillies, R. (2008). The effects of cooperative learning on junior high school students' behaviours, discourse and learning during a science-based learning activity. *School Psychology International, 29*(3), 328–347.

Gillies, R. M., & Ashman, A. F. (1998). Behaviour and interactions of children in cooperative groups in lower and middle elementary grades. *Journal of Educational Psychology*, *90*(4), 731-746.

Gillman, L. (1991). The limit of x sin x/x. *Teaching of Mathematics*, *1*(2), 346-352.

Gömleksiz, M. N. (2007). Effectiveness of cooperative learning (jigsaw II) method in teaching English as a foreign language to engineering students (Case of Firat University, Turkey). *European Journal of Engineering Education*, *32*(5), 613-625.

Guba, E. G. (1981). Criteria for assessing the trustworthiness of naturalistic inquiries. *Educational Resources Information Center Annual Review Paper, 29*(5), 75-91.

Gur, H. (2009). Trigonometry learning. *New Horizons in Education*, *57*(1), 67-80.

Guskey, T. R. (2002). Does it make a difference? Evaluating professional development. *Educational Leadership*, *59*(6), 334-345.

Digitized by Sam Jonah Library

© University of Cape Coast https://ir.ucc.edu.gh/xmlui

Hammond, L. D., Austin, K., Orcutt, S., & Rosso, J. (2001). *The learning classroom: Theory into Practice A Telecourse for Teacher Education and Professional Development.* Standford University School of Education.

Harder, H. (2012). Explanatory case study research. In A. J. Mills, G. Durepos & E. Wiebe (Eds.). *Encyclopedia of case study research.* SAGE Publications, Inc. London, UK.

Hew, K. F., & Brush, T. (2007). Integrating technology into K-12 teaching and learning: Current knowledge gaps and recommendations for future research. *Educational Technology Research and Development*, *55*(3), 223-252.

Hinton, P.R., McMurray, I., & Brownlow, C. (2014). *SPSS explained.* London: Routledge Taylor and Francis Group.

Ho, A., Watkins, D., & Kelly, M. (2001). The conceptual change approach to improving teaching and learning: An evaluation of a Hong Kong staffs development programme. *Higher Education*, *42*(2), 143-169.

In 'am, A., & Sutrisno, E. S. (2021). Strengthening students' self-efficacy and motivation in learning mathematics through the cooperative learning model. *International Journal of Instruction, 14*(1), 395-410.

Ismail, M. (2000). The effects of cooperative learning strategy of TGT on the attitude of year four students toward mathematics in SRK Sekaan *Journal of Education*, 99(2) 67-77.

Javed, A., Saif, N., & Kundi, G. M. (2013). The Study of effectiveness of cooperative learning approach in teaching of mathematics at secondary levels in Pakistan. *Mathematical Theory and Modelling*, *3*(10), 24-33.

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Jenkins, J. R., Antil, L. R., Wayne, S. K., & Vadasy, P. F. (2003). How cooperative learning works for special education and remedial students. *Exceptional children*, *69*(3), 279-292.

Johnson, D. W., & Johnson, R. T. (2002). Cooperative learning and social interdependence theory. In *Theory and research on small groups* (pp. 9-35). Springer, Boston, MA.

Johnson, D. W., Johnson, R. T., & Smith, K. (2007). The state of cooperative learning in postsecondary and professional settings. *Educational Psychology Review*, *19*(1), 15-29.

Johnson, D. W., Johnson, R. T., & Smith, K. A. (1998). Cooperative learning returns to college what evidence is there that it works? *Change: The Magazine of Higher Learning*, *30*(4), 26-35.

Johnson, D., Johnson, R., Holubec, E., & Roy, P. (2015). *Cooperation in the classroom.* Edina, MN: Interaction Books.

Johnson, D.W., Johnson, R.T., & Smith, K.A. (1991). *Active learning: Cooperation in the college classroom.*, MN: Interaction Book Company Edina.

Jong, O. D., & Brinkman, F. (1997). Teacher thinking and conceptual change in science and mathematics education, *European Journal of Teacher Education*, *20*(2), 121-124.

Kazemi, E., & Hubbard, A. (2008). New directions for the design and study of PD: Attending to the coevolution of teachers' participation across contexts. *Journal of Teacher Education, 59(5),* 428-441.

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Kelly, A. E. (2010). When is design research appropriate? In T. Plomp, & N. Nieveen (Eds.). *An introduction to educational design research*, Enschede: Netzodruk.

Kendal, M., & Stacey, K. (2003). Tracing learning of three representations with the differentiation competency framework. *Mathematics Education Research Journal*, *15*(1), 22-41.

Keppell, M., Au, E., Ma, A., & Chan, C. (2005). *Development-based research and professional development*. Retrievedfromhttp://www.ascilite. org. au /conferences/brisbane05/blogs/proceedings/39_Keppell.pdf

Khan, F., & Ahmad, S. R. (2013). Polysaccharides and their derivatives for versatile tissue engineering application. *Macromolecular Bioscience*, *13* (4), 395-421.

Koh, J. H. L. (2013). A rubric for assessing teachers' lesson activities with respect to TPACK for meaningful learning with ICT. *Australasian Journal of Educational Technology*, *29*(6), 12-23.

Kothari, C. R. (2004). *Research methodology: Methods and techniques*. New Age International. New York City, USA.

Koutselini, M. (2008). Teacher misconceptions and understanding of cooperative learning: An intervention study. *The Journal of Classroom Interaction*, *43*(2), 34-44.

Kyndt, E., Dochy, F., & Cascallar, E. (2013). Students' approaches to learning in higher education: The interplay between context and student. In *Learning Patterns in Higher Education* (pp. 265-288). Routledge.

Digitized by Sam Jonah Library

© University of Cape Coast https://ir.ucc.edu.gh/xmlui

Landis, J. R., & Koch, G. G. (1977). An application of hierarchical kappa-type statistics in the assessment of majority agreement among multiple observes. *Biometrics*, *3*(2), 363-374.

Lehrer, R., & Schauble, L., (2000). The development of model-based reasoning. *Journal of Applied Developmental Psychology, 21*(1), 39-48.

Lincoln, Y. S., & Guba, E. G. (1985). Establishing trustworthiness. *Naturalistic Inquiry*, *289*(331), 289-327.

Lopata, C., Miller, K. A., & Miller, R. H. (2003). Survey of actual and preferred use of cooperative learning among exemplar teachers. *The Journal of Educational Research*, *96*(4), 232-239.

Mehta, S., & Kulshrestha, A. K. (2014). Implementation of cooperative learning in science: A developmental-cum-experimental study. *Education Research International*, *9*(8), 50-57.

Mercer, C. D., Lane, H. B., Jordan, L., Allsopp, D. H., & Eisele, M. R. (1996). Empowering teachers and students with instructional choices in inclusive settings. *Remedial and Special Education*, *17*(4), 226-236.

Mercer, N., Wegerif, R., & Dawes, L. (1999). Children's talk and the development of reasoning in the classroom. *British Educational Research Journal*, *25*(1), 95-111.

Merriam, S. B. (1998). *Qualitative research and case study applications in education: Revised and expanded from* case study research in education. Jossey-Bass Publishers, San Francisco.

Miles, M. B., & Huberman, A. M. (1994). *Qualitative data analysis,* (2nd ed.). Newbury Park, CA: Sage.

Digitized by Sam Jonah Library

© University of Cape Coast https://ir.ucc.edu.gh/xmlui

Miller, S. (2001). Understanding transformations of periodic functions through art. *Mathematics Teacher, 94(8),* 632–635.

Mingfong, J., YamSan, C., & Ek Ming, T. (2010). Unpacking the design process in design-based research. *Educational Media International, 47*(4), 263-275.

Ministry Of Education. (2010). *Teaching Syllabus for Core Mathematics (SHS)*. Accra: CRDD.

Mishra, P., & Koehler, M. J. (2006). Technological pedagogical content knowledge: A framework for teacher knowledge. *Teachers' College Record, 108*(6), 1017-1054.

Mishra, P., & Koehler, M. J. (2013). What is technological pedagogical content knowledge (TPACK)? *Journal of Education, 193*(3), 13-19.

Molla, E., & Muche, M. (2018). Impact of cooperative learning approaches on students' academic achievement and laboratory proficiency in biology subject in selected rural schools, Ethiopia. *Education Research International*, 2018, https://doi.org/10.1155/2018/6202484

Morell, L., Buxeda, R., Orengo, M., & Sánchez, A. (2001). After so much effort: Is faculty using cooperative learning in the classroom? *Journal of Engineering Education, 90*(3), 357-362.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: National Council of Teachers of Mathematics.

Nieveen, N. (2010). Formative evaluation in educational design research. In T. Plomp, & N. Nieveen (Eds.), *An introduction to educational design research* (pp. 89-101). Enschede: Netzodruk.

 Digitized by Sam Jonah Library

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Nieveen, N., McKenney, S., & van den Akker, J. (2006). Educational design research: The value of variety. In J. van den Akker, K. Gravemeijer, S. McKenney, & N. Nieveen (Eds.), *Educational design research* (pp. 125-136). London: Routledge.

Nihuka, K. A., & Voogt, J. (2011). Instructors and students' competences, perceptions and access to e-learning technologies: Implications for e-learning implementation at the open university of Tanzania. *International Journal on E-learning*, *10*(1), 63-78.

Nurhuda, Lukito, A., & Masriyah, (2018). Effectiveness of cooperative learning instructional tool with predict –observe –explain strategy on the topic of cuboid and cube volume J. physics. *Conf Series*. *947*(1), 98-118.

Oloyede, E. O., Adebowale, O. F., & Ojo, A. A. (2012). The effects of competitive, cooperative, and individualistic classroom interaction models on learning outcomes in mathematics in Nigerian senior secondary schools. *International Scholarly Research Notices*, *9*(1). 61-81.

Opfer, V. D., & Pedder, D. (2011). Conceptualising teacher professional learning. *Review of Educational Research, 81*(3), 376-407.

Opitz, C. F., Ewert, R., Kirch, W., & Pittrow, D. (2008). Inhibition of endothelin receptors in the treatment of pulmonary arterial hypertension: Does selectivity matter? *European Heart Journal*, *29*(16), 1936-1948.

Orhun, E., & Demiralp, Ç. (2001). Usability evaluation for a set of cognitive tool software. *2nd International Conference on Technology in*

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© University of Cape Coast https://ir.ucc.edu.gh/xmlui

Teaching and Learning in Higher Education (pp. 249-254), Samos. Greece.

Orhun, N. (2004). Students' mistakes and misconceptions on teaching of trigonometry. *Journal of Curriculum Studies*, *32*(6), 797-820.

Ormrod, J. E. (2004). *Human learning* (4th ed.). Columbus, Ohio: Pearson Merrill Prentice Hall.

Othman, H., Asshaari, I., Bahaludin, H., Tawil, N. M., & Ismail, N. A. (2012). Student's perceptions on benefits gained from cooperative learning experiences in engineering mathematics courses. *Procedia-Social and Behavioural Sciences*, *60*(1), 500-506.

Ozsoy, N., & Yildiz, N. (2004). The effect of learning together technique of cooperative learning method on student achievement in mathematics: Teaching 7th class of primary school. *Turkish Online Journal of Educational Technology-TOJET*, *3*(3), 49-54.

Penuel, W. R., Fishman, B. J., Yamaguchi, R., & Gallagher, L. P. (2007). What makes professional development effective? Strategies that foster curriculum implementation. *American Educational Research Journal, 44*(4), 921-958.

Phipps, M., Phipps, C., Kask, S., & Higgins, S. (2001). University students' perceptions of cooperative learning: Implications for administrators and instructors. *Journal of Experiential Education*, *24*(1), 14-22.

Plomp, (2009). Educational design research: An introduction T. Plomp, N. Nieveen (Eds.), *An introduction to educational design research, Enschede,* The Netherlands (SLO).

 Digitized by Sam Jonah Library

© University of Cape Coast https://ir.ucc.edu.gh/xmlui

Plomp, T. (2010). Educational design research: An introduction. In Plomp, T. & N. Nieveen, (Eds.), *An introduction to educational design research*, (pp. 1-35). Enschede: Netzodruk.

Prawat, R. S., & Floden, R. E. (1994). Philosophical perspectives on constructivist views of learning. *Educational Psychologist*, *29*(1), 37-48.

Pritchard, L., & Simpson, A. (1999). *The role of pictorial images in trigonometry problems*. In PME Conference (pp. 4-81), Haifa, Israel.

Reda, T. A. (2015). Attitude of students towards cooperative learning methods (the case of wolaita sodo university psychology department second year students). *International Journal of Sciences: Basic and Applied Research (IJSBAR)*, *24*(2), 33-44.

Richey, R. C., & Klein, J. D. (2005). Developmental research methods: Creating knowledge from instructional design and development practice. *Journal of Computing in Higher Education*, *16*(2), 23-38.

Richey, R. C., & Nelson, W. A. (1996). Developmental research. *Handbook of research for educational communications and technology*, (pp. 1213-1245), Lawrence Erlbaum Associates, New Jersey.

Roger, E.W., Olsen, B. & Kagan, S. (1992). *About cooperative learning: A teacher's resource book.* In K. Kesler (Ed.). Cooperative language learning. Prentice-Hall Inc.

Roger, T., & Johnson, D. W. (1994). An overview of cooperative learning. *Creativity and collaborative learning*, *1*(11), 1-21.

Rohrbeck, R., & Schwarz, J. O. (2013). The value contribution of strategic foresight: Insights from an empirical study of large European

Digitized by Sam Jonah Library

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companies. *Technological Forecasting and Social Change*, *80*(8), 1593-1606.

Romme, A. G. L., & Putzel, R. (2003). Designing management education: Practice what you teach. *Simulation & Gaming*, *34*(4), 512-530.

Rose, S., Spinks, N., & Canhoto, A. I. (2015). *Management research: Applying the principles*. Routledge: New York.

Ross, J., Bruce, C., & Sibbald, T. (2011). Sequencing computer-assisted learning of transformations of trigonometric functions. *Teaching Mathematics and its Applications*, *30*(5), 120 – 137.

Shafiuddin, M. (2010). Cooperative learning approach in learning mathematics. *International Journal of Educational Administration*, *2*(4), 589-595.

Sharan, Y. (2010). Cooperative learning for academic and social gains: Valued pedagogy, problematic practice. *European Journal of Education*, *45*(2), 300–313.

Shenton, A. K. (2004). Strategies for ensuring trustworthiness in qualitative research projects. *Education for Information*, *22*(2), 63-75.

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher*, *15*(2), 4-14.

Slavin, R. E. (1985). An introduction to cooperative learning research. In R. Slavin, S. Sharan, S. Kagan, R. Lazarowitz, C. Webb & Sc R. Schmuck (Eds.), *Learning to Cooperate, Cooperating to Learn.* New York: Plenum Press.

Slavin, R. E. (1991). Synthesis of research of cooperative learning. *Educational Leadership*, *48*(5), 71-82.

 Digitized by Sam Jonah Library

© University of Cape Coast https://ir.ucc.edu.gh/xmlui

Slavin, R. E. (2010). Instruction based on cooperative learning. In R. Mayer (Ed.), *Handbook of Research on Learning and Instruction* (pp. 344–360). London: Taylor and Francis.

Slavin, R. E. (2014). Cooperative learning and academic achievement: Why do groupwork work? *Anales de Psicología, 30*(3), 785–791.

Slavin, R. E., & Karweit, N. L. (1981). Cognitive and affective outcomes of an intensive student teams learning experience. *Journal of Experimental Education*, *50*(1), 29-34.

Stein, M. K., Silver, E. A., & Smith, M. S. (1998). *Mathematics reform and teacher development: A community of practice perspective*. In J. G. Greeno & S. V. Goldman (Ed.). Thinking practices in mathematics and science learning (pp. 17-52). Mahwah, NJ: Lawrence Erlbaum.

Stepanovienė, A. (2017). Applying case study method while developing communicative competence of professional English. *Public Security and Public Order*, (18).

Summers, J. J., Gorin, J. S., Beretvas, S. N., & Svinicki, M. D. (2005). Evaluating collaborative learning and community. *The Journal of Experimental Education*, *73*(3), 165-188.

Tall, D., & Thomas, M. (1991). Encouraging versatile thinking in algebra using the computer. *Educational Studies in Mathematics*, *22*(2), 125-147.

Teddlie, C., & Tashakkori, A. (2006). A general typology of research designs featuring mixed methods. *Research in the Schools*, *13*(1), 12-28.

Tellis, W. (1997). Application of a case study methodology. *The Qualitative Report*, *3*(3), 1-19.

Digitized by Sam Jonah Library

© University of Cape Coast https://ir.ucc.edu.gh/xmlui

The West African Examinations Council [WAEC], (2011). *Chief Examiners' Report. Core Mathematics Programme:* May/June West African senior school certificate examination. Accra: WAEC.

The West African Examinations Council [WAEC], (2012). *Chief Examiners' Report. Core Mathematics Programme:* May/June West African senior school certificate examination. Accra: WAEC.

The West African Examinations Council [WAEC], (2013). *Chief Examiners' Report. Mathematics (core):* May/June West African senior school certificate examination. Accra: WAEC.

The West African Examinations Council [WAEC]. (2018). *Chief Examiners' Report. Mathematics Programme*: *May/June West African senior school certificate examination*. Accra: WAEC.

The West African Examinations Council [WAEC]. (2020). *Chief Examiners' Report. Mathematics Programme*: *May/June West African senior school certificate examination*. Accra: WAEC.

Thompson, P. W. (2008). Conceptual analysis of mathematical ideas: Some spadework at the foundations of mathematics education. In *Proceedings of the annual meeting of the International Group for the Psychology of Mathematics Education* (pp. 31-49). PME Morelia, Mexico.

Tran, V. D. (2014). The effects of cooperative learning on the academic achievement and knowledge retention. *International Journal of Higher Education*, *3*(2), 131-140.

Trauth-Nare, A., & Buck, G. (2011). Assessment for learning. *The Science Teacher*, *78*(1), 34.

 Digitized by Sam Jonah Library

© University of Cape Coast https://ir.ucc.edu.gh/xmlui

Udu, D. A. (2018). Comparative effects of individualised and cooperative learning instructional strategies on senior secondary school students' academic achievement in Organic Chemistry. *The Electronic Journal for Research in Science & Mathematics Education*, *22*(2), 25-39.

Van den Akker, J. (2010). Building bridges: How research may improve curriculum policies and classroom practices. In S. M. Stoney (Ed.), *Beyond Lisbon 2010: Perspectives from research and development for education policy in Europe* (pp. 177-195). Slough Berkshire: National Foundation for Educational Research.

Vaughan, W (2002). Effects of cooperative learning on achievement and attitude among students of colour. *The Journal of Educational Research*, *95*(6), 359-364.

Visscher-Voerman, I., & Gustafson, K. L. (2004). Paradigms in the theory and practice of education and training design. *Educational Technology Research and Development, 52*(2), 69-89.

Von Glasersfeld, E. (1974). Piaget and the radical constructivist epistemology. *Epistemology and education*, *2*(1), 9-24.

Wang, F., & Hannafin, M. J. (2005). Design-based research and technology-enhanced learning environments. *Educational Technology Research and Development, 53*(4), 5-23.

Watson, A. (2009). *Modelling, problem-solving and integrating concepts: In key understandings in mathematics learning*. London: Nuffield Foundation.

Digitized by Sam Jonah Library

© University of Cape Coast https://ir.ucc.edu.gh/xmlui

Watson, A., Jones, K., & Pratt, D. (2013). *Key ideas in teaching mathematics. Research-based guidance for ages 9-19* (1st ed.). Oxford: Oxford University Press.

Webb, N. M., & Farivar, S. (1999). Developing productive group interaction in middle school mathematics. *Cognitive Perspectives on Peer Learning, 7*(5), 117-149.

Webb, N. M., & Mastergeorge, A. (2003). Promoting effective helping behavior in peer-directed groups. *International Journal of Educational Research*, *39*(2), 73-97.

Weber, K. (2005). Students' understanding of trigonometric functions. *Mathematics Educational Research Journal*, *17*(3), 91–112.

Weiner, B. (1980). A cognitive (attribution)-emotion-action model of motivated behavior: An analysis of judgments of help-giving. *Journal of Personality and Social psychology*, *39*(2), 171-186.

Westbrook, J, Durrani, N., Brown, R., Orr, D., Pryor, J., Boddy, J., & Salvi, F. (2013). Pedagogy, curriculum, teaching practices and teacher education in developing countries. *Education Rigorous Literatures Review*. Retrieved from http://eppi.ioe.ac.uk/cms/Portals/0/PDF.

Whicker M. K., Linda B., & John A. N. (1997). Cooperative learning in the secondary mathematics classroom, *The Journal of Educational Research, 91*(1), 42-48.

Wilkinson, D., & Birmingham, P. (2003). *Using research instruments: A guide for researchers*. London: Routledge.

Yin, R. K. (2003). *Case study research: Design and methods* (2nd ed.). Thousand Oaks.CA: Sage.

Digitized by Sam Jonah Library

© University of Cape Coast https://ir.ucc.edu.gh/xmlui

Yin, R. K. (2009). *Case study research: Design and methods* (4th ed.). Thousand Oaks, CA: Sage.

Zainal, Z. (2007). Case study as a research method. *Jurnal kemanusiaan*, (9), 1-6.

Zakaria, E., Chin, L. C., & Daud, M. Y. (2010). The effects of cooperative learning on students' mathematics achievement and attitude towards mathematics. *Journal of Social Sciences*, 6(2), 272-275.

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APPENDICES

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APPENDIX A

IMTs Semi-Structured Interview Guide

(For Pre-During and Post Instructional Assessment)

A: Background information

1. Name: _____

2. Gender _____

3. Number of years teaching _____

4. Topic taught: _____

Planning and Preparation of Lesson

1. What motivated you to use these instructional learning materials in the lesson you taught?
2. Where did you obtain the instructional learning materials used in the lesson?
3. How did the instructional learning materials support your preparation of the lesson?
4. How did the training workshop influence your preparation to teach the content of the lesson?
5. Which aspects of the training workshop helped in preparing the lesson?

During the lesson

6. Do you think the learners liked this new method of teaching (CTLIA)? Why or why not?
7. What aspects of this new method of teaching (CTLIA) went well and supported student learning? Please explain.
8. What aspects of this new method of teaching (CTLIA) needed improvement?

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9. Did you experience any difficulty in using this new method of teaching (CTLIA) during the lesson? Please If yes explain and if no explain
10. Did you observe any changes in the behavior of your students when you used this new method of teaching (CTLIA)? If yes explain and if no explain.
11. What were the difficulties in guiding learners to learn trigonometric concepts with this new method of teaching (CTLIA)?
12. What was your role in using this new method of teaching (CTLIA)?
13. Describe the learners' attention in this lesson? Were they engaged? Did they act differently than in regular lessons?
14. Describe how you engaged the learners using this new method of teaching (CTLIA)
15. What did you observe about the mood of the students in the lesson?

Lesson Activities

16. Was the activity sheet useful? Why and why not? Please explain.
17. How will you describe the lesson activities? Learner-centred or teacher-centred? Please explain.
18. Which of the elements (positive interdependence, face to face, individual accountability, group processing and social skills) was more prevalent in your lesson? Why and how?
19. In what way did the lesson activities encourage learners to construct the concept they were learning?
20. What elements in your lesson activities do you think promoted cooperative based learning?

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Post – review of teaching

21. Was the incorporation of this new method of teaching (CTLIA) helpful in teaching the trigonometry in a cooperative based manner in the lesson? Please explain.
22. What preparation do you think would be needed to teach another lesson that incorporates this new method of teaching (CTLIA)?
23. Will you teach other trigonometric concepts using this new method of teaching (CTLIA)? If yes why? And if no why?
24. Is it feasible in using this new method of teaching (CTLIA) to teach other contents in mathematics at the SHS?
25. Would you recommend more cooperative based-oriented professional development programmes for the teaching and learning of mathematics?
26. What general comment can you make about this new method of teaching (CTLIA) as a medium of instruction in the SHS mathematics class?

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APPENDIX B

SHS Mathematics Students Semi-Structured Interview Guide (For Post Instructional Assessment)

1. How will you assess today's lessons?
2. How different is today's lessons from your usual math lesson?
3. How did the sharing of ideas with your colleagues help you to understand the topic during the lesson?
4. How did the learning together in a group help you to solve questions by yourself?
5. How did the idea of putting you into groups with some assigned duties help you to understand the topic at the end of the lesson?
6. How did the Dos and Don'ts observed during the group activities help you to understand the topic at the end of the lesson?
7. How did the group presentation on the chalkboard promote your understanding of the topic at the end of the lesson?
8. What is your general impression of the method of teaching you were introduced to in class today?

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APPENDIX C

IMTs Pre- and Post-CTLA Self-Assessment Questionnaire

(Pre and Post Instructional Assessment)

Introduction

Dear Respondent,

This questionnaire seeks to solicit your views on the method of teaching you introduced in class. Kindly complete the sets of questions in the table below by indicating against each set of question the response you deem appropriate. The questionnaire is divided into two parts. The first part seeks to obtain your background information and the second part is a list of statements for which you are required to indicate whether you agree or disagree by circling in the appropriate box. Your participation is totally voluntary and that you may withdraw from this study at any time without penalty. Further, your responses will be kept strictly confidential. Under no circumstances will your individual responses be released to the school authorities.

Thank you for taking time to participate.

Demographic instrument

Part I: Background Information

1. Gender: Male { } Female { }

2. Age: Between 16-20 { } Between 21-25 { }

Other (please, specify): .....

3. Form: SHS1 { } SHS 2 { } SHS 3 { }

Read the following statements carefully and indicate whether you agree or disagree by circling in the appropriate box. **Note: Strongly Disagree (SD) =**

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1, Disagree (D) = 2, Not Sure (NS) = 3, Agree (A) = 4, Strongly Agree (SA) = 5,

S/N	Statements	SD	D	NS	A	SA
1	In my class, specific responsibilities are assigned to students in groups to enable them find solution to questions during lessons	1	2	3	4	5
2	In my lesson, students are encouraged to share ideas with their team mates on the concept under instruction.	1	2	3	4	5
3	My lessons are participatory enough.	1	2	3	4	5
4	I provide opportunities for students to interact with their group members throughout the teaching period	1	2	3	4	5
5	Through the use students' worksheet and cooperation of group members realistic problems are solved during the task stage of the lesson	1	2	3	4	5
6	I involve students in activities that enable them to share ideas on the topic during the lesson.	1	2	3	4	5
7	My role in the lesson is more of offering support and guidance to the students.	1	2	3	4	5
8	I put in place disciplinary measures during group activities to enable the students solve problems in teams in the classroom.	1	2	3	4	5
9	Students are able to understand some mathematical concepts through feedback from peers in my lesson.	1	2	3	4	5
10	In my lesson, students are encouraged to see their group members as helpful mates in the learning process rather than competitors in class	1	2	3	4	5
11	In my class I put measures in place to enable students understand better when questions are solved together as a group rather than as individuals.	1	2	3	4	5
12	My fairness in applying the rules to ensure orderliness during group activities serves	1	2	3	4	5

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	as motivation for students to work together with one another in solving questions during lessons.					
13	I promote respect for each other's opinion during group work activities in class.					
14	Through the group's activities students learn to solve questions on their own without relying on other team mates always during individual evaluations.	1	2	3	4	5
15	In order for students to develop problem-solving skills groups are requested to present their solutions on the chalkboard at the end of group activities in class.	1	2	3	4	5
16	In my lessons the dos and don'ts for a successful group activity are formulated by the students in class.	1	2	3	4	5
17	I ensure students views/contributions were respected by their mates during group discussions in the lesson.	1	2	3	4	5
18	I clearly define the lesson's learning objectives to guide students during the teaching in the lesson	1	2	3	4	5
19	In my class, I assess continually individual students' contributions during the group's activities throughout the lesson	1	2	3	4	5
20	In class, I provide clarity to students' questions as a means of helping them to understand the concept under instruction during lesson.	1	2	3	4	5

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APPENDIX D

SHS Mathematics Students Perceived Effectiveness of the Intervention Questionnaire (For Post Assessment)

Dear student,

This is to seek your views on the lessons taught today. Kindly complete the sets of questions in the table below by indicating against each set of question the response you deem appropriate. The questionnaire is divided into two parts. The first part seeks to obtain your background information and the second part is a list of statements for which you are required to indicate whether you agree or disagree by circling in the appropriate box. Your participation is totally voluntary and that you may withdraw from this study at any time without penalty. Further, your responses will be kept strictly confidential. Under no circumstances will your individual responses be released to the school authorities.

Thank you for taking time to participate.

Part I: Background Information

1. Sex: Male { } Female { }

2. Age: .....

3. Form.....

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For each statement indicate whether you **Strongly Disagree (SD) = 1, Disagree (D) = 2, Not Sure (NS) = 3, Agree (A) = 4, Strongly Agree (SA) = 5, by circling in the appropriate box.**

S/N	Statements	SD	D	NS	A	SA
1	The specific responsibilities assigned to me in the group helped me to find solution to the question in class.	1	2	3	4	5
2	The lesson allowed me to share ideas with my colleagues in the group on the concept taught in class.	1	2	3	4	5
3	The lesson was participatory enough in class.	1	2	3	4	5
4	In this lesson, I got the chance to interact with my group members on the topic throughout the teaching period	1	2	3	4	5
5	Through the use of students' worksheet and cooperation received from my group members, I was able to solve realistic problems during the task stage of the lesson	1	2	3	4	5
6	I was involved in activities that enabled me to share ideas on the topic during the lesson.	1	2	3	4	5
7	The teacher's role in the lesson was more of offering support and guidance to me.	1	2	3	4	5
8	The disciplinary measures that were put in place during group activities helped me to solve problems with my group members in the classroom.	1	2	3	4	5
9	I understood some mathematical concepts through feedback from my colleagues in	1	2	3	4	5

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	the lesson.					
10	I see my group members as helpful mates in the learning process rather than competitors in class	1	2	3	4	5
11	I understood better in the questions we solved together as a group.	1	2	3	4	5
12	My teacher's fairness in applying the rules in the lesson motivated me to work together with my group members in solving questions.	1	2	3	4	5
13	There was respect for each other's opinion during group work activities.	1	2	3	4	5
14	Through the group's activities I have learnt to solve questions on my own without relying on my colleagues in the group always	1	2	3	4	5
15	The group presentation experiences in the lesson helped me to develop problem-solving skills.	1	2	3	4	5
16	The rules for a successful group activity were formulated by myself and colleagues during the lessons.	1	2	3	4	5
17	my views and contributions were respected by my mates during group discussions in the lesson.	1	2	3	4	5
18	The teacher clearly defined the lesson's objectives to guide me during the lesson	1	2	3	4	5
19	Teacher continually assesses my individual contributions during the group's activities throughout the lesson	1	2	3	4	5
20	The teacher provides clarity to my questions which helped me to understand the lesson.	1	2	3	4	5

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APPENDIX E

SHS Mathematics Students Experience with Intervention (CTLA)

Questionnaire

(For Post Instructional Assessment)

Introduction

Dear student,

This is to seek your views on the lessons taught today. Kindly complete the sets of questions in the table below by indicating against each set of question the response you deem appropriate. The questionnaire is divided into two parts. The first part seeks to obtain your background information and the second part is a list of statements for which you are required to indicate whether you agree or disagree by circling in the appropriate box. Your participation is totally voluntary and that you may withdraw from this study at any time without penalty. Further, your responses will be kept strictly confidential. Under no circumstances will your individual responses be released to the school authorities.

Thank you for taking time to participate.

Part I: Background Information

1. Sex: Male { } Female { }

2. Age: .....

3. Form.....

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For each statement indicate whether you **Strongly Disagree (SD) = 1, Disagree (D) = 2, Not Sure (NS) = 3, Agree (A) = 4, Strongly Agree (SA) = 5, by circling in the appropriate box.**

Experiences with the intervention	SD	D	NS	A	SA
1. With the activities I experienced in the lesson, I now need little assistance to learn mathematics.	1	2	3	4	5
2. The friendly atmosphere created by my group members has enhanced my participation in class activities.	1	2	3	4	5
3.The sharing of ideas with my group has motivated me to learn	1	2	3	4	5
4. The reliance on one another in for a solution to a trigonometric task during group activities improved my thinking skills	1	2	3	4	5
5. Inviting students from a group to justify their solution on the chalkboard improved my understanding on the concepts.	1	2	3	4	5
6. The teacher's use of personal and group scores in grading students in the lesson has enabled me to become independent in classroom.	1	2	3	4	5
7. The ideas I shared with my group members during the lesson has increased my passion to learn mathematics.	1	2	3	4	5
8. The rules and regulations regarding the conduct of group activities in the lesson made the lesson sociable.	1	2	3	4	5
9. The opportunity for students to make an input in the presentations offered by other groups made the lesson interesting.	1	2	3	4	5
10. I understood concepts during the activities in the lesson that I found difficulty to understand before.	1	2	3	4	5

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11. The activities in the lesson helped me to understand the concept better.	1	2	3	4	5
12. The use of the worksheet made the lesson interesting.	1	2	3	4	5
13. With the support of the group members and under the guidance of the teacher I can relate the concepts in the lesson to real life application.	1	2	3	4	5
14. The assistance or support received from my mates made the content of the lesson understandable	1	2	3	4	5
15. The group interactions enhanced my understanding of the lesson in class.	1	2	3	4	5
16. The respect for each other's opinion contributed to the group's motivation to find solution to the task in class.	1	2	3	4	5
17. The examples on the student worksheet enhanced my understanding of the lesson.	1	2	3	4	5
18. My friends in the team helped me to understand better	1	2	3	4	5
19. The various contributions from the groups made the lesson exciting.	1	2	3	4	5
20. There was clarification of some difficult concepts.	1	2	3	4	5
21. Due to clarity received from my mates in the group I can do my assignment with ease.	1	2	3	4	5
22. The guidance from the rules in the group helped me to learn better.	1	2	3	4	5
23. The feedback from the teacher motivated my learning.	1	2	3	4	5
24. Group presentations on the chalkboard were exciting.	1	2	3	4	5

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APPENDIX F

IMTs Focus Group Discussion Guide (For Post Instructional Assessment)

1. How will you assess today's lesson; was it a success or failure?
2. Were the instructional materials used in the lessons appropriate for the topics?
3. To what extent did the following activities promote cooperative based teaching and learning approach in the lessons?

   -engaging students to share ideas on a subject matter.

   - ensuring students learn together and independently.
   - putting students into groups of specific assigned responsibilities.
   - urging students to brainstorm on the norms in the lesson.
   - promoting evaluation in respect of the subject matter.

4. What were the weaknesses perceived in using the underlisted activities in promoting cooperative based teaching and learning approach in the lessons?

   -engaging students to share ideas on a subject matter.

   - ensuring students learn together but performs alone.
   - putting students into groups of specific assigned responsibilities.
   - urging students to brainstorm on the Dos and Don'ts (norms and ethics) in the lesson.
   - promoting evaluation in respect of the subject matter.

5. What strengths in the cooperative based teaching approach lessons do you think should be maximized for effective teaching of mathematics?
6. What can be done to improve the cooperative based teaching approach?

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APPENDIX G

Cooperative Learning Observation Protocol (CLOP)

(For During Instructional Assessment)

Introduction

The instrument (CLOP) is designed to evaluate the overall effectiveness of cooperative learning skills (intervention) used by students as they engage in cooperative tasks in the classroom. For the purpose of this study, the instrument was adapted to evaluate students' engagement in the skills necessary for effective group and individual student learning in a mathematics classroom learning environment.

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TIME INTERVAL	ELEMENTS	CHECK LIST	H	M	L	N	NOTES
	P. I	The lesson provides students with the opportunity to be ✓ Assigned specific task ✓ Put into groups ✓ Elected into positions Leader Recorder Time keeper					
	I. A	The lesson includes a wide range of activities where students ✓ Participate ✓ Contribute ✓ Engage ✓ Put together individual work to form one group work					
	G. P	The lesson provides students with the opportunity to ✓ Critique each group's solution ✓ Do group presentation on the Chalkboard ✓ Make inputs into other's work ✓ Do summary of the lesson's key highlights by the IMT					
	F.F	The lesson provides students with the opportunity to ✓ Share ideas on the task ✓ Do appropriate interruption during group activities ✓ Stimulate each other to draw out new ideas					

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	S.S	The lesson includes a wide range of activities where students ✓ Manage Conflict during group activities ✓ Respect the views of the other students in the group ✓ Harmonize the dos and don'ts in class ✓ Ensure orderliness in the group discussion					

H – High (3), M – Medium (2), H – High (1), N – Non-existent (0), Adapted from Kern, Moore, and Akillioglu (2007)

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APPENDIX H

UNIVERSITY OF CAPE COAST
INSTITUTIONAL REVIEW BOARD SECRETARIAT
TEL: 0558093143 / 0508878309
E-MAIL: irb@ucc.edu.gh
OUR REF: UCC/IRB/A/2016/1155
YOUR REF:
OMB NO: 0990-0279
IORG #: IORG0009096

11TH NOVEMBER 2021

Mr. Richard Asamoah Kyere
Department of Mathematics and ICT Education
University of Cape Coast

Dear Mr. Kyere,

ETHICAL CLEARANCE – ID (UCCIRB/CES/2021/75)

The University of Cape Coast Institutional Review Board (UCCIRB) has granted Provisional Approval for the implementation of your research titled **Using Cooperative Teaching-Learning Instructional Approach to Improve Students' Learning of the Concept on Trigonometry.** This approval is valid from 11th November 2021 to 10th November, 2022. You may apply for a renewal subject to submission of all the required documents that will be prescribed by the UCCIRB.

Please note that any modification to the project must be submitted to the UCCIRB for review and approval before its implementation. You are required to submit periodic review of the protocol to the Board and a final full review to the UCCIRB on completion of the research. The UCCIRB may observe or cause to be observed procedures and records of the research during and after implementation.

You are also required to report all serious adverse events related to this study to the UCCIRB within seven days verbally and fourteen days in writing.

Always quote the protocol identification number in all future correspondence with us in relation to this protocol.

Yours faithfully,

Samuel Asiedu Owusu, PhD
UCCIRB Administrator

ADMINISTRATOR
INSTITUTIONAL REVIEW BOARD
UNIVERSITY OF CAPE COAST

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APPENDIX TI
(Intervention 1)

Subject	Core Mathematics	Class/Form	SHS 2
Topic	The Sine, Cosine and Tangent of an acute angle	Duration	120 mins (2 periods)
Overview of the lesson: In this lesson, the following support materials: **I. Lesson plan;** a step-by-step organization of the content to be taught; **II. Student worksheet**- details a comprehensive student' activities and questions in relation to the lesson. Thus, the activity sheets are designed in line with the lesson objectives and aimed at promoting cooperative learning-instructional approach to teaching. **III. Assignment sheet**-provides feedback and consequently ensures modification of the students' grasp of the key concepts.			
Instructional Objectives	By the end of the lesson, the student will: 1define the three main trigonometric ratios tangent, sine and cosine of an acute angle of a right-angle triangle in degrees.		
Cooperative learning Objectives	explore the three main basic trigonometric ratios (tangent, sine and cosine of an acute angle) in degrees. define the tangent, sine and cosine of an acute angle in degrees.		
Skills	Students will use analytical, critical and observation skills to explain the concept of the three main trigonometric ratios and apply them in real life situations		
Knowledge	Students will learn how to: define the three main trigonometric ratios (tangent, sine and cosine of an acute angle) in degrees.		
Attitude	critical thinking and active participation Corporation: students to interact, respect depend on each other with the ultimate aim of finding solution to problems.		
Relevant Previous knowledge	Students are able to: identify and construct tangles such as 30°, 45° and 60° using a protractor and a rule. identify the sides of right-angled triangle; opposite, adjacent and hypotenuse		
Teaching and Learning Materials	calculator Activity sheets for groups work four -figure table protractor, Rule.		

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Lesson Plan		
	Teacher Activity	*Student Activity*
Introduction stage (15 minutes)	• The teacher presents some test items (pre-test) on trigonometry to students at the beginning of the lesson. In order to prepare the students for the main concept in the lesson, the teacher • reviews students RPK on differences between the various angles of 30°, 45° and 60° in a right-angle triangle by chalkboard illustration • guides students to construct the angles of 30°, 45° and 60°. of an acute angle with the help of the protractor and a rule • presents a right-angle triangle on the board and guides students to distinguish between the various lengths of the angles involved as opposite, adjacent and hypotenuse around which the three ratios revolve and • shares the lesson objectives with the students.	• Students answer the test items based on their prior knowledge on trigonometry. • Students respond to the question by identifying 30°, 45° and 60° in right angled triangle • Students construct angles of 30°, 45° and 60°on the worksheet. • Students describe the characteristics of lengths of a right-angle triangle as opposite side, adjacent side and hypotenuse side to the angle in the right-angle triangle • Students write the lesson objectives in their books.
Main lesson stage (50 minutes)	*Activity 1 Definition of Sine, Tangent and Cosine of an Acute angle in a right-angle triangle (20 minutes)* In order for students to acquire their own practical understanding of the concept of Sine, Tangent and Cosine of an angle in a right-angle triangle, the teacher • puts students into groups of four • assigns students into roles of responsibilities • presents an activity of the processes leading to the exploration of the comparison of the ratios of length in a right-angle triangle and guides students to • measure and record the lengths of **adjacent *a* (cm)** to the angle • measure and record the lengths of **hypotenuse *b* (cm)** to the angle • measure and record the lengths of **opposite *c* (cm)** to the angle of	• students are randomly assigned into groups of three • students elect themselves into the following roles • **Leader**- leads all discussions in • **Groups Recorder**- records all proceedings and present solutions on behalf of the group • **Timer**- keeps the group in checks of time group discussion. • Students explore the activities as guided by

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	measurement and finally, • with the help of the calculator find the ratios of the measured lengths as indicated in the respective tables in the activity sheet 1.	the teacher in groups and respond to the question on activity sheet (*under activity 1*) based on their observations and newly constructed knowledge. ***Some discussion points expected from students:*** $SinA \frac{opposite\ side\ to\ angle\ A}{hypotenuse\ side\ to\ angle\ A}$ A $= 30º$ $CosA \frac{adjacent\ length\ to\ angle\ A}{hypotenuse\ length\ to\ angle\ A}$ A $= 30º$ $TanA \frac{opposite\ length\ to\ angle\ A}{adjacent\ length\ to\ angle\ A}$ A $= 30º$ $f(x) = Sinx = \frac{opposite}{hypotenuse}$ $f(x) = Cosx = \frac{adjacent}{hypotenuse}$ $f(x) = Tanx = \frac{opposite}{adjacent}$
Task stage (40min)	In order for students to apply their own understanding of activity 1 to find solution to practical questions in trigonometry, the teacher • guides students to establish rules needed for a small group interaction • fosters whole group discussions on the construction of the list of items as a common standard practice expected of each member in the groups • draws the attention of the students to the grading system as a	• Students reminiscent of think pair share model co-develop guidelines for group discussions to ensure interactions within each group • Students assist in harmonizing the guidelines collectively developed in class • Students are made aware of the

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	composition of product score =40% and process scores =60% • presents the task (questions) as activity 2 to the students • attends to the various groupings in the class for the purposes of ensuring successful group activities	composition of the overall marks • Students in each group cooperate with each other in finding solutions to task by discussing their respective individual work out put/solutions with their colleagues in each group to the questions under *activity 2* (see activity sheet) • students' pieces together the most appropriate solution to the problem as a collective solution reflective of the groups
Conclusion stage (15 min)	In order to provide clarity and correct errors with the aim of helping students to grasp concept taught, the teacher • calls to the board, the group's recorder to present solutions on behalf of the group • fosters whole group discussions on the various presentations on the solutions to the problem • solicits alternate methods/inputs of finding solution to the questions • Finally, sums up the whole lesson by highlighting the key concepts discussed as follows: $f(x) = Sinx = \frac{opposite}{hypotenuse}$ $f(x) = Cosx = \frac{adjacent}{hypotenuse}$ $f(x) = Tanx = \frac{opposite}{adjacent}$ The above statements of the three basic trigonometric ratios hold at the varying the values of the angle A (A=30°, 45° and 60°). Hence the definitions of $SinA, TanA, CosA$.	• Group leaders lead discussions on the solutions to the problems after the group's recorder presentation of the solution on the board • students provide constructive feedback their peers on the solutions presented on the board • students provide alternate solutions to the solutions presented on the board • Students listen and take notes of the core points as the teacher provides clarity to some questions. • Students provide feedback responses to teacher's questions and then take assignment sheets to be solved as homework.

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	Teacher then brings closure to the lesson by • inviting questions from students based on the set instructional objectives of the lesson • giving assignment sheets to students	

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ASSIGNMENT:

CORE MATHEMATICS (SHS 2)

TRIGONOMETRIC RATIOS

Assignment Sheet

Trigonometric application demanding questions such as below:

Given that sin x = 0.6 and $0^o \leq x \leq 90^o$, find 1-tan x, leaving your answer in the form $\frac{a}{b}$ where 'a' and 'b' are integers. In (a) candidates were to find 1 - tan x in form $\frac{a}{b}$ where a and b are integers given that Sin x = 0.6 and $0^o \leq x \leq 90^o$.

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THE TRIGONOMETRIC RATIOS

Activity sheet

Introduction

In this activity sheet, you have been provided with two activities that is; activity 1 and activity 2. Activity 1 has further been sub divided into three different activities. These activities have been developed to equip you with the prerequisite skills and knowledge needed to define and compute the concept of the three main basic trigonometric ratios. You will also be able to state the trigonometric equation connecting the angles and their respective ratios of length as well as calculate the value of the missing parameters in trigonometric equations.

Activity 1: Definition of sine, cosine and tangent of an acute angle.

Instruction

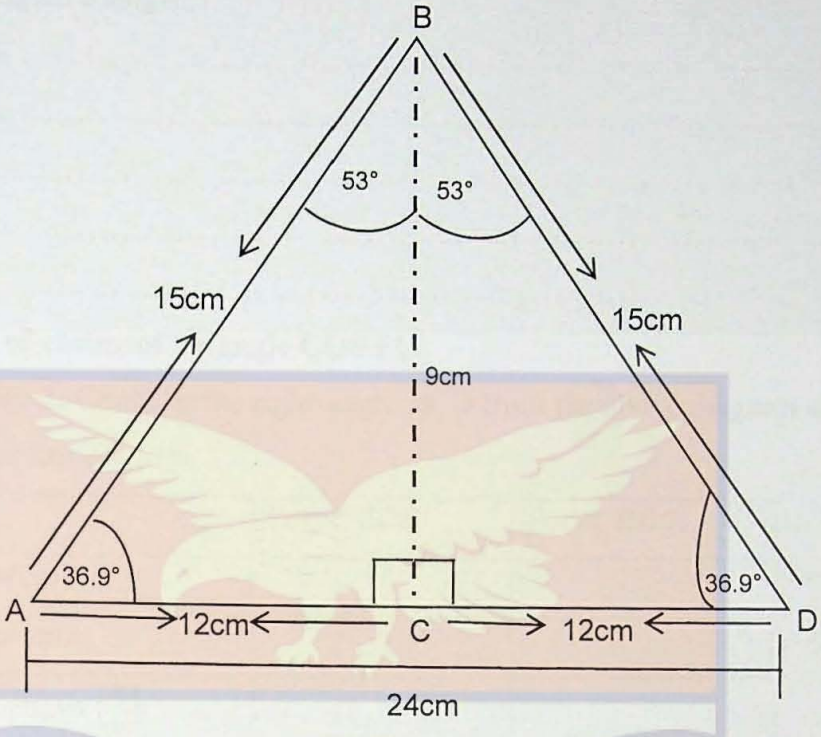
In this activity, measure and record on the activity sheets 1, the lengths of the dimensions of the right-angle triangle – **adjacent a(cm), - hypotenuse b(cm) and – opposite c(cm)** to the angle of measurement and finally, with the help of the calculator find the ratios of the measured lengths as indicated in the respective tables below as well as provide responses to the questions that follows.

Note: these activities are to be done in groups.

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Definition of sine of an angle SIN (A)



Sub Activity 1: Consider the right-angle BCD from the above diagram and complete the table below.

	For $< CDB$	For $< DBC$
Length = c(cm)		
Length = b(cm)		
Ratio of Length $\left(\frac{c}{b}\right)$		
$\sin^{-1}\left(\frac{c}{b}\right)$		
Angle measured	$< CDB = 36.9°$	$< DBC = 53°$

1.0 What is the relationship between the sine inverse of the ratios of the lengths as calculated in the table above to the angle CDB and angle DBC as measured in the diagram?

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2.0 From the above activities how would you define the Sine of an acute angle in a right-angled triangle?

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Definition of cosine of an angle COS (A)

Sub Activity 2: Consider the right-angle BCD from the above diagram and complete the table below.

	For $< CDB$	For $< DBC$
Length = a(cm)		
Length = b(cm)		
Ratio of Length $\left(\frac{a}{b}\right)$		
$\cos^{-1}\left(\frac{a}{b}\right)$		
Angle measured	$< CDB = 53°$	$< DBC = 36.9°$

1.0 What is the relationship between the cosine inverse of the ratios of the lengths as calculated in the table above to the angle CDB and angle DBC as measured in the diagram?

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2.0 From the above activities how would you define the Cosine of an acute angle in a right-angled triangle?

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Definition of tangent of an angle TAN (A)

Sub Activity 3: Consider the right-angle BCD from the above diagram and complete the table below.

	For $< CDB$	For $< DBC$
Length = c(cm)		
Length = a(cm)		
Ratio of Length $\left(\frac{c}{a}\right)$		
$\tan^{-1}\left(\frac{c}{a}\right)$		
Angle measured	$< CDB = 36.9°$	$< DBC = 53°$

1.0 What is the relationship between the tan inverse of the ratios of the lengths as calculated in the table above to the angle CDB and angle DBC as measured in the diagram?

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2.0 From the above activities how would you define the tangent of an acute angle in a right-angled triangle?

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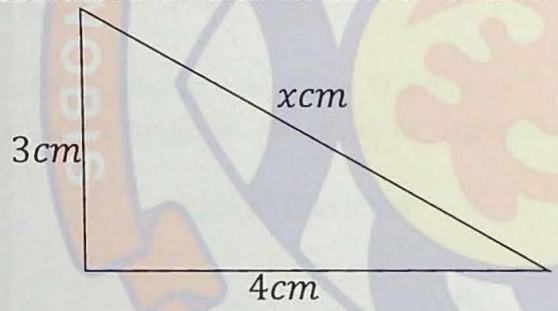
APPENDIX T2
Intervention 2

Subject	Core Mathematics	Class/Form	SHS 2
Topic	The Sine, Cosine and Tangent of an acute angle	Duration	120 mins (2 periods)
Overview of the lesson: In this lesson, the following support materials: **I. Lesson plan;** a step-by-step organization of the content to be taught; **II. Student worksheet**- details a comprehensive student' activities and questions in relation to the lesson. Thus, the activity sheets are designed in line with the lesson objectives and aimed at promoting cooperative learning-instructional approach to teaching. **III. Assignment sheet**-provides feedback and consequently ensures modification of the students' grasp of the key concepts			
Instructional Objectives	By the end of the lesson, the student will: 1. Determine the appropriate trigonometric ratio in right angle triangle • tangent, • sine and • cosine of an acute angle in degrees. • Calculate angles of elevation • apply trigonometric ratios to calculate distances and heights		
Cooperative learning Objectives	Through the use of the cooperative learning-instructional approach in the lesson, students will be able to: • provide the appropriate diagrammatic representation of trigonometric ratio in right angle triangle • find angles of elevation in the right-angle triangle • apply trigonometric ratios to find angles of elevation in real life applications.		
Skills	Students will use analytical, critical and observation skills to explain the angles of elevation and apply them in real life situation.		
Knowledge	Students will learn how to: ▪ Determine the appropriate trigonometric ratio in right angle triangle ▪ Calculate angles of elevation. ▪ apply the trigonometric ratios to find solution to angles of elevation in real life problems.		
Attitude	▪ critical thinking and active participation ▪ Corporation: students to interact, respect depend on each other with the ultimate aim of finding solution to problems.		
Relevant Previous knowledge	Students are able to: • Determine the three trigonometric ratios in a given right angle triangle • Apply the Pythagoras theorem in a given right angle triangle.		

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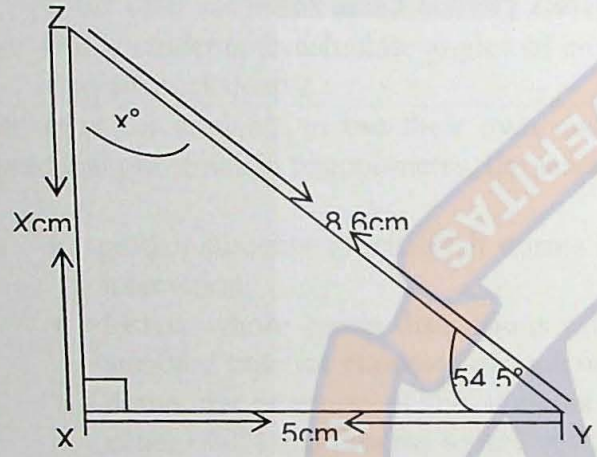
Teaching and Learning Materials	Calculator Copies of activity sheets for each student or groups of students four -figure table Protractor, Rule.

Lesson Plan

Teacher Activity		*Student Activity*
Introduction stage (15 minutes)	• The teacher presents some test items (pre-test) on trigonometry to students at the beginning of the lesson. In order to prepare the students for the main concept in the lesson, the teacher • begins the lesson by asking students to determine the three basic trigonometric ratios in a given right angle triangle as projected on the chalkboard. • presents a right-angle triangle on the board and guides students to apply the Pythagoras theorem to find the missing lengths (xcm) of the triangles involved as opposite, adjacent and hypotenuse around which the three ratios revolve.  • shares the lesson objectives with the students.	• Students answer the test items based on their prior knowledge on trigonometry. • Students strives for the solution to the question by sharing ideas on the appropriate trigonometric ratio to be used in the given angle • Students apply Pythagoras theorem to find the missing lengths as projected on the board • Students write the lesson objectives in their books.
Main lesson stage (50 minutes)	*Activity 1: Determining the appropriate trigonometric ratio in a right-angle triangle (20 minutes)* In order for students to determine the appropriate trigonometric ratio to calculate angles of elevation and depression, the teacher • puts students into groups of four	• Students are randomly assigned into groups of three • students elect into groups of assigned roles

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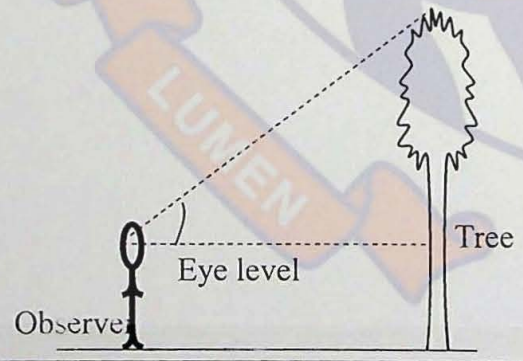
- assigns students into roles of responsibilities
- presents activity 1 as the processes leading to the determination of the appropriate trigonometric ratio in right-angle triangle and
- guides students to respond to the question on activity sheet (*under activity 1*)



Activity 2: *Calculate angles of elevation (20 minutes)*

- Teacher introduces the angle of elevation as depicted below for the purposes whole class discussion.

Angle of Elevation



- **Leader**- leads all discussions in
- **Groups Recorder**- records all proceedings and present solutions on behalf of the group
- **Timer**- keeps the group in checks of time and conditions pe-requisite for a successful group discussion.

- Students explore the activities as guided by the teacher in groups and respond to the question on activity sheet (*under activity 1*) based on their observations and newly constructed knowledge.

Some discussion points expected from students:

- The angle elevation of an object B, from an observer at A, who is below the level of B is the angle which AB makes with the horizontal line.

- Students exhibit the Diagrammatic representation of the angles as presented in the case scenarios

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	• Through group discussion, teacher guides students to provide a diagrammatic representation of the case scenarios under activity 2 on the students' worksheet and consequently • Guide students to calculate angles of elevation in the right-angle triangle as presented on the student worksheet 2	under activity 2 on the students' worksheet.
Task stage (40min)	In order for students to use their own understanding of activities 1 and 2 to find solution to practical questions in trigonometry, the teacher • guides students to establish norms and ground rules needed for a fruitful small group interaction • fosters whole group discussions on the construction of the list of items as a common standard practice expected of each member in the groups • draws the attention of the students to the grading system as a composition of product score =40% and process scores =60% • attends to the various groupings in the class for the purposes of ensuring successful group activities. • Teacher guides students in each group to find solution to task after a successful group work on activity 3.	• Students reminiscent of think pair share model co-develop guidelines for group discussions to ensure healthy interactions within each group • Students assist in harmonizing the guidelines collectively developed in class • Students are made aware of the composition of the overall marks • students discuss their understanding of the materials (questions) and develop a solution to the problem • Students in each group cooperate with each in finding solutions to task by discussing their respective individual work out put/solutions with their colleagues in each group to the questions under *activity 2* (see activity sheet)

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		• students piece together the most appropriate solution to the problem as a collective solution reflective of the groups
Conclusion stage (15 min)	In order to provide clarity and correct errors with the aim of helping students to grasp concept taught, the teacher • calls to the board, the group's recorder to present solutions on behalf of the group • fosters whole group discussions on the various presentations on the solutions to the problem • solicits alternate methods/inputs of finding solution to the questions • Finally, sums up the whole lesson by highlighting the key concepts discussed as follows: (G.P) $f(x) = Sinx = \frac{opposite}{hypotenuse}$ $f(x) = Cosx = \frac{adjacent}{hypotenuse}$ $f(x) = Tanx = \frac{opposite}{adjacent}$ The above statements of the three basic trigonometric ratios hold at the varying the values of the angle A (A=30°, 45° and 60°). Hence the definitions of $SinA, TanA, CosA$. Teacher then brings closure to the lesson by • inviting questions from students based on the set instructional objectives of the lesson • giving assignment sheets to students	• Group leaders lead discussions on the solutions to the problems after the group's recorder presentation of the solution on the board • students provide constructive feedback their peers on the solutions presented on the board • Students listen and take notes of the core points as the teacher provides clarity to some questions. • Students provide feedback responses to teacher's questions and then take assignment sheets to be solved as homework

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ASSIGNMENT: students perform task as sanctioned on the assignment sheet and on the progress report sheet.

CORE MATHEMATICS (SHS 2)

TRIGONOMETRY (ANGLES OF ELEVATION)

Assignment Sheet

The angle of elevation of a bird in the air from a hunter standing on the ground is 35°. If the hunter is 1.4m tall and the bird is 81.7m away from the hunter's line of sight, how high, correct to 1 decimal place is the bird from the ground?

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6. A bird on a tree 15m high was observed at an angle of elevation of 75° from the same horizontal ground as the foot of the tree. Calculate, correct. to the nearest whole number, the distance of the observer from the foot of the tree

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CORE MATHEMATICS (SHS 2)

THE ANGLES OF ELEVATION

Student's Activity worksheet sheet

Introduction

In this activity sheet, you have been provided with four activities that is; activity 1,2,3 and 4. These activities have been developed in such a way that, after going through the activities, you will be equipped with the necessary prerequisite skills and knowledge needed to determine the appropriate trigonometric ratio to calculate angles of elevation and depression and apply the use of trigonometric ratios to calculate distances and heights.

Note: all activities are to be done in groups.

Activity 1: Determining the appropriate trigonometric ratio in a right-angle triangle.

Instruction

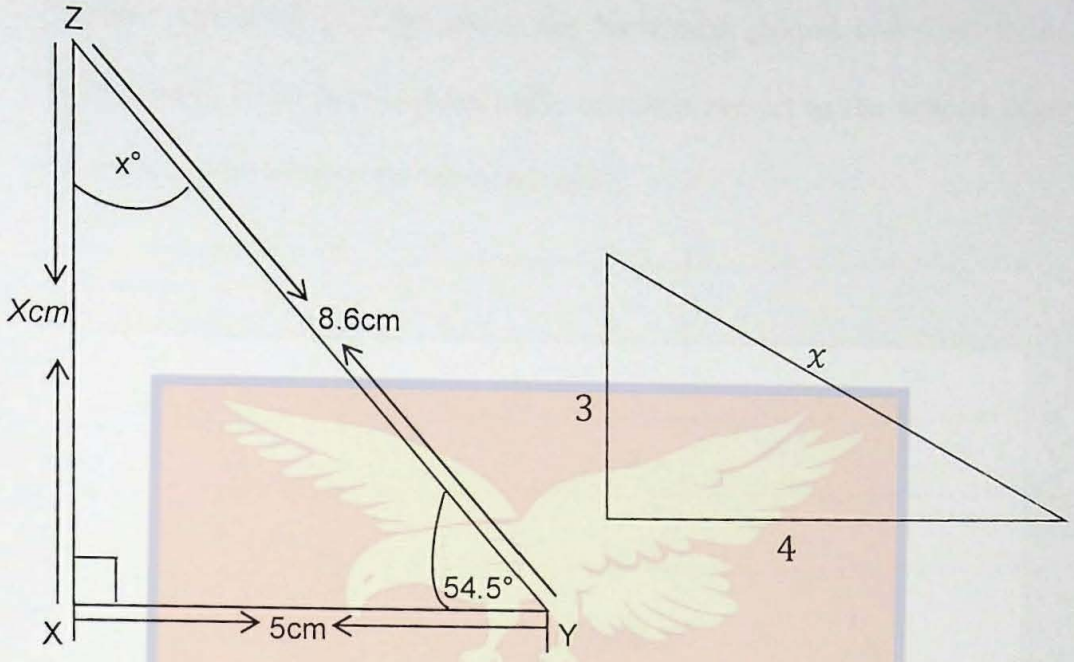
In this activity, you are presented with following diagrams to enable you share ideas in your groups as to the appropriate trigonometric ratio needed in finding the missing parameters in the angles below. You are kindly requested to read carefully and answer the question that follows.

Note: all activities are to be done in groups

- 1.0 Determine the appropriate trigonometric ratio in finding the missing parameters in the following diagrams below.

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Activity 2: Calculate angles of elevation

Instruction

In this activity, you are presented with following case scenarios with the view of consulting each other in the group as to the appropriate computation and diagrammatic representation of the angle of elevation in each of the case scenario as provided below.

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CASE 1

A man's eye level is 1.7m above the horizontal ground and 13m from a vertical pole. If the pole is 8.3m high, calculate correct to the nearest degree the angle of elevation of the top of his eye?

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CASE 2

A man's eye level is 1.7m above the horizontal ground and 13m from a vertical. If the pole is 8.3m high, calculate correct to the nearest degree, the angle of elevation of the top of the pole?

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Activity 3: *application of trigonometric ratios to calculate distances and heights*

Instruction

In this activity, the following case scenarios affords you an opportunity to assess the extent to which the three basic trigonometric ratios could be applied in determining the distances and heights in problems of trigonometry as provided in each case scenario below. You are kindly requested to carefully analyse and answer the questions for the purposes of electing a representative from each group to present the group's solution on the chalkboard

QUESTION

Solve the question.

(a) If $\tan x = \frac{15}{8}$, $0^{\circ} \leq x \leq$ find the value of Cos x.

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