UNIVERSITY OF CAPE COAST

IN-SERVICE MATHEMATICS TEACHERS' UNDERSTANDING AND TEACHING OF TRIGONOMETRY: IMPLICATION FOR PROFESSIONAL DEVELOPMENT

SANDRA ODJER

2021

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BY

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Thesis submitted to the Department of Mathematics and Information and Communication Technology Education of the College of Education Studies, in Partial Fulfilment of the Requirements for the Award of Master of Philosophy Degree in Mathematics Education

AUGUST 2021

DECLARATION

Candidate's Declaration

I hereby declare that this thesis is a result of my own original research and that no part of it has been presented for another degree in this university or elsewhere.

Candidate's Signature…………………… Date:…………………………

Name: Sandra Odjer

Supervisor's Declaration

I hereby declare that the preparation and presentation of the thesis were supervised following the guidelines on supervision of thesis laid down by the University of Cape Coast.

Supervisor's Signature………………..…… Date……………………………

Name: Mr. Alexander Asare-Inkoom

ABSTRACT

The study develops insight into In-Service Mathematics Teachers' (ISMTs') understanding and teaching of trigonometry concepts in SHS. Mixed method was used. Population was primarily ISMTs. A sample size of 220 ISMTs were used. Stratified sampling was used in selecting the schools and ISMTs. Data collection was done with questionnaire. The study revealed that knowledge of ISMTs on trigonometry content was not encouraging. The few who answered the questions performed better on Elective Mathematics Trigonometry questions. For identification of errors from hypothesized student's solutions, majority did not find anything wrong with the solutions presented. On causes of the errors, they mainly attributed it to lack of understanding of either a procedure, or concept but not both. On correcting the errors and helping students understand the concepts, majority displayed insufficient knowledge of content and students. Regarding self-efficacy, ISMTs were much more confident to teach Core Mathematics related items compared to Elective, and identified some items they have least and most confidence in teaching, with reasons. On challenges, the major ones were; lack of confidence, inadequate instructional resources, and difficulty and problem of teaching trigonometry. On teacher professional development needs, support is needed to teach trigonometry in: teacher self-improvement in trigonometric content and pedagogy; preparation and utilization of teaching materials; and use of ICT and the others. It is recommended that advisory services, school heads and principals should organize and encourage teachers to attend professional development courses to refresh their trigonometry contents and become abreast with new developments.

KEY WORDS

Constructivism

Development

Errors

In-Service

Knowledge

Misconceptions

Needs

Pedagogical

Professional

Radical

Self-Efficacy

Trigonometry

ACKNOWLEDGEMENTS

I want to extend my sincere gratitude to Mr. Alexander Asare-Inkoom, my supervisor, for his advice and supervision in making this research a success. The work has been finished by posing smart queries, recommendations, remarks, and directives. Additionally, I would like to express my sincere gratitude to Dr. Ernest Ampadu of the Department of Learning at the School of Engineering and Industrial Management of the Royal Institute of Technology, Stockholm, University of Central Florida, Sweden, and Mr. Benjamin Sokpe, and Dr. Christopher Yarkwah of the Department of Mathematics and ICT Education, UCC for their assistance and cooperation in pursuing this programme. Again, my appreciation goes to all the lecturers of the Department of Mathematics and ICT Education, for their supports and cooperation during the programme. I also express my gratitude to the In-Service Mathematics Teachers who took time out of their hectic schedules to complete the questionnaire. For their help and collaboration, I thank all my colleagues and the nonacademic personnel at the Department of Mathematics and ICT Education. I value the many ways that they each assisted me in finishing this programme. Additionally, I want to express my gratitude to Mr. Theophilus Asamoah for his unwavering backing, words of inspiration, astute observations, and helpful recommendations, as well as for sticking with me throughout the entire programme. God bless you abundantly, Kwaku. Also deserving of my appreciations are Emmanuel Nti-Asante, John Erabekyere, and Stephen Atepor. Finally, I am singularly thankful to all my family members for their cooperation throughout the programme.

DEDICATION

To My Lovely Family, Particularly, Mr. David Apim Odjer and Mrs. Elizabeth Apim

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LIST OF ABBREVIATIONS

AEM	Awareness of Errors and Misconceptions
ANOVA	Analysis of variance
CCK	Common Content Knowledge
CER	Chief Examiner Reports
CK	Content Knowledge
COVID-19	Corona Virus Disease-2019
CRDD	Curriculum Research and Programme Development
EFA	Exploratory Factor Analysis
HCK	Horizon Content Knowledge
ICT	Information and Communication Technology
ISMTs	In-Service Mathematics Teachers'
KCC	Knowledge of Content and Curriculum
KCS	Knowledge of Contents and Student
KCT	Knowledge of Content and Teaching
KMO	Kaiser-Meyer-Olkin
MKT	Mathematical Knowledge for Teaching
MOESS	Ministry of Education, Science and Sports
NCTM	National Council of Teachers of Mathematics
NSC	National Senior Certificate
PCK	Pedagogical Content Knowledge
PD	Professional Development
RCT	Radical Constructivism Theory
SCK	Specialized Content Knowledge
SMK	Subject Matter Content Knowledge

SHS	Senior High School
SMK	Subject Matter Knowledge
SPSS	Statistical Package for Social Sciences
STIN-LP	Science Teachers Inventory Needs-Limpopo Province
TAT	Trigonometry Assessment Test
TCK	Trigonometrical Content Knowledge
TCKD-T	Trigonometry Content Knowledge and Diagnostic Test
TIMSS	Trends in International Mathematics and Science Study
TPD	Teacher Professional Development
TPQ	Trigonometry Perception Questionnaire
TQ	Trigonometry Questionnaire
TSE	Trigonometry Self-Efficacy
TTTES	Teacher Trigonometry Teaching Efficacy Scale
UCC	University of Cape Coast
WASSCE	West African Secondary Schools Certificate Examinations

CHAPTER ONE

INTRODUCTION

Background to the Study

As indicated by Doyran (2012), the impact of any educational system can only be as powerful, effective, and quality as the teachers, making the teacher factor an important area of concern. Teachers constitute the most central component in the delivery of quality education as mediators of curriculum implementation, since they are ultimately responsible for deciding what and how students should learn mathematics (Berk, 2005). What teachers know is one of the most significant variables that impacts what is done in the classroom (Mohd-Rustam, 2016). Without a doubt, teachers' mastery of key mathematical ideas has a substantial impact on their capacity to effectively teach mathematics, and it has one of the most remarkable effects on students' mathematics engagement (Attard, 2011). This is because, first, teachers' personal understanding (SCK) of mathematical concepts and ideas establishes the most immediate source of what they plan and expect students to learn, and what they know about how these concepts and ideas can develop. Also, their pedagogical understanding (PCK) of how to make the topic exhaustive for the students through effective teaching techniques is equally important. Moreover, as cited in Lovett (2016), the collection of teacher pieces of knowledge (Shulman, 1986), and beliefs (Wilson et al., 1989), of mathematical concepts and approaches for assisting students' acquisition of these concepts, have a critical impact on what they can learn and how effectively they learn in any instructional situation. Echoing comparable sentiments, a Chinese proverb says that: "if you want to give the student one cup of water, you (teacher) should have one bucket of water of your own" (Kandjinga, 2018, p. 2, cited from An et al., 2004). This Chinese saying infers that teachers should know more than what they are to provide for students regarding subject matter knowledge. Consequently, today's mathematics educational environment necessitates a teacher who is knowledgeable about teaching and learning, has good-based mathematical knowledge, as well as a teacher who can connect real-life situations to the mathematics curriculum (Montoro, 2012). In this respect, teachers have "a need, as never before, to update and improve their skills through professional development" (Craft, 2000).

Among the contents in mathematics, Garden, Lie, Robitaille, Angell, Martin, and Mullis (2008) reported that trigonometry was included in the innovative mathematics curriculum of all the 48 nations examined by TIMSS in 2007. Thus, trigonometry has an important place in the mathematics curriculum in several nations, including Ghana, even though its meaning may vary from one country to another. From the MOESS, the senior high school Core and Elective curricula specify seven and eight content areas respectively that any learner or teacher of mathematics in Ghana must acquire (MOESS, 2010). More essentially, teachers' content and pedagogical knowledge, efficacy, and competency to teach these seven and eight content areas are very crucial. Whereas all the seven and eight content areas are very necessary and essential in the acquisition of mathematical competencies' in Ghana, trigonometry is the most important one. This is due to the fact that trigonometry's inclusion in the curriculum offers a suitable setting for investigating, connecting, exploring, and relating mathematical concepts as well as for the meaningful integration of various scientific disciplines, and that students' mathematical abilities have been closely linked to their levels of trigonometric understanding (Nabie, et. al., 2018).

A branch of physical mathematics called trigonometry is concerned with the comprehension and applications of the idea. Angles, angle measurement, triangles, and their interactions are among its topic areas (Orhun, 2010). It combines geometric, algebraic, and graphical reasoning to provide a space where problems involving triangles, trigonometric terminology, and graphs can be solved. An understanding of the trigonometry concept is a prerequisite to learning calculus and is used in investigating real-world phenomena (MOESS, 2010). This is why it is a significant school subject in mathematics, as well as having both application and a prerequisite for advanced studies in Geography, Architecture, and Astronomy. It is also utilized in other disciplines, including survey, cartography, geometry, maritime, optics, and physics (Tuna, 2013). Understanding trigonometry helps students improve their cognitive skills by providing a framework for organizing concepts such as angles, angle and length measurement, forms and similarities, vectors, polar coordinates, and parametric curves (CRDD, 2010), making it impossible to ignore in the mathematics context of education, more specifically, in Ghana. It encompasses trigonometric concepts, ideas, processes, and problem-solving applications, and mastering it at the secondary school level provides a solid basis for learning mathematics at tertiary institutions (MOESS, 2010).

Because of these relative significances of trigonometry, mathematics educators and researchers from many countries (CER, 2012, 2014, 2015, 2016, 2018, for Core Mathematics; 2012, for Elective Mathematics; Nabie et al., 2018; Bosson-Amedenu, 2017; Mensah, 2017; Gur, 2009) have conducted and continue to investigate ways in students learn and understand trigonometry. These investigations consistently documented low understanding, errors, and misconceptions of these trigonometric ideas among a diverse populace across various settings. Several studies (Nabie et al., 2018; Bosson-Amedenu, 2017; Mensah, 2017; Gur, 2009), including the CER (2012-2018), recommended that the conceptual understanding of trigonometric ideas ought to be stressed over procedures, computations, and algorithmic skills with an explanation of the concept. That is, authentic teaching must not only involve the teacher establishing how to find the right answer, but it also implies that a teacher should discover how to improve the subject comprehended by the students.

However, ISMTS' must develop a conceptual understanding of trigonometrical content to direct their practices towards such recommendations. This is because if teachers do not possess the appropriate understanding of trigonometric concepts, they are unlikely to help students learn effectively. Hence, this current study examines the personal and pedagogical understandings of teachers and provides implication for professional development for teaching trigonometry in SHS.

Statement of the Problem

a) Find the acute angle from the given trigonometry equation,

$\tan(\theta + 25°) = 5.145$, $0° \le \theta \le 90°$, *[WASSCE May/June 2018]*

b) *Given that* $5\cos\left(x + 8.5°\right) - 1 = 0$, $0° \le x \le 90°$, *calculate, correct to the nearest degree, the value of x. [WASSCE May/June 2012]*

According to the CER in 2018 and 2012, $(\theta + 25°) = 5.145$ and $\cos x + \cos 8.5 = 0.2$ respectively, were how the majority of the candidates expanded the equations in the WASSCE Core Mathematics Paper 2. These statements fill a mathematics researcher with apprehension and lead to the question: how did the way mathematics teachers taught trigonometry led to such glaring errors and misconceptions?

Regardless of the overall significance of trigonometry, there is increasing disappointment with and concerns about students' misconceptions, errors, and difficulties in understanding trigonometric concepts as reported in various studies (Bosson-Amedenu, 2017, Mensah, 2017). It is generally uncovered in literature, both globally and locally, that students have issues with the concept, as well as the process, and precept (a combination of process and concept) in learning trigonometry (Nabie et al., 2018; CER, 2018; Mensah, 2017; Gür, 2009). When students were asked about major challenges in mathematics learning, and why some topics such as trigonometry were regarded as difficult, most students pointed to the mathematics teacher as the number one challenge and factor, with others including; word problems, ineffective instructional techniques, lack of motivation, the abstract nature of trigonometric concepts, lack of understanding of trigonometric language and basic concepts, insufficient utilization of manipulatives to improve picturing capacities, and a few teachers intentionally skipping clarifications on triangles and the deductions of trigonometry formulae (Bosson-Amedenu, 2017; Mutodi & Ngirande, 2014).

Trigonometry is a potential examinable topic for each student sitting for the WASSCE. In Ghana, it is reported that there is persistent skipping of trigonometry and its related content questions in both the Core and Elective Mathematics by candidates in the WASSCE. The CER (2018, 2016, 2014, and

2012) indicated that students routinely opt not to attempt trigonometry problems at the completion of secondary school education, and even the few candidates who attempt such questions do not answer them satisfactorily. Students at the secondary level have consistently demonstrated misunderstandings of topics such as Pythagoras' theorem, trigonometric ratios, functions, identities, and equations, the concept of limits, as well as having weak visualization abilities (CER, 2018, 2016, and 2014).

Additionally, students experienced difficulty with the geometrical representation of trigonometric functions and could not fully understand how to interpret and find solutions to graphs of trigonometric functions. This is evident in CER for WASSCE May/June 2018 Core Mathematics Paper 2. An evidence-based sample in this paper is in the compulsory part, Question 1(a), where candidates were required to "find the value of $\boldsymbol{\theta}$, correct to one decimal place, in the trigonometric equation;

$$\tan(\theta + 25°) = 5.145, \ 0° \le \theta \le 90°,"$$

After the analysis of individual questions, the CER (2018) revealed that the candidates were not able to find the acute angle from the given trigonometric equation. Hence, most of them rather expressed it as $(\theta + 25^0) = 5.145$ instead of $\theta + 25^0 = \tan^{-1}(5.145)$.

In the optional section, the following question was posed;

"11b) The angle of elevation of the top, X, of a vertical pole from a point, W, on the same horizontal grounds as the foot, Z, of the pole is 60°. If W is 15km from X and 12km from a point Y on the pole,

(a) illustrate this information with a diagram;

(b) calculate, correct two decimal places, the:

(i) the angle of elevation of Y from W;

(ii) length, XY."

The CER (2018) expressed that:

> *"most of the candidates did not attempt this question on trigonometry using the concept of angle of elevation and those who attempted it could not show the information with a diagram which would enable them to answer the subsequent questions" (p. 12).*

It was observed that while a large portion of the candidates responded to Question 1 (a) because it was in the compulsory part, Question 11(b) was not attempted since it was optional. The CER (2018) has it that candidates performed ineffectively on both questions as they messed up the entire calculations.

To help solve the problems identified above, the chief examiner and researchers proffered numerous suggestions and recommendations around a set of basic teaching practices and principles to direct teachers' judgment and the knowledge required for teaching mathematics subjects generally and trigonometry in particular. While the CER (2018, 2016, 2014, & 2012) and Kagenyi (2016) suggested that teachers should adopt appropriate instructional methods and strategies and relate the teaching of mathematics to real-life problems, Mensah (2017) recommended that teachers must be well prepared and capable of analyzing students' learning errors in order to come up with alternate solutions to students' problems, particularly in solving trigonometric ratios. The chief examiner also recommended that teachers should stop specializing in teaching topics they are familiar with. In the views of Magbanua (2018) and Niranjan (2013), trigonometry instruction should involve more hands-on investigations like the use of manipulatives, for example, self-guiding worksheets that will effectively engage students to develop a practical understanding of trigonometry concepts.

Thematically, all the fundamental recommendations from these researches were directed at teachers though the main subjects of the study were students. This implies that the need for teachers to teach to the understanding of students is still the hope for conceptual understanding of trigonometry. Hence, research into this topic from the standpoint of teachers is imperative, as it is well-established in the literature that what instructors know determines what and how they teach, which in turn influences what and how students learn (Beausaert, Segers & Wiltink, 2013). If so, a significant question that begs for an answer is what ISMTs understanding and teaching of trigonometry are. Given this, Davis (2005) reported that trigonometry and the different ways it is taught in the classroom have received limited attention. This means that issues students experience in learning trigonometry may emerge more from how it is taught.

Along with Borko (2004), in order "to foster students' conceptual understanding, teachers must have a rich and flexible knowledge of the subjects they teach" (p. 5). He again argued that to teach and engage students, teachers require sufficient knowledge of the content to be taught, as well as appropriate methodological techniques to transfer the content so that the learners can understand it with ease. Similarly, Kapenda and Kasanda (2015) stated that without enough subject knowledge, little can be communicated to the learners, which may result in poor understanding and mastery of content. Thus, teachers should have sufficient trigonometry knowledge to maximize learning.

However, in Ghana, studies on in-service mathematics teachers' trigonometry concepts have been negligible and literature has shown that no study has been conducted regarding the ISMTs' understanding of trigonometry. Nevertheless, a few studies were conducted in Ghana that concentrated predominantly on students' (SHS, undergraduate) and reported that students have insufficient content knowledge of trigonometry and are susceptible to errors and misconceptions.

Yet, little consideration has been paid to the personal and pedagogical understanding and challenges experienced by ISMTs as well as their professional development needs in teaching this topic, since their misunderstanding of trigonometry may have detrimental effects on students' understanding of trigonometry. Hence, consulting ISMTs' to ascertain their trigonometrical content knowledge, knowledge of trigonometrical errors and misconceptions, self-efficacy, and the challenges of the topic area is necessary. This work, therefore, seeks to examine ISMTs' understanding and teaching of trigonometry to determine their needs for effective professional development activities that can improve learners' academic performance in trigonometry.

Purpose of the Study

The study's purpose is to gain an insight into ISMTs' understanding and teaching of trigonometry concepts in SHS. Given these, the following specific objectives guided the study. To;

1. Determine the trigonometry content knowledge level of ISMTs for teaching senior high school.
2. Examine ISMTs' awareness of trigonometrical errors and misconceptions.

3. Determine ISMTs' trigonometry self-efficacy for teaching senior high school.

4. Identify some challenges faced by ISMTs' in the teaching of senior high school trigonometry.

5. Assess and describe the ISMTs professional development needs for teaching senior high school trigonometry.

Research Questions

The study was guided by the following research questions;

1. What is the trigonometrical content knowledge of ISMTs?
2. What is ISMTs' awareness of trigonometrical errors of students?
3. What is ISMTs' self-efficacy for teaching the senior high school trigonometry?
4. What are some of the challenges ISMTs encounter in the teaching of trigonometry?
5. What are the ISMTs' professional development needs for teaching trigonometry?

Research Hypothesis

There was a need to determine the various levels that the ISMT's were doing well, so in support of the research objectives (iii), research hypothesis was used to determine if any significant differences existed between their self-efficacy for Core and Elective mathematics trigonometry. Hence, the following hypothesis was formulated to guide the study:

1. $H_{01:}$ There is no significant difference between ISMTs' Trigonometry Self-Efficacy.

Significance of the Study

An inquiry into teachers' understanding and teaching would offer potential insights into ways of promoting better teaching. Results from investigating personal and pedagogical understandings and identification of the professional development (PD) needs of teachers for teaching trigonometry are expected to have a strong impact in several ways and on different groups, comprising policymakers, curriculum developers, PD developers, education researchers, school administrators, and classroom teachers to formulate a policy that will strengthen the trigonometry understandings and teaching of In-Service Mathematics teachers in Ghana. The findings can be used by those who create training programmes for math teachers to establish and improve future chances that have a good chance of altering participants' knowledge and beliefs as well as having a significant influence on their practices. Thus, the research's results and conclusions may be applied to the creation of an in-service training programme that aims to improve the caliber of trigonometry education in particular as well as mathematics instruction in general. Hence, enabling teachers to make the teaching of trigonometry lively and interesting for students enables them to appreciate the topics in the syllabus as prescribed by the chief examiner.

In addition, despite the fact that this study's focus was on in-service teachers, its findings offer information that can significantly strengthen the trigonometry teacher education program. This is due to the fact that professional knowledge has been regarded as a vital component in teacher education programs (Ball & McDiarmid, 1990). The results of this study may help in determining which trigonometry subtopics, as well as instructional strategies and modules, should be covered or included in pre-service and in-service teacher education programs, aiding in the creation of practical strategies to train current and aspiring teachers to have a deeper understanding of teaching trigonometry. The results might also add to the field of research in the Ghanaian context, since this is the first study of its kind that examines the in-service mathematics teachers' personal and pedagogical understanding of trigonometry in Ghana.

Delimitations of the Study

The study was delimited to ISMTs' in selected public SHS in Ghana. Again, with all the scope of content areas in SHS mathematics, only trigonometry was considered. Attention was only paid to the personal and pedagogical understanding, self-efficacy, challenges, and professional development needs of the selected ISMTs'.

Definition of Terms

Core Trigonometry: it covers SHS trigonometry content which includes, trigonometry ratios and rules, angle of elevation, simple trigonometry graphs and their application. Thus, Trigonometry I and II.

Elective Trigonometry: it covers SHS further trigonometry content, which includes trigonometric ratios and rules, compound and multiple angles, and functions, equations, and graphs.

In-Service Mathematics Teacher: a Mathematics teacher currently teaching mathematics in the classroom.

Mathematical Knowledge for Teaching: In order to effectively teach the subject of trigonometry, teachers must possess the necessary mathematical skills to carry out engaging classroom exercises.

Pedagogical Understanding: refers to teachers' awareness of students' trigonometrical errors, misconceptions, and difficulties and how to help correct them.

Personal Understanding: while addressing or resolving SHS trigonometry issues, is the accurate application of trigonometrical ideas, facts, and processes, the justification for trigonometrical methods, and the link between trigonometrical concepts.

Teacher Understanding: refers to both the personal (subject matter content knowledge) and pedagogical content knowledge of teachers.

Trigonometry Content Knowledge: refers to knowledge of the trigonometry content in the SHS Core and Elective Mathematics Curriculum.

Trigonometry Self-Efficacy: refers to one's ability to confidently achieve any of the stated objectives for teaching SHS trigonometry.

Trigonometry: refers to SHS trigonometry content, consisting of angles, measurement of angles, triangles, and their relationships, as well as trigonometry ratios and rules, functions, equations, and graphs and their applications.

Organization of the Study

There are five chapters in the research study. The study's background, problem statement, purpose, research questions, significance, delimitation, limits, and organizational structure were all introduced in Chapter One. A review of pertinent studies and related literature linked to the issue this study sought to solve was offered in Chapter Two. Research methods were covered in Chapter Three and included research design, study population, sample size and sampling techniques, research instruments, and administering of research instruments. The results and analyses of the information acquired through the use of the research instruments were the main emphasis of Chapter Four. The researcher's results were summarized, their implications for practice were discussed, and suggestions for further research were made in Chapter Five.

CHAPTER TWO

LITERATURE REVIEW

Overview

The study's purpose is to gain an insight into ISMTs' understanding and teaching of trigonometry concepts in SHS. The chapter begins with the definition of the concepts of trigonometry and their importance, as well as the place of trigonometry in the Ghanaian mathematics curriculum. It continues with the theoretical framework that underpins this research. Besides, the study provides literature regarding mathematics teachers' Subject Content Knowledge (SCK) and Pedagogical Content Knowledge (PCK). Another supporting theory, radical constructivism, and its application to the study are carried out. In terms of the empirical reviews, they include a review of teachers' understanding, knowledge types for teaching mathematics and trigonometry in particular, awareness of errors and misconceptions, self-efficacy, challenges, and needs assessment. The literature review has been conceptualized into a framework as presented in Figure 2.

Concepts and History of Trigonometry

This section gives an overview of trigonometry's history and definition, its importance, and the place of trigonometry in the Ghanaian mathematics curricula, as well as the concept of need assessment.

Definition of Trigonometry

"Trigonometry is a Greek word which means tri-three, gono-angle, and metry-measurement" (Talkokul, 2017, p. 1). The Greek mathematician Hipparchus began the development of trigonometry to apply geometry to astronomical studies circa 120 B.C., hence, a branch of geometry (Rouse-Ball, 2010 referred from Walsh, 2015). Trigonometry has a long history, emerging from the practical measuring techniques and surveying of land by ancient Egyptians and Babylonians using triangle trigonometry, which centers on the ratios between the side lengths of a right-angled triangle (Wiest & Lamberg, 2011).

Also, circle trigonometry was used by the Greek Astronomers, focusing on the chords of a circle and their associated arcs to find the longitude and latitude of stars as well as the size and distance of the moon and sun (Wiest & Lamberg, 2011). It is an exploratory part of mathematics with connections to history, culture, music, art, and design, including its interconnections with these important human entities, and provides ways to make trigonometry lessons more interesting. This conforms to what the English-born revolutionary Thomas Paine said in Walsh (2015, p. 29) that "trigonometry is the soul of science. It is an eternal truth. It contains the mathematical demonstration of which man speaks and the extent of its uses is unknown". This quote explains that trigonometry has applications in almost every area of work and life.

In this study, 'trigonometry' refers to trigonometric ratios, rules and identities, compound and multiple angles and formulae, functions, equations, and graphs (CRDD, 2010), as opposed to 'spherical trigonometry' or 'periodicity trigonometry', which not only requires trigonometric knowledge but also knowledge of working with coordinates on a two-dimensional (2-D) set of axes and circles. These added concepts render trigonometry more complex and require a more accomplished manner of thinking.

Importance of Trigonometry in the School Mathematics Curriculum

Since the dawn of time, trigonometry has been a crucial component of the mathematics curriculum in schools. Beginning with the requirement for humans to designate amounts, measure figures, land, and earth, the sun and moon, and construct maps, it has played a very significant role in people's lives. It is a mathematical subject taught in schools, and it is a crucial idea that is used in subjects like geometric, algebraic, and graphical reasoning (Sarac & Tutak, 2017). Literature (Phonapichat, Wongwanich, and Sujiva, 2014) asserts that trigonometry instruction aids in the development of cognitive strategies such as reasoning, proofing, and visualizing skills.

A strong foundation in trigonometric functions also strengthens the learning of various mathematical topics, for instance, the Fourier series, limits, complex numbers, derivatives, integrals, and understanding of calculus (NCTM, 2010). Furthermore, conceptual understanding of trigonometry content at the SHS level offers students an in-depth mathematical knowledge which provides the basis for meaningful learning of mathematics in many colleges and university programs (MOESS, 2010). Hence, making trigonometry so important that it could not have been left out of any school's curriculum throughout the world in general.

Place of Trigonometry in the Ghanaian SHS Mathematics Curricula

The SHS core and elective mathematics curricula are based on the notion that an effective mathematics curriculum is the result of a series of key judgments about three inextricably related elements: content, instruction, and assessment (MOESS, 2010). Consequently, the core mathematics syllabus is designed to place a strong emphasis on the acquisition and application of fundamental mathematical knowledge and abilities. On the other hand, Elective Mathematics builds on the SHS Core Mathematics and is a core requirement for students interested in engineering, scientific research, and a variety of advanced mathematics programs at institutions of higher learning. The major contents covered in the SHS core and elective mathematics classes are:

Table 1: Trigonometry Contents in the Ghanaian SHS Mathematics Curricula

Elective Mathematics Curriculum	Core Mathematics Curriculum
Algebra	Numbers and Numeration.
Coordinate Geometry	Plane Geometry
Vectors and Mechanics	Mensuration
Logic	Algebra
Trigonometry	Statistics and Probability
Calculus	Trigonometry
Matrices and Transformation	Vectors and Transformation in a Plane
Statistics and Probability	

Source: MOESS (2010)

Lastly, problem-solving and its application (mathematical processes). “Problem solving and application has not been made a topic by itself in the syllabus since nearly all topics include solving word problems as an activity. It is hoped that teachers and textbook developers will incorporate appropriate problems that will require mathematical thinking rather than mere recall and use of standard algorithms” (MOESS, 2010, p. iv) and is also the methodological approach to teaching these scopes of content. One out of the 7 and 8 major content areas in the core and elective syllabi respectively is trigonometry, comprising of trigonometric ideas, processes, and their applications to problem-solving (MOESS, 2010). Due to its significance in complex mathematical reasoning, trigonometry is a crucial and integral topic in the SHS mathematics curriculum. Trigonometry themes are taught at two different levels in the Core (Trigonometry I, SHS 2, and Trigonometry II, SHS 3), and (SHS 2) in Elective

Mathematics, according to the CRRD (2010), utilizing the spiral method to teaching and learning. The curriculum specified these trigonometric levels along with the content's unique learning and achievement goals for students, and also included teaching and learning activities that teachers could employ to help students better understand the material.

In the Ghanaian SHS Core Mathematics curriculum, second-year students study Trigonometry I with structured teaching and learning activities which include the teachers' ability to guide students to use appropriate diagrams to define trigonometric ratios, assist students to draw an equilateral triangle of dimensions (e.g. 2-units) and use it to derive the trigonometric ratios for 30^0 and 60^0. Assist students to draw a square of side one unit, draw one of the diagonals, and use the diagonal and two sides to derive the value of the trigonometric ratios of 45^0 and 60^0, and also assist students to use their calculators to find trigonometric ratios for given angles from 0^0 and 360^0. And to guide students to find the inverse of given trigonometric ratios using tables or calculators, finding the inverse of given trigonometric ratios using tables or calculators.

Finally, teachers should utilize illustrations to explain to students what angles of elevation and depression are, as well as hands-on exercises like puzzles that simulate real-world situations and require students to employ trigonometric ratios. Furthermore, in the third-year core mathematics content, students are exposed to trigonometry II, with the objective of being able to draw simple trigonometric function graphs and identify maximum and minimum values and use them to solve simple trigonometric equations. Here too, the teaching and learning activities include the teachers' ability to guide students to prepare tables for given trigonometric functions of the form $f(x) = a\sin x$ and $f(x) = b\cos x$ in a range $0^0 \le x \le 360^0$.

And also assist students with using their tables to draw the graphs of the functions and find the maximum and minimum values. Guide students to draw simple graphs of trigonometric functions of the form: $(fx) = a\sin x + b\cos x$ in the range $0^0 \le x \le 360^0$. The teacher should also guide students to use their graphs to solve equations of the form: $a\sin x + b\cos x = 0$ and $a\sin x + b\cos x = k$, etc. Also, one out of the 8 major content areas in the Elective mathematics syllabus is trigonometry, which is done in the third year. This year, specific objectives include the student's ability to determine basic trigonometric ratios and their reciprocals, convert angles into radians, state and use sine and cosine rules. Students should also be able to state and apply simple trigonometric identities to determine trigonometric ratios for compound angles as well as use simple trigonometric identities to obtain trigonometric ratios for a variety of angles. These skills are related to solving bearing problems using the sine and cosine laws.

In addition, you should be able to solve trigonometric equations, graph trigonometric functions, and determine the maximum and lowest points of a given trigonometric function. The requirements of the curriculum do not exclude teachers. With their role as implementing agents, there are teaching and learning activities designed and dedicated to guiding and advising them on the various activities to be employed for effective teaching of the stated objectives.

Concept of Needs Assessment

According to Ekşi (2010, p. 8), as cited from (Witkin & Altshuld, 1995), "a need is generally considered to be a discrepancy or gap between 'what is', or the present state of affairs concerning the group and situation of interest, and 'what should be', or a desired state of affairs". In the context of this study, a need is defined as a desire or interest felt by a person or a group to fill a void. Altschuld and Kumar (2000) cited in Ekşi (2010, p. 24) emphasized that "two conditions ('what is and what should be') must be assessed and the difference between them would identify the need". In the context of education, needs assessment refers to the method of acquiring and analyzing information resulting from the stated needs of teachers, learners, and perhaps other involved experts in communities (Mohamed, 2013). In all these definitions of needs assessment, Ekşi (2010) emphasized the collection of information within a particular context. In order to design training programmes that are both effective and successful, need assessment is required to assemble data about the settings in which the program participants work.

A need assessment is a crucial stage in the organization of professional development programs since it provides useful information on a variety of themes. A comprehensive needs assessment reveals inconsistencies, depicts the current situation, encourages proper decision-making for change and improvement, prioritizes development needs, and gives teachers a sense of involvement in the program. That is, teachers' present performance is assessed and compared to the capabilities and skills that they will need to acquire to execute their profession. As indicated by Witkin, Altschuld and Witkin (2000), they changed the process model of needs assessment. The revised model includes three phases: pre-assessment, assessment, and post-assessment.

The purpose of the pre-assessment phase is to gather and organize available knowledge on the subject. This first step informs investigators of what kind of data to gather in the second phase, assessment. In the second phase, new data is collected to identify needs and prioritize them. Post-assessment is the final phase, which entails identifying and implementing solutions for high-priority needs. The first two phases aligned themselves with this study. The researcher investigated teachers' understanding and teaching of trigonometry, identified and described their needs for teaching trigonometry, which is expected to inform TPD programs (third phase; post-assessment). A needs assessment in this study is the process of determining areas in which teachers wish to be helped in teaching trigonometry. Thus, gathering information about the present situation of the ISMTs to identify needs, strengths, shortcomings, and opinions to compare to the ideal situation in order to rectify, update, or improve with regards to trigonometric concepts is an important part of the research process. This is an essential step in designing a TPD program.

Theoretical Framework

Theories and constructions are similar to eyeglasses that enable the researcher to view the subject of interest more clearly. The goal of a theory is to give the investigation a focus, to limit information fragmentation by organizing it, to supply tools for data interpretation, to provide theoretical justifications and a deeper knowledge of the issue under study. The Mathematical Knowledge for Teaching (MKT) paradigm and radical constructivism theory serve as the study's guiding principles. The MKT has been adapted from Ball, Thames, and Phelps (2008) to examine Content Knowledge (CK), Awareness of Errors and Misconceptions (AEMs), Self-Efficacy, and finally, identify challenges and TPD needs of ISMTs. The radical constructivism theory, on the other hand, explains the necessity of studying individual ISMTs to identify their strengths, weaknesses, and TPD needs for teaching trigonometry.

Mathematical Knowledge for Teaching

For the past 40 years, teachers' mathematics knowledge for teaching has become an area of study. Shulman (1986) was among the first studies to focus on the field of teaching knowledge separately from a certain subject knowledge. This means that teachers must not only understand the subject they teach, but also other aspects of the subject that other professionals in the same sector are not required to know. Shulman suggested a framework for teacher SMK, consisting of CK, PCK, and Curricular Knowledge, in his presidential presentation to the American Education Research Association in 1986. He well-described CK as "the amount and organization of knowledge per se in the mind of the teacher" while PCK is defined as "knowledge which goes beyond SMK per se to the dimension of SMK for teaching" (p. 9).

PCK, he claims, has become essential for researchers and teacher educationalists. It is "the category most likely to distinguish the understanding of the content specialist from that of the pedagogue" (Shulman, 1987, p. 8). It covers "the most useful forms of representation of those ideas, the most powerful analogies, illustrations, examples, explanations, and demonstrations. In other words, the most useful ways of representing and formulating the subject that make it comprehensible to others. It also includes the understanding of what makes the learning of specific topics easy or difficult: the conceptions and preconceptions students of different ages and backgrounds bring with them to the learning of the most frequently taught topics and lessons" (Shulman, 1987, p. 9).

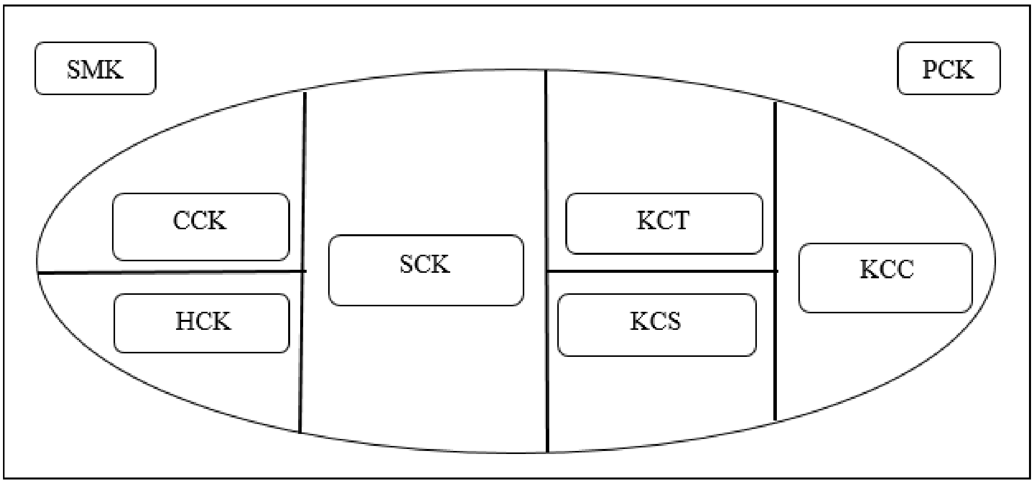
Curricula Knowledge, the third element, represents horizontal and vertical curriculum knowledge for a topic. It deals with both the teacher's understanding of existing content as well as the tasks meant to assist students in learning the content. Shulman's idea of the CK and PCK had a significant impact on educational research. He labeled the lack of emphasis on CK and PCK as the *missing paradigm.* According to Shulman, PCK is not just lacking in schools but also in educational research. Shulman's views on teaching decisions were largely concerned with generic pedagogy and practice, rather than with the types of teacher activities that are specialized for a particular discipline (Ball et. al., 2008). Thus, teachers require knowledge tailored toward the instruction of a specific subject and topic at a particular grade level (Shulman, 1986).

Therefore, Ball and her team built a theoretical framework based on Shulman's work by looking at how Shulman's concepts can be operationalized in mathematics education settings (González, 2014). This theoretical framework is termed Mathematical Knowledge for Teaching (MKT). They broke up MKT teaching into two parts, SMK and PCK, which is different from the PCK discussed by Shulman. Here, it is vital to comprehend what these two main MKT domains imply. SMK deals with the mathematics knowledge of teachers. It is concerned not just with procedural knowledge but also with the underlying mathematical principles which govern that behavior. It also looked at how different mathematics topics are interconnected. PCK might be evident when a teacher uses his or her SMK in teaching. When a teacher anticipates students' reactions to activities, examines what tasks would be good to introduce a new topic, and considers what materials are available to assist students with their mathematics studies, he or she is using PCK.

The MKT model was adopted in the current study due to its emphasis on what teachers need to know about trigonometry and how such content knowledge can be transformed into learners' understanding. Furthermore, in discussing the impact of the MKT model, Koponen (2017, p. v) concludes, “MKT and its measurements have been successfully applied as one of the most promising frameworks to describe the knowledge needed for teaching mathematics in several countries”, for instance, Ghana (Cole, 2012); South Korea (Kwon, Thames & Pang, 2012); Norway (Fauskanger, Jakobsen, Mosvold & Bjuland, 2012); Malawi (Kazima, Jacobsen, & Kasoka, 2016). The model has been used in numerous studies (Sunzuma & Maharaj, 2019; Amiruzzaman, 2016; Malambo, 2015; Sibuyi, 2012) in the analysis of teachers' reasoning processes on mathematics tasks like central tendencies, quadratics, trigonometry, and geometry. Hence it provides a vital framework of reference for the current study. The model is useful in the sense that it helped in determining how much or little teachers understand and can confidently teach trigonometry, and informed us about the challenges faced and the TPD needs required for teaching effectively.

Components of the MKT Model

Referring to Figure 1, SMK is categorized into three: Common Content Knowledge (CCK), Horizon Content Knowledge (HCK), and Specialized Content Knowledge (SCK). Besides, PCK also consists of three parts: Knowledge of Contents and Students (KCS), Knowledge of Contents and Teaching (KCT), and Knowledge of Contents and Curriculum (KCC). Ball et al. (2008) categorized and defined the components of these two fields of knowledge.



Source: Ball et al. (2008)

Figure 1: Framework for Mathematics Knowledge for Teaching

Teacher Content Knowledge

SMK deals with teachers' mathematics knowledge. SMK is concerned not just with procedural knowledge but also with the underlying mathematical concepts which govern that behavior. It also looked at how mathematics topics are interconnected. Teachers, according to Shulman, are expected to gain competency in the subject-matter in addition to standard knowledge of teaching (classroom organization methods, incentive policies, and awareness of students' characteristics). Ball et al. (2008) depicted Shulman's (1987) category of teacher content knowledge as an overarching SMK category. SMK has been clearly partitioned into CCK, SCK, and HCK within the MKT framework, as described in the proceeding sessions.

Common Content Knowledge (CCK)

Teachers need to have a thorough grasp of "pure mathematics," or the mathematical concepts, theories, definitions, conclusions, procedures, rules, justifications, and symbols used in the many branches of mathematics. This sort of information may be applied in any environment outside of the classroom, including computing, problem-solving, and other general mathematical knowledge that is not only for teachers (Ball et al., 2008). CCK includes abilities like understanding how to follow a method and defining concepts. These characteristics are crucial in other fields, like engineering and mathematics, and hence, they have been denoted as CCK (Ball et al., 2008). Despite the fact that CCK is an important part of teachers' knowledge, several researchers believe it is insufficient for teaching (Kaiser & Blömeke, 2013; Ball et al., 2008).

Specialized Content Knowledge (SCK)

This is a form of mathematical knowledge that is peculiar to the teaching profession. SCK is defined by Hill, Ball, and Schilling (2008), as mentioned in Koponen (2017, p. 36), as a competency that "allows teachers to engage in particular teaching tasks, including how to accurately represent mathematical ideas, provide mathematical explanations for common rules and procedures, and examine and understand unusual solution methods to problems." The SCK assesses teachers' ability to explain and represent mathematical concepts, as well as analyse and understand non-routine solutions. Specifically, Etkina (2010, p. 1) emphasized that "teachers of a specific subject should possess special understanding and abilities that integrate their knowledge of the content of the subject that they are teaching".Teaching is a multifaceted activity that necessitates the integration of a wide range of specialized knowledge (Mishra & Koehler, 2006). Therefore, teachers must acquire this knowledge in order to assess learners' understanding of mathematics. This knowledge is distinct from that of students or pedagogy (Hill et al., 2005). A teacher with SCK has the ability to convey mathematical information to pupils in a way that makes sense, as opposed to only teaching mathematical processes and techniques. Contrary to conceptual knowledge, which is rich in relationships and has to be taught in a meaningful way, procedural knowledge is mostly composed of rules and algorithms (Herber, 2013). According to Bair and Rich (2011), "it is the unique mathematical knowledge for teaching mathematics with comprehension," which is in line with this point of view on SCK (p. 295). According to these writers, mathematics teachers have unique needs when it comes to some mathematical criteria, such as decompressing the subject matter (Ball et al., 2008).

Horizon Content Knowledge (HCK)

Understanding the connections between current student experiences, subject matter, noteworthy mathematical activities, and key disciplinary notions and structures that are on the horizon for mathematics constitutes this component. It addresses those aspects of mathematics that are not part of the curriculum but are nevertheless important to the learning of current students, shedding light on and providing an understandable idea of the larger importance of what may only be partially visible in contemporary mathematics (Ball & Bass, 2009). In addition, it is important to keep in mind that, unlike CCK and SCK, HCK does not enforce a dominating behavior in a particular mathematical way. Because the MKT framework refers to what teachers ought to comprehend as effective, Ball et al. (2008)'s Subject Matter Content Knowledge is employed in this study.

In summary, CCK, HCK, and SCK are SMK domains that do not require any prior knowledge of students or pedagogy. Only CCK and SCK have been included in this study. HCK will not be explored because the objective of this study is not to examine the connections between mathematical topics in the curriculum (Bair & Rich, 2011).

Pedagogical Content Knowledge (PCK)

PCK is evident whenever a teacher employs his or her SMK in teaching. This is because PCK is an amalgamation of a teacher's whole knowledge base that is individually built by him or her. When a teacher predicts students' reactions to activities, examines what tasks would be good to introduce a new topic, and evaluates what materials are available to assist students in their mathematics studies, he or she is using PCK. The teacher must have an idea of potential students' conceptions about the topic to develop clarifications that will help to eradicate or strengthen those concepts as needed. Additionally, as PCK is an amalgamation of a teacher's whole knowledge base that was personally constructed by him or her, it is impossible to examine the parts of a teacher's PCK in isolation from any of its other components. It is thus individualistic since it is the confluence of pedagogy, students’ conceptions, and SMK (Mishra & Koehler, 2006). Ball et al. (2008) present a refined division of PCK, comprising of KCS, KCT, and KCC, as described below.

Knowledge of Content and Students (KCS)

KCS is a representation of a teacher combined knowledge of students and mathematics. This is knowledge about how students learn and grasp mathematics. It requires the teacher’s familiarity with and anticipation of feasible student thinking trajectory, envisaging students’ challenges while dealing with certain mathematical concepts or procedures, as well as hearing and construing students' thinking for a given content (Ball et al., 2008).

Furthermore, having this knowledge allows a teacher to anticipate what will be difficult or simple for students and also be able to analyse patterns in students' errors as they learn a specific content area. Ball et al. (2008) specify that at the core of KCS is the teacher's awareness of common conceptions, misconceptions, and errors about a particular mathematics content. To put it another way, a key component of KCS is being aware of the many ideas, mistakes, and misconceptions that students are likely to have about a subject as well as the challenges that they could run into when studying a certain piece of information.

Teachers must not only understand the material but also be familiar with students' mathematical reasoning and frequent student errors in order to engage in these activities successfully. Hill, Ball, and Schilling (2008) conclude that KCS is a major component of Shulman's PCK. For the reason that one portion of Shulman's PCK is "an understanding of what makes the learning of specific topics easy or difficult: the conceptions and preconceptions that students of different ages and backgrounds bring with them to the learning of those most frequently taught topics and lessons" (Hill, Ball &Schilling, 2008, p. 375). In the current study, teachers are expected to have the ability to identify and diagnose students' errors and misconceptions about how solutions to questions are presented.

Knowledge of Content and Curriculum (KCC)

KCC means having knowledge of "the full range of programs designed for the teaching of particular subjects and topics at a given level" (Shulman, 1986, p. 10). Thus, teachers' awareness of how certain topics and concepts are instructed in the curriculum at specific grade levels, as well as the grade-wise links between them. The KCC domain clarifies instructors' knowledge of mathematics content with respect to teaching resources and programs (Bair & Rich, 2011). These comprise both learning aims and objectives, along with instructional resources to help students achieve these targets and objectives. Meaning, teachers should know the types of teaching resources available and how they might be used (for example, textbooks, boards), and technological tools (computers, software, smartboards, calculators, etc.). Employing these resources and technological tools in instruction requires integrated SMK, PCK, and knowledge of equipment. All of these features of knowledge can be summarized in terms of content and curriculum knowledge (Jankvist, Mosvold, Fauskanger & Jakobsen, 2015).

Knowledge of Content and Teaching (KCT)

KCT discusses the knowledge of how to create instructive lessons that foster students' use of quantitative reasoning. The ability to choose which examples to use as a starting point and which examples to use to guide students deeper into the material is required. Teachers evaluate the advantages and disadvantages of different representations used to teach a certain subject, as well as the advantages and disadvantages of other techniques and processes (Ball et al., 2008). For each setting and topic, teachers must select appropriate strategies. Pedagogical thinking is required for all aspects of teaching, including planning lessons, selecting effective techniques, structuring classrooms, stimulating engagement, and communicating with students. KCT is made up of mathematical and instructional knowledge (Ball et al., 2008). Sleep (2009) identifies the categories of KCS, KCT, and KCC as the types of knowledge that necessitate a unified set of SMK and PCK. In this study, only two of the components, KCS and KCC were used. As reported by Ball et al., Table 2 summarizes the six components of MKT.

Table 2: The Six Components of Ball Framework

Components	Definition
Common Content Knowledge	Mathematical knowledge and skills used in settings other than teaching
Specialised Content Knowledge	Mathematical knowledge unique to teaching (e.g. looking for a pattern in students' errors, examining a nonstandard approach).
Horizon Content Knowledge	Knowledge of how mathematical topics are related throughout mathematics and trigonometry
Knowledge of Content and Students	Knowledge to anticipate students thinking and misconceptions in mathematics
Knowledge of Content and Teaching	Knowledge needed to sequence instruction and appropriate teaching practices for mathematics
Knowledge of Content and Curriculum	Vertical and horizontal knowledge of mathematics content and curriculum

Source: Ball et al. (2008)

Application of the MKT Model in the Current Study

In this research, two of the SMK components (CCK and SCK) in the MKT model are employed. In this study, a combination of CCK and SCK is termed as *personal understanding* and operationalized as Trigonometrical Content Knowledge (TCK). TCK refers to the knowledge that teachers must demonstrate when teaching and interacting with students. Thus, the CCK reflects the mathematical knowledge and abilities that teachers must have to correctly solve trigonometric content test items, know how to carry out a technique, and understand the definition of a concept. Alluding to Hill et al. (2008)'s descriptions of the MKT domains, SCK, on the other hand, demonstrates their capacity to accurately express mathematical ideas, provide explanations for conventional rules and processes, as well as investigate and comprehend novel problem-solving approaches. Specifically, it describes a teacher's ability to explain the underlying algorithm and make representations of each trigonometry test item. Both CCK and SCK answer the question: Is the teacher capable of providing an acceptable, correct response and explaining the trigonometric tasks that have been assigned? CCK and SCK cover the subjective understanding of trigonometry concepts and ideas. Therefore, a form of trigonometrical knowledge that ISMTs must have to help learners understand the topic effectively. As said earlier, only descriptors of CCK and SCK are integrated into this study as these descriptors represent the personal understanding which forms the TCK. Despite the fact that HCK (Figure 1) is a component of content knowledge, it is not included in the current study because the goal is not to examine how mathematics themes/topics are linked in a curriculum.

PCK as an MKT knowledge base is a composition of three others: KCS, KCT, and KCC (Ball, et al., 2008). The study aligns itself with only the KCS and KCC aspects of the PCK, herein referred to as pedagogical understanding in the conceptual framework (Figure 1). Appraisal of teachers' acquaintance with and anticipating of students' mathematical reasoning and understanding of a specific content is required to assess ISMTs' awareness of trigonometrical errors and misconceptions. KCS demonstrates how teachers may track their students' thinking, explain the underlying method/procedure, and assess the correctness of their solutions (Dubinsky & Wilson, 2013).

In a specific task in this study, a teacher should be able to demonstrate evidence of recognizing the most possible reasons for students' common solutions and difficulty with trigonometrical concepts. So, if teachers can identify more often than not why students are having trouble understanding trigonometric concepts, or the cause of errors and misconceptions they will be capable of preventing such problems and assisting learners in overcoming such hurdles. In assessing teachers' knowledge of errors, teachers are required to give reasons as to why a learner solved a question in a particular manner. Teachers have to analyze learners' thinking and reason about the prior knowledge and backgrounds they bring with them, resulting in such a solution. ISMTs must have this type of trigonometrical knowledge in order to help students understand the topic more effectively. Generally, evaluating all these leads to measuring ISMTs' knowledge of content and students (Shulman, 1986), as cited in Koponen (2017).

The second component of the PCK, KCC, parallels with the study. The national curriculum usually lays out the guidelines and objectives for teachers, such as the kinds of mathematics topics that should be covered and the set of objectives for teaching at different levels. Teachers must be cognizant of the various standards, aims, and objectives that come with teaching mathematics topics. Such proof of teachers' comprehension of and adherence to the educational objectives and intentions of the official curriculum documents regarding the teaching of the trigonometrical contents present in the given task, as well as trigonometry in general, becomes entangled with their knowledge and influences their feelings of readiness to teach (Thompson, 1992, as seen in Lovett, 2016).

Asserting this, Grossman, Wilson, and Shulman (1989), as shown in Gencturk, 2012, p. 8, stated that no examination of teacher knowledge would be complete without a discussion of teacher belief, since the two can be difficult to distinguish at times. Similarly, in synthesizing various forms of research (Sarac & Aslan-Tutak, 2017), the subject of 'teacher belief' influences the other components of PCK, and it is stressed to have an inseparable link with teachers' knowledge. Teachers' self-perceptions reflect self-efficacy ideas in teaching a given subject. Teachers' attitudes about the subject matter, teaching, and learning appear to impact what and how they teach. Therefore, in this study, self-efficacy is also connected to KCC because when there is harmony in one's curriculum knowledge, he or she appreciates the adequacy of teaching, which results in an increased confidence level. As a result, such teachers' self-efficacy will be measured not just by their ability to teach the procedures they want their students to learn, but also by the ideas they want them to have (Thompson, Carlson, & Silverman, 2006).

Teachers' beliefs, specifically self-efficacy, refer to their in-depth understanding of the objectives and ideas of topics. Trigonometry self-efficacy represents a teacher’s ability to confidently achieve the stated objectives and ideas for teaching the SHS trigonometry content outlined in the curriculum (CRDD, 2010). All of these aspects of KCS and KCC conceptualize pedagogical understanding of ISMTs in teaching (Jankvist et al., 2015). Essentially, *pedagogical understanding* consists of ISMTs’ awareness of errors and misconceptions, and self-efficacy.

In summary, it is expected that ISMTs will have the ability to accurately describe mathematical ideas, explain basic principles and procedures, and investigate as well as comprehend unusual problem-solving strategies. Specifically, it is anticipated that ISMTs will be able to explain the underlying algorithm and make representations of each trigonometry test item. Also, ISMTs are expected to be aware of trigonometrical errors and misconceptions and be self-efficacious. Therefore, obtaining an insight into ISMTs' understanding and teaching of trigonometry is deemed vital (Figure 2).

The Radical Constructivism Theory (RCT)

The final theoretical foundation for this research is radical constructivism's core stance that each person's "knowledge is self-constructed and considered fundamentally unknowable to any other individual" (Moore, 2009, p. 2). This theory by von Glasersfeld (1974) defines a specific way of viewing knowledge and knowing as subjectively resident in an individual (Steffe & Thompson, 2000). The central assumption is that individual cognizing agents construct their knowledge based on their experiences (von Glasersfeld, 1995). Moore (2009) elaborated on the central premise of this theory, stating that a researcher must take into account each individual's knowledge of themselves. As a result, each teacher's knowledge is essentially self-constructed and unknowable to others.

Its two key principles are: "knowledge is not passively received either through the senses or by way of communication, but it is actively built up by the cognizing subject; and the function of cognition is adaptive and serves the subject's organization of the experiential world, not the discovery of an objective ontological reality" (von Glasersfeld, 1995, cited in Liu, 2005, p.56). This is why von Glasersfeld (1995) said, radical constructivism starts from the assumption that knowledge, no matter how it is to be defined, is in the heads of persons. So, from the radical constructivist perspective, each individual constructs his or her own conceptual schema as a result of his or her cognition. In this situation, the radical constructivist believes that knowledge resides in individuals and that conceptual understanding and the specific needs of every teacher to instruct more effectively are resident in the individual teacher.

Correspondingly, Fi (2003) asserts that knowledge is unique to each individual. He explained the personality nature of knowledge by considering if a:

> "Cognizing individual should engage in inter-agent social interaction mediated through a common shared language. At the end of the interaction, at the point where the agents dissociate into intra-agent reasoning, at the point when the social interaction is no more; if knowledge was constructed, then it must reside in the individual cognizing agents, albeit, each individual carries with him or her idiosyncratic versions of the knowledge … the locus of knowledge is in individuals and not in social interaction" (p. 32).

Thus, investigating teachers' understanding, which includes pedagogy, learner conceptions, and SMK, among other things, is individualistic. Each ISMT's knowledge and understanding of trigonometry is unique to them.

Agreeing with the subjective views of the radical constructivist, constructs like teachers' understanding, self-efficacy, knowledge of errors, and misconceptions are subjective. If this is the case, how should I investigate both teachers' conceptual and pedagogical knowledge of trigonometry? Or, what is the justification for any study aimed at understanding a teacher's trigonometry knowledge? To answer these questions, the researcher must step into the larger picture and investigate these constructs from the perspective of a teacher, because it is widely accepted in the literature that what teachers know influences what and how they teach, which in turn influences what and how students learn (Beausaert, Segers & Wiltink, 2013; Sadler et al., 2013). As a result, there is a need to investigate the concept's comprehension since trigonometric knowledge is unique and particular.

In summary, the theory supports the study because it explains that there is no need to propose a system that approaches teachers' conceptual constructs or builds feasible models of teachers' understanding of a specific mathematical concept, which will allow us to make judgments about teachers' understanding of the idea (Liu, 2005). Since, the radical constructivism theory asserts that knowledge is built based on individual experiences and that the mind has certain powers that facilitate the construction of concepts and their relationships. Close interaction and interrogation of teachers will reveal their knowledge base, errors, and misconceptions, self-efficacy, and challenges they encounter, as well as TPD needs for teaching trigonometry. The next section discusses the empirical basis underpinning the study.

Empirical Reviews

The empirical reviews of the study are based on the following themes; Trigonometrical Content Knowledge (TCK) of Teachers; Awareness of Errors and Misconceptions (AEMs); Teacher Self-Efficacy (TS-E); Challenges in Teaching Mathematics (Trigonometry); and Teacher Professional Development (TPD) Needs.

Trigonometrical Content Knowledge (TCK) of Teachers

Nabie et al. (2018) explored teachers' perceptions and knowledge of trigonometric concepts. The Trigonometry Perception Questionnaire (TPQ) and Trigonometry Assessment Test (TAT) were used for data collection. The data was analyzed using descriptive statistics, and the findings revealed that teachers thought of trigonometry as esoteric, challenging, and monotonous to learn, and they had only a rudimentary understanding of basic trigonometric ideas and concepts. As a result, Nabie et al. (2018) found that more than half of the teachers were not able to form and reconstruct the schemas necessary for meaningful knowledge to answer simple trigonometry problems. Hence, these difficulties reveal a knowledge gap and inadequate comprehension of trigonometric conceptual relations on the part of teachers, despite their high regard for trigonometric concepts. They concluded that teachers lacked the necessary SMK and PCK in trigonometry, and this is a problem that should be addressed in teacher education programs.

Koyunkaya (2016) studied the comprehension of trigonometric ratios among nine (seven females, two men) graduate students in mathematics education at a famous public university in the Midwest. Five of the master's degree participants were also high school mathematics teachers, while the other four intended to pursue a doctorate in mathematics education to work as researchers in this subject. These adult learners solved and finished three exercises using right triangle trigonometry and trigonometric ratios utilizing sine and cosine. Participants were expected to create diagrams, generalize ideas, and defend their thinking while completing the assignments. Koyunkaya (2016) described the participants' trigonometric ratio knowledge via the lenses of

Skemp's (2006) instrumental and relational understanding. Tall and Vinner's (1981) concept definition illustrations.

The findings of Koyunkaya (2016) highlighted the difficulty that adult learners experienced in responding to trigonometry-related tasks. Most participants, in particular, struggled to reason about trigonometric ratio problems, which need a more flexible understanding than ones that just require memorization of rules. They struggled to understand trigonometric ratios in a relational sense due to a lack of underlying knowledge of the idea of angles. Despite having studied trigonometry and related subjects and having taught in high schools, all of these adult learners struggled with trigonometric ratio exercises. So, according to Koyunkaya (2016), adult learners will continue to struggle with trigonometry and trigonometry-related ideas until they obtain fundamental understanding of angles and angles measurement. On the same note, Jadama (2014) adds that teachers need to have a thorough understanding of the subject matter they teach learners in order to select the optimal pedagogy to help learners understand the subject matter.

Thompson et al. (2007) discovered that many teachers have a difference of opinion that is anchored in their commitment to trigonometry curricula knowledge, and that this knowledge is focused on memorizing procedures rather than teaching concepts to students. Most teachers' challenges are due to a lack of basic trigonometry knowledge, according to the conclusions of the study. They appeared to have a rudimentary understanding of the concept of a unit circle, and their understanding of angle measures was primarily dependent on degrees. According to Thompson et al. (2007), if teachers do not have sufficient understanding of the topic they are instructing, they will be unable to recognize the students' deficiency or comprehension of these concepts. Not only Thompson et al., but also Fi (2006) on teachers' trigonometry knowledge report that teachers lack the content knowledge required to facilitate their students' trigonometric learning. Teachers are frequently confined to explaining trigonometric functions in the context of a right triangle while establishing only superficial links to circle contexts. Besides, the results from these studies revealed the narrow, limited, and entrenched understanding of trigonometry that teachers' had when teaching the topic.

Awareness of Errors and Misconceptions (AEMs)

Chigonga (2016) explored the types of errors students make while solving trigonometric equations, the likely reasons, and how this knowledge may be used to plan instructional interventions from the perspective of instructors. According to the instructors' results, students misunderstood sine, cosine, and tangent of an angle when their values were negative, failed to identify crucial quadrants, and formed inaccurate assumptions. In summation, Chigonga (2016) claims that students make mistakes when simplifying trigonometric equations, that teachers have difficulty teaching the same concept, and that some errors are gleaned from teachers' responses. Many of the errors identified in teachers' responses stem from inadequate understandings of the fundamentals and underlying competence taught in their early levels of education.

Furthermore, Zuya (2014) evaluated mathematics teachers' abilities to assess students' cognitive processes about specific algebraic topics. The purpose of the study was to evaluate math teachers' skills to notice, discuss, and forecast students' mistakes and misunderstandings, as well as their thought processes regarding algebraic topics. Math instructors (156) from public secondary schools in Bauchi State, Nigeria, were chosen at random. Data was collected via an open-ended questionnaire, which was subsequently analyzed using qualitative and expository approaches. Teachers were given hypothetical student responses to the variable concept and instructed to offer questions that would assist the students uncover their inaccuracies in one phase of the questionnaire. Most teachers, according to the data, are unable to ask competent questions that can help in measuring students' cognitive processes. Another interesting discovery was that the professors themselves failed to understand the material, which led to them asking unrelated questions. Teachers were unable to notice the learners' flaws or inaccuracies in the hypothetical answers presented in the questionnaires because they failed to comprehend the difficulties, despite the fact that the questions were simple.

Kilic (2011) discovered that having a strong SMK is necessary for being a competent teacher, but it is not enough for teaching effectively in his study on teachers' understanding of their students. He collected data through classroom observations, organized interviews, questionnaires, and journals. The findings revealed that teachers lack sufficient awareness of students' conceptions and "when the teachers were given examples of learners' errors and asked how to address them, the teachers tended to repeat how to carry out the procedures or explain how to apply a rule or mathematical fact to solve the problem instead of explaining the correct concepts that would help eliminate the learners' errors" (p. 23). Similarly, Sibuyi (2012) used lesson plan analysis, observations, and interviews as data collection techniques to investigate instructors' awareness of learners' concepts, SMK, and knowledge of teaching strategies. The study also showed that teachers have limited and weak knowledge. This is due to their inability to identify learners' misconceptions by analyzing their (learners') solutions to questions.

Moreover, Prediger (2010) discovered that teachers have difficulty accurately assessing their students' responses to diagnosing misconceptions in a study on the diagnostic competence of teachers on learners' misconceptions and difficulties. Klein (2015) also examined students' trigonometric skills, which helped them grasp connected topics. The findings indicate that uncovering students' prior knowledge and making explicit knowledge-in-action leads to a transformation in attitude. Bukova-Güzel (2010) complemented this by analyzing mathematics instructors' pedagogical topic knowledge using tangible objects and discovered that the teachers did not recognize probable student errors. Semi-structured interviews, assessments of lesson plans generated by the students' teachers, and video recordings of instructional applications were used to collect data for the study. Turnuklu and Yesildere (2007) discovered in another study that teachers' mathematics understanding was subpar, and so they were unable to assist their students with the misconceptions they demonstrated. Likewise, Mwadzaangat (2017) found out that most teachers used a procedure-based approach instead of a conceptual-based approach in analyzing students' errors. This was noted in the findings when the teachers only pointed out one mistake, even in students' hypothesized solutions, which contained several mistakes.

Furthermore, the theme of Chick, Pham, and Baker's (2006) research focused on teachers' PCK when teaching the subtraction algorithm. The findings clearly demonstrated that while the teachers delivered an effective lesson, they lacked knowledge in how to recognize and correct students' errors.

Nevertheless, Holmes, Miedema, Nieuwkoop, and Haugen (2013) assert, there is a lot of potential for boosting students' conceptual knowledge when teachers are skilled at determining and remedying misconceptions. This suggests that the interpretive position of teachers is critical in the process of correcting discovered errors and misconceptions (Peng, 2010). Yet, according to Chigonga (2016), there is a paucity of literature describing teachers' interpretative viewpoints on students' trigonometry errors.

So, if the ability of teachers to recognize, discuss, and forecast students' errors, misconceptions, and thinking processes is critical and cannot be underestimated as investigated in many mathematics topics and other fields (Zuya, 2014), then equally, the significance of teachers' knowledge of errors and misconceptions in identifying and discussing trigonometric errors needs to be investigated to have effective and meaningful trigonometry teaching. Hence, the reason for this study. That is why ISMTs' knowledge of trigonometry errors is being investigated in this study.

Trigonometry Self-Efficacy (TS-E)

Sarac and Aslan-Tutak (2017) evaluated sixteen (16) teachers' trigonometry teaching efficacy in South Africa using the Teacher Trigonometry Teaching Efficacy Scale (TTTES), which queried instructors about their confidence level in teaching trigonometry. Rather than finding the answers to the problems on the instrument, teachers were asked to choose a number that best described their degree of confidence level in solving the given trigonometry exercises. The findings demonstrate that the efficacy of each of the 16 participants was high. For virtually all tasks, teachers obtained a maximum grade for their efficacy in teaching trigonometry. Therefore the researchers were unable to distinguish between them. Sarac and Aslan-Tutak (2017) improved the method of analyzing teacher-efficacy levels with individual interviews for more information on these teachers. Teachers were invited to describe their teaching approaches and processes throughout the interviews, with the assumption that teaching experience was the most important marker of one's teaching efficacy. Sarac and Aslan-Tutak (2017) deduced the participants' enthusiasm for teaching trigonometry as well as their feelings about the teaching process from the participants' expressions.

In his study on pedagogical factors affecting the learning of trigonometry, Kagenyi (2016) writes to say, “if the teachers’ attitudes are negative towards trigonometry, this, in turn, will affect their teaching of the topic and is reflected in pupils’ performance” (p. 40). Besides, Nadelson et al. (2012) also discovered that once teachers feel uneasy with the content they are teaching, they prefer to avoid going beyond the superficial layer or even skip teaching it entirely. Therefore, when teachers' trigonometry self-efficacy improves, they will be more willing to study and implement more innovative teaching techniques to efficiently impart the content.

Haynes and Stripling (2014) investigated Wyoming agriculture education teachers' mathematics efficacy and professional development requirements in relation to instructing contextually relevant mathematics. In terms of personal mathematics teaching efficacy as well as mathematics teaching results, Wyoming agriculture education instructors were fairly effective. Following Briley's (2012) findings that mathematical self-efficacy is a statistically significant predictor of mathematical teaching efficacy, trigonometry self-efficacy can be a statistically significant predictor of assessing teachers' trigonometrical teaching efficacy in SHS.

Challenges in Teaching Mathematics (Trigonometry)

A research done in Ghana's second cycle of education (Appiahene, Opoku, Akweittey, Adoba, and Kwarteng, 2014) found a negative attitude toward mathematics education and its abstract character as problems in teaching and studying the subject. Furthermore, Gafoor and Kurukkan (2015) discovered that a lack of prior knowledge is the primary cause of mathematics being difficult to teach and learn, and this item is mentioned by both students and instructors. Klein (2015) used the significant learning theory and the conceptual field theory to assess students' understanding of trigonometry and how effectively it helped them grasp related ideas and concepts. The findings show that identifying students' prior knowledge and making explicit knowledge-in-action leads to a transformation in their attitudes. As a result, a teacher with poor pre-requisite integrity will be unable to predict cognitive issues that students may face. There's also the possibility that high school instructors with low pre-requisite integrity may have a poor understanding of the structural links between trigonometric concepts. Thus, teachers' ability to deliver trigonometry content will be hindered, and students will not be subjected to a comprehensive and meaningful body of trigonometry content. Also, from Handelsman et al. (2004), some teachers feel scared of the prospect of learning novel teaching practices and hence oppose making any changes in their classrooms.

Mutodi and Ngirande (2014) explored the influence of students' attitudes on mathematics achievement in a few South African secondary schools. As constructs of perceptions that affect students' performance, mathematical strengths and limitations, teacher support/learning aids, family background and support, interest in mathematics, difficulties or challenges in studying mathematics, self-confidence, and myths and beliefs about mathematics are identified. Poudel (2015) performed research on the challenges that secondary mathematics teachers confront. He came to the conclusion that the majority of the issues stem from a lack of student cooperation in mathematics classes, insufficient training, a lack of opportunities to attend mathematical workshops, symposiums, and other initiatives, a lack of effective instructional practices, a lack of administration support for mathematics subjects, and also a teacher's lack of self-esteem and preparedness. On a more specific note, in assessing students' views on the most challenging mathematical concepts in SHS, trigonometry was also determined to be one of the problematic areas for students (Bosson-Amedenu 2017; Mensah, 2017; Kagenyi, 2016). The findings of Bosson-Amedenu (2017) indicated that students' difficulties with this and other topics are attributable to a lack of interest on their behalf and teachers' intentional omitting of some mathematics topics. Kagenyi (2016) conducted a study on pedagogical factors affecting the teaching and learning of trigonometry. Learners' difficulties in solving trigonometry problems are identified as a result of teachers' reactions to challenges they have in teaching and learning trigonometry content. During the observation segment, Kagenyi recognized some of the obstacles influencing the teaching and learning of the topic, such as teachers' teaching approaches not encouraging active engagement of learners and ill-developed mathematical knowledge for teaching.

Many of the errors gleaned from teachers' responses to trigonometry questions could be the result of a poor understanding of the fundamentals and underlying knowledge taught in their prior grades, according to Chigonga (2016)'s findings from a trigonometry study in South Africa. As a result, they have difficulty teaching the same content. Participants in the study emphasized, “the reason for this could be that many teachers are not confident about some contents in the National Senior Certificate (NSC)..., and also, it appears that many teachers are struggling to teach learners how to solve trigonometric equations, especially finding solutions within a given interval” (p. 169).

Luneta and Makonye (2010) investigated student errors in elementary analysis in a South African grade 12 class. The study's goal was to examine students' differential calculus errors, identify errors produced in response to calculus problems, and demonstrate how students' errors in calculus are connected to their misconceptions. They argue that poor understanding of a mathematics topic can be owed to language and terminological problems associated with the topic. This is because their case study research on a grade 12 class on student errors in elementary analysis found that some pupils demonstrated an insufficient grasp of calculus terms, such as mixing turning points with axial intercepts. This report implies that the kind of language and terminologies of a topic can be a challenge for teachers.

Similarly, according to Brodie and Berger (2010), students' errors demonstrate that they are thinking and applying prior information to new settings. This means that teachers need to be cognizant of their students' relevant prior knowledge to construct learning strategies that will help them bridge any gaps in their understanding of new concepts and the already assimilated. Such incompetence of teachers to predict and link students' backgrounds may be a challenge to teachers. Furthermore, according to Aguele and Usman (2007), mathematics education in the twenty-first century faces a multitude of challenges. Some of these issues are: integrating new scientific and technological innovations into mathematics, accelerating programs for teacher professional development, and the necessity for math teachers to develop new assessment tools that match the changing demands. The research of Appiahene et al. (2014) set out to explore the issues of mathematics teaching and learning in various second-cycle schools.

Despite the fact that 400 questionnaires were administered, only 360 people answered, with 100 teachers and 260 students making up the sample. It was discovered that some of the issues include: fear of the subject (16.67%), bad teaching methods used by teachers (15.28%), use of abstract concepts in teaching (12.5%), lack of good learning materials (11.11%), and laziness on the part of teachers (8.33%), incompetent teachers (6.5%), and others (29.61%). Etsey (2005) identified inadequacy of teaching and learning materials as one of the main causal factors of low academic performance among students in some Ghanaian schools.

Teacher Professional Development (TPD) Needs

According to Rakumako and Laugksch's (2010) study on demographic characteristics and perceived INSET needs of secondary Mathematics teachers, when participants were asked to specify their utmost professional need from the Science Teachers Inventory Needs-Limpopo Province [STIN-LP] survey instrument, 40% outlined teaching skills, followed by content knowledge (24%) and class discipline (21%), with assessing learners being rated as the least (12%) need. They looked at the unique INSET criteria for Math teachers (in Section B of STIN-LP). All needed items were categorized into dualistic sets of those regarded as needed, and those not regarded. To facilitate this categorization, “not familiar” was joined to the response “great need”. The categories "no need" and "small need" were combined into a single category "no need," and the categories "moderate need" and "great need" were combined into a single category "need." Using these new categories and assuming a 50% anticipated frequency for each answer category, goodness-of-fit chi-square tests were performed to establish the significance of the degree of need for each item. Each requirement was regarded crucial. Mathematics professors desired all of the items. The item "Use a computer to aid organize instruction" was seen as a need by 89% of the instructors (the highest proportion), while 64% identified the need to "Update your understanding of the history of mathematics" (least percentage).

To summarize, statistics from Rakumako and Laugksch (2010) show that teachers who answered the survey believe they need assistance with all the INSET items mentioned in STIN-LP. Motivating students, on the other hand, appeared to be their most pressing need. They contended that until teachers' most pressing needs are met first, even the best-designed and high-quality INSET programs might not be able to attract them.

Mohamed (2013) investigates the professional development needs of secondary school mathematics teachers in Zanzibar, Tanzania. Data on seven professional development sub-scales was collected using the Needs Analysis questionnaire. These sub-scales are: Planning Mathematics Instruction, Delivering Mathematics Instruction, Managing Mathematics Instruction, self-improvement in content knowledge, self-improvement in pedagogical knowledge, communication skills in the English Language, and the use of ICT in mathematics teaching. The needs analysis instrument used in the Mohamed study was a modified and adapted STIN by Abu Bakar (1984). He gathered information by administering a questionnaire to a total of 191 mathematics teachers in Zanzibar, accounting for 54.14% of the population. Interviews were also conducted. The research found that all constructs are within fair need, with an average value ranging from 3.16 to 3.49, with the exception of the construct for teacher self-improvement in content knowledge, which indicated a great need. This was followed by the use of ICT in mathematics teaching, and the delivery of mathematics instruction was one of the least needed subscales.

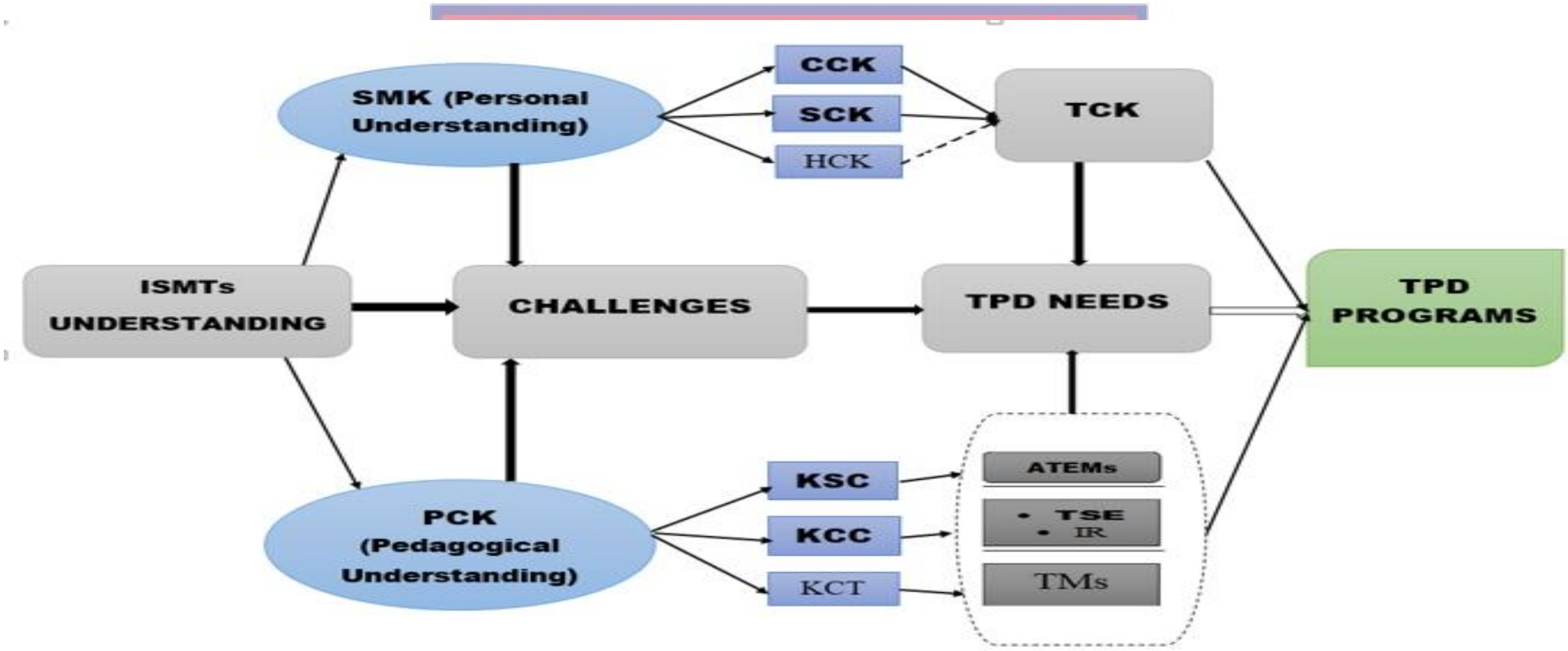
As shown in a study on Malaysian teachers' needs assessment, instructors' lack of teaching experience and technical weaknesses are identified as professional development requirements (Abubakar & Tarmizi, 1995, cited in Mohamed, 2013). Furthermore, Quan-Baffour (2007) argues that it is vital to equip teachers with ongoing professional development at the school level so that they can teach effectively and raise student achievement. This is due to the inadequate knowledge and skills received by most pre-service teachers in developing countries, and also to the fact that additional knowledge renders what was acquired a few years ago obsolete. With concerns, Quan-Baffour conducted a survey to determine the specific needs of individual instructors in their distinct settings. This was conducted on forty educators to reveal their pedagogical deficiencies. The results presented showed that teachers' professional development needs were fixated on effective instructional approaches, classroom organization, alternate methods of evaluating students, and preparing lessons for effective instruction. He concluded that the results obtained were used as a basis for training programmes in some schools.

Conceptual Framework

The conceptual framework defines the relationship between the essential components of a study. It is logically organized to help in the visual portrayal of how several themes in a research study relate to one another (Grant & Osanloo, 2014). According to Miles and Huberman (1994, p. 18), conceptual frameworks might be "graphical or narrative in style, displaying the major variables or constructs to be explored and the hypothesized connections between them." A conceptual framework acts as a mirror for the research. It helps the researcher comprehend and develop his or her own viewpoints on the issue under investigation, for example (Grant & Osanloo, 2014).

To inform meaningful TPD, Wessels and Nieuwoudt (2010) proposed that teachers should be profiled based on their SMK in general and the topic in particular. As posited by Wessels and Nieuwoudt (2010), the components of the conceptual framework are centered on the MKT and radical constructivism's theoretical frameworks. This is to help gain comprehensive insights into the state of individual ISMTs' trigonometry understanding and teaching. And also, identify their trigonometrical TPD needs to deepen the need for teachers to access regular and appropriate programs that will improve their professional competencies as well as learners' academic performance in trigonometry and its related content.

Since the MKT components of teacher knowledge are not age-specific or age-dependent, it suggests that teachers' competence is dependent on in-depth personal and pedagogical understanding [CK and PCK] (Tsafe, 2013) of trigonometry.



Source: Researcher's Construct (2020)

Figure 2: Conceptual Framework

Legend:

CCK-Common Content Knowledge; HCK-Horizontal Content Knowledge; ISMTs In-Service Mathematics Teachers; KCC-Knowledge of Content and Curriculum; KCT-Knowledge of Content and Teaching; ATEMs-Awareness of Trigonometrical Errors and Misconceptions; KCS-Knowledge Content and Student; PCK-Pedagogical Content Knowledge (Pedagogical Understanding); SCK-Special Content Knowledge; SMK-Subject Matter Knowledge (Personal Understanding); TCK-Trigonometry Content Knowledge; TMs-Teaching Methods; TPD-Teacher Professional Development; TSE -Trigonometry Self-Efficacy; IR-Instructional Resources.

Dependent Variables: *TPD Programs*

Independent Variables: *Challenges**;** ATEMs; TSE**;** TCK**;** TPD Needs*

The study is conceptualized into the main domains of personal and pedagogical understanding. These are subdivided into variables used in the objectives (content knowledge, awareness of errors, self-efficacy, challenges, and TPD needs). As a result, this study singled out ISMTs' trigonometry content knowledge, awareness of trigonometrical errors and misconceptions, self-efficacy, and challenges for teaching the topic as the main factors that may influence the trigonometry TPD needs.

Figure 2 defines that teachers' trigonometry content knowledge, awareness of trigonometrical errors, and trigonometry self-efficacy are expected to make the teacher understand trigonometry concepts and ideas. For example, if a teacher has the required content knowledge, self-efficacy, and the ability to detect, diagnose, and correct errors or learning problems that learners might encounter, then the teacher will be capable of preventing such issues and assisting students in overcoming obstacles. Similarly, they (ISMTs) will also be able to encourage students and make lessons more engaging and stimulating. Understanding all these constructs will then help them discover the challenges they face and the TPD needs to be needed. This study explores teachers' understanding and teaching of trigonometry as well as identifying the challenges and TPD needs to inform professional development in trigonometry.

Explanation to Various Themes in the Conceptual Framework

The conceptual framework makes it easier to describe Ghanaian ISMTs' understanding and teaching of SHS trigonometry in the research. In this framework, ISMTs' understanding of trigonometry, as prescribed in the Ghanaian SHS mathematics curricula, is divided into two components: personal understanding (referred to as CK or SMK) and pedagogical understanding (referred to as PCK). As indicated in Figure 2, personal understanding comprises Trigonometry Content Knowledge (TCK), while pedagogical understanding is broken down into three descriptors (awareness of trigonometry errors and misconceptions; and trigonometry self-efficacy. These specific descriptors of personal and pedagogical understanding are used to assess and explore the overall ISMTs' understanding of trigonometry. From these, challenges, as well as identification of trigonometry professional development needs, are ascertained, which is expected to inform an effective TPD program for teachers. The components of ISMTs' understanding of trigonometry are described below.

Teacher Understanding

Influential research programs on teacher knowledge (for example, Ball et al., 2008 & Shulman, 1987 referred to by González, 2014) identified several knowledge bases, such as SMK, PCK, and curricula knowledge, among others, that teachers must use to instruct effectively. SMK and PCK are the two knowledge bases that are most closely connected to teachers' understanding of a content or topic. The wealth of knowledge that teachers require for teaching is made up of these foundations. Hence, SMK and PCK, which stand for Personal and Pedagogical Understanding respectively, are the subjects of this study. Shulman (1987) stated that the teaching of mathematics topics commences with the teacher's knowledge of what (subject content) needs to be taught and how (pedagogy) it should be instructed to students. In particularly, Etkina (2010, p. 1) emphasizes that "teachers of a specific subject should possess special understandings and abilities that integrate their knowledge of the content of the subject that they are teaching as well as knowledge of the learners who are learning the content". Based on the literature on teacher knowledge, this part starts with an explanation of teacher content knowledge to define what it implies to understand a subject matter (trigonometry). A discussion on teachers' awareness of errors and misconceptions, self-efficacy, challenges, and needs assessment for an effective TPD for teaching trigonometry ensues.

Personal Understanding (Subject Matter Knowledge)

Personal understanding refers to SMK. It mainly denotes the two categories of SMK, which are CCK and SCK (Ball et al., 2008) to explore ISMT TCK. According to Malambo (2015) and Ball, Hill, and Bass (2005), CCK involves "knowledge that is used in the work of teaching in ways that are common with how it is used in many other professions or occupations that also use mathematics" (p. 39). Its descriptors involve solving questions, provision of accurate definitions and characteristics of the concept, precise recognition of examples and non-examples, as well as correct application of rules, principles, theorems, symbols, and notations. Therefore, teachers generally require this knowledge to be able to calculate and solve questions about mathematical topics. In particular, the study suggests that ISMTs who have CCK in trigonometry should be able to handle issues with these descriptors. However, mathematics teachers require more than CCK to proficiently teach. Hence, the need for SCK, which provides the ability to explain and justify reasoning. SCK refers to specialized mathematics knowledge that allows teachers to "unpack" subject content (Ball et al., 2008). The unraveling of the subject content assists teachers in conducting the complex enterprise of teaching and separates them from normal professions which utilize mathematics in compressed form but are not necessarily teachers (Nyikahadzoyi, 2013). The CCK and SCK descriptors are crucial in the development of the data gathering instrument. For example, the trigonometry test items require calculations, explanations, justifying of reasons, and translating of real-life situations into mathematics. This seeks to assess and provide a description of how comprehensively ISMTs understands trigonometry.

Pedagogical Understanding (PCK)

Pedagogical understanding in this study stands for PCK. Sibuyi (2012, p. 9), indicates that "Shulman (1987) suggests that PCK forms a unique and distinct knowledge domain of teacher cognition. Thus, PCK emphasizes how teachers relate SMK (what they know about what they teach to their pedagogical knowledge), to what they know about teaching, how their learners learn and the learners conceptions and how SMK is part of the process of pedagogical reasoning." Shulman (1986, p. 9) further elaborated on PCK as "the most useful form of representation of those ideas, the most powerful analogies, illustrations, examples, explanations, and demonstrations-in a word, the ways of representing and formulating the subject that make it comprehensible to others".

Moreover, Carpenter, Fennema, Peterson, and Carey (1988) cited from Sibuyi (2012, p. 10) saw PCK as teachers' awareness of the "conceptual and procedural knowledge that students bring to the learning of a topic, the misconceptions about the topic that they may have developed, and the stages of understanding that they are likely to pass through in moving from a state of having little understanding of the topic to mastery of it. It also includes knowledge of techniques for assessing students' understanding and diagnosing their misconceptions". These key components in Shulman's and Carpenter et al's definition of PCK correspond to the three PCK domains expounded by Ball et al. (2008), which are; knowledge of contents and teaching; knowledge of contents and students; and knowledge of contents and curriculum in the MKT framework underpinning the study.

Trigonometric Content Knowledge (TCK)

The most basic component of teaching knowledge is content knowledge (González, 2014), which is key for influencing student performance (Kim, 2007). This signifies the teacher having a profound knowledge of the subject contents. Mishra and Koehler (2006) found that content knowledge of teachers of any subject is essential for teaching. They argue that content knowledge means understanding of the topic or content that will be taught and learned. Teachers must have a reasonable understanding of the topic matter in order to successfully transfer information to pupils. Furthermore, Mishra and Koehler (2006) asserted that teachers must know and understand the mathematics they teach, including knowledge of central facts, concepts, theories, and procedures within a given topic; knowledge of explanatory frameworks that organize and connect ideas; and knowledge of evidence and proof rules (p. 1026). Teachers' knowledge of the trigonometry content in the senior high mathematics curriculum differs from that of those in non-teaching occupations. As a result, the purpose it serves must also be examined. This is because teachers' in-depth and precise knowledge of mathematics, as reported by Hill, Rowan, and Ball (2005), improves teaching efficacy.

Considering the significance of trigonometry and the problems students and teachers' experience, according to Davis (2005), the topic and its several ways of teaching in the classroom have received less attention. Thus, little exploration has been done into the degree of understanding of trigonometry required for teaching. This calls for the need to investigate the trigonometrical content knowledge that ISMTs are passing down to students. Research carried out on students' understanding of trigonometry constantly reveals that students perceive trigonometry as abstract, challenging and are susceptible to errors and misconceptions in the discipline.

Trigonometry is an integral part of the curriculum and if students are to excel, they must be well prepared. This necessitates the use of teachers who are knowledgeable in trigonometry to assist students. The development of other components of trigonometrical knowledge for teaching is built on the foundation of TCK. In this study, trigonometry content knowledge denotes the knowledge of the trigonometry content as described in the Ghanaian SHS mathematics curriculum (CRDD, 2010). ISMTs need to have trigonometrical knowledge across all the forms in the core and electives mathematics in the senior high school. The trigonometry content knowledge would be measured using an achievement test covering the SHS Core and Elective Mathematics trigonometry content.

Awareness of Errors and Misconceptions (AEMs)

According to Zuya (2014), the mathematics teacher is a crucial component in changing mathematics teaching and learning, and teachers' knowledge structures show their type. Adler (2005) considers teachers' understanding of mistake analysis to be a component of mathematics for teaching in South Africa. This suggests that involvement in mistake analysis, a type of classroom evaluation, is both a professional obligation and an important element of instructors' expertise. Tsafe (2013) agrees with Adler that teachers need have excellent PCK in order to deconstruct mathematics into discrete pieces and explain a concept or technique at a level that incorporates the steps essential for learners to make sense of reasoning (p. 37). Thus, they should know the most effective way to represent and explain various concepts, as well as detect learners' misconceptions about particular content. Such knowledge, according to Hill, Ball, and Schilling (2008), might help teachers to design instructions that will address issues such as preconceptions, errors, or misconceptions that students carry into the classroom. If teachers' understanding and detection of common students' errors and misconceptions are effective strategies and important aspects of mathematics teaching to help students avoid them (Chick & Baker, 2005), then this calls for the need to investigate the awareness of errors and misconceptions in trigonometry from a teacher's perspective.

In this study, ISMTs' awareness of errors and misconceptions answers the question: 'Is the teacher able to analyze solutions that students might present for a task and explain in a clear and acceptable manner what mathematical or trigonometrical steps likely produce such responses from students, and why?'. This teachers' knowledge of students' trigonometrical errors and misconceptions in this study resonates with the theoretical framework subdivision of PCK, which is the knowledge of contents and students by Ball et al (2008).

Teacher Self-Efficacy

Swackhamer et al. (2009) asserted that teachers must possess specific competences in order to carry out important responsibilities such as achieving objectives and fostering effective and long-term learning throughout the mathematics teaching process. They also identified self-efficacy belief as the most important competency among those in the mathematics discipline. To Bandura (1994), self-efficacy is described as “people’s beliefs about their capabilities to produce designated levels of performance that exercise influence over events that affect their lives” (p. 1). Also, teacher self-efficacy, according to Woolfolk (2007), is “a teacher’s belief that he or she can reach even difficult students to help them learn” (p. 334). Woolfolk further stated that it “appears to be one of the few personal characteristics of teachers correlated with student achievement” (p. 334) and a pronounced contributor to teacher effectiveness (Briley, 2012). Mathematics teaching efficacy is determined by a teacher's self-efficacy with mathematical material, personal views, and past experience with the subject's substance (Briley, 2012). That is, self-efficacy refers to one's conviction in 'I can do' or 'I cannot do.' According to the idea, however, judgements of one's own efficacy are task-specific talents, according to Schweinle and Mims (2009). And this assumption is consistent with the study's goal of investigating teachers' TSE, which is topic-specific.

Nonetheless, it was discovered that students struggled with several basic trigonometry concepts and were uninterested in the topic (Akkoc, 2008). And since teaching efficacy is connected to instructional procedures, students’ learning, and interest, it would be enlightening to examine teachers’ trigonometry self-efficacy to understand its relationship with students’ academic achievement and motivation. Hence, it is crucial to study ISMTs TSE across the core and elective mathematics in the Ghanaian SHS curriculum. In this study, the TSE of ISMTs corresponds to the theoretical framework subdivision of PCK, which is the knowledge of KCC by Ball et al (2008). Curricula are designed to identify certain teaching goals. KCC refers to teachers' understanding of how various subjects and concepts are taught in the school curriculum at various grade levels, as well as their grade-level connections.

Furthermore, because one's confidence in teaching is contingent on evaluating one's teaching in relation to the national curriculum, teachers' beliefs are identified as one of the ingredients that combine to form PCK (Grossman, 1990). As a result, knowing the curriculum contents, as well since the purposes and objectives of the themes in the mathematics curriculum, is critical for instructors, as this goes a long way toward forecasting their self-efficacy level. TSE in this study refers to a teacher's capacity to confidently meet the stated objectives for teaching senior high school trigonometry content in the SHS mathematics curriculum provided (RDD, 2010). This was determined by administering a questionnaire that addressed the stated objectives for teaching senior high school core and optional mathematics trigonometry.

Teacher Challenges in Teaching

Difficulty in mathematics teaching and learning is found to be a common and significant problem throughout most countries (Bichi, Ibrahim & Ibrahim, 2018). The problems that normally occur in mathematics teaching-learning processes are relatively higher as compared to those in other subjects. Students' negative attitude and disinterest towards mathematics is a concern acknowledged worldwide (Singha, Goswami, & Bharali, 2010). In some instances, factors affecting mathematics teaching and learning are associated with issues, which include, availability of appropriate mathematics textbooks and instructional resources such as manipulatives and technological tools, preparing and training of pre-service and in-service teachers, understanding of mathematical terminologies, and poor attitudes by both teachers and students (Mulwa, 2015).

Needs Assessment for Professional Development Programmes

Teachers' Professional Development (TPD) is a strategy that comprises all activities that help teachers progress in their careers (Tecle, 2006), as well as formal and informal exposures throughout a teacher's profession (Arends et al. 1998). Needs assessment, in light of this explanation, is an approach for determining areas where teachers need and desire assistance (Mohammed, 2013). The effectiveness of teacher professional development is a motivating factor for teacher professionalism around the world (Bantwini, 2012). This is because it offers prospects for teachers to experience new tasks, learn new skills, improve their knowledge, beliefs, and practices, and expand their horizons as instructors and individuals to encourage active learning and better academic achievements. The importance of effective

PD programmes for teachers cannot be underrated. However, Zakaria and Daud (2009) suggested that for a PD activity to be effective and benefit teachers, a needs assessment has to be done to know the teachers' actual areas of need for that specific PD. Therefore, it is critical for ISMTs to identify these perceived needs. Ajani, Govender, and Maluleke (2018) expressed that this demands consultations with teachers to ascertain their teaching obstacles and requirements. Some inquiries regarding teachers' needs for PD have been organized in numerous settings (Mohamed, 2013; Rakumako & Laugksch, 2010). The findings of these surveys were utilized to establish in-service teacher training programs or as a precursor to national policymaking. This research is significant in the sense that it is one of its kind needs assessment study on the PD of ISMTs for teaching trigonometry, though some studies focused on mathematics in general. It provides significant information about the effectiveness of trigonometry instruction in SHS, and therefore, an educational development program could be carried out accordingly, focusing on trigonometry.

CHAPTER THREE

RESEARCH METHODS

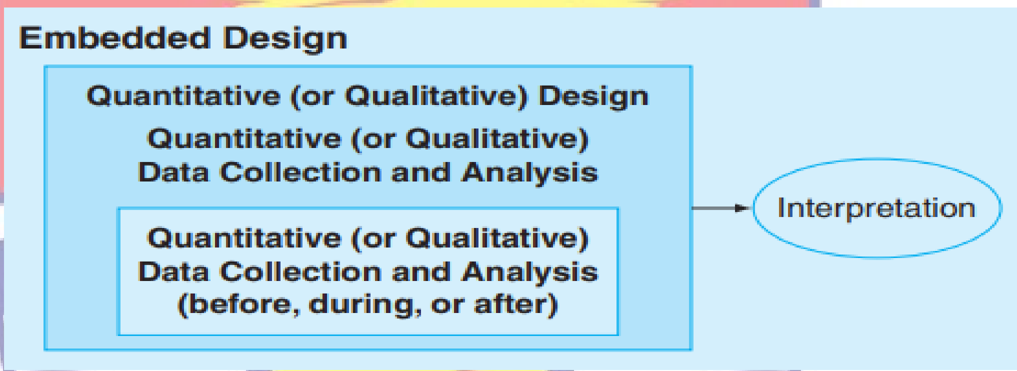
Overview

The study's purpose is to gain an insight into ISMTs' understanding and teaching of trigonometry concepts in SHS. The chapter outlined the methodologies used to explore ISMTs' understanding and self-efficacy, identify some challenges, and identify their professional development needs in teaching trigonometry. It also considered research design, study area, population, sample and sampling techniques, data collection instruments and procedures, data analysis and presentation, ethical considerations, reliability and validity of research instruments.

Research Design

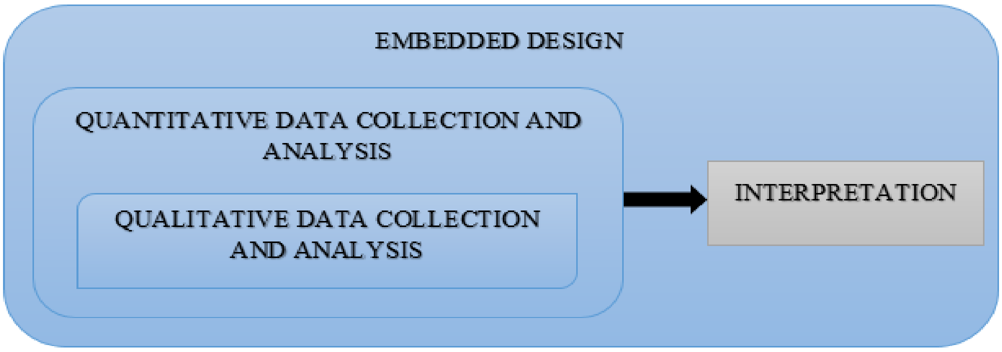
The mixed methods approach is utilized to obtain insight into teachers' conceptual and pedagogical understanding of effective trigonometry teaching, as well as to offer a foundation for effective TPD. This strategy incorporates quantitative and qualitative data and draws conclusions from both sets of information (Creswell, 2015). To this end, the embedded design is employed in aligning the hypotheses, questions, and purposes of capturing constructs of content knowledge, knowledge of errors and misconceptions, self-efficacy, challenges, and needs of ISMTs for teaching trigonometry. An embedded design, according to Creswell (2012), is one in which the researcher gathers both quantitative and qualitative data concurrently, yet one kind of data serves as a support for the other (Figure 3).

Therefore, for the purpose of this study, quantitative data served as the major form of data collected to achieve the objectives, while qualitative data was gathered to augment or provide additional sources of information. It usually answers a different question that was not asked for and was not answered by the primary data source. Though qualitative data were collected, the overall design still emphasizes quantitative approaches. During the study, both types of data were collected simultaneously. The modified or adapted form of the embedded research design by Creswell (2012) is presented in Figure 4. This focused on the actual methods of data collection used.



Source: Creswell (2012)

Figure 3: Embedded Research Design



Source: Adapted From Creswell (2012)

Figure 4: Adapted Embedded Research Design

Study Area

The study is confined to selected SHS in the Greater Accra Metropolis, Ghana, where SHS and ISMTs were selected. This study area was chosen for the research because of the assurance of obtaining the relevant information regarding teachers understanding and teaching of the trigonometry content.

Population

The study's population was limited to public secondary schools in the Greater Accra Metropolis of Ghana's Greater Accra Region. There are now 61 public secondary schools in the city. Private SHS were excluded from the research because they do not get central government funding to acquire teaching and learning materials or attend professional trainings. Furthermore, these private schools do not receive human resource help from the Ghana Education Service. The respondents were ISMTs in public SHS in the Greater Accra Metropolis. There were about 15 ISMTs in each of these SHS and a total of about 915 ISMTs served as the total population for the study. However, not all the public SHS and ISMTs were used in the study. The sample sizes and sampling procedures are discussed in the preceding section.

Sample Size and Sampling Procedures

A sample is a subset of a larger population chosen for a survey (Cohen, Manion & Morrison, 2007). The ISMTs that participated in the study were chosen using a stratified sampling technique. Stratified sampling, according to Creswell (2014), is a probability sampling approach in which the researcher separates a population of the same characteristics into strata and then utilizes a basic random sampling method to obtain a sample from each stratum. The stratified sampling approach was chosen to provide a fair representation of all school types in the sample, and this is owing to the categorizing of schools in Ghana (MoE, 2019). The public SHS have already been grouped by the MoE into categories "'A', 'B', 'C', and 'D'" based on accessible school resources and achievements (MoE, 2019). For this reason, the researcher adopted stratified random sampling in selecting the schools. All the categories of schools are available in the metropolis. That is, there are 5 Category ‘A’ schools, 8 Category ‘B’, 16 Category ‘C’ and 32 Category ‘D’ schools (MoE, 2019).

The schools in each category were sampled using a proportionate stratified sampling procedure. The proportion of categories of schools (A, B, C, and D) in the Greater Accra Metropolis is 5:8:16:32 for A, B, C, and D respectively. Due to the vast variations in the number of schools in each category, the proportionate stratified sampling technique was used, applying a fixed ratio of one-third (Kabutey, 2016). And this helped in getting a sample that is representative of the different categories of schools in the population. Hence, 2 schools were selected from category A, 3 from B, 5 from C, and 10 from D, giving a total of 20 schools. The random number generating approach was used to choose the sampled schools from each category using a simple random sampling procedure. The schools in each category were assigned random integers, 1 to N (N_i is the aggregate number of schools within each category). Sample size, n (n_i is the sample size taken from each category).

For example, ‘Category A’ has N= 5 schools and n= 2 were selected. Random integers were generated between 1 and 5 inclusive, using a computer (scientific calculator, 991ES). When a random integer is repeated in the generation process by the calculator, it is discarded and the next integer is considered. Hence, with the random number generation procedure, the numbers, e.g. 2 and 4, were generated for category ‘A’ from the range of 1 to 5 inclusive. This process was followed until all sample categories of schools had been covered. On average, each school has about 15 ISMTs. A total of 300 ISMTs from the sampled schools took part. The number of schools and ISMTs is presented in Table 3.

Table 3: Number of schools and participants per circuits

Category	Total	(33% of the total of schools) Sample size	The average of participants in each school	The average of participants in each school
A	5	2	15	30
B	8	3	15	45
C	16	5	15	75
D	32	10	15	150
Total	61	20		300

Source: Researcher's Computation (2020)

Data Collection Instruments

In this stud6y, a questionnaire (i.e. Trigonometry Questionnaire) was used to examine the trigonometry content knowledge, errors and misconceptions, self-efficacy, challenges, and needs that would inform TPD for teaching trigonometry. The instruments were developed using the study's specific objectives as a basis. Data were gathered utilizing both quantitative and qualitative approaches. Data were gathered by means of the following:

Trigonometry Questionnaire (TQ)

There are five sections of the TQ for teachers: A, B, C, D, and E. Section A looked at the demographic features of participants. Section B is an open-ended Trigonometry Content Knowledge and Diagnostic Test (TCKD-T) that provides participants with a chance to express their thoughtfulness about trigonometric concepts. The participants constructed responses to demonstrate understanding, application, and the ability to identify and diagnose basic trigonometric concepts. The TCKD-T is in two parts. Part I (Knowledge of Mathematics tasks) specifically measures ISMTs' content knowledge, and this is to check teachers' awareness of different ways of solving the problem (Krauss et al., 2008a). In measuring content knowledge, TCKD-T items were developed by the researcher based on past WASSCE questions and adopted trigonometry questions in several mathematics textbooks. This covered the trigonometry content in the Ghanaian SHS Core and Elective Mathematics curricula.

Teachers' competence to identify and diagnose errors and misconceptions was assessed in Part II (Knowledge of students' errors and misconceptions). ISMTs were given hypothetical student solutions to trigonometric concepts. The student samples written by the researcher were based on common students' misconceptions and errors. The teachers evaluated the solutions to identify, analyze (provide cognitive explanations for comprehension issues), or predict a common student error and challenge (Krauss et al., 2008a). This is because being able to analyze students' thinking is a vital element of their pedagogical skills for teaching trigonometry. All ten (10) items of the TCKD-T covered relevant content areas where learners usually did not perform well, as per the Examiner reports.

Section C measures the ISMTs' confidence levels in teaching trigonometry. It is denoted as the TSE, which is a 36 closed-ended items divided into five categories (I-V) developed by the researcher to be given to ISMTs. The first 11 items cover Core Mathematics Forms 2 and 3 Trigonometry I and II respectively, and the remaining 25 cover the SHS Form 3 Elective Mathematics Trigonometry objectives. The items in this section were adopted from the Ghanaian Core and Elective Mathematics syllabi and covered the stated objectives for teaching SHS Trigonometry (MOESS, 2010). Part I of the TSE has 7 items and Part II consists of 4 items covering the SHS Forms 2 and 3 Core Mathematics Trigonometry I and II objectives respectively. Also, Part

III contains 13 items, Part IV has 4 items, and Part V contains 8 items, which respectively cover the subscales: trigonometric ratios and rules, compound and multiple angles, and trigonometry functions, equations, and graphs in the SHS Form 3 Elective Mathematics Trigonometry objectives (CRRD, 2010).

In determining ISMTs' TSE levels, they completed TSE items by assessing their confidence in teaching trigonometry on a scale of 1 to 5, where, "1 = not at all confident, 2 = only a little confident, 3 = somewhat confident, 4 = confident, 5 = very confident". The findings were interpreted as follows: the lowest possible score is 1, indicating a very low degree of confidence in teaching trigonometry, and the highest possible score is 5, indicating a great level of confidence in teaching trigonometry. ISMTs were asked to pick an item that they felt least confidence in teaching and an item that they felt most confident in teaching for the open-ended component in each of the five TSE segments, as well as offer explanations for their choices. For each TSE Part I–V, there are two open-ended questions.

One asked the respondent about each TSE, *"Choose one trigonometry item that you indicated feeling LEAST CONFIDENT about teaching high school students. Think about the reason(s) you feel this way. Use the space below to identify the item number and explain your reason(s)"* and *"Choose one trig item that you indicated feeling MOST CONFIDENT about teaching high school students. Think about the reason(s) you feel this way. Use the space below to identify the item number and explain your reason(s)"* (Harrell-Williams et al., 2014). Therefore, a respondent who answers the entire open-ended questions is expected to have five least confident and five most confident responses. This qualitative data was gathered to support the quantitative or close-ended response of the TSE. Section D looked at some challenges ISMTs encounter in teaching trigonometry. This was made up of 12 items, and developed by the researcher based on the literature reviewed.

Finally, Section E examined ISMTs' TPD needs for teaching trigonometry. This is a modified version of the Laugksch, Rakumako, Manyelo, and Mabye (2005) instrument (STIN-LP) used in their study, "In-service training needs of secondary mathematics, physical science, and biology teachers in the Northern Province – a survey of teachers' views". Despite the fact that the original STIN was determined to be reliable and valid in other nations (Abu Bakar & Rubba, 1985; Zurub & Rubba, 1983), it was updated in 1994, resulting in Baird et al (1994)'s STIN-3. The STIN-3 instrument was renamed the STIN-LP when it was revised and validated for use in South Africa, specifically in the Limpopo province. It contains 98 items (Laugksch, Rakumako, Manyelo & Mabye, 2005).

After administering the instrument, the alpha coefficient of the items and the instrument's Guttman split-half reliability were determined to obtain information on the instrument's reliability. This instrument's alpha coefficient reliability was 0.97 and adjusted Guttmann split-half reliability was 0.83. These coefficients have a high value, indicating that the STIN-LP is reliable in predicting ISMTs' perceived trigonometry INSET needs. Section B, referred to as the need category of the original STIN-LP, consisted of 47 items assessing teachers' INSET needs in seven categories. These are as follows: improving personal competence (6 items), specifying instructional objectives (2 items), diagnosing and evaluating learning (3 items), planning instruction (14 items), delivering instruction (5 items), managing instruction (7 items), and administering instructional facilities and equipment (10 items). The response options ranged from "not familiar" to "very important." Following a thorough examination and evaluation of needs assessments (Mohamed, 2013; Laugksch, Rakumako, Manyelo & Mabye, 2003; Abu Bakar & Rubba, 1985; Zurub & Rubba, 1983), the STIN-LP items were cautiously and jointly constructed to mirror the present needs of Ghanaian SHS mathematics teachers and the trigonometry content in the curriculum and re-named as Mathematics Teachers Inventory Needs for Teaching Trigonometry [MTIN-TRIG] to align to the purpose of this study.

The modification of Section B of the original STIN-LP resulted in 60 items for the trigonometry needs of teachers. As on the original STIN-LP, MTIN-TRIG response choices went from "not familiar to great need." These needs were categorized into eight distinct dimensions: Planning Trigonometric Instructions (11 items); Delivering Trigonometric Instructions (14 items); Managing Trigonometric Instructions (4 items); Teacher Self-Improvement in Trigonometric Content (4); Teacher Self-improvement in Trigonometric Pedagogy (6 items); Use of ICT (12 items); Preparation and Utilization of Teaching Materials/Aids [Manipulatives] (5 items); Diagnosing and Evaluating Learning (4 items).

Data Collection Procedures

Before the data collection, consent letters were sought from the IRB, and the Head of the Department of Mathematics and ICT Education, University of Cape Coast. Two weeks prior to the data collection exercise, these approval letters were presented to the heads of the schools selected. Furthermore, approval was obtained from ISMTs through the offices of the heads of the sampled schools' mathematics departments. The researcher then created a good relationship with ISMTs, and the study's objective was explained to them. Appointments for data collection were then made between the ISMTs and the researcher. The participants were informed and assured that the data they submitted would be kept private. The data collection was done after school hours, with the support of field assistants, to guarantee that it would not interfere with academic activity.

Fortunately, the research was conducted at the beginning of the students' end-of-term examinations in December, before the Christmas break, when schools closed earlier, and this made it easy for the participants to stay behind to complete the research instruments. With the help of the field assistants, the researcher collected data from sampled schools in the same vicinity on the same day, and this was done to make sure there was no leakage of information. Most participants completed the questionnaire unsupervised in their staff room for about 2 hours and 30 minutes. A period of two weeks was used for data collection.

Data Processing and Analysis

Data screening were conducted to ensure it was properly recorded. Quantitative and qualitative approaches are used in data analysis. Data was coded and processed using the software packages SPSS (V. 25.0) and Minitab (v. 18.0) to create a trigonometric database that was used to describe, analyze, and present data using descriptive and inferential statistics. Specifically, in analyzing TCK, summary statistics for ISMTs' scores on the TCKD-T Parts I & II assessment of overall mean score and standard deviations were generated. Boxplots were drawn for the performance of sub-scores and overall scores. For performance in terms of the forms and sub-scores, a paired sample t-test was used to test mean differences in various levels of trigonometry. TCKD-T Part II was also examined by looking at how teachers spotted problems in a hypothetical student's response and what they advised doing to correct the issue. The PCK category of the teacher's assessment of students' thinking led the analysis. They involve detecting students' mistakes, determining the causes of the mistakes and misunderstandings, and developing effective techniques for dealing with them (Krauss, Neubrand, Blum & Baumert, 2008).

Following data collection, substantial effort was spent on the data by reviewing the inputted data back against the original replies for correctness in order to determine the origins of the mistakes and misconceptions and design effective methods for correcting the issues. This was accomplished by reading and re-reading the data and observing opening concepts. The goal was to make it easier to analyze the data by facilitating close reading and interpretative skills (Lapadat & Lindsey, 1990). This was done manually by taking notes on the texts being analysed.

Also, for trigonometry self-efficacy (TSE), Exploratory Factor Analysis (EFA), and summary statistics for ISMTs' self-confidence scores on TSE items overall, as well as the various forms in the SHS Core and Elective Mathematics syllabi. T-test was used to determine whether the mean scores differed significantly between the various forms to indicate whether ISMTs' confidence scores differ significantly as the trigonometrical sophistication of items increased. The data for the TSE open-end responses was analysed using thematic analysis. This is a technique for detecting, analyzing, and recording data features. It normally organizes and describes the various aspects of the data set in great detail (Boyatzis, 1998). Analysis stands between description and interpretation, and it involves the cautious process of systematically discovering key characteristics and relationships.

Using this procedure, five phases that evolve throughout time (Ely et al., 1997) were followed. This includes familiarity with data, code generation, considering themes, reviewing the themes, defining and identifying the themes. After data collection, much time was spent on the data by checking the entered data back against the original responses for accuracy, by reading and re-reading, and noticing primary thoughts. The goal was to make it easier to analyze the data by facilitating close reading and interpretative skills (Lapadat & Lindsey, 1990). After that, codes were generated. The generating of codes involves creating initial codes from data. The codes highlighted data features the researcher found intriguing and referred to the most fundamental section, or part, of the raw data that could be considered in an expressive way (Boyatzis, 1998). This was done by hand, by taking notes on the texts being analysed. Themes were then searched for and reviewed afterward. The themes were then defined and named.

On the challenges ISMTs face in teaching trigonometry, data were subjected to EFA and descriptive statistical analysis by computing means and standard deviations (SDs) for the items.

Likewise, data were subjected to EFA and descriptive statistical analysis when assessing the TPD needs of ISMTs, with the means and SDs of various items under each type of need or construct computed. For each of the eight categories of TPD needs, overall averages and standard deviations were determined. Because the items are on a 5-point Likert scale, the identification of these factors is done through EFA and Ranking Analysis. A model adopted by Wang and Yuan (2011) in their analysis of the factors affecting risk attitudes. To do this, means, standard deviations, and the ranking of each and factor loadings are utilized to screen them from the sample data. Drawing from previous studies (Ikediashi et al., 2012; Wang & Yuan, 2011), a mean value of 3.0 is recognized as the benchmark. Risk factors are classified as critical when the mean values are greater than or equal to the benchmark (3.0). This benchmark has been adopted in the current study. Hence, items with the least standard deviation are considered more critical in the event where two or more items have the same mean value (Wang & Yuan, 2011). This is because the items under each of the categories are on a 5-point scale, where 1 is the least score associated with the item and 5 is the supreme. Also, the EFA takes into consideration the factor loadings and Cronbach Alpha values. For this analysis, only items with factor loadings of 0.50 or more were retained for further analysis. Thus, items that recorded or received factor loadings of less than 0.50 were discarded.

These variables, after examination, provided insights into ISMTs' understanding and teaching of trigonometry, confidence, challenges, as well as their professional development needs. The results obtained could be used as a basis for designing better TPD programs for them. It is therefore expected to help ISMTs make the teaching of trigonometry lively and interesting for students to appreciate the topic. This is a very critical issue considered as far as the syllabi and the teacher who is the implementer are concerned, as suggested by the chief examiner in his reports over the years.

Reliability and validity of Research Instruments

In establishing the content validity, two experienced mathematics tutors (mathematics teachers with a minimum of 5 years and more in SHS) and three experts in educational research from the Department of Mathematics & ICT Education, UCC, revised the items. They analyzed unclear, biased, and deficient items and evaluated sections where items had been placed. Their recommendations aided in determining the items' both face and content validity. After comparing it to the SHS Trigonometry content, the tutors concluded that it satisfied ISMTs' criteria and cleared it for administration. To guarantee the questionnaire's reliability and validity, a pilot test was performed to 20 ISMTs from Edinaman SHS and Oguaa Senior Technical School in the Central Region to ensure the items were not inexcusable and were relevant.

The instrument was piloted in another region rather than the Greater Accra Region to minimize information leaking. Cronbach's alpha was used to measure the instrument's dependability. Cronbach's alpha was used to measure the instrument's dependability. The pilot study yielded an overall dependability coefficient of 0.886 for the research instrument. According to (Hair et al., 2010; Mugenda & Mugenda, 2003), a research instrument is considered very dependable if its reliability coefficient is 0.50 or more. As a result, the entire questionnaire was trustworthy. The trigonometry content knowledge and diagnostic test and trigonometry self-efficacy on the questionnaire, in particular, earned a reliability value of 0.738 and 0.972, indicating a good, dependable item. The error analysis items have a value of 0.868. Furthermore, a reliability coefficient of 0.530 and 0.942 was obtained for the challenge items and the Trigonometry TPD needs, and this also showed strong and reliable items.

In conclusion, the instrument was deemed reliable and good for the study because the overall questionnaire and the various components or sections all recorded reliability values of more than the threshold of 0.50 required.

Ethical Considerations

According to Creswell (2014, p. 160), "data collection should be ethical and it should respect the site" of data collection. Therefore, researchers should seek the necessary permission from relevant authorities and respondents before a study begins (Resnik, 2010). Before the study was conducted, introductory letters were obtained from the Department of Mathematics and ICT education, and the UCCIRB, to convince respondents of the purpose of the study. Approval was also sought from the headmasters/mistresses and heads of the departments of the selected schools. These include informed consent, confidentiality, and anonymity. The objectives of the study were communicated to have a good understanding of the study and to willingly participate. Lastly, anonymity was ensured. Hence, the names of participants did not appear on either instrument, and the names of schools were substituted with codes.

Chapter Summary

This chapter covered research design, study area, population, sample and sampling techniques, research instruments and data collecting procedures, data processing and presentation methods, ethical issues, and the reliability and validity of research instruments.

CHAPTER FOUR

RESULTS AND DISCUSSION

Overview

The study develops insight into In-Service Mathematics Teachers' (ISMTs') understanding and teaching of trigonometry concepts in SHS. Thus, to encourage teachers to prompt and redirect their instructional goals, objectives, practices, strengths, and weaknesses in teaching trigonometry. This chapter defines the findings and discussions. Data presentations are grounded in the objectives postulated to guide the study, and SPSS and Minitab are used in data analysis. Thus, Preliminary Analysis (Reliability Tests); demographic characteristics, ISMTs' Understanding and Teaching of Trigonometry, and ISMTs' Trigonometry Self-Efficacy. Others include; Challenges Faced by ISMTs in Teaching Trigonometry, and Assessment and Description of Overall Trigonometrical Professional Development needs. Counts, percentages, means, and standard deviations (SD) were used. Means and SD were used for the quantitative part while the qualitative part relied on frequencies and percentages. Tables were used in the presentation of results. The respondents were ISMTs at the selected SHS in the Greater Accra Metropolis of Ghana.

In all, 300 ISMTs from 20 SHS were used. This is because the number of SHS is large and with the issue of the COVID-19 pandemic to deal with. However, 220 of the 300 surveys were returned correctly. This represented a response rate of 73.33%, which is regarded extremely high. This is because Mugenda & Mugenda (2003) said that a 50% response rate is sufficient, a 60% response rate is good, and a response rate of more than 70% is considered extremely high. The current study's response rate of 73.33% is quite good and satisfactory. Before beginning the analysis, the reliability of the study's numerous constructs must be verified. The Cronbach's reliability test is used to do this.

Preliminary Analysis

Table 4 displays Cronbach's alpha, a scale reliability metric for the variables under discussion. The quality and consistency of the researcher's data collecting and analysis techniques are referred to as dependability in this context.

Table 4: Reliability Test

Constructs	Reliability	N
Understanding and Teaching of Trigonometry:	-	-
• *Trigonometry Content Knowledge*	0.713	10
• *Misconceptions and Errors Committed*	0.825	14
ISMTs' Trigonometry Self-Efficacy	0.952	36
Challenges Faced in Teaching Trigonometry	0.530	12
Teacher Professional Development Needs	0.825	60

Source: Researcher's Computation (2020)

Table 4, which presents the psychometric information of the various constructs on the questionnaire, shows that the reliability measures recorded are at least 0.50. These values fall into the recommended (0.50) of Hair et al. (2010) and can therefore be relied upon for further analysis. The Cronbach's alpha for demographic characteristics, ISMTs' Understanding and Teaching of Trigonometry, ISMTs' Trigonometry Self-Efficacy, Challenges Faced by ISMTs in Teaching Trigonometry, and Assessment and Description of Overall Trigonometrical Professional Development Needs (PDNs) of ISMTs are: 0.713, 0.825, 0.952, 0.530, and 0.825 respectively. These measures are significant because they fall within the standard Cronbach alpha threshold of 0.500, indicating that the constructs have adequate internal consistency and can be designated as robust for analysis.

Demographic Characteristics of Participants

In this study, information was collected on the demographic variables of the respondents. Information was collected on sex, age distribution, highest academic qualifications, highest professional ranks, and teaching experience. Other variables in this section include; In-Service Training Workshop Attended, Classes taught by ISMTs, and Specific Mathematics Subject Taught. The result is presented in Table 5. Concerning age distribution, it was observed that 67.3% were between 20-30 years old, while 30.9% were between 31 and 40 years old. The least were those between 41 and 50 years, with only 1.8%. Concerning the highest academic qualifications of teachers, it was observed that 65.5% hold Bachelor of Education (B.Ed) certificates while 20% also hold Bachelor of Science (B.Sc) certificates. The least were those with a Master of Philosophy Education (MPhil. Ed) certificate with 3.6%.

Table 5: Demographic Features of Respondents

Demographic Variables	Frequency	Percentage (%)
Sex		
Male	184	83.6
Female	36	16.4
Age Distribution		
20-30 years	148	67.3
31-40 years	68	30.9
41-50 years	4	1.8
Highest Academic Qualifications		
B.Ed.	144	65.5
B.Sc.	44	20.0
M.Ed.	24	10.9
MPhil. Ed.	8	3.6
Highest Professional Ranks		
Principal Superintendent	140	63.63
Assistant Director I	65	29.54
Assistant Director II	15	6.83
Teaching Experience		
1-5years	76	34.5
6-10years	80	36.4
Above 10 years	64	29.1
In-Service Training Workshop Attendance		
Yes	60	27.3
No	160	72.7

Source: Researcher's Computation (2020)

However, 10.9% were Master of Education (MPhil. Ed) certificate holders. Furthermore, the researcher sought to examine the highest professional ranks of teachers. The result also revealed that 63.63% were Principal Superintendents. This was followed by 29.54% for those who were Assistant Director I. The least were those with Assistant Director II. In terms of teaching experience, 36.4 % and 34.5 % of the teachers in the study have been teaching for 6 to 10 years and 1 to 5 years, respectively. The least was 29.1% for those who had taught for over 10 years. It was also seen that, regarding the question of whether the teachers had attended any in-service training workshops before or not, the majority (72.7%) of them had never attended any such training programs, while the remaining proportion (27.3%) had attended in-service workshops.

Table 6: Classes taught by ISMTs

	Responses		Percent of
Classes	N	Percentage (%)	Cases (%)
Teachers Currently Teaching Form One Classes	124	34.8	57.4
Teachers Currently Teaching Form Two Classes	128	36.0	59.3
Teachers Currently Teaching Form Three Classes	104	29.2	48.1
Total	356	100.0	164.8

Source: Researcher's Computation (2020)

Most of the teachers were teaching Form Two classes, as presented in Table 6, while in Table 7, it was seen that most teachers teach both Core and Elective subjects.

Table 7: Specific Mathematics Subject Taught by ISMTs

Subjects	Responses		Percent of Cases (%)
	N	Percentage (%)	
Teachers Who teach Core Mathematics Only	96	43.6%	43.6%
Teachers who Teach Elective Mathematics Only	24	10.9%	10.9%
Teachers who Teach Both Core and Elective Mathematics	100	45.5%	45.5%
Total	220	100.0%	100.0%

Source: Researcher's Computation (2020)

The researcher further examines the conceptual and pedagogical understanding of ISMTs in the teaching of trigonometry. The results presentation is based on the research questions and hypotheses that guided the study.

ISMTs Understanding and Teaching of Trigonometry

The performance of students relates to their teachers' understanding of a particular subject. Therefore, concerning ISMTs' personal and pedagogical understanding and teaching of trigonometry, the researcher examines this from two perspectives: ISMTs' trigonometry content knowledge and the identification of errors and misconceptions from hypothetical students' solutions.

RQ 1: What is the TCK level of ISMTs'?

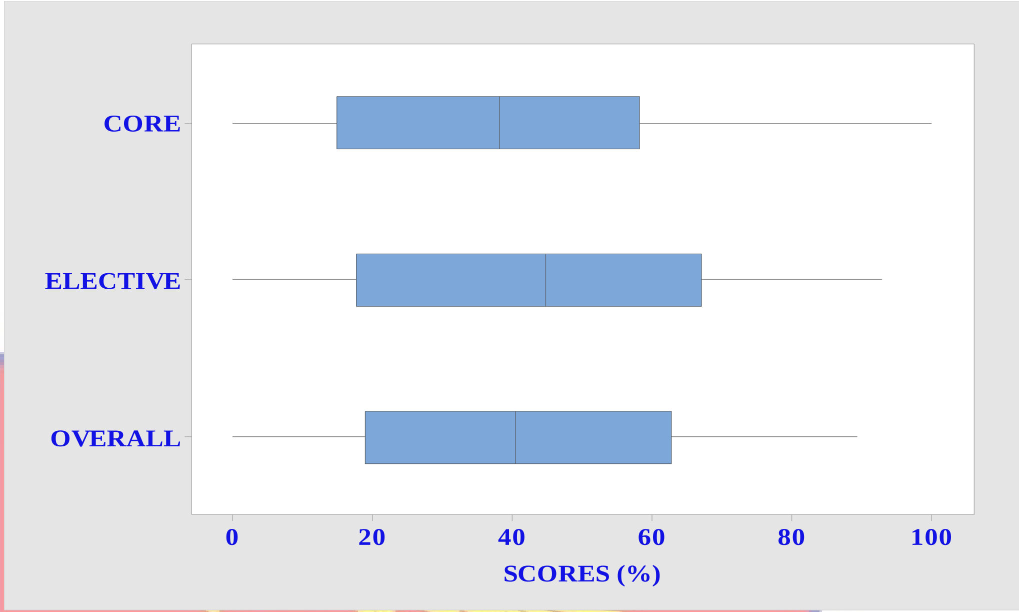
The researcher examined the trigonometry content knowledge (TCK) of ISMTs. In doing this, summary statistics for scores on the Trigonometric Content Knowledge Diagnostics Test (TCKD-T) were generated. Boxplots were also used to examine the performance of ISMTs. Also, the researcher carried out analyses of individual TCKD-T scores and participants' TCKD-T scores per question. Ten (10) questions were asked. Four of the questions are on Core Mathematics (2, 3, 4, & 9) and the rest are on Elective. It is to be noted here that, because the questions have different weights, there is a need to standardize the scores and, therefore, all the scores obtained were converted to 100%. Table 8 summarizes the findings.

Table 8: Descriptive Statistics of ISMTs TCK

TCK-T Questions	Mean (%)	SD (%)
Core Mathematics (Trigonometry I):		
TCK-T2	32.42	36.93
TCK-T4	43.64	49.71
TCK-T9	41.55	42.97
Core Mathematics (Trigonometry II):		
TCK-T3	35.45	39.02
Overall	38.27%	25.5%
Elective Mathematics:		
TCK-T1	48.00	34.05
TCK-T5	40.80	43.32
TCK-T6	55.09	42.86
TCK-T7	65.80	41.84
TCK-T8	17.10	30.67
TCK-T10	28.05	44.36
Overall	42.47%	27.7%

Source: Researcher’s Computation (2020)

ISMTs appear to have performed better on Elective Mathematics related trigonometry questions than in Core Mathematics. Thus, on a whole, they scored higher marks on Elective Mathematics (42.47%) than on Core Mathematics (38.27%) related questions. This is further explained by Figure 5 (Performance of ISMTs on Core and Elective Mathematics Questions and its overall).



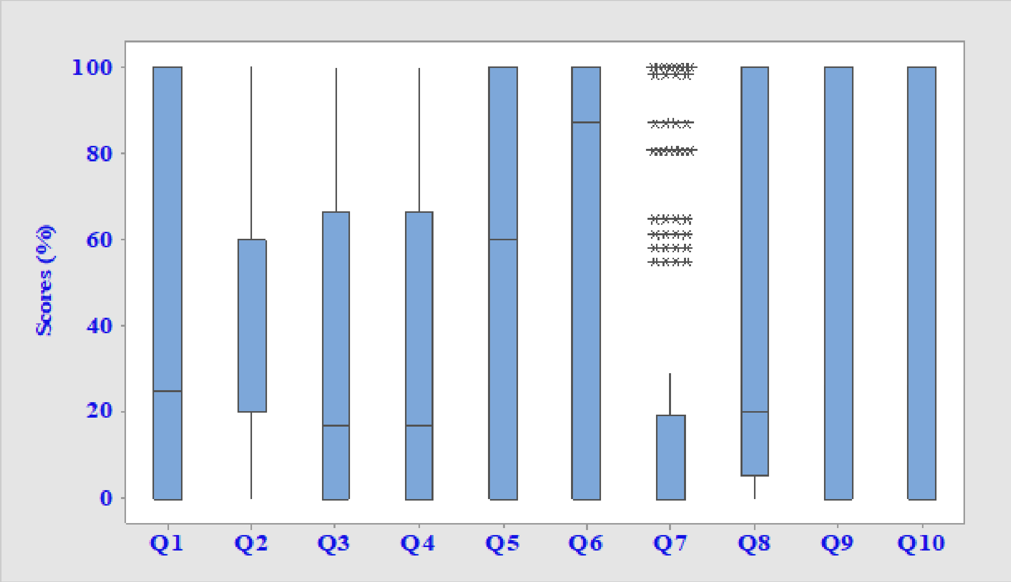
Source: Researcher's Computation (2020)

Figure 5: **Performance in Core, Elective and overall Trigonometry Questions**

ISMTs' mean score is averagely higher in Elective Mathematics items than in the Core Mathematics, demonstrating a quite strong knowledge of Elective Trigonometry than the Core. Hence, this suggests that ISMTs TCK is weaker as the items demand more conceptual understanding than formulae. The performance of ISMTs on individual questions in both Core and Elective Mathematics in presented in Figure 6.

Performance on Individual Questions (Core & Elective Trig test)

The analysis of ISMTs trigonometry content knowledge test scores per question is presented in Figure 6.



Source: Researcher's Computation (2020)

Figure 6: **Performance of ISMTs on Individual Questions**

It was observed that ISMTs performed slightly well in Question 6. This was followed by Question 5. Some extremely high marks were recorded in questions 1, 5, 6, 8, 9 and 10, but most questions (Q1, Q3, Q4, Q5, Q6, Q9 & Q10) also scored zero marks out of 100 marks in the trigonometry content knowledge test items. The ISMTs perform woefully in Questions 7 and 8 as a whole. It was seen that most of the high marks were in the Elective Mathematics trigonometry related questions as compared to the Core Mathematics ones. In particular, most of the participants had difficulties reasoning about the tasks on trigonometric concepts that demanded more flexible cognition (for example, TCK-T 2, 3, 8) than those that simply required recall of rules (for example, TCK-T 6, 7, see Appendix D, Section B). Although the results look faintly satisfactory, the ISMTs were limited to a certain extent. Less than 50% were recorded on most TCK questions, and the overall average score was 40.6%. The scores were not good enough since the test was based on the basic competencies that were indicated in the Ghanaian Mathematics syllabi that they teach students, as well as WASSCE past questions. If teachers are unsuccessful in some questions based on what they teach to students, then it can be debated that their trigonometrical content knowledge is limited. Thus, it can be further argued that the ISMTs teachers have fair Trigonometry Content knowledge.

RQ 2: What is ISMTs' awareness of trigonometrical errors of students?

The main aim of Part II of the TCKD-T is to investigate the ISMTs' PCK: awareness of student errors, misconceptions, and difficulties. The analysis of Pedagogical Understanding components focused on secondary school mathematics teachers' knowledge of trigonometry errors using hypothetical students' solutions to some trigonometry questions. This was about how students responded to some trigonometric questions, and ISMTs were supposed to comment on their solutions to see whether the students answered them correctly or not, and what they planned to do to address them if they noticed any errors or misconceptions. The PCK category of the teacher's assessment of students' thinking led the analysis. They included detecting pupils' faults, determining the reasons of the issues, and developing suitable ways for dealing with such mistakes (Krauss, Neubrand, Blum & Baumert, 2008). Hence, in this study, ISMTs were presented with two tasks. Each task had two hypothetical students' solutions (Student A & B) to trigonometric concepts. Each student solution was examined by ISMTs to identify, evaluate, or detect a common student error or a particular understanding challenge (Krauss et al., 2008a). This is because the ability to analyze students' thinking is a vital element of teachers' pedagogical skills. ISMTs were given three sub-questions for each task (see Appendix D, Part II of Section B) on the three aspects of PCK (Krauss et al., 2008a). The sub-questions under each task were as follows:

a) Do you see anything wrong with the student solution? Yes/NO, and identify the errors (if any)?

b) Suggest reasons or causes for student's errors and misconceptions (if any)

c) How would you help the students to understand the concepts if there are errors and present the correct solutions (if any)?

These three PCK components were used as themes to examine how a teacher analyzed students' thinking on each activity. Table 9 shows the findings of the question concerning whether the ISMTs detected anything wrong with the question, and Table 10 shows their responses to the individual problems found.

Table 9: Hypothetical Student Solutions

	Students A Solution		Students B Solution	
Task 1	Yes	No	Yes	No
Do you see anything wrong with the student Solution?	96 (43.6%)	124 (56.4%)	76 (34.5%)	144 (65.5%)
Tasks 2				
Do you see anything wrong with the student Solution?	112 (50.9%)	108 (49.1%)	92 (41.8%)	128 (58.2%)

Source: Researcher's Computation (2020)

The findings from Table 9 showed that ISMTs reported on different issues during analysis of the students' solutions. In answering the question, *"Do you* *see anything wrong with the students' solution?"*, the results indicated that Task 1 was poorly answered with 43.6% and 34.5% of the ISMTs being able to answer it correctly for Student A and B Solutions respectively. For Task 2, 50.9% and 41.8% of the participants responded in the affirmative. The findings appeared to show that most of the ISMTs found nothing wrong with both solutions presented to them. For the few who noticed that there was somethings wrong with both solutions, in explaining or identifying specifically where students made mistakes, only pointed out one mistake even though the solutions contained several errors (see Table 10). The results also showed there were several statements not clearly identified as being wrong by all the ISMTs. For example, in Task 1 (see Appendix D, Part II of Section B), most of them only recognized the use of the 'Sin' as a common factor in Student A and the incorrect use of the distributive property of the Trigonometric function in Student B.

The ISMTs did not comment on the statement of how Student A changed "1/Sin" into "Sin^{-1}" and rendered the answer not judicious. Likewise, in Task 2 (see Appendix D, Part II of Section B), for "Student A", most of the ISMTs recognized the misapplication of the distributive property to expand the cosine of compound angles, while the majority of them did not identify the step where $\cos\theta + \cos\theta \neq \cos 2\theta$ and also the step where the student divided both sides of the equation using "2" in the cosine double angle $(\cos 2\theta)$ resulting in an incorrect answer. The results demonstrate that these teachers were only able to identify errors in statements that were easy to understand and had difficulty determining learners' errors and misconceptions about trigonometry. Refer to Appendix H for the categorization of errors in hypothesized students' solutions.

Table 10: Identification of Errors

		Categorization of Errors					
		Error 1		Error 2		Error 3	
Tasks	Students	Yes (%)	No (%)	Yes (%)	No (%)	Yes (%)	No (%)
Task 1	Student A	21 (38.2)	34 (61.8)	20 (36.4)	35 (63.6)	*	*
	Student B	36 (65.5)	19 (34.5)	7 (12.7)	48 (87.3)	1 (1.8)	54 (98.2)
Task 2	Student A	28 (50.9)	27 (49.1)	4 (7.3)	51 (92.7)	1 (1.8)	54 (98.2)
	Student B	17 (30.9)	38 (69.1)	7 (12.7)	48 (87.3)	*	*

Source: Researcher's Computation (2020)

In terms of the reasons of the errors, the findings indicated that the teachers either described what the student was thinking or what information the student lacked. The teachers went on to discuss the causes of the mistakes based on how they were detected. The findings revealed that during solution analysis utilizing a procedural-based approach, teachers primarily ascribed the causes of students' failures to a lack of grasp of either a process or a concept, but not both. The teachers' claimed causes are connected to a lack of comprehension of trigonometric topics such as trigonometry inverse and compound angles. They thought the pupil comprehended the principles but used the incorrect technique. For example, some teachers explained wrongly that, in Task 1 [i.e., $\sin(\theta+1)=0.5$] of "Student's A" solution, the cause of the mistake is lack of knowledge of using the distributive property. That is, they expected students to use 'sin' as a monomial factor and distribute or separately apply it to each term of the binomial factor (θ^0+1). Others also simply stated *"improper expansion"* and *"inappropriate procedure".* This might imply that in some of the solutions, the teachers were not able to recognise whether the mistake was a result of a dearth of understanding of either the concept or procedure, since some of the teachers gave inaccurate suggestions or solutions to the problems. The findings revealed that ISMTs have difficulties in assessing learners' errors.

Also, on how to correct the errors and help the students understand the concepts, since most ISMTs did not find anything wrong with both solutions as presented in Table 10 and 11, surprisingly, for that matter, they did not even bother correcting the errors to help the students. Besides, the ISMTs, instead of suggesting correct alternative explanations that would make the student understand the problem and eliminate the errors, generally ignored them or responded with how the students should have solved the problem. Interestingly, analysis of the overall performance of ISMTs in presenting the correct solution to the questions (Tasks 1 & 2) revealed their difficulty in providing different representations for trigonometrical situations. This is because the mean score on Task 1 was 43.65% for PCK questions, while Task 2 recorded the lowest (28.05%). This means the performance of ISMTs is not good because they were unable to reach half of the scores (50%) of the knowledge of trigonometry errors and misconceptions which described their PCK. This result also seems to suggest that ISMTs have insufficient knowledge of content and students, which might affect their pedagogical understanding required to teach the SHS trigonometry in the Ghanaian core and elective mathematics syllabi.

On the performance of ISMTs on Tasks 1 and 2 (Task 1 is on Core Mathematics and Task 2 is on Elective Mathematics trigonometry). The mean and SD scores are presented in Table 11.

Table 11: Tasks 1 and 2 Descriptive Statistics

Tasks	Mean	SD
Task 1	43.64	49.71
Task 2	28.05	44.36

Source: Researcher's Computation (2020)

The result shows that the ISMTs performed woefully on the two tasks (Tasks I & II). The high variabilities recorded stress more on the unsuccessful performance of the ISMTs on the Trigonometric Diagnostic Knowledge Tests.

RQ 3: What is In-Service Mathematics Teachers TSE for SHS?

The aim of this research question was to determine the Trigonometry Self-Efficacy (TSE) of ISMTs in their bid to accomplish the specified objectives for teaching the SHS Trigonometry content. In accomplishing this, they were asked to rate their level of confidence on a 5-point Likert scale questionnaire "(1 = Not At All Confident; 2 = Only a Little Confident; 3 = Somewhat Confident; 4 = Confident; 5 = Very Confident)." The ISMTs were provided with statements describing the TSE components in the Mathematics Curriculum (Elective & Core) from a broader perspective. However, to scale down the enormous set of items to a smaller but more controllable size while maintaining as much of the original information as possible (Field, 2013), Exploratory Factor Analysis (EFA) was used. This is because the TSE items were many and there was a need for scale construction.

Table 12 illustrates the results of the KMO and Bartlett's Sphericity tests. This measures the suitability of the various constructs for EFA. KMO values of 0.50 and above (Mumford, Ferron, Hines, Hogarty, & Kromrey, 2003) and Bartlett's test of Sphericity (Tabachnick & Fidell, 2007) were used to assess the EFA's suitability for the items, both of which reached statistical significance (P=0.000). The results of the EFA take into consideration the TSE components, factor loadings, as well as Cronbach's Alpha (reliability) values of the constructs. For this analysis, only items with factor loadings of 0.50 or more were retained for further analysis. Thus, items that recorded or received factor loadings of less than 0.50 were discarded.

Table 12: KMO and Bartlett's Tests

Constructs		Statistic
Trigonometry I:		
KMO Measure of Sampling Adequacy		0.790
Bartlett's Test of Sphericity	Approx. Chi-Square	943.775
	Degrees of Freedom	21
	p-value	0.000
Trigonometry II:		
KMO Measure of Sampling Adequacy		0.718
Bartlett's Test of Sphericity	Approx. Chi-Square	487.040
	Degrees of Freedom	6
	p-value	0.000
Trigonometric Ratios and Rules:		
KMO Measure of Sampling Adequacy		0.820
Bartlett's Test of Sphericity	Approx. Chi-Square	1902.554
	Degrees of Freedom	78
	p-value	0.000
Trigonometric Compound and Multiple Angles:		
KMO Measure of Sampling Adequacy		0.796
Bartlett's Test of Sphericity	Approx. Chi-Square	511.206
	Degrees of Freedom	6
	p-value	0.000
Trigonometric Functions, Equations and Graphs:		
KMO Measure of Sampling Adequacy		0.840
Bartlett's Test of Sphericity	Approx. Chi-Square	1381.464
	Degrees of Freedom	28
	p-value	0.000

Source: Researcher's Computation (2021)

In all, 5 subscales were used. The variables (TSE Components) in the study are; Trigonometry I and II (Core Mathematics Trigonometry); Trigonometric Ratios and Rules; Trigonometric Compound and Multiple Angles; and Trigonometric Functions, Equations, and Graphs (Elective Mathematics Trigonometry). All 36 items were chosen as having high loadings on extracted factors. The results of the TSE components, factor loadings, and the Cronbach Alpha values of the 5 factors are presented in Tables 13 to 17.

Table 13: Reliability for Trigonometry I

Items	Factor Loading	Reliability
Assist pupils in defining trigonometric ratios using appropriate diagrams.	0.865	
Instruct pupils to create an equilateral triangle with two dimensions (e.g., two units) and use it to calculate the trigonometric ratios for 30o and 60^{o}.	0.766	
Assist students with drawing a square with one unit side, one diagonal, and using the diagonal and two sides to calculate the value of trigonometric ratios of 45^{0}.	0.847	
Instruct pupils to use their calculators to calculate trigonometric ratios for specified angles between 0^{0} and 360^{o}.	0.781	0.860
Use tables or calculators to help pupils discover the inverse of specified trigonometric ratios.	0.791	
Using graphics, explain to pupils what angles of elevation and angles of depression are.	0.697	
Set real-life challenges utilizing trigonometric ratios for pupils to solve.	0.572	

Source: Researcher's Computation (2020)

Table 14: Reliability for Trigonometry II

Items	Factor Loading	Reliability
Assist pupils in creating tables for the following trigonometric functions	0.876	
Assist pupils with drawing graphs of functions and determining maximum and minimum values using their tables.	0.799	0.833
Assist pupils in creating basic graphs of trigonometric functions	0.937	
Encourage pupils to use their graphs to answer problems	0.719	

Source: Researcher's Computation (2020)

Table 15: Reliability (Trig Ratios &Rules)

Items	Factor Loading	Reliability
Help pupils review the three fundamental trigonometric ratios: sine, cosine, and tangent.	0.629	
Show pupils how to use quadrants to get fundamental trigonometry ratios.	0.726	
Assist pupils in determining the reciprocals of trigonometric ratios.	0.804	
Assist pupils in converting trigonometric ratios to Cartesian coordinates of a circle point (x, y).	0.740	
Assist pupils in determining trigonometric identities.	0.757	
Assist pupils in developing the understanding of negative angles and establishing the following relationships.	0.678	
Encourage pupils to use the calculator to confirm the relationships listed in 17 above.	0.742	0.910
Using the connection, assist students in obtaining radian equivalents for angles in degrees and vice versa.	0.771	
Assist pupils in determining the sine rule.	0.715	
Instruct pupils on how to apply the sine rule to solve related issues.	0.715	
Assist students with deriving the cosine rule and applying it to triangle situations.	0.696	
Assist pupils in using the cosine rule to solve triangle issues.	0.795	
Assist pupils in using the sine and cosine laws to solve bearing difficulties (real-life problems)	0.620	

Source: Researcher's Computation (2020)

Table 16: Reliability for Trigonometric Compound and Multiple Angles

Items	Factor Loading	Reliability
Assist pupils in determining the identities of compound angles.	0.721	
Assist pupils in using their identities to solve difficulties.	0.891	
Assist students with determining the double angle identities for Sin2A, Cos2A, and Tan2A and using them to develop identities for Sin3A and Cos3A in terms of sin A and cos A.	0.910	0.871
Encourage pupils to check these relationships using the calculator and particular values.	0.886	

Source: Researcher's Computation (2020)

Table 17: Reliability for Trigonometric Functions, Equations, and Graphs

Items	Factor Loading	Reliability
Examine sin x and cos x graphs.	0.789	
Assist pupils with drawing the tan x graph and comparing it to the sine and cosine graphs.	0.869	
Encourage pupils to study the nature of trigonometric function graphs using a calculator and a computer.	0.750	
Assist pupils in locating solution sets for trigonometric equations up to quadratic equations.	0.873	
Encourage pupils to utilize the calculator and computer to graph trigonometric functions and solve them (graphical approach)	0.698	0.906
Examine trigonometric function graphs of the type,	0.884	
Instruct pupils on how to express the trigonometric function.	0.677	
Assist pupils in calculating the maximum and lowest points of the function using the result.	0.827	

Source: Researcher's Computation (2020)

From Tables 14 to 18, none of the items have been discarded because they all recorded factor loadings of more than 0.50, with very high reliability for each construct or subscale. Table 18 shows the descriptive statistics of the 5 constructs.

Distribution of ISMTs' Responses on TSE

The assessment of the TSE of ISMTs using these critical TSE components was done through Ranking Analysis, a model adopted by Wang and Yuan (2011) in their analysis of factors affecting risk attitudes. To classify and establish a list of the TSE components of ISMTs to assess their level of confidence or effectiveness in teaching trigonometry to SHS students, means, standard deviations (SDs), and rankings of each item were used to screen them from the sample data obtained. Drawing from previous studies (Ikediashi et al., 2012; Wang & Yuan, 2011), a mean value of 3.0 was acknowledged as the benchmark where risk factors are classified as critical when the mean values are greater than or equal to the benchmark. This benchmark has been adopted in the current study. Hence, a TSE component with the least SD is considered more critical in the event where at least two TSE components have the same mean values (Wang & Yuan, 2011). This is because the items are on a five-point Likert scale. The result is in Table 18.

Table 18: Distribution of ISMTs' Responses on SHS TSE

Subscales/Constructs	Mean	SD	Ranking
Trigonometry I	4.387	0.627	1
Trigonometry II	4.341	0.709	2
Trigonometric Ratios and Rules	4.287	0.561	3
Trigonometric Compound and Multiple Angles	3.777	0.907	4
Trigonometric Functions, Equations and Graphs	3.739	0.848	5
Overall TSE	4.106	0.742	

Source: Researcher's Computation (2020)

Table 18 revealed that ISMTs are confident in teaching Trigonometry I (M=4.39; SD=0.84), Trigonometry II (M=4.34; SD=0.86), Trigonometric Ratios and Rules (M=4.29; SD=0.80), Trigonometric Compound and Multiple Angles (M=3.78; SD=1.07), and Trigonometric Functions, Equations, and Graphs (M=3.74; SD=1.09). This is because the mean value of each of these items is greater than 3.0. However, the ISMTs appear to be much more efficacious when it comes to teaching students Trigonometry I and II, and Trigonometric Ratios and Rules. This is because they recorded the highest mean. The least confident TSE components are Trigonometric Compound and Multiple Angles and Trigonometric Functions, Equations, and Graphs. The overall TSE (M=4.11, SD=0.93) further explained that the majority of the ISMTs seemed confident in accomplishing the stated objective of teaching both Core and Elective SHS trigonometry content. In other words, these high confidence levels of ISMTs are anticipated to help them teach or guide students to understand most of the SHS trigonometry content. Nevertheless, it was also be observed that, as the level of TSE components increased from Trigonometry I and II (Core Mathematics Items) through to Trigonometric Functions, Equations and Graphs (Elective Mathematics Items), the confidence level of ISMTs decreased on a whole. Thus, ISMTs seemed much more confident in teaching Core Mathematics Trigonometry-related items compared to Elective Trigonometry.

H 1: There is no statistically significant difference between ISMTs TSE Levels

To examine whether the differences in efficacy levels were statistically significant, and the particular efficacy level at which ISMTs were highly efficacious, Analysis of Variance (ANOVA), a post-hoc test, was used to identify the particular items that teachers were least and most confident in teaching and their reasons for such choices. The result is presented in Table 19.

Table 19: Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Factor	4	90.160	22.540	40.940	0.000
Error	1095	602.840	0.551		
Total	**1099**	**693.000**			

Source: Researcher's Computation (2020)

As mentioned earlier, the ISMTs appeared to have a higher efficacy or confidence level on some components than others, and this has been confirmed by the ANOVA results. Thus, (F-value =40.940; df_1=4; df_2=1095; p-value <0.000). These values indicate that there are differences in the effectiveness of the ISMTs on the various components of TSE. To get the particular components of the TSE that the ISMTs are highly efficacious at teaching, the post-hoc test by Fisher was used. The result is presented in Table 20.

Table 20: Grouping Information Using the Fisher LSD Method

Factor	N	Mean	Groupings	
Trigonometry I	220	4.387	A	
Trigonometry II	220	4.341	A	
Trigonometric Ratios and Rules	220	4.287	A	
Trigonometric Compound and Multiple Angles	220	3.777		B
Trigonometric Functions, Equations and Graphs	220	3.739		B

Source: Researcher's Computation (2020)

NB: Means that do not share a letter are significantly different.

Table 19 revealed there were statistically significant differences in ISMT TSE levels. This confirmed the earlier assertion that as the level of TSE components increased, the confidence level of ISMTs decreased as a whole. Thus, ISMTs were much more confident in teaching Core Mathematics Trigonometry-related items compared to Elective Mathematics.

Mathematics teachers generally expressed high confidence in their ability to teach all Trigonometry topics. However, it was observed in Table 18 that ISMTs exhibited a decline from Core TSE to Elective TSE. This stressed the need to carry out a detailed inquiry to determine the specific trigonometry objectives or contents in the Core and Elective Mathematics syllabi where ISMTs may have weak or strong confidence in teaching and assisting students. These specific analyses are presented in Table 21. It was seen that ISMTs were less efficacious at teaching trigonometry by relating it to real-life problems in both Core and Elective.

Table 21: Least and Most Confident TSE Objectives

Subscales	Least Confident Items	%	Mean	SD	Most Confident Items	%	Mean	SD
				Core Mathematics (Trigonometry)				
Trigonometry I	Pose problems of real life situations involving trigonometric ratios for students to solve.	41.8	3.96	1.01	Discuss with students what angles of elevation and angles of depression are using diagrams.	20.0	4.52	0.66
Trigonometry II	Guide students to use their graphs to solve equations such as: $a\sin x + b\cos x = 0$, $a\sin x + b\cos x = k$, etc	34.5	4.24	1.01	Guide students to prepare tables for given trigonometric functions for $y = a\sin x$ and $y = b\cos x$, where, $0^0 \le x \le 360^0$	45.5	4.32	0.91
				Elective Mathematics (Trigonometry)				
Trigonometric Ratios and Rules	Assist students to apply the sine and cosine rules to solve problems involving bearings (real life problems)	27.3	4.146	0.885	Assist students to revise the three basic trigonometric ratios; sine, cosine and tangent	18.2	4.509	0.712
Trigonometric Compound and Multiple Angles	Guide students to derive the compound angles identities: $\sin(A \pm B) = \sin A\cos B \pm \sin B\cos A$ $\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$ $\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$	56.4	3.382	1.139	Encourage students to use the calculator and specific values to verify Sin2A, Cos2Aand Tan2A and for Sin3A and Cos3A	50.9	4.055	1.037
Trigonometric Functions, Equations and Graphs	Guide students to express the trigonometric function, $f(x) = a\sin x + b\cos x$ in the form, $R\cos(x \pm a)$ or $R\sin(x \pm a)$, where, $0^0 \le a \le 90^0$.	41.8	3.400	1.187	Revise graphs of sin x and cos x	27.3	4.109	1.005

Source: Researcher's Computation (2020)

The reasons given by the ISMTs for their least confidence level on these items include; difficult/challenging/confusing/time-consuming, lack of in-depth knowledge of the subject matter, little or no expertise with trigonometrical technology, no/limited previous experience teaching trigonometry to students, as well as limited knowledge of students misconceptions. Specifically, difficult/challenging/confusing/time-consuming is reflected by the following comments from some selected teacher. For instance, ISMT 4 selected items 7 and 25 as difficult and specifically gave the following reason. For item 7, he stated that,

> *"I have not tried using real life situations to teach trig because I find it difficult to do so".*

For that of item 25, he again stated that,

> *"Because their proving are not direct application of distributive property of operations, hence, making it confusing".*

Another teacher (ISMT 29) also selected item 35 as a challenging because he was of the view that,

> *"Teaching Trig. ratios in SHS is challenging so we teach in the instrumental way".*

For lack of in-depth knowledge of the subject matter as one of the major themes, ISMT 12 also selected item 11 as the one he lacks in-depth trigonometry content knowledge in carrying out, and provided the following reason,

> *"Because it requires knowledge of other topics which I sometimes find it difficult to learn. Also, I don't feel confident with graphs of trig, where we compare the given equation and*

original equation. That is, guiding students to use their graphs drawn to solve other equations"

Another teacher (ISMT 27) chose item 33 and gave the following reason,

"Using computer/calculator is a little challenging for me"

This reason reflected the thematic area of little or no expertise with trigonometrical technology. Furthermore, no or limited previous experience teaching trigonometry to students has also been identified as one of the themes. With respect to this, two teachers chose two different items but provided same reasons to that effect. ISMT 1 selected item 7 while ISMT 28 selected item 35. However, both of them gave the following reason,

"I don't normally teach it / I have not done enough preparations on it for sometime"

Finally, regarding the theme, limited knowledge of students misconceptions, ISMT 53 selected item 25 with the following reason,

"Proving of the compound angle which is a lengthy formula with students makes the class objective harder to achieve at the end of the day because you won't know what to link it to for them to understand"

That is, the above are specific excerpts from the ISMTs on the reason they selected certain items as less confident in teaching students.

Also, the reasons given by the ISMTs for their choices of some of the items as the ones they are much more efficacious at teaching students include; ease of teaching due to experience, engagement with technology, in-depth trigonometrical content knowledge, view of trigonometry as a set of procedures as well as extended experiences with content from training. However, some specific statements on the reasons why ISMTs selected such items as most confident in teaching are presented below. For instance, the statement,

> *"I have been teaching all that (ITEM 19) for some years now (about 5 years*)"

was stated by ISMT 123 as the reason why he chose item 19 as most confident he can teach students. This reflected the theme, easiness from teaching experience. Also, regarding engagement with technology as a theme, ISMT 6 opted for item 4 with the simple reason he gave that,

> *"I will go with number 4 because I will be able to take students through comprehensive usage of calculators including how to use calculator to find trig ratios"*

In a similar fashion, ISMT 91 also selected item 28 under the same theme and gave this reason,

> *"I am competent in the use of the calculator, beside, the process is less tedious since students will be required to verify the relations themselves"*

Another theme identified is in-depth trigonometrical content knowledge, and for this, ISMT 91 again selected item 1, and gave a reason that,

> *"I possess the content knowledge in carrying out ITEM 1"*

Furthermore, concerning the theme on viewing of trigonometry as a set of procedures, ISMT 53 again chose item 27 and provide the following reason,

> *"I can assist students to derive the double angle because it follows simple application of compound angles"*

Extended experiences with content from training (in-service or personalized) was the final theme on the reasons given by ISMTs for choosing such items as most important. On this theme, ISMT 205 for instance opted for item 21 with the reason stated below,

> *"I always use the diagram to explain the formula to students and how it is applied because I have done enough preparations on them for some time now"*

That is, the above are specific excerpts from the ISMTs on the reason they selected certain items as most confident in teaching students.

In summary, it was observed that ISMTs performed better on Elective Mathematics trigonometry questions than in the core. On the contrary, their self-efficacy was high (efficacious) on the Core Mathematics trigonometry items than in electives. These results aligned with the common belief that people generally perceive Core Mathematics content to be easy to comprehend and, hence, believe they can easily accomplish any required tasks. This could explain why ISMTs expressed high efficacy in the Core Mathematics trigonometry content than in electives. The Core Mathematics content is such that it requires actual conceptual understanding of the underlying principles, properties, relationships, and the inter-connectedness of all valid concepts, theorems, and algorithms. The memorization of formulas and uninformed following of already existing procedures is not a meaningful strategy to effectively learn and understand the concepts of Core Mathematics.

Although ISMTs expressed high confidence in the Core Mathematics trigonometry content areas, their low mean scores could be explained by their weak conceptual understanding of the required concepts. This means, ISMTs generally believed in their high ability to the Core Mathematics content, hence they recorded high efficacy, but their actual conceptual abilities were the contrary, as evident in their TCK scores. The Core Mathematics content may be perceived as easy, but it requires deeper conceptual abilities to perform better in it. Therefore, ISMTs' self-efficacy and achievements (conceptual abilities) were indirectly proportional, since they expressed being very efficacious but revealed abysmal content abilities due to low understanding of the underlying principles, properties, and relationships of the core mathematics trigonometry concepts.

On the other hand, ISMTs expressed low self-efficacy in the Elective Mathematics trigonometry content but recorded fairly high TCK mean scores. Inasmuch as concepts of Elective Mathematics equally require conceptual understanding to excel, there is an appreciably good degree of formulas and procedural computations that may enable one to record high scores in its content but with low comprehension and, hence, low self-efficacy.

Therefore, the results explained the true reflection of ISMTs' perceived confidence in the Core and Elective Mathematics trigonometry content and their knowledge levels of the concepts. The ISMTs were very confident in their abilities to solve Core Mathematics trigonometry problems, but their true knowledge levels were very low. One needs good conceptual knowledge to excel in it, since it is requires fewer formulae and procedural computations, and that was not realized from the responses of the ISMTs. Also, ISMTs may have expressed low self-efficacy in Elective Mathematics trigonometry since it is often perceived to be a more cognitively demanding field, but recorded fairly high TCK mean scores as a result of its formulae-oriented, rigorous procedures nature, which ISMTs could easily follow to perform tasks without understanding the actual conceptual relationships.

RQ 4: What are the challenges ISMTs encounter in teaching trigonometry?

A lot of factors affect ISMTs, especially in the teaching of Trigonometry. It is therefore practical on the part of the researcher to consider some of these critical challenges for better and more pragmatic solutions, hence the aim of this research question. Overlooking some of these challenges identified through this study may deepen the woes of ISMTs as well as students concerning their performance on trigonometry-related questions. The ISMTs were provided with statements describing some of these challenges. In accomplishing this, they were to indicate on a 5-point Likert scale questionnaire whether they “(1 = Strongly Disagree; 2 = Disagree; 3 = Neutral; 4 = Agree; 5 = Strongly Agree)”. However, Exploratory Factor Analysis (EFA) was used to reduce the enormous set of items to a smaller but more controllable size while retaining as much of the original information as possible (Field, 2013). This is because the items were many and there was a need for scaled construction. The results of the KMO and Bartlett’s tests are presented in Table 22. These tests measure the appropriateness of the various constructs of the challenges for the EFA. This appropriateness is measured by the KMO values of 0.50 or more (Mumford, Ferron, Hines, Hogarty, & Kromrey, 2003) and Bartlett’s Test of Sphericity (Tabachnick & Fidell, 2007), all reaching statistical significance.

Table 22: KMO and Bartlett's Test

Tests		Statistic
KMO Measure of Sampling Adequacy		0.634
Bartlett's Test of Sphericity	Approx. Chi-Square	896.305
	Degrees of Freedom	66
	p-value	0.000

Source: Researcher’s Computation (2020)

The results of the EFA take into consideration the challenges, factor loadings, and Cronbach’s Alpha values of the constructs. For this analysis, only items with factor loadings of 0.50 or more were retained for further analysis. Thus, items that recorded or received factor loadings of less than 0.50 were discarded. The same applies to reliability values. Table 22 summarizes the results of the KMO and Bartlett tests.

Table 23: Reliability for Challenges Constructs

Constructs/ subscales	Items	Factor Loadings	Reliability
Difficulty and Problem of Teaching Trigonometry	Problems of implementing new teaching strategies	0.505	0.726
	Inadequate pre-service training towards the teaching of trigonometry	0.518	
	Difficulty in teaching trigonometry because students do not have the necessary relevance previous knowledge (RPK)	0.505	
	The inability of teachers to envisage trigonometry errors students may commit	0.514	
	The inability of teachers to link students RPK to trigonometry topics	0.555	
	Trigonometrical Knowledge and terminologies are difficult to understand	0.656	
Inadequate Instructional Resources	Inadequate teaching resources	0.644	0.805
	Insufficient funds for purchasing equipment and supplies needed in teaching Trigonometry	0.649	
	Inadequate in-service training on trigonometry concepts	0.668	
Limited Interest and Attention	Lack of teachers' interest in teaching trigonometry	0.552	0.305
	Students' viewing trigonometry as difficult, abstract, and boring	0.570	
Lack of confidence	Teachers not confident about some trigonometry content in the curriculum	0.921	-

Source: Researcher's Computation (2020)

In all, 4 subscales were used. The latent variables identified were; Difficulty and Problem of Teaching Trigonometry, inadequate instructional resources, limited interest and attention, and lack of confidence. All the 12 items were chosen as high loadings on the factors after an EFA. The results of the 4 latent factors are presented in Table 23. It was observed that none of the items had been discarded because they all recorded factor loadings of more than 0.50. However, the construct on limited interest and attention, reflected by *"Lack of teachers' interest in teaching trigonometry"* and *"Students' viewing trigonometry as difficult, abstract, and boring"*, recorded a reliability value of less than the 0.50 threshold and has therefore been discarded.

Distribution of ISMTs' Responses on Challenges Faced

Table 24 shows the descriptive statistics of the subscales. The evaluation of the challenges ISMTs encounter in teaching trigonometry is done through ranking analysis.

Table 24: Distribution of ISMTs' Responses on Challenges Faced

Subscales	Items	Mean	SD	Ranking
Lack of confidence	1	4.346	6.06	1
Inadequate Instructional Resources	3	4.188	0.928	2
Difficulty and Problem of Teaching Trigonometry.	6	3.346	1.086	3
Overall		3.960	2.691	

Source: Researcher's Computation (2020)

Table 24 revealed that ISMTs encountered the following challenges in teaching trigonometry; lack of confidence (M=4.35, SD= 6.06), inadequate instructional resources (M=4.19, SD= 0.93, and difficulty and problems with teaching trigonometry (M=3.35, SD= 1.09). This is because the mean value of each of these constructs is greater than 3.0. However, the ISMTs faced challenges of *confidence and inadequate instructional resources* when it came to teaching trigonometry. This is because they recorded the highest mean values and corresponding SDs. The least of the challenges was *difficulty and problem of teaching trigonometry*. The overall (M=3.960, SD=2.691) further explained that teachers faced a varying degree of challenges in teaching their students trigonometry and its concepts.

In summary, the study showed that lack of confidence, inadequate instructional resources, and difficulty and problems with teaching trigonometry were the major challenges ISMTs faced when it comes to teaching trigonometry.

RQ 5: What are the ISMTs' PD needs for teaching trigonometry?

The goal of this research question was to determine the Teacher Professional Development (TPD) needs for teaching trigonometry. In achieving this, ISMTs were to indicate on a 5-point Likert scale questionnaire "(Not Familiar =1; No Need=2; Little Need=3; Moderate Need=4; and Great Need=5)". The ISMTs were provided with statements describing the TPD needs for teaching trigonometry effectively. However, EFA was used to reduce the enormous set of TPD needed items to a smaller but more controllable size while maintaining as much of the original information as possible (Field, 2013). The results of the KMO and Bartlett's test of Sphericity are presented in Table 25. These measure the suitability of the various constructs for EFA. KMO values of more than 0.50 (Mumford, Ferron, Hines, Hogarty, & Kromrey, 2003) and Bartlett's Test of Sphericity (Tabachnick & Fidell, 2007) were used, both of which reached statistical significance. The results of the EFA take into consideration the TPD needs, constructs, factor loadings, and Cronbach Alpha values of the constructs. For this analysis, only items with factor loadings of

0.50 or more were retained for further analysis. Thus, items that recorded factor loadings of less than 0.50 were discarded. The results are presented in Table 25.

Table 25: KMO and Bartlett's Tests

Constructs/Subscales		Statistic
Planning Trigonometric Instructions		
KMO Measure of Sampling Adequacy		0.851
	Approx. Chi-Square	1992.206
Bartlett's Test of Sphericity	Degrees of Freedom	55
	p-value	0.000
Delivering Trigonometric Instructions		
KMO Measure of Sampling Adequacy		0.832
	Approx. Chi-Square	2332.816
Bartlett's Test of Sphericity	Degrees of Freedom	91
	p-value	0.000
Managing Trigonometric Instructions		
KMO Measure of Sampling Adequacy		0.696
	Approx. Chi-Square	324.103
Bartlett's Test of Sphericity	Degrees of Freedom	6
	p-value	0.00
Teacher Self-Improvement in Trigonometric Content		
KMO Measure of Sampling Adequacy		0.875
	Approx. Chi-Square	1117.214
Bartlett's Test of Sphericity	Degrees of Freedom	15
	p-value	0.000
Use of Information and Communication Technology		
KMO Measure of Sampling Adequacy		0.734
	Approx. Chi-Square	456.303
Bartlett's Test of Sphericity	Degrees of Freedom	6
	p-value	0.000
Teacher Self-Improvement in Trigonometric Pedagogy		
KMO Measure of Sampling Adequacy		0.823
	Approx. Chi-Square	2003.966
Bartlett's Test of Sphericity	Degrees of Freedom	66
	p-value	0.000
Preparation and Utilization of Teaching Materials/Aids		
KMO Measure of Sampling Adequacy		0.693
	Approx. Chi-Square	183.860
Bartlett's Test of Sphericity	Degrees of Freedom	10
	p-value	0.000
Diagnosing and Evaluating Learning		
KMO Measure of Sampling Adequacy		0.766
	Approx. Chi-Square	355.144
Bartlett's Test of Sphericity	Degrees of Freedom	6
	p-value	0.000

Source: Researcher's Computation (2021)

In all, 8 constructs were developed and used: The variable's (TPD Needs) constructs in the current study are; *Planning Trigonometric*

Instructions; Delivering Trigonometric Instructions; Managing Trigonometric Instructions; Teacher Self-Improvement in Trigonometric Content; Teacher Self-improvement in Trigonometric Pedagogy; Use of ICT; Preparation and Utilization of Teaching Materials/Aids (Manipulatives); Diagnosing and Evaluating Learning. The results of the TPD Need constructs, factor loadings, and Cronbach's Alpha values are presented in Tables 26 to 33.

Table 26: Planning Trigonometric Instructions

Items	Factor Loadings	Reliability
Creating trig instructions based on student readiness data	0.771	
Material selection for teaching	0.756	
Creating an adequate teaching-learning environment, techniques, and resources	0.893	
Determine learning objectives (i.e., outcomes) that outline the information required by Trigonometry students.	0.842	
Determine learning objectives (i.e., outcomes) that outline the attitudes that students must adopt regarding trigonometry.	0.844	
Determine learning objectives (i.e., outcomes) that outline the abilities that students must gain in Trigonometry.	0.863	0.928
Determine acceptable learning objectives (i.e., outcomes) for encouraging multicultural approaches of learning in Trigonometry.	0.768	
Create lesson plans (learning activities) that include the history of trigonometry.	0.780	
Create lesson plans (learning activities) that integrate Trigonometry with other subjects.	0.792	
Choose commercially produced instructional resources for Trigonometry (e.g., textbooks, charts, models, etc.).	0.672	
Create lesson plans (i.e., learning activities) for Trigonometry subtopics.	0.833	

Source: Researcher's Computation (2020)

Table 27: Delivering Trigonometric Instructions

Items	Factor Loadings	Reliability
Encourage students to study trigonometry.	0.779	
In Trigonometry, use an inquiry/discovery teaching strategy (i.e., method).	0.845	
In the Trigonometry educational lessons, use hands-on teaching approaches.	0.794	
In Trigonometry, demonstrate process abilities (e.g., generalizing, defining, etc.).	0.786	
In Trigonometry, demonstrate manipulative abilities (e.g., measuring).	0.864	
Apply Trigonometry ideas to the learners' daily lives (i.e., to real-life situations)	0.701	
Conduct a field excursion to help students study Trigonometry more effectively.	0.751	
In Trigonometry, use teaching approaches (i.e., procedures) that allow you to focus on educating individuals rather than the entire class.	0.705	0.909
For teaching big courses in Trigonometry, use instructional methodologies (i.e., methods).	0.819	
In Trigonometry, use instructional methods that require students to teach each other (i.e., peer tutoring).	0.790	
To enhance Trigonometry instruction, use audio-visual technology (e.g., overhead projector, cassette or video recorder, etc.).	0.849	
Trigonometry may be taught using computers.	0.782	
In your Trigonometry lesson, maintain learner discipline.	0.744	
Evaluate your teaching efficiency as a Trigonometry instructor.	0.673	

Source: Researcher’s Computation (2020)

Table 28: Managing Trigonometric Instructions

Items	Factor Loadings	Reliability
Maintain learner discipline in your class	0.856	
Evaluate your teaching effectiveness as a teacher	0.885	
Organize and manage physical space (e.g., learner desk placement, etc.) in the Trigonometry classroom.	0.790	0.780
In trigonometry, use a computer to aid organize teaching (e.g., storing student records).	0.576	

Source: Researcher's Computation (2020)

Table 29: Teacher Self-Improvement in Trigonometric Content

Items	Factor Loadings	Reliability
Update your understanding about Trigonometry-related job options for students.	0.763	
In Teaching Trigonometry, refresh your understanding of successful teaching techniques (i.e., strategies).	0.873	
Update your understanding on Trigonometry-related societal issues (economics, electrification, etc.)	0.929	0.928
Update your understanding of how students learn trigonometry in a global culture.	0.895	
Update your learning expertise to include a constructivist approach to Trigonometry learning.	0.884	
Update your understanding of how Trigonometry is applied in society.	0.809	

Source: Researcher's Computation (2020)

Table 30: Teacher Self-improvement in Trigonometric Pedagogy

Items	Factor Loadings	Reliability
Improve your ability to identify and rectify frequent trigonometry misunderstandings and mistakes.	0.796	
Improve your Trigonometry content understanding.	0.905	0.843
Update your knowledge and skills in Trigonometry	0.901	
Update your knowledge of the history of Trigonometry	0.693	

Source: Researcher's Computation (2020)

Table 31: Use of ICT

Items	Factor Loadings	Reliability
To enhance Trigonometry instruction, use audio-visual technology (e.g., overhead projector, cassette or video recorder, etc.).	0.860	
Computers are used to teach Trigonometry.	0.832	
In Trigonometry, use a computer to aid organize teaching (e.g., keeping student records).	0.846	
Choose supplementary materials (such as library and reference books, films, and so on) for teaching trigonometry.	0.868	
Internet use (selection of suitable websites, user groups/discussion, etc.)	0.747	
Presentation software such as PowerPoint	0.788	0.914
Spreadsheet such as Microsoft Excel for plotting statistical graphs	0.785	
Graphical calculator use	0.806	
Scientific calculator use	0.804	
Multimedia operation course (using digital video and/or audio devices in trigonometry)	0.842	
Subject-specific training using learning software to achieve particular subject maths objectives (e.g. tutorials, simulations, etc.)	0.737	
Course on pedagogical concerns with the incorporation of ICT into teaching and learning.	0.689	

Source: Researcher's Computation (2020)

Table 32: Preparation and Utilization of Teaching Materials/Aids

Items	Factor Loadings	Reliability
Develop own teaching materials/aids for Teaching Trigonometry	0.694	
Use hands-on teaching methods in the Trigonometry instructional lessons	0.755	
Demonstrate manipulative skills (e.g., use trigonometric models) in Trigonometry	0.628	0.685
Identify free and locally available Trigonometry teaching materials.	0.695	
Choose supplementary materials (such as library and reference books, films, and so on) for teaching trigonometry.	0.559	

Source: Researcher's Computation (2020)

Table 33: Reliability for Diagnosing and Evaluating Learning

Items	Factor Loadings	Reliability
Use performance records to diagnose trigonometric learning	0.791	
Interpret performance record to determine students readiness for trigonometric instruction	0.882	
Design assessment items that validly assess trigonometric instructions	0.842	0.835
Arrange informal assessment situations in trigonometric teaching	0.754	

Source: Researcher's Computation (2020)

It can be seen that none of the items have been discarded because they all recorded factor loadings of more than 0.50, which lies within the acceptable region or threshold. Table 34 shows the descriptive statistics of the 8 subscales or constructs.

Distribution of ISMTs' Responses on TPD Needs

The evaluation of the TPD needs of ISMTs for teaching trigonometry was done through ranking analysis. The result is presented in Table 34.

Table 34: Distribution of ISMTs' Responses on TPD Needs

Subscales	Mean	SD	Ranking
Teacher Self-improvement in Trigonometric Content	4.309	0.852	1
Teacher Self-improvement in Trigonometric Pedagogy	4.175	0.924	2
Preparation and Utilization of Teaching Materials/Aids	4.102	0.851	3
Use of ICT	4.046	1.044	4
Delivering Trigonometric Instructions	3.960	1.022	5
Planning Trigonometric Instructions	3.942	1.009	6
Managing Trigonometric Instructions	3.864	1.426	7
Diagnosing and Evaluating Learning	3.850	0.934	8
Overall TSE	4.031	1.008	

Source: Researcher's Computation (2020)

Table 34 reveals that ISMTs were in dire need of professional development programs in the following areas: Teacher Self-improvement in

Trigonometric Content (M=4.31; SD=0.85), Teacher Self-improvement in Trigonometric Pedagogy (M=4.18; SD=0.92); Preparation and Utilization of Teaching Materials/Aids (M=4.10; SD=0.85); Use of ICT (M=4.05; SD=1.04); Delivering Trigonometric Instructions (M=3.96; SD=1.02); Planning Trigonometric Instructions (M=3.94; SD=1.01); Managing Trigonometric Instructions (M=3.86; SD=1.43); and Diagnosing and Evaluating Learning (M=3.55; SD= 0.93). This is because the mean value of each of these constructs was greater than 3. The overall (M=4.03; SD=1.01) further explained that the ISMTs need assistance concerning the 8 professional development program areas for teaching trigonometry and its concepts.

Drawing from Table 34, the results indicated that all the 8 constructs were considered critical and were therefore maintained since they received mean values above the established yardstick (3.0): Thus, *"Teacher Self-improvement in Trigonometric Content", "Teacher Self-improvement in Trigonometric Pedagogy", "Preparation and Utilization of Teaching Materials/Aids" and "Use of Information* and *Communication Technology"* were considered as the top 4 constructs that received very high scores ranging from 4.10 to 4.31. These top 4 factors recorded relatively high scores exceeding the average value of all the mean values (4.03) and were therefore considered as key areas of TPD needs of ISMTs to help improve their teaching of trigonometry to learners. This construct, "*Teacher Self-improvement in Trigonometric Content knowledge,"* rates highly, owing to the vital function it plays in the development of other parts of trigonometrical knowledge for instruction. Thus, a teacher ought to be acquainted with subject matter knowledge to enable him/her to comprehensively transfer it to students.

Otherwise, the consequence may be that students perceive the topic as abstract, boring, and difficult, and this will hinder their performance in trigonometry and its related topics.

The second most critical TPD need construct that followed "Teacher Self-improvement in Trigonometric Content" was *"Teacher Self-improvement in Trigonometric Pedagogy",* as it recorded the next highest mean value. This was followed by *"Preparation and Utilization of Teaching Materials/Aids", "Use of ICT", "Delivering Trigonometric Instructions", "Planning Trigonometric Instructions", "Managing Trigonometric Instructions",* and "*Diagnosing and Evaluating Learning"* with their corresponding means and SDs. When it comes to TPD needs, ISMTs highly need support to effectively teach trigonometry in: Teacher Self-improvement in Trigonometric Content; Teacher Self-improvement in Trigonometric Pedagogy; Preparation and Utilization of Teaching Materials/Aids; and Use of ICT and others.

Discussion of Findings

Trigonometry Content Knowledge (TCK) of ISMTs

Regarding trigonometry content knowledge (TCK), ISMTs' scores as indicated in Figure 6 and Table 8 appeared not sufficient since the TCK test questions were set based on the basic competencies of the Ghanaian Mathematics syllabi ISMTs teach the students and WASSCE Past Questions. The researcher anticipated that all ISMTs would score better than the results in Figure 6 and Table 8. Therefore, it can be argued that ISMTs have satisfactory TCK but an inadequate conceptual understanding of trigonometry. That is, they had not sufficiently mastered the trigonometry content they taught to students, and this culminated in their inability to tackle questions relating to content knowledge of trigonometry.

The findings showed that most teachers' difficulties were grounded in fundamental trigonometric understanding, since their performance in Core Mathematics Trigonometry, which serves as the foundational knowledge for Advanced Mathematics Trigonometry (Elective), was not good enough. Some ISMTs acknowledged that they lack confidence to teach and explain some trigonometry and related concepts. As a result, as indicated in Table 24, this may be one of the underlying factors for poor TCK among the ISMTs. If teachers themselves acknowledge that they are struggling to teach and explain some topics to learners, as indicated in Table 24, then, by implication, it may suggest that they are perhaps not sufficiently prepared to teach the SHS trigonometry in the Ghanaian Core and Elective Mathematics syllabus. This argument is supported by their responses in Table 34, as the majority indicated that they first need support and require training in teacher self-improvement in trigonometric content.

Insufficient TCK among ISMTs should be considered as a concern because teachers with limited content mastery will not be in a position to simplify aspects of the content in a manner that learners can learn with ease (Nabie et al., 2018; Koyunkaya, 2016). Moreover, teachers might also not sufficiently make the content interesting and relevant by linking it to actual contexts. Wu (2002) states that teachers are required to fully understand the subject matter that they teach to learners so that they can enable them to select the best pedagogy required to help learners understand the subject matter.

Referring to Ball et al. (2008), "teachers must know the subject well... further, teachers who know the content well are likely to have a good knowledge of learners and help them understand the subject content" (p. 404). Buttressing Ball et al. (2008) argument, Jadama (2014) states that teachers' ability to understand the subject content thoroughly means they are able to teach the subject content in an efficient manner. However, if a teacher has inaccurate information about the subject content, he/she might keep on passing on such information to the learners, which might negatively affect their academic performance. Kandjinga (2018) adds that teachers ought to be more knowledgeable in the content areas that they are teaching learners.

He further elaborated that "if teachers fail some questions based on what they teach to the learners, then it can be argued that their SCK is limited" (p. 60). In the same way, Black (2009) argues, "it will be difficult for the teacher to teach learners about a subject if the teacher does not know the content himself/herself" (p. 2). It then makes it easier to conclude from the views of (Kandjinga, 2018; Black, 2009; Ball et al. 2008) that teachers need to know and understand the underlying principles, foundational ideas behind trigonometry concepts and procedures for getting correct answers and transferring such knowledge to learners.

Awareness of Students Trigonometrical Errors and Misconceptions

Hypothetical Students' solutions were used to determine ISMTs' awareness of trigonometrical errors and misconceptions. The results of this question revealed the ISMTs' understanding of content and students as a component of pedagogical understanding. The findings were shown in Tables 10 to 12, which seemed to suggest that the ISMTs have an insufficient understanding of students' conceptions. The study indicated that the majority of the ISMTs found nothing wrong with either of the solutions presented to them. Even though the responses had multiple faults, the handful who were able to discover something incorrect only brought out one mistake in explaining or pinpointing particularly where pupils were defective. Son (2011) refers to this as a procedural-based approach. He added that in this method of analyzing student work, teachers describe single concepts that are correct or erroneous without tying them to the overall answer. This might imply that ISMTs failed to recognize that the claims were incorrect. Because of their approach to solution analysis, these ISMTs may have missed a mistake in the statements.

Because the ISMTs were only recognizing one incorrect statement in each solution, it is reasonable to believe that they stopped analyzing the solution after identifying the first inaccuracy. Also, corresponding to this assertion, Zuya (2014) explored "mathematics teachers' ability to evaluate students' thinking process about some algebraic concepts". According to the findings, "most teachers are unable to ask competent questions that can aid in the evaluation of students' cognitive processes". This stresses the fact that ISMTs are unable to identify errors and misconceptions when it comes to the way students' present solutions. These findings appear consistent with those of Sibuyi (2012) and Kilic (2011). Sibuyi (2012) used interviews, observations, and lesson plan analysis as data collection procedures to investigate instructors' understanding of learners' concepts, SMK, and knowledge of teaching techniques. Because many teachers lacked awareness of the nature, characteristics, and interpretations of the problems to be solved, the study revealed that teachers have limited and weak knowledge. This is due to their failure to identify learners' misconceptions by examining learners' answers to the questions.

Also, on how to correct the errors and help students understand the concepts, since most ISMTs found nothing wrong with either solutions, surprisingly, they did not even bother correcting the errors to help the students in that regard. Thus, instead of suggesting ways that would make the students understand the problem, they just worked out the problem. Son (2011) noted that teachers who use procedural-based techniques in judging students' faults tend to believe that a student understands the ideas but does not grasp the procedure. Similarly, Kilic (2011) also documented limited PCK among mathematics teachers. He found that teachers had difficulties assessing learners' errors and providing different representations for mathematical situations. Further, Kilic's findings revealed that teachers lacked sufficient awareness of students' conceptions and argued that "when teachers were given examples of students' errors and asked how to address them, the teachers tended to repeat how to carry out the procedures or explain how to apply a rule or mathematical fact to solve the problem instead of explaining the correct concepts that would help eliminate the learners' errors" (p. 23).

Tsafe (2013) agreed with Kilic (2011) that teachers need strong PCK, first in knowing subject content, the most effective ways of depicting and explaining diverse topics, and concepts for identifying learners' misconceptions about a particular content. Evidently, ISMTs investigated in the study had insufficient PCK and this culminated in their inability to identify students' errors and misconceptions and knowledge to identify alternative ways of overcoming such mistakes. Bukova-Güzel (2010) also used solid objects to investigate mathematics teachers' pedagogical subject understanding and discovered that teachers give no consideration to likely student misconceptions. This is due to teachers' mediocre mathematical knowledge, which prevented them from assisting their students (Turnuklu & Yesildere, 2007).

Interestingly, analyses of the overall performance of ISMTs in presenting correct solutions to Tasks 1 and 2 questions indicated that they performed woefully. This is because the mean score on PCK questions of Task 1 was 43.65% while Task 2 recorded the lowest (28.05%), implying that the performance of ISMTs was not good enough since they were not able to reach half of the scores (50%) on errors and misconceptions. As a result, the ISMTs' insufficient understanding of content and students might affect the pedagogical understanding they require to teach SHS trigonometry in the Ghanaian Core and Elective Mathematics syllabi.

The low PCK component among the ISMTs was mainly due to the fact that most ISMTs indicated that they were not confident about some trigonometry content in the curriculum (Table 24), which may be one of the causal factors of the poor PCK component among the ISMTs. If teachers themselves acknowledge that they are struggling to teach and explain some topics to learners, then this should be considered as a critical concern because, by implication, it means they will not adequately make the content comprehensive for the learners. Therefore, managers and curriculum developers at institutions of higher learning should ensure that their programmes are designed to adequately prepare teachers to impart accurate subject content to students in a comprehensive manner. Indeed, this argument is supported by their responses in Table 34, as the majority indicated teacher self-improvement in trigonometric pedagogy as their second greatest professional development need that they require training and support in.

Thompson et al. (2007) argued in support of this, arguing that if teachers do not have adequate understanding of the issues they are teaching, they will be unable to notice the students' shortcomings or, for that matter, their mastery of these ideas. He recommends instructors, echoing French (2005), to have knowledge and abilities to detect students' errors and misconceptions, as well as expertise to correct such preconceptions, misconceptions, and errors.

Trigonometry Self-Efficacy (TSE) of ISMTs

Regarding ISMTs' TSE, the study revealed that as the level of TSE components increased from Trigonometry I and II (Core Mathematics Items) through to Trigonometric Functions, Equations and Graphs (Elective Mathematics Items), the confidence level of ISMTs decreased on a whole (Table 18). The difference in the TSE of ISMTs between Core (Trig I & II) and Elective (Trigonometric Ratios and Rules, Trigonometric Compound and Multiple Angles, and Trigonometric Functions, Equations, and Graphs) were tested in Hypothesis One. From Table 19, there were statistically significant differences observed in efficacy levels of ISMTs. This means they seemed much more confident in teaching Core Mathematics Trigonometry related items compared to electives. It is then appropriate to know the core and elective specific trigonometry objectives that ISMTs exhibited very weak or strong self-efficacy.

Finally, ISMTs identified some specific trigonometry items as those they have the least and most confidence in teaching students for reasons. From Table 21, all the least confident items of the ISMTs received mean more than

3.0. This shows that they exhibited very low confidence in achieving those stated objectives of teaching Core and Elective SHS trigonometry. Thus, *pose problems involving real life situations involving trigonometric ratios for students to solve and guide students to use their graphs to solve equations such as:* $a\sin x + b\cos x = 0$, $a\sin x + b\cos x = k$, *etc*, for core trigonometry and also assist students to "*apply the sine and cosine rules to solve problems involving bearings (real life problems), guide students to derive the compound angles identities, and guide students to express the trigonometric function,* $f(x) = a\sin x + b\cos x$ *in the form,* $R\cos(x \pm a)$ or $R\sin(x \pm a)$, *where,* $0^0 \leq a \leq 90^0$ " in Elective Mathematics Trigonometry.

The result showed that most ISMTs cannot use real-life scenarios (practical activities) to teach trigonometry since it is selected in both Core and Elective Mathematics. This seemed to suggest that ISMTs could not guide students to use practical activities, including the use of trigonometric diagrams and models, as well as word problems. Hence, the overall mean for the least confidence items (M=3.826, SD=.719) indicates very weak efficacy. Thus, teachers do not demonstrate strong confidence in their ability to teach trigonometry by relating it to real life problems. This is contrary to the suggestion by the Chief Examiner (WAEC WASSCE May/June, 2012-2018) that teachers should take the necessary steps to make the teaching of mathematics more practical and relate it to real-life problems, as it will help students to appreciate the topics being taught. This finding suggests that ISMTs are confidently weak at posing problems in real-life situations involving trigonometry, and this may not help them to teach or assist learners to know and appreciate the topic. Since understanding of real-life situations is an important source of students' high performance and good attitude to mathematics, this recommends the necessity for professional development developers to help ISMTs get over this weakness.

On the other hand, the overall TSE for the Most Confident items (M=4.306, SD=0.596) for teaching trigonometry, goes further to explain that the majority of the ISMTs are confident in achieving the stated objectives (Table 21) for teaching the SHS Core and Elective trigonometry content. In other words, these high confidence levels of ISMTs would help them teach or assist students to understand most of the Core and Elective Mathematics trigonometry content.

Having established that ISMTs have more or less confidence in some trigonometry content, they stated specific reasons for being least or most confident. Some reasons given by the ISMTs for their least confident level include; difficulty/challenging, time-consuming, lack of in-depth content knowledge, little or no experience with trigonometrical technologies, and no/limited previous experience teaching trigonometry to students. In support of these findings are the results of studies (Kagenyi, 2016; Nadelson et al., 2012).

Kagenyi (2016), in his study on "pedagogical factors affecting the learning of trigonometry", has this to say, "if the teachers' attitudes are negative towards trigonometry, this, in turn, will affect their teaching of the topic and be reflected in the pupils' performance" (p. 40). This is exactly what has been observed in Table 18. That is, as the trigonometrical sophistication of questions advanced, so did ISMTs TSE. Nadelson et al. (2012) observed that when teachers feel uncomfortable with the topic or subject they are teaching, they tend to avoid teaching it beyond the surface layer or even cease teaching it altogether, causing students harm in the end. Furthermore, Chigonga (2016) discovered in a trigonometry study that many of the errors gleaned from teachers' responses to trigonometry questions may have their origins in a lack of understanding of the fundamentals and foundational competencies taught in their earlier grades, resulting in problems teaching the same content. A participant in Chigonga (2016) emphasized that, "the reason for this could be that many teachers are not confident about some contents in the National Senior Certificate (NSC) and also, it appears that many teachers are struggling to teach learners how to solve trigonometric equations, especially finding solutions within a given interval" (p. 169)

Also, the reasons given by the ISMTs for their choice of the items as the ones they are much more efficacious at teaching are; ease of teaching due to experience, engagement with technology, in-depth trigonometrical content knowledge, and viewing of trigonometry as a set of procedures. The finding is congruent with the study by Sarac and Aslan-Tutak (2017). They investigated 16 teachers' trigonometry teaching efficacy in South Africa, asking them about their level of confidence in teaching trigonometry for a specific problem. The findings revealed that each of the 16 respondents had a high level of efficacy.

Challenges Faced by ISMTs' in Teaching Trigonometry

On the subject of challenges faced by ISMTs in teaching trigonometry, the study showed that lack of confidence, inadequate instructional resources, deficiency of attention and difficulty, and the problem of teaching trigonometry were the major challenges teachers face when it comes to teaching trigonometry. These challenges may have contributed mainly to their low trigonometry content knowledge, limited knowledge of identifying learners' misconceptions, as well as the decline in TSE as trigonometrical sophistication items increase. Corresponding to the findings are the results of Appiahene et al. (2014) in Ghana. The survey highlighted, among other obstacles, a negative attitude toward mathematics and its abstract nature, a shortage of teaching and learning resources, a poor attitude toward the study of mathematics, unsatisfactory teaching methodologies employed by teachers, and the majority of students' dread of the subject.

Etsey (2005) recognized insufficiency of teaching and learning resources as one main factor of low academic performance of children in several Ghanaian institutions. Perhaps, the preparedness and the level of motivation to learn mathematics are not encouraging on the part of students, and this is attributable to the challenges teachers encounter in ensuring that students understand trigonometry and its concepts very well.

ISMTS Professional Development Needs for Teaching Trigonometry

When it comes to TPD needs, ISMTs highly need support in teacher self-improvement in trigonometric content, teacher self-improvement in trigonometric pedagogy, preparation and utilization of teaching materials/aids, use of information and communication technology, delivering trigonometric instructions, planning trigonometric instructions, managing trigonometric instructions, and diagnosing and evaluating learning. Teacher Self-improvement in Trigonometric Content rated high among these categories, most likely because it is the most fundamental component of teaching knowledge (Ball & McDiarmid, 1990) and hence the most significant knowledge with the largest influence on student achievement. In addition, Kraus et al (2008b) asserted that it is not likely that a teacher will make the subject content reachable to learners if he or she does not understand the basic strands of the subject content. So, since it is Content Knowledge that comprises of the principles, laws, and concepts of a particular subject or topic, ISMTs therefore see it as a necessity to be knowledgeable about the trigonometry content as indicated in the curriculum. Hence, teachers need programmes on the content knowledge necessary to support their students' learning of trigonometry.

The second most critical construct after "Teacher Self-improvement in Trigonometric Content" is "Teacher Self-improvement in Trigonometric Pedagogy", as it recorded the next highest mean value, with the least being diagnosing and evaluating learning. Teacher Self-improvement in Trigonometric Pedagogy as the second most critical trigonometry need by ISMTs aligns with Baurmert et al. (2010) results which revealed that "PCK is inconceivable without a substantial level of Specialized Content Knowledge (SCK), but that SCK alone is not a sufficient basis for teachers to deliver cognitively activating instructions that, at the same time, provide individual support for student learning" (p.164). This indicated that having a strong PCK is the most effective approach for displaying and explaining diverse topics and concepts to learners from a developmental standpoint. In addition, it was established by Tsafe (2013, p. 37) that "teachers with good pedagogical knowledge understand where learners may have trouble learning the subject matter and should also be able to represent mathematical concepts in a way that their learners can comprehend their structure and avoid any difficulties." Hence, in conformance to why 'Pedagogy' is the second TPD need indicated by the ISMTs.

However, ISMTs need support in all the 8 areas. Consistent with this is the findings of Mohamed (2013), who investigated “professional development needs of secondary school mathematics teachers in Zanzibar, Tanzania" and reported that all the subscales were moderately accepted by teachers, but the “teacher self-improvement of content knowledge” sub-scale indicated strong demand, and the most important need as shown. This indicates the needs of teachers in teaching a particular topic or subject can be regional or country-specific. In this study, “self-improvement in content knowledge” was the most important subscale of professional development needs in this survey, followed by ICT usage in mathematics teaching. The delivery of mathematics instruction was one of the least needed subscales. Thus, teachers needed support in the areas presented to them to be able to teach trigonometry very well and effectively, as always suggested by the chief examiner over the years.

Also, Rakumako and Laugksch (2010) studied the demographic profile and perceived INSET needs of mathematics teachers in the Limpopo Province of South Africa. From a STIN-LP survey instrument, respondents were asked to identify their greatest professional needs, and 40% chose teaching skills, preceded by content knowledge and class discipline, with assessing learners rated as the least requirement. Another needs analysis survey of 40 educators highlighted their pedagogical weaknesses, revealing that teachers require professional development in areas such as effective instruction approaches, classroom management, substitute means of assessing learners, and lesson planning for effective instruction (Quan-Baffour, 2007).

In conclusion, the needs identified by the teachers should be seriously identified and implemented by professional development developers as teachers are in dire need, since Table 5 indicates that 160 out of 220 in-service mathematics teachers have never attended an in-service training workshop on trigonometry. In this study, the empirical evidence obtained showed that professional development is imperious to improve their capabilities in teaching trigonometry.

Summary of Key Findings

Regarding trigonometry content knowledge, on a whole, the knowledge of ISMTs about the content was not encouraging as most of them scored lower on the content knowledge questions. But for the few who answered these questions, significant differences were observed in performance on Core and Elective Mathematics trigonometry questions. Thus, ISMTs performed better on Elective Mathematics trigonometry questions than on the core.

For the identification of errors from hypothetical students' solutions, the results indicated that the majority of the ISMTs found nothing wrong with the student solutions presented to them. For the few who noticed something wrong with the solutions, in explaining or identifying specifically where students were flawed, they only pointed out one mistake, even though the solutions contained several errors.

Regarding ISMTs TSE, the study revealed that they are much more confident in teaching Core Mathematics trigonometry-related items compared to electives, and identified some items as those they have the least and most confidence in teaching students, for reasons.

On the subject of some of the challenges faced by ISMTs in the teaching of trigonometry, the study that revealed lack of confidence, inadequate instructional resources, and the difficulty of teaching trigonometry are the major challenges ISMTs face when it comes to teaching trigonometry.

On TPD needs, ISMTs highly need support to effectively teach trigonometry in: Teacher Self-Improvement in Trigonometric Content; Teacher Self-Improvement in Trigonometric Pedagogy; Preparation and Utilization of Teaching Materials/Aids; and Use of ICT and others.

CHAPTER FIVE

SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

Overview

The study's purpose is to get insight into In-Service Mathematics Teachers' (ISMTs) understanding and teaching of trigonometry concepts in Senior High Schools (SHS). As a result, teachers are encouraged to talk over and redirect their educational goals, objectives, practices, strengths, and weaknesses when teaching trigonometry. This chapter is the last of a series of five chapters which present summary, conclusions, recommendations, and suggestions for further studies. The study was designed to:

1. Determine ISMTs trigonometry content knowledge
2. Examine the awareness of misconceptions and errors by ISMTs in teaching and solving trigonometric questions
3. Determine ISMTs trigonometry self-efficacy
4. Ascertain the challenges faced by these ISMTs in the teaching of trigonometry
5. Assess and describe the level of overall trigonometrical professional development needs of ISMTs.

The following research hypothesis guided the study:

1. H_{01}: There is no statistically significant difference between ISMTs Core and Elective Mathematics TSE.

The study employed 300 ISMTs in total, and data was obtained using a questionnaire. However, 220 questionnaires were successfully filled and returned. The questionnaires were physically inspected for completeness and accuracy of filling, missing values, and then coded for entry into SPSS and

Minitab software packages for analysis. Counts, percentages, means, and standard deviations (SD) were used. Means and SD are used for the quantitative part, while the qualitative part relies on frequencies and percentages. Tables are used in the presentation of results. In testing or verifying the stated hypotheses, paired t-test were used. The summary of findings is presented below.

Summary of Findings

According to the survey, the majority of ISMTs (83.6%) were males, with the remaining fraction being females. This is not surprising as most mathematics programs are undertaken by males in various institutions of higher education. Another (67.3%) of the ISMTs were between the ages of 20 and 30 years, with the least being those between 41 and 50 years. Also, most (65.5%) of the ISMTs were Master of Philosophy Education (MPhil. Ed.) Certificate holders. Moreover, the majority (63.63%) were Principal Superintendents. Besides, most (70.9%) of them have been teaching at various secondary schools for at least 10 years. Another majority (72.7%) had never attended any in-service training before. This assertion is expected to influence their Trigonometry Content Knowledge (TCK) scores. Most (59.3%) of them teach Form Two classes, while another majority (45.5%) teach both Core and Elective subjects.

Regarding trigonometrical content knowledge, on a whole, the knowledge of the ISMTs is not encouraging as most of them scored lower marks on these questions. For the few who answered these questions correctly, significant differences were observed in performance on Elective and Core mathematics trigonometry questions. Thus, ISMTs performed better on Elective Mathematics trigonometry questions than the core ones.

For the identification of errors from hypothetical students' solutions, the study indicated that the majority of the ISMTs found nothing wrong with the solutions presented to them. Even though the responses had multiple flaws, the handful who spotted something wrong with them only pointed out one issue in explaining or pinpointing particularly where pupils were defective. The ISMTs clarified the causes of the errors by explaining either what the learner was thinking or the information the student lacked. They went on to discuss the causes in the order in which they were discovered. According to the data, teachers ascribed the causes of students' errors to a lack of comprehension of either a process or a concept, but rarely both.

To end, on how to correct the errors and help students understand the concepts, since most ISMTs did not find anything wrong with either solution, they did not bother correcting the errors to help the students. Thus, instead of suggesting ways that would make the students understand the problem, they just worked out the problem. Interestingly, in analyzing the overall performance of the ISMTs in presenting the correct solutions to the questions, they performed woefully. This is because the mean score on Task 1 was 43.65% on PCK questions, while Task 2 recorded the lowest (28.05%). This means the performance of the ISMTs is not good, because they were unable to get half of the scores (50%) for their solutions marked. Thus, the ISMTs have insufficient knowledge of content and students, which might affect their pedagogical understanding required to teach SHS trigonometry.

Regarding ISMTs TSE, the study revealed that as the level of TSE components increased from Trigonometry I and II (Core Mathematics Items) through to Trigonometric Functions, Equations and Graphs (Elective

Mathematics Items), the confidence level of ISMTs decreased. Thus, ISMTs seemed much more confident in teaching Core Mathematics trigonometry related items compared to electives. Thus, statistically significant differences existed in ISMT TSE levels. This means ISMTs are much more confident in teaching Core Mathematics trigonometry related items. Finally, the ISMTs identified some items as those they have the least and most confidence in teaching students for reasons. The reasons given for the least confidence items are: difficult/challenging/confusing/time-consuming, absence of in-depth content knowledge, little or no experience with technologies, and no/limited previous experience teaching trigonometry to students. Those with the most confidence are ease of teaching due to experience, engagement with technology, in-depth trigonometrical content knowledge, and a perception of trigonometry as a set of processes.

On the subject of some challenges faced by ISMTs in teaching trigonometry, the study revealed a lack of confidence, inadequate instructional resources, and the difficulty of teaching trigonometry as major challenges. Finally, the study revealed that when it comes to TPD needs, ISMTs highly need support to effectively teach trigonometry in: Teacher Self-Improvement in Trigonometric Content; Teacher Self-Improvement in Trigonometric Pedagogy; Preparation and Utilization of Teaching Materials/Aids; and Use of ICT and others.

Conclusions

1. Regarding trigonometry content knowledge, on a whole, the knowledge of ISMTs on the topic was not encouraging because most of them did

not perform well in the questions. Thus, on average, ISMTs scored 40.6% on TCK questions, which signifies limited TCK.

2. Regarding the awareness of ISMTs about students' trigonometrical errors and misconceptions, most of them were unable to identify, suggest reason or causes, and could not provide remedies to students' trigonometrical errors. Thus, indicating insufficient awareness of ISMTs knowledge of content and students.
3. Regarding ISMTs TSE, they are much more confident in teaching Core Mathematics Trigonometry related items compared to elective, and identified some items as those they have least and most confidence in teaching students.
4. The challenges faced in teaching trigonometry are lack of confidence, inadequate instructional resources, and the difficulty of teaching trigonometry are major challenges ISMTs face when it comes to teaching trigonometry.
5. On TPD needs, ISMTs need support to effectively teach trigonometry in: Teacher Self-improvement in Trigonometric Content; Teacher Self-improvement in Trigonometric Pedagogy; Preparation and Utilization of Teaching Materials/Aids; and Use of ICT and others.

Recommendations

The ensuing commendations were made based on the results to strengthen ISMTs' TCK and facilitate students' conceptualization, build confidence, and resolve challenges, and enhance TPD needs for teaching trigonometry:

1. Teachers displayed a sense of ill-developed trigonometrical knowledge for teaching. Since ISMTs' content knowledge is mainly developed through formal training at teacher training institutions and off-campus teaching practices, it is recommended that pre-service teachers should be taught the Trigonometry content that they will be teaching in high school during their final year, so that they will be able to make the subject content comprehensible for the learners. In addition, it is recommended that, educational stakeholders could provide in-service courses and seminars on specialized pedagogical content knowledge on trigonometry.
2. It is also suggested that teachers should be aware of students' conceptions, misconceptions, and errors that constitute the background knowledge of learners to construct learning activities that can bridge any existing learning gaps. Also, teachers should use students' solutions as a resource to deal with misconceptions and errors, since they reveal the learners' thoughts of trigonometry. This means that teachers will be better positioned to help learners reconstruct understanding in line with more integrated mathematical knowledge if they become more aware of typical errors.
3. Teachers' exhibition of confidence during mathematical lessons motivates students to perform better in mathematics. Students, in particular, draw on tutors' attitudes to build their own, which can influence their learning results in core and elective mathematics. Heads of departments should organize and encourage teachers to attend workshops, conferences, and professional development courses in order

to refresh their teaching methods and become abreast of new developments in educational matters. Hence, increasing their efficaciousness in teaching the topic content in both core and elective mathematics as a whole.

4. The study showed that lack of confidence, inadequate instructional resources, limited interest and attention, and difficulty in teaching trigonometry are the major challenges teachers face when it comes to teaching trigonometry. It is therefore recommended that all the stakeholders involved come up with measures and strategies to curb these situations to enable teachers to focus and teach this topic to the understanding of students.
5. The study revealed that ISMTs highly need support to effectively teach trigonometry to students, as proposed by the chief examiner over the years. That is, teachers must have access to ongoing professional development via in-service programs, short-term conferences, and workshops. This is anticipated to provide a chance to equip themselves with new knowledge, skills, and modern ways of how to handle and help their students, as well as make the topic comprehensive for the learners. Moreover, consultative teachers for mathematics should visit schools on a regular basis to identify teachers' needs in terms of specific mathematics topics, content knowledge, and PCK so that they can address such needs in workshops and conferences.

Suggestions for Further Studies

The study focused on examining ISMTs' understanding and teaching of trigonometry and its implications for TPD within the Greater Accra Metropolis of the Greater Region of Ghana. It is therefore suggested that future research on the subject should widen its reach to cover more metropolises and regions, if not the whole country. This can help in collecting facts from such studies to develop better techniques and suggestions to serve the communities and policymakers by improving the overall performance of ISMTs.

A larger sample size might be used in a comparable study so that the results can be made general to a greater population. Finally, teachers teach mathematics to pupils based on their idea of what mathematics is and how it should be taught. As a result, more study is needed to determine whether there is a link between teachers' content understanding of certain mathematical topics and their mathematical beliefs. More significantly, mathematics education and research organizations should prioritize instructors and trigonometry instruction above students and trigonometry learning. The majority of existing studies have primarily supported student learning, interventions, and systems-oriented curriculum above teacher learning and practice of mathematics as a discipline, particularly trigonometry. It is really important not to ignore the teaching side of teaching-and-learning.

The current study had a few drawbacks. To begin with, it only included ISMTs and a limited number of schools; thus, the results cannot be applied to other schools or regions in other areas. Second, because the respondents came from a single metropolis, the results were constrained by the sampling location. Despite these flaws, this research provides valuable insight into ISMTs' understanding and teaching of trigonometry, as well as the consequences for TPD.

REFERENCES

Abu Bakar, J. H. A., Rubba, P. A., Tomera, A. N., & Zurub, A. R. (1988). Jordanian and Malaysian science teachers' prominent perceived professional needs: A comparison. *Journal of Research in Science Teaching*, *25*(7), 573-587.

Abu Bakar, K. H. (1984). *A comparison of the perceptions of Malaysian secondary science teachers and teacher educators regarding the science teaching needs of Malaysian secondary teachers*. (Doctoral dissertation, Southern Illinois University at Carbondale).

Abu Bakar, K. H., & Rubba, P. A. (1985). The construct validity of the science teacher inventory of need: Recommendations and modifications. *Educational and psychological measurement*, *45*(3), 699-704.

Adler, J. (2005). Mathematics for teaching: What is it and why is it important that we talk about it? Pythagoras, *62,* 2–11.

Aguele, L. I., & Usman, K. O. (2007). Mathematics education for dynamic economy in Nigeria in the 21st century. *Journal of Social Sciences*, *15*(3), 293-296.

Ajani, O. A. (2018). Needs for in-service professional development of teachers to improve students' academic performance in Sub-Saharan Africa. *Arts Social Sci J*, *9*(330), 2.

Ajani, O. A., Govender, S., & Maluleke, N. (2018). Teachers' in-service professional development: Gateway to improved classroom practices in Nigerian schools. *African Journal of Gender, Society & Development*, *7*(3), 21-40.

Akkoc, H. (2008). Pre-service mathematics teachers' concept images of radian. *International Journal of Mathematical Education in Science and Technology*, *39*(7), 857-878.

Altschuld, J. W., & Kumar, D. D. (2010). *Needs assessment: An overview*. Thousand Oaks, CA: SAGE.

Altschuld, J. W., & Witkin, B. R. (2000). *From needs assessment to action: Transforming needs into solution strategies.* Thousand Oaks, CA: Sage.

Appiahene, P., Opoku, M., Akweittey, E., Adoba, E., & Kwarteng, R. (2014). Assessing the challenges of learning and teaching of mathematics in second cycle institutions in Ghana. *International Journal of Innovation and Applied Studies*, *6*(3), 362-368.

Attard, C. (2011). "If you like the teacher, you'll get mathematics more: student talk about good mathematics teachers. *Curriculum Leadership: An Electronic Journal for Leaders in Education*, *9*(6), 5-23.

Bair, S. L., & Rich, B. S. (2011). Characterizing the development of specialized mathematical content knowledge for teaching in algebraic reasoning and number theory. *Mathematical Thinking and Learning*, *13*(4), 292-321.

Baird, W. E. (1993). A comparison of Alabama secondary science and mathematics teachers: Demographics and Perceived Needs. *School Science and Mathematics*, *93*(4), 175-82.

Baird, W. E., Prather, J. P., Finson, K. D., & Oliver, J. S. (1994). Comparison of perceptions among rural versus non rural secondary science teachers: A multistate survey. *Science Education*, *78*(6), 555-576.

Ball, D. L., & Bass, H. (2009). *With an eye on the mathematical horizon: Knowing mathematics for teaching to learners' mathematical futures*.

Paper presented at The 2009 Curtis Center Mathematics and Teaching Conference. Retrieved from http://www.mathematik.u-dortmund.de/ieem/cms/media/BzMU/BzMU2009/Beitraege/Hauptvortraege/BALL _Deborah_BASS_Hyman_2009_Horizon.pdf

Ball, D.L., & Hill, H.C. (2008). Mathematical knowledge for teaching (MKT) measures. Mathematics released items 2008. Retrieved September 12, 2010, from http://sitemaker.umich.edu/lmt/files/LMTsample_items.pdf.

Ball, D. L., & McDiarmid, G. W. (1990). The subject-matter preparation of teachers. In W. R. Houston (Ed.), *Handbook of research on teacher education*, 437-449. New York: Macmillan

Ball, D. L., Hill, H. C., & Bass, H. (2005). Knowing mathematics for teaching: Who knows mathematics well enough to teach third grade, and how can we decide? *American Educator*, *Fall 2005*, 14-17, 20-22, 43-46.

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it Special? *Journal of Teacher Education*, *59*(5), 389–407, Doi:10.1177/0022487108324554

Bandura, A. (1994). Self-efficacy. In V. S. Ramachaudran (Ed.). Encyclopedia of human behavior. *New York: Academic Press*, *4*, 71-81.

Bantwini, B. D. (2012). Primary school science teachers' perspectives regarding their professional development: Implications for school districts in South Africa. *Professional development in education*, *38*(4), 517-532.

Beausaert, S. A., Segers, M. S. R., & Wiltink, D. P. (2013). The influence of teachers' teaching approaches on students' learning approaches: The student perspective. *Educational Research*, *55*(1), 1-15.

Berk, D. (2005). Educational policy as a vehicle for teacher learning. In *Proceedings of the 27th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*.

Bichi, A. A., Ibrahim, R. H., & Ibrahim, F. B. European Journal of Psychology and Educational Research. *Educational Research*, *1*(2), 53-59.

Borko, H. (2004). Professional development and teacher learning: Mapping the terrain. *Educational Researcher*, *33*(8), 3-15.

Bosson-Amedenu, S. (2017). Archbishop porter girls' senior high school students' perception of difficult concepts in senior high school further mathematics curriculum in Ghana. *Asian Research Journal of Mathematics*, *4*(3), 1-14.

Bosson-Amedenu, S. (2017). Remedial students' perception of difficult concepts in senior high school core mathematics curriculum in Ghana. *Asian Research Journal of Mathematics*, *3*(3), 1-13.

Boyatzis, R. E. (1998). Transforming qualitative information: Thematic analysis and code development. Sage.

Briley, J. S. (2012). The relationships among mathematics teaching efficacy, mathematics self-efficacy, and mathematical beliefs for elementary pre-service teachers. *Issues in the undergraduate mathematics preparation of school teachers*, *5*.

Brodie, K., & Berger, M. (2010). Toward a discursive framework for learner errors in mathematics. In *18th annual meeting of the Southern African Association for Research in Mathematics, Science and Technology Education, Durban*.

Bukova-Güzel, E. (2010). An investigation of pre-service mathematics teachers' pedagogical content knowledge, using solid objects. *Scientific Research and Essays*, *5*(14), 1872-1880.

Carpenter, T. P., Fennema, E., Petersen, P., & Carey, D. (1988). Teachers' pedagogical content knowledge of students' problem solving in elementary arithmetic. *Journal for Research in Mathematics Education*, *19*, 385-401

Chick, H. L., & Baker, M. K. (2005). Investigating teachers' responses to student misconceptions. In *Proceedings of the 29th Conference of the International Group for the Psychology of Mathematics Education*, *2*, 249-256.

Chick, H. L., Pham, T., Baker, M., & Cheng, H. (2006). Probing teachers' pedagogical content knowledge: Lessons from the case of the subtraction algorithm. *Identities, cultures and learning spaces*, 139-146.

Chigonga, B. (2016). Learners' errors when solving trigonometric equations and suggested interventions from grade 12 mathematics teachers.

Cohen, L., Manion, L., & Morrison, K. (2007). Observation. *Research methods in education*, *6*, 396-412.

Cole, Y. (2012). Assessing elemental validity: the transfer and use of mathematical knowledge for teaching measures in Ghana. ZDM, *44*(3), 415-426.

Craft, A. (2000). *Continuing professional development: A practical guide for teachers and schools* (2nd ed.). London: Routledge Falmer.

Creswell, J. W. (2012). Educational research: Planning, conducting and evaluating quantitative and qualitative research (4th ed.). Boston, United States of America: Pearson Education.

Creswell, J. W. (2014). Educational research: Planning, conducting and evaluating quantitative and qualitative research (4th ed.). London, England: Pearson Education Limited.

Creswell, J. W. (2015). Revisiting mixed methods and advancing scientific practices. In *The Oxford handbook of multimethod and mixed methods research inquiry*.

Cumming, C. (2011). CPD: support strategies for professional learning, national initiatives and major curriculum reform. *Improving schools*, *14*(2), 145-155.

Curriculum Research and Development Division (2010). Core and Elective Mathematics syllabus for Senior High School. Accra: Ministry of Education.

Davis, J. (2005). Connecting procedural and conceptual knowledge of functions. Mathematics Teacher, *99*(1), 36-39.

Doyran. F. (2012). Research on teacher education and training. Athens: Athens Institute for Education and Research. *Educational Review*, *57*, 1-23

Ekşi, G. (2010). An assessment of the professional development needs of English language instructors working at a state university (Master's thesis).

Ekşi, G., & Aydın, Y. C. (2013). English instructors' professional development need areas and predictors of professional development needs. *Procedia-Social and behavioral sciences*, *70*, 675-685.

Etkina, E. (2010). Pedagogical content knowledge and preparation of high school physics teachers. *Physical Review Special Topics-Physics Education Research*, 6(2), 020110.

Fauskanger, J., Jakobsen, A., Mosvold, R., & Bjuland, R. (2012). Analysis of psychometric properties as part of an iterative adaptation process of MKT items for use in other countries. *ZDM*, *44*(3), 387–399.

Fi, C. (2006, November). Preservice secondary school mathematics teachers' knowledge of trigonometry: Cofunctions. In *Proceedings of the 28th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*, *2*, 833-834.

Field, A. (2013). Discovering statistics using IBM SPSS statistics. Sage.

Gafoor, K. A. & Kurukkan, A. (2015). Why high school students feel mathematics difficult? An exploration of affective beliefs. Paper presented at the national seminar on pedagogy of teacher education trends and challenges, Kerala, India, 18–19 August. Available at https://files.eric.ed.gov/fulltext/ED56 0266.pdf. Accessed 1 February 2020. Ganser, Tom. "An ambitious vision of professional development for teachers." *NASSP bulletin 84*(618), 6-12.

Garden, R. A., Lie, S., Robitaille, D. F., Angell, C., Martin, M. O., & Mullis, I. (2008). VS, Foy, P. & Arora, A. (2006). *TIMSS Advanced*.

Gencturk, Y. (2012). Teachers' mathematical knowledge for teaching, instructional practices, and student outcomes (Doctoral dissertation, University of Illinois at Urbana-Champaign).

Glasersfeld, E. (1996). Aspects of radical constructivism and its educational recommendations. In L. P. Steffe, P. Nesher, P. Cobb, G. A. Goldin, &

B. Greer (Eds.), *Theories of mathematical learning*, 307-314. New Jersey: Lawrence Erlbaum Associates, Publishers.

Glasersfeld, E. V. (1995). Radical Constructivism: A Way of Knowing and Learning. London, Washington, DC.

González, O. (2011). Secondary school teachers' statistical knowledge for teaching and espoused beliefs on teaching and learning of variability-related concepts. In *Proceedings of the Conference of the 35th International Group for the Psychology of Mathematics Education, Ankara, Turkey*.

Graham, K. J. & Fennell, F. (2001). Principles and standards for school mathematics and teacher education: Preparing and empowering teachers. *School science and mathematics, 101*, 319-327.

Grant, C., & Osanloo, A. (2014). Understanding, selecting, and integrating a theoretical framework in dissertation research: Creating the Blueprint for' House'. *Administrative Issues Journal*: Connecting Education, Practice and Research, 12-22.

Gür, H. (2009). Trigonometry learning. *New Horizons in Education, 57*(1), 67-80.

Guskey, T. R. (2002). Professional development and teacher change. *Teachers and Teaching: Theory and Practice, 8*(3), 381-391.

Hair, J. F., Sarstedt, M., Pieper, T. M., & Ringle, C. M. (2012). The use of partial least squares structural equation modeling in strategic management research: a review of past practices and recommendations for future applications. *Long range planning, 45*(5-6), 320-340.

Handelsman, J., Ebert-May, D., Beichner, R., Bruns, P., Chang, A., DeHaan, R., Gentile, J., Lauffer, S., Stewart, J., Tilghman, S. M., & Wood, W. B. (2004). Scientific teaching. Science, New Series, *304*(5670), 521-522.

Harrell-Williams, L. M., Sorto, M. A., Pierce, R. L., Lesser, L. M., & Murphy, T. J. (2014). Validation of scores from a new measure of preservice teachers' self-efficacy to teach statistics in the middle grades. *Journal of Psychoeducational Assessment*, *32*(1), 40-50.

Haynes, J. C., & Stripling, C. T. (2014). Mathematics efficacy and professional development needs of Wyoming agricultural education teachers. *Journal of Agricultural Education*, *55*(5), 48-64.

Hill, H. C., Ball, D.L., & Schilling, S.G. (2008). Unpacking pedagogical content knowledge: Conceptualizing and measuring teachers' topic specific knowledge of students. *Journal for Research in Mathematics Education*, *39*, 372-400.

Hill, H.C., Rowan, B & Ball, D.L. (2005). Effects of teachers' mathematical knowledge for teaching on student achievement. *American Educational Research Journal*, *42*(2), 371-406.

Holmes, V. L., Miedema, C., Nieuwkoop, L., & Haugen, N. (2013). Data-driven intervention: correcting mathematics students' misconceptions, not mistakes. *The Mathematics Educator*, *23*(1), 22–44. http://www.scribd.com/doc/54223801/TEACHER-KNOWLEDGE-OF-Students'-Mathematical.

Ikediashi, D. I., Ogunlana, S. O., Boateng, P., & Okwuashi, O. (2012). Analysis of risks associated with facilities management outsourcing. *Journal of Facilities Management*.

Jadama, L. M. (2014). Impact of subject matter knowledge in teaching and learning process. *Middle Eastern & African Journal of Education Research*, *7*, 21-29. Retrieved from www.majersite.org

Jankvist, U. T., Mosvold, R., Fauskanger, J., & Jakobsen, A. (2015). Analysing the use of history of mathematics through MKT. *International Journal of Mathematical Education in Science and Technology*, *46*(4), 495-507.

Kabutey, D. T. (2016). *Resources available for teaching mathematics in Senior High Schools in the Western Region of Ghana* (Doctoral dissertation, University of Cape Coast).

Kagenyi, D. G. (2016). Pedagogical factors affecting the learning of trigonometry (Doctoral dissertation, Kenyatta University).

Kaiser, G., & Blömeke, S. (2013). Learning from the Eastern and the Western debate: The case of mathematics teacher education. *ZDM*, *45*(1), 7-19.

Kandjinga, J.J. (2018). Investigating the grade 12 teachers' subject and pedagogical content knowledge of Mathematics in some selected public schools in the Khomas education region, Namibia (Doctoral dissertation, University of Namibia).

Kasanda, C. D., & Kapenda, H. M. (2015). School learners' knowledge and views of traditional medicinal plant use in two regions in Namibia. *Indigenous Knowledge of Namibia; Chinsembu, KC, Cheikhyoussef, A., Mumbengegwi, DR, Kandawa-Schulz, M., Kasandra, CD, Kazembe, L., Eds*, 135-156.

Kazima, M., Jakobsen, A., & Kasoka, D. N. (2016). Use of mathematical tasks of teaching and the corresponding LMT measures in the Malawi context. *The Mathematics Enthusiast*, *13*(1), 171-186.

Kilic, H. (2011). Preservice secondary mathematics teachers' knowledge of students. *Turkish Online Journal of Qualitative Inquiry*, *2*(2), 17-35.

Klein, M. E. (2015). Teaching trigonometry based on the meaningful learning theory and the conceptual field theory. *Konrad Krainer; Na da Vondrov a CERME 9-Ninth Congress of the European Society for Research in Mathematics Education,* 3169.

Klein, M. E. Z. (2015). Teaching trigonometry based on the meaningful learning theory and the conceptual field theory. In *CERME 9-Ninth Congress of the European Society for Research in Mathematics Education*, 3169.

Komba, W. L., & Nkumbi, E. (2008). Teacher professional development in Tanzania: Perceptions and practices. *Journal of international cooperation in education*, *11*(3), 67-83.

Koponen, M. (2017). Investigating mathematical knowledge for teaching and mathematics teacher education (Doctoral dissertation, Itä-Suomen yliopisto).

Krauss, S., Neubrand, M., Blum, W. & Baumert, J (2005). The professional knowledge of German secondary mathematics teachers: investigations in the context of COACTIV Project: *Journal of Educational Psychology*, *100*(3), 716-725

Krauss, S., Neubrand, M., Blum, W., & Baumert, J. (2008). The professional knowledge of German secondary mathematics teachers: Investigations in the context of the COACTIV project. *The 11th*

International Congress on Mathematics Education. Monterrey, Mexico. ICME.

Kwon, M., Thames, M. H., & Pang, J. (2012). To change or not to change: adapting mathematical knowledge for teaching (MKT) measures for use in Korea. ZDM, *44*(3), 371-385.

Lapadat, J.C. & Lindsay, A.C. (1999). Transcription in research and practice: from standardization of technique to interpretive positioning. Qualitative Inquiry 5, 64/86.

Laugksch, R., Rakumako, A., Manyelo, T., & Mabye, D. (2005). Development and validation of the Science Teacher Inventory of Needs for Limpopo province (STIN-LP). *South African journal of education*, *25*(4), 273-278.

Lieberman, A., & Pointer Mace, D. H. (2008). Teacher learning: The key to educational reform. *Journal of teacher education*, *59*(3), 226-234.

Liu, Y. (2005). Teachers' understandings of probability and statistical inference and their implications for professional development (Doctoral dissertation). Vanderbilt University, Nashville, TN.

Lovett, J. N. (2016). The preparedness of preservice secondary mathematics teachers to teach statistics: a cross-institutional mixed method study.

Luneta, K., & Makonye, P. J. (2010). Learner Errors and Misconceptions in Elementary Analysis: A Case Study of a Grade 12 Class in South Africa. *Acta Didactica Napocensia*, *3*(3), 35-46.

Luse, A., Mennecke, B., & Townsend, A. (2012). Selecting a research topic: a framework for doctoral students. *International Journal of Doctoral Studies*, *7*, 143-152.

Magbanua, M. A. U. (2018). Self-guiding worksheets: strategy in improving students' performance in trigonometry. *International Journal of Innovation in Science and Mathematics*, *6*(6), 201-203.

Malambo, P. (2015). Exploring Zambian Mathematics student teachers' content knowledge of functions and trigonometry for secondary schools (Doctoral dissertation, University of Pretoria).

Mbugua, Z. K., Kibet, K., Muthaa, G. M., & Nkonke, G. R. (2012). Factors contributing to students' poor performance in mathematics at Kenya certificate of secondary education in Kenya: A case of Baringo County, Kenya.

Mensah, F. S. (2017). Ghanaian senior high school students' error in learning of trigonometry. *International Journal of Environmental and Science Education*, *12*, 8.

Miles, M. B., & Huberman, A. M. (1994). *Qualitative data analysis: An expanded sourcebook*. Sage.

Ministry of Education, Science and Sports. (2010). *Teaching syllabus for mathematics (Senior High School).* Accra: Curriculum Research and Development Division (CRDD)

Ministry of Education. (2019). Computerised school selection and placement system (CSSPS), Guidelines for selection of schools for placement. Accra, Ghana: MoE.

Mishra, P., & Koehler, M. (2006). Technological pedagogical content knowledge: A framework for teacher knowledge. Teachers college record, *108*(6), 1017-1054.

Mishra, P., & Koehler, M. J. (2007). Technological pedagogical content knowledge (TPCK): Confronting the wicked problems of teaching with technology. In Society for Information Technology & Teacher Education International Conference, 2214-2226. Association for the Advancement of Computing in Education (AACE).

Mkhwanazi, S. (2014). Crossing the past political boundary: Teacher professional learning in a South African reform context. *Mediterranean Journal of Social Sciences*, *5*(10), 425.

Mohamed, S. H. (2013). Professional development needs analysis of secondary mathematics teachers in Zanzibar, Tanzania.

Mohd-Rustam, M. R. (2016). Challenges in mathematics learning: a study from school students' perspective. *Research on Educational Studies, Universiti Teknologi Malaysia, 1*.

Montoro, V. (2012). "Professional development, teacher learning, and national standards: a mixed-method multiple-case study of the professional learning experiences of evangelical Christian school teachers" dissertations. 585. https://digitalcommons.andrews.edu/dissertations/585.

Moore, K. C. (2009). Trigonometry, technology, and didactic objects. In *S. Swars, D. Stinson, & S. Lemons-Smith (red.), Proceedings of 31st Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*, *5*, 1480-1488.

Mugenda, O. M., & Mugenda, A. (2003). *Research Methods in Education*.

Mulwa, E. C. (2015). Difficulties Encountered by Students in the Learning and Usage of Mathematical Terminology: A Critical Literature Review. *Journal of Education and Practice*, *6*(13), 27-37.

Mumford, K. R., Ferron, J. M., Hines, C. V., Hogarty, K. Y., & Kromrey, J. D. (2003). Factor retention in exploratory factor analysis: A Comparison of Alternative Methods.

Mutodi, P. & Ngirande, H. (2014). The influence of students' perceptions on mathematics performance: a case study of a selected high school in South Africa. *Mediterranean Journal of Social Sciences*, *5*(3), 431-445.

Mwadzaangati, L. E. N. (2017). An exploration of mathematical knowledge for teaching geometric proofs. *Unpublished doctoral dissertation, University of Malawi, Zomba, Malawi. Retrieved from http://repository. cc.ac.mw, 8080.*

Nabie, M. J., Akayuure, P., Ibrahim-Bariham, U. A., & Sofo, S. (2018). Trigonometric Concepts: Pre-Service Teachers' Perceptions and Knowledge. *Journal on Mathematics Education*, *9*(2), 169-182.

Nadelson, L. S., Seifert, A., Moll, A. J., & Coats, B. (2012). i-STEM summer institute: An integrated approach to teacher professional development in STEM. *Journal of STEM Education: Innovation and Outreach*.

National Council of Teachers of Mathematics. (2010). Principles and standards for school mathematics. Reston, VA: Author.

Niranjan, C. (2013). Using artefacts to support an embodied approach to learning trigonometry: a case study of grade 10 learners (Doctoral dissertation).

Nyikahadzoyi, M. R. (2013). Teachers' Knowledge of the concept of a function: A theoretical framework. *International Journal of Science and Mathematics Education*, 1-23. https://doi.org/10.1007/s10763-013-9486-9

Orhun, N. (2010). The gap between real numbers and trigonometric relations. *Quaderni di Ricerca in Didattica, 20*, 175-184

Peng, A. (2010). Teacher knowledge of students' mathematical errors.

Phonapichat, P., Wongwanich, S., & Sujiva, S. (2014). An analysis of elementary school students' difficulties in mathematical problem solving. *Procedia-Social and Behavioral Sciences*, *116*, 3169-3174.

Poudel, M. (2015). Problem Faced by Mathematics Teacher at Higher Secondary Level (Doctoral dissertation, Department of Mathematics Education Central Department of Education).

Quan-Baffour, K. P. (2007). Continuous professional development: a strategy to enhance quality of teaching at the school level. *International Journal of Learning*, *14*(1). 183-189.

Rakumako, A. & Laugksch, R. (2010). Demographic profile and perceived INSET needs of secondary Mathematics teachers in Limpopo province. *South African Journal of Education*, *30*(1), 139-152.

Resnik, D. B. (2010). *What is Ethics in Research & Why Is It Important? N*IEHS. http://www.niehs.nih.gov/research/resources/bioethics/whatis.cfm.

Sadler, P. M., Sonnert, G., Coyle, H. P., Cook-Smith, N., & Miller, J. L. (2013). The influence of teachers' knowledge on student learning in middle school physical science classrooms. *American Educational Research Journal, 50(5),* 1020-1049.

Sarac, A., & Tutak, F. A. (2017). The relationship between teacher efficacy, and students' trigonometry self-efficacy and achievement. *International Journal for Mathematics Teaching and Learning*, *18*(1).

Schweinle, A., & Mims, G. A. (2009). Mathematics self-efficacy: Stereotype threat versus resilience. *Social Psychology of Education*, *12*, 501-514.

Shulman, S. (1986). Those who understand: knowledge growth in teaching. *Educational Researcher*, *15*(20), 4-14.

Sibuyi, C. D. (2012). Effective teachers' pedagogical content knowledge in teaching quadratic functions in mathematics (Doctoral dissertation, University of Pretoria).

Singha, K. G., Goswami, M., & Bharali, R. K. (2012). Study of various problems faced by the students and teachers in learning & teaching mathematics and their suggestive measures. *International Journal of Advanced Research in Management and Social Sciences*, *1*(2), 195-201.

Steffe, L. P. and P. W. Thompson, Eds. (2000). Radical constructivism in action: Building on the pioneering work of Ernst von Glasersfeld. London, Falmer Press.

Sunzuma, G., & Maharaj, A. (2019). In-service teachers' geometry content knowledge: Implications for how geometry is taught in teacher training institutions. *International Electronic Journal of Mathematics Education*, *14*(3), 633-646.

Tabachnick, B. G., & Fidell, L. S. (2007). *Experimental designs using ANOVA* (p. 724). Belmont, CA: Thomson/Brooks/Cole.

Talkokul, E. N. Research on the Trigonometry as a Main Function of Sine, Secant, Tangent and Formula of Tan and Cot are Inverse.

Tecle, T. (2006). The potential of professional development scenario for supporting biology teachers in Eritrea. *Enschede: Print Partners IPS Kamp*.

Tsafe, A. K. (2013). Teacher pedagogical knowledge in mathematics: A tool for addressing learning problems. *Scientific Journal of Pure and Applied Sciences*, *2*(1), 1-7.

Tuna, A. (2013). A conceptual analysis of the knowledge of prospective mathematics teachers about degree and radia. *World Journal of Education*, *3*(4), 1-9.

Turnuklu, E. B., & Yesildere, S. (2007). The pedagogical content knowledge in mathematics: pre-service primary mathematics teachers' perspectives in Turkey. *Issues in the Undergraduate Mathematics Preparation of School Teachers*, *1*.

Von Glslsersfield, E. (1995). Sensory experience, abstraction, and teaching. In L. P. Steffe & Gale, J. (Eds.). *Constructivism in education*, 369-383. Hillsd & NJ: Lawrence Erlbaum Associates, Inc.

Von Glasersfeld, E. (1974). Piaget and the radical constructivist epistemology. *Epistemology and education*, *1*, 24.

Walsh, R. (2015). A purpose-built model for the effective teaching of trigonometry: a transformation of the van Hiele model.

Wang, J., & Yuan, H. (2011). Factors affecting contractors' risk attitudes in construction projects: Case study from China. *International Journal of Project Management*, *29*(2), 209-219.

Wessels, H., & Nieuwoudt, H. (2010). Teacher knowledge and confidence in grade 8 and 9 data handling and probability. In *Proceedings of the 8th International Conference on Teaching Statistics*.

West African Examination Council. (2012). West African Senior School Certificate Examinations Chief Examiner's Report for Elective Mathematics.

West African Examination Council. (2012). West African Senior School Certificate Examinations Chief Examiner's Report for Core Mathematics.

West African Examination Council. (2014). West African Senior School Certificate Examinations Chief Examiner's Report for Core Mathematics.

West African Examination Council. (2015). West African Senior School Certificate Examinations Chief Examiner's Report for Core Mathematics.

West African Examination Council. (2016). West African Senior School Certificate Examinations Chief Examiner's Report for Core Mathematics.

West African Examination Council. (2017). West African Senior School Certificate Examinations Chief Examiner's Report for Core Mathematics.

Wiest, L. R., & Lamberg, T. (Eds.). (2011). Proceedings of the 33rd Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education. Reno, NV: University of Nevada, Reno.

Wilkins, J. L. (2008). The relationship among elementary teachers' content knowledge, attitudes, beliefs, and practices. *Journal of Mathematics Teacher Education, 11*(2), 139-164

Witkin, B. R., & Altschuld, J. W. (1995). *Planning and conducting needs assessments: A practical guide*. Thousand Oaks, CA: Sage Publications.

Wood, M. B., Turner, E. E., Civil, M., & Eli, J. A. (Eds.). (2016). Proceedings of the 38th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education. Tucson, AZ: The University of Arizona.

Woolfolk, A. (2007). *Educational psychology*. Pearson: New York.

Woolfolk, A. E., & Hoy, W. K. (1990). Prospective teachers' sense of efficacy and beliefs about control. *Journal of Educational Psychology, 82*(1), 81-91.

Wu, H. (2002). Optimal designs for first-order trigonometric regression on a partial cycle. *Statistica Sinica*, 917-930.

Yiğit, K. M. (2016). Mathematics education graduate students’ understanding of trigonometric ratios. *International Journal of Mathematical Education in Science and Technology*, *47*(7), 1028-1047.

Zakaria, E., & Daud, M. Y. (2009). Assessing mathematics teachers’ professional development needs. *European Journal of Social Sciences*, *8*(2), 225-231.

Zurub, A. R., & Rubba, P. A. (1983). Development and validation of an inventory to assess science teacher needs in developing countries. *Journal of Research in Science Teaching*, *20*(9), 867-873.

Zuya, E. H. (2014). Mathematics teachers' ability to investigate students' thinking processes about some algebraic concepts. *Journal of Education and Practice*, *5*(25), 117-122.

APPENDICES

Appendix A: Response to, and Permission to Use Instrument

University of Cape Town

Rondebosch

South Africa.

2nd March, 2020

Dear Ms. Odjer,

RE: REQUEST FOR RESEARCH INSTRUMENT

Thank you for your email and interest in my work! As requested, I am attaching a copy of the STIN-NP as well as the original STIN-3 on which the South African version is based. For your records, I am also attaching a copy of the paper that describes the development and validation of the STIN-NP (called STIN-LP in the final version).

Good luck with your research – please keep me posted on your progress!

With best wishes and kind regards,

Rudiger C. Laugksch, PhD

Associate Professor (Science Education)

School of Education

Director: Postgraduate Studies & Funding

Faculty of Humanities

University of Cape Town

Rondebosch, South Africa

Appendix B: Ethical Clearance from IRB

UNIVERSITY OF CAPE COAST

INSTITUTIONAL REVIEW BOARD SECRETARIAT

TEL: 0558093143 / 0508878309
E-MAIL: irb@ucc.edu.gh
OUR REF: UCC/IRB/A/2016/845
YOUR REF:
OMB NO: 0990-0279
IORG #: IORG0009096

27TH NOVEMBER, 2020

Ms. Sandra Odjer
Department of Mathematics and I.C.T Education
University of Cape Coast

Dear Ms. Odjer,

ETHICAL CLEARANCE – ID (UCCIRB/CES/2020/80)

The University of Cape Coast Institutional Review Board (UCCIRB) has granted **Provisional Approval** for the implementation of your research titled **In-Service Mathematics Teachers' Understanding and Teaching of Trigonometry: Implication for Professional Development of Mathematics Teachers.** This approval is valid from 27TH November, 2020 to 26th November, 2021. You may apply for a renewal subject to submission of all the required documents that will be prescribed by the UCCIRB.

Please note that any modification to the project must be submitted to the UCCIRB for review and approval before its implementation. You are required to submit periodic review of the protocol to the Board and a final full review to the UCCIRB on completion of the research. The UCCIRB may observe or cause to be observed procedures and records of the research during and after implementation.

You are also required to report all serious adverse events related to this study to the UCCIRB within seven days verbally and fourteen days in writing.

Always quote the protocol identification number in all future correspondence with us in relation to this protocol.

Yours faithfully,

Samuel Asiedu Owusu, PhD
UCCIRB Administrator

ADMINISTRATOR
INSTITUTIONAL REVIEW BOARD
UNIVERSITY OF CAPE COAST

Appendix C: Consent Letter from Head of Department

UNIVERSITY OF CAPE COAST
COLLEGE OF EDUCATION STUDIES
FACULTY OF SCIENCE AND TECHNOLOGY EDUCATION
DEPARTMENT OF MATHEMATICS AND I.C.T EDUCATION

Telephone: 0332096951
Telex: 2552, UCC, GH
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Email: dmicte@ucc.edu.gh

University Post Office
Cape Coast, Ghana

Your Ref:

Our Ref: DMICTE/P.3/V.1/081

Date: 27th July, 2020

TO WHOM IT MAY CONCERN

Dear Sir/Madam,

RESEARCH VISIT

I write as the Head of Department to introduce our student Ms Sandra Odjer, with registration number ET/MDP/18/0026 an MPhil (Mathematics Education) student of the Department of Mathematics and ICT Education, College of Education Studies, University of Cape Coast.

As part of the requirement for the award of a master's degree, she is required to undertake a research on the topic "**IN-SERVICE MATHEMATICS TEACHERS' UNDERSTANDING AND TEACHING OF TRIGONOMETRY: IMPLICATIONS FOR PROFESSIONAL DEVELOPMENT**".

I would be grateful if you could give her the necessary assistance she may need.

Thanks for your usual support.

Yours faithfully,

Dr. Kofi Ayebi-Arthur
HEAD

Appendix D: Questionnaire for Data Collection

UNIVERSITY OF CAPE COAST

DEPARTMENT OF MATHEMATICS AND ICT EDUCATION

Research Questionnaire @ 2020

Dear Respondents,

The purpose of this study is to investigate In-Service Mathematics Teachers' (ISMTs) comprehension and teaching of trigonometry, as well as the implications for professional development. The purpose of this research is to learn more about ISMTs' personal and pedagogical comprehension of trigonometry ideas in SHS. Thus, to get an understanding of the challenges, both conceptual and pedagogical, that instructors face in order to effectively teach trigonometry in the classroom, establishing a foundation for good professional development. Because the study is academic in nature, you may be confident that your replies will be utilized only for the purposes described above. You are cordially asked to read and comprehend the items on this questionnaire before responding to them in order to improve the quality of the study. Responses that are objective will be much welcomed. Please read the instructions under each area of the questionnaire to help you answer the questions. Your replies are entirely optional and private.

Thank you so much for your willingness to participate in this study

<u>INSTRUCTIONS</u>

PLEASE TICK (√) THE APPROPRIATE RESPONSES AND PROVIDE ANSWERS WHERE NECESSARY

SECTION A: DEMOGRAPHIC CHARACTERISTICS

1. Sex:

   Male [ ]

   Female [ ]

2. Age (years):

   20-30 [ ]

   31-40 [ ]

   41-50 [ ]

   51-60 [ ]

3. Highest academic qualification

   B.Ed. [ ]   B.A [ ]   B.Sc [ ]   M.Ed [ ]

   M.A [ ]   MSC [ ]   MPhil Ed [ ]   PhD. [ ]

   Other (*please specify*)……………………………………………………………

4. Highest Professional Rank Superintendent [ ]   Senior Superintendent [ ]

   Principal Superintendent [ ]   Director II [ ]   Assistant Director I [ ]

   Other (*Please specify*)……………………………………………………………

5. For how long have you been teaching mathematics?

   1-5 years [ ]   6-10 years [ ]   Above 10 years [ ]

6. Which year group(s) do you teach in senior high school presently? (*tick as many as applicable*) Form One [ ]   Form Two [ ]   Form Three [ ]

7. Which of the following aspects do you teach?

   Core Mathematics [ ]   Elective Mathematics [ ]

8. Have you ever attended In-Service Training on Trigonometry?

   Yes [ ]

   No [ ]

SECTION B: TRIGONOMETRY CONTENT KNOWLEDGE

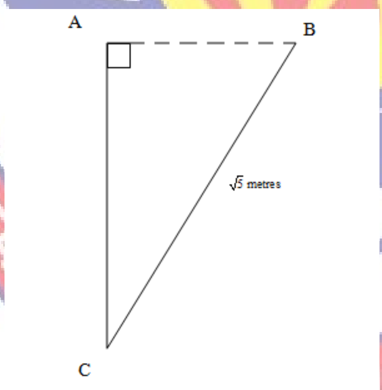
Instructions:

This test contains ten (10) questions about knowledge on trigonometry. You may use calculator to verify your answers if you choose. An answer booklet will be provided to clearly show workings to the free-response questions. ***Please attempt all questions in the instrument.***

PART I

1. Although $\sin(-\theta) = -\sin\theta$ and $\tan(-\theta) = -\tan\theta$, $\cos(-\theta) = \cos\theta$, for $0^0 < \theta < 90^0$. Explain with relevant diagrams.

2. The triangle ABC below shows the markings done by a carpenter on a plywood. The carpenter's goal was to measure and cut out exact length of $\sqrt{5}$ metres for a purpose. Study the markings and answer the questions that follow.



Which of the sides of triangle ABC is referred to as the:

i. Opposite? Explain.
ii. Adjacent? Explain.
iii. Hypotenuse? Explain.
iv. To be able to achieve the exact length of $\sqrt{5}$ metres (as shown in the diagram of ΔABC), what should be the lengths of the other two sides, $|AB|$ and $|AC|$?

3. The tangent of positive angles between 0^0 and 90^0 are well defined. Also, the tangent of positive angles between 90^0 and 270^0 are well defined. However, the tangent of 90^0 and 270^0 in particular remain undefined.
   i) Use the sketch of an appropriate graph to explain why it is so.
   ii) Write down the mathematical expression for the statement "θ is the acute angle whose tangent is 1"
4. A small stone is tied to a point P vertically above it by an elastic string 102cm long. If the string is moved such that it is inclined at an angle of 50^0 to the vertical, how high does the stone rise? [Correct your answer to two decimal places].
5. Simplify: $\frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}-\frac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta}$ and express the answer in terms of $\tan\theta$
6. Solve $\cos 2\theta+5\cos\theta=2$ for $0^0 \le \theta \le 360^0$
7. Express $\cos\phi+2\sin\phi$ in the form of $R\sin(\phi+\alpha)$ where α is an acute and find the maximum and minimum values for the expression giving the values of ϕ between 0^0 and 360^0 for which they occur.
8. The sides of triangle ABC are $|AB|$ =8.6 cm $|BC|$ =12.7 cm and $|AC|$ =13.9 cm. Calculate the height of the perpendicular from A to BC

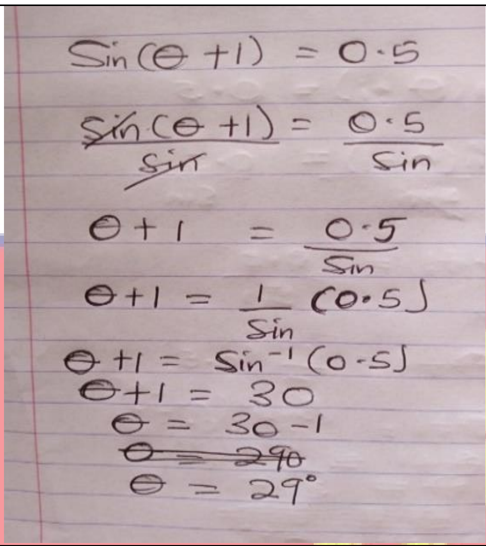
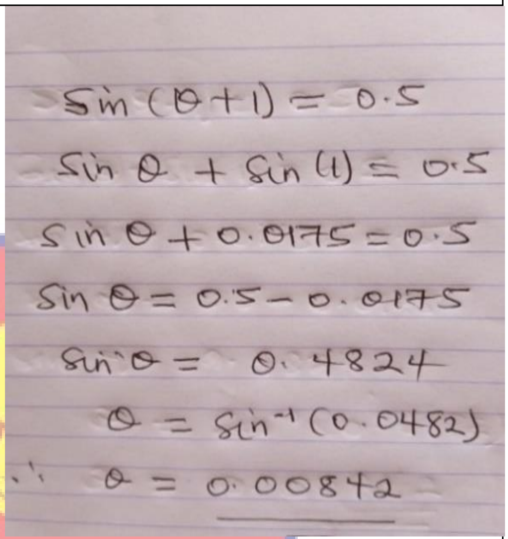
PART II

Instructions:

Below are the step-by-step presentations of solution made by two students. Study the solutions carefully and comment on them

TASK 1

9. Given that $\sin(\theta+1)=0.5$, find the value of θ if $0^0 < \theta < 90^0$.

Student A	**Student B**
 $\sin(\theta+1) = 0{\cdot}5$ $\frac{\sin(\theta+1)}{\sin} = \frac{0{\cdot}5}{\sin}$ $\theta+1 = \frac{0{\cdot}5}{\sin}$ $\theta+1 = \frac{1}{\sin}(0{\cdot}5)$ $\theta+1 = \sin^{-1}(0{\cdot}5)$ $\theta+1 = 30$ $\theta = 30-1$ $\cancel{\theta = 29\theta}$ $\theta = 29^\circ$	 $\sin(\theta+1) = 0{\cdot}5$ $\sin\theta + \sin(1) = 0{\cdot}5$ $\sin\theta + 0{\cdot}0175 = 0{\cdot}5$ $\sin\theta = 0{\cdot}5 - 0{\cdot}0175$ $\sin\theta = 0{\cdot}4824$ $\theta = \sin^{-1}(0{\cdot}0482)$ $\theta = 0{\cdot}00842$
a) Do you detect any flaws in the student's solution? Yes/No. Indicate the mistakes (if any). b) Can you provide causes or justifications for the student's solution? (if any). c) What would you do to remedy any errors and assist the learner in understanding the concepts? If you have a solution, please offer it.	a) Do you detect any flaws in the student's solution? Yes/No. Indicate the mistakes (if any). b) Can you provide causes or justifications for the student's solution? (if any). c) What would you do to remedy any errors and assist the learner in understanding the concepts? If you have a solution, please offer it.

TASK 2

10. Solve the following equation for values of θ, if $0^0 < \theta < 360^0$.

$\cos\left(\theta + 30^0\right) + \cos\left(\theta - 30^0\right) = \cos 30^0$

Student A	**Student B**
$\cos(\theta+30^\circ) + \cos(\theta-30^\circ) = \cos 30^\circ$ $\Rightarrow \cos\theta + \cos 30^\circ + \cos\theta - \cos 30^\circ = \cos 30$ $\Rightarrow \cos\theta + \cos\theta = \cos 30^\circ$ $\frac{\cos \cancel{2}\theta}{\cancel{2}} = \frac{\cos 30^\circ}{2}$ $\Rightarrow \cos\theta = \frac{\cos 30^\circ}{2}$ but $\cos 30^\circ = \sqrt{3}/2$ $\Rightarrow \cos\theta = \sqrt{3}/4$ $\theta = \cos^{-1}\frac{\sqrt{3}}{4}$ $\theta = 63.34^\circ$	$\cos(\theta+30^\circ) + \cos(\theta-30^\circ) = \cos 30^\circ$ $\cos\theta + \cos 30^\circ + \cos\theta - \cos 30^\circ = \cos 30^\circ$ $2\cos\theta = \cos 30^\circ$ but $\cos 30^\circ = \frac{\sqrt{3}}{2}$ $2\cos\theta = \frac{\sqrt{3}}{2}$ $\cos\theta = \frac{\sqrt{3}}{4}$ $\theta = \cos^{-1}\left(\frac{\sqrt{3}}{4}\right)$ $\theta = 64.34^\circ$
a) Do you detect any flaws in the student's solution? Yes/No. Indicate the mistakes (if any). b) Can you provide causes or justifications for the student's solution? (if any). c) What would you do to remedy any errors and assist the learner in understanding the concepts? If you have a solution, please offer it.	a) Do you detect any flaws in the student's solution? Yes/No. Indicate the mistakes (if any). b) Can you provide causes or justifications for the student's solution? (if any). c) What would you do to remedy any errors and assist the learner in understanding the concepts? If you have a solution, please offer it.

SECTION C: TRIGONOMETRY SELF-EFFICACY

On a scale of 1 to 5, 1 = not at all confident, 2 = only a little confident, 3 = slightly confident, 4 = confident, and 5 = extremely confident. Please indicate your level of confidence in teaching pupils the abilities required to accomplish the work effectively by checking [√] your choice. For the open-ended questions please include as much detail as you feel comfortable sharing.

N	PART I: TRIGONOMETRY I	1	2	3	4	5
1	Assist pupils in defining trigonometric ratios using appropriate diagrams.					
2	Instruct pupils to create an equilateral triangle with two dimensions (e.g., two units) and use it to calculate the trigonometric ratios for 30o and 60o.					
3	Assist students with drawing a square with one unit side, one diagonal, and using the diagonal and two sides to calculate the value of trigonometric ratios of 450.					
4	Instruct pupils to use their calculators to calculate trigonometric ratios for specified angles between 00 and 360o.					
5	Using tables or calculators, assist students in determining the inverse of specified trigonometric ratios.					
6	Using graphics, explain to pupils what angles of elevation and angles of depression are.					
7	Pose real-life scenarios utilizing trigonometric ratios for pupils to solve.					

Open-Ended Question: Please review your responses to items 1-7

a) Choose an item from items 1-7 (on this page only) that you indicated feeling ***LEAST CONFIDENT*** about teaching high school students. Think about the reason(s) you feel this away. Use the space below (and the back of this paper, if necessary) to find the item number and explain your reason(s).

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b) Choose an item from items 1-7 (on this page only) that you indicated feeling ***MOST* CONFIDENT** about teaching high school students. Think about the reason(s) you feel this away. Use the space below (and the back of this paper, if necessary) to find the item number and explain your reason(s).

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Using a scale of 1 to 5, where 1 = not at all confident, 2 = only a little confident, 3 =somewhat confident, 4 = confident, and 5 = very confident.

N	PART II: TRIGONOMETRY II	1	2	3	4	5
8	Guide students to prepare tables for given trigonometric functions for: $y = a\sin x$ and $y = b\cos x$, where, $0^0 \le x \le 360^0$					
9	Guide students to use their tables to draw the graphs of the functions and find the maximum and minimum values.					

10	Guide students to draw simple graphs of trigonometric functions of the form: $f(x) = a\sin x + b\cos x$, where, $0^0 \le x \le 360^0$					
11	Guide students to use their graphs to solve equations such as: $a\sin x + b\cos x = 0$, $a\sin x + b\cos x = k$, etc					

Open-Ended Question: Please review your responses to items 8-11

a) Choose an item from items 8-11 (on this page only) that you indicated feeling ***LEAST CONFIDENT*** about teaching high school students. Think about the reason(s) you feel this away. Use the space below (and the back of this paper, if necessary) to find the item number and explain your reason(s).

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b) Choose an item from items 8-11 (on this page only) that you indicated feeling ***MOST CONFIDENT*** about teaching high school students. Think about the reason(s) you feel this away. Use the space below (and the back of this paper, if necessary) to find the item number and explain your reason(s).

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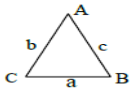
Using a scale of 1 to 5, where 1 = not at all confident, 2 = only a little confident, 3 =somewhat confident, 4 = confident, and 5 = very confident.

N	PART III: TRIGONOMETRIC RATIOS AND RULES	1	2	3	4	5
12	Assist students with reviewing the three fundamental trigonometric ratios: sine, cosine, and tangent.					
13	Show pupils how to use quadrants to get fundamental trigonometry ratios.					
14	Assist pupils in determining the reciprocals of trigonometric ratios.					
15	Assist students to relate trigonometric ratios to Cartesian co-ordinates of the point (x, y) on the circle: $x^2 + y^2 = r^2$					
16	Assist students to derive the trigonometric identities					
17	Assist students to form the concept of negative angles and to establish the following relations: $\sin(-\theta) = \sin(360 - \theta) = -\sin\theta$ $\cos(-\theta) = \cos(360 - \theta) = \cos\theta$ $\tan(-\theta) = \tan(360 - \theta) = -\tan\theta$					
18	Encourage students to use the calculator to verify the relations in 17 above					
19	Assist students to obtain radian equivalents for angles in degrees and vice versa using the relation $\left[\pi \; radian = 180^0\right]$					

20	Assist students to deduce the sine rule $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ 					
21	Guide students to use the sine rule to solve related problems					
22	Assist students to deduce the cosine rule $a^2 = b^2 + c^2 - 2bc\cos A$, etc and use it to solve problems on triangles					
23	Assist students to use the cosine rule to solve problems on triangles					
24	Assist students to apply the sine and cosine rules to solve problems involving bearings (real life problems)					

Open-Ended Question: Please review your responses to items 12-24

a) Choose an item from items **12-24** (on this page only) that you indicated feeling ***LEAST CONFIDENT*** about teaching high school students. Think about the reason(s) you feel this away. Use the space below (and the back of this paper, if necessary) to find the item number and explain your reason(s).

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b) Choose an item from items **12-24** (on this page only) that you indicated feeling ***MOST CONFIDENT*** about teaching high school students. Think about the reason(s) you feel this away. Use the space below (and the back of this paper, if necessary) to find the item number and explain your reason(s).

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Using a scale of 1 to 5, where 1 = not at all confident, 2 = only a little confident, 3 =somewhat confident, 4 = confident, and 5 = very confident.

N	PART IV: COMPOUND AND MULTIPLE ANGLES	1	2	3	4	5
25	Guide students to derive the compound angles identities $\sin(A \pm B) = \sin A \cos B \pm \sin B \cos A$, $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$ and $\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$					
26	Assist pupils in using their identities to solve difficult questions					
27	Assist students with determining the double angle identities for Sin2A, Cos2A, and Tan2A and using them to develop identities for Sin3A and Cos3A in terms of sin A and cos A.					
28	Encourage pupils to check these relationships using the calculator and particular values.					

Open-Ended Question: Please review your responses to items 25-28

a) Choose an item from items ***25-28*** (on this page only) that you indicated feeling ***LEAST CONFIDENT*** about teaching high school students. Think about the reason(s) you feel this away. Use the space below (and the back of this paper, if necessary) to find the item number and explain your reason(s).

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b) Choose an item from items ***25-28*** (on this page only) that you indicated feeling ***MOST CONFIDENT*** about teaching high school students.

	Think about the reason(s) you feel this away. Use the space below (and the back of this paper, if necessary) to find the item number and explain your reason(s). …………………………………………………………………………… ……………………………………………………………………………

Using a scale of 1 to 5, where 1 = not at all confident, 2 = only a little confident, 3 =somewhat confident, 4 = confident, and 5 = very confident.

N	PART V: TRIGONOMETRIC FUNCTIONS, EQUATIONS AND GRAPHS	1	2	3	4	5
29	Examine sin x and cos x graphs.					
30	Assist pupils with drawing the tan x graph and comparing it to the sine and cosine graphs.					
31	Encourage pupils to study the nature of trigonometric function graphs using a calculator and a computer.					
32	Assist students to find solution sets to trigonometric equations up to quadratic $2\sin^2 x - \sin x - 3 = 0$, $0^0 \le x \le 360^0$					
33	Encourage students to use the calculator and computer to draw graphs of trigonometric functions and find their solutions (graphical approach)					
34	Revise graphs of trigonometric functions of the form,					

	$f(x) = a\sin x + b\cos x$					
35	Guide students to express the trigonometric function, $f(x) = a\sin x + b\cos x$ in the form, $R\cos(x \pm a)$ or $R\sin(x \pm a)$, where, $0^0 \leq a \leq 90^0$.					
36	Assist students to use the result to calculate the maximum and minimum points of the function					

Open-Ended Question: Please review your responses to items 29-36

a) Choose an item from items ***29-36*** (on this page only) that you indicated feeling ***LEAST CONFIDENT*** about teaching high school students. Think about the reason(s) you feel this away. Use the space below (and the back of this paper, if necessary) to find the item number and explain your reason(s).

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b) Choose an item from items ***29-36*** (on this page only) that you indicated feeling ***MOST CONFIDENT*** about teaching high school students. Think about the reason(s) you feel this away. Use the space below (and the back of this paper, if necessary) to find the item number and explain your reason(s).

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SECTION E: CHALLENGES TEACHERS FACE IN TEACHING TRIGONOMETRY

The following are statements about challenges teachers face in teaching trigonometry. Kindly indicate your level of agreement or disagreement with the statements by ticking [√] in the spaces provided, where; strongly Disagree= SD, Disagree= D, Agree=A and Strongly Agree =SA

S/N	Statements	SD	D	N	A	SA
1	Inadequate teaching resources					
2	Problems of implementing new teaching strategies					
3	Insufficient funds for purchasing equipment and supplies needed in teaching Trigonometry					
4	Lack of teachers' interest in teaching trigonometry					
5	Students' viewing trigonometry as difficult, abstract and boring					
6	Inadequate in-service training on trigonometry concepts					
7	Inadequate pre-service training towards the teaching of trigonometry					
8	Difficulty in teaching trigonometry because students do not have the necessary relevance previous knowledge (RPK)					
9	Inability of teachers to envisage trigonometry errors students may commit					
10	Teachers not confident about some trigonometry content in the curriculum					
11	Inability of teachers to link students RPK to trigonometry topics					

SECTION F: MATHEMATICS TEACHERS' INVENTORY NEEDS FOR TEACHING TRIGONOMETRY [MTIN-TRIG]

Dear Teacher,

You have been chosen to take part in a research to determine the professional needs of Senior High School In-Service Mathematics Teachers (ISMTs) in the teaching of Trigonometry. Perhaps you believed that no one cared or desired to assist you in improving or acquiring abilities to further increase your teaching effectiveness in this area. The questionnaire included might be the first step towards helping you.

Please take some time out of your busy schedule to help us determine what ISMTs most need to increase the quality of their trigonometry understanding and teaching (Please note that the study is anonymous: you do not need to give your name). The findings of this study are likely to have a beneficial impact on the design and delivery of appropriate, effective, and ongoing In-Service Education and Training (INSET) programs for the teaching and learning of trigonometry in Senior High Schools. INSET activities must recognize your professional needs as you view them in order to be effective; hence, your involvement in this study is vital.

Thank you for helping us to help you!

Miss Sandra Odjer (Researcher)

Sandra.odjer@stu.ucc.edu.gh

INSTRUCTIONS AND GUIDELINES

This portion of the questionnaire has been developed to assist you in expressing your needs as a Mathematics classroom teacher (Trigonometry). The statements explain the duties that teachers must complete before to, during, and after trigonometry education. Please rate your need for assistance on a 5-point scale. For items in this portion of the questionnaire, use the following scale: (1 = Not Familiar; 2 = No Need; 3 = Little Need; 4 = Moderate Need; 5 = Great Need). When answering to the items, you must mark the one number on the questionnaire that best represents your level of need for assistance with that job.

TPD Needs

S/N	Tasks	**1**	**2**	**3**	**4**	**5**
1	Creating trig instructions based on student readiness data					
2	Material selection for teaching					
3	Creating an adequate teaching learning environment, techniques, and resources					
4	Determine learning objectives (i.e., outcomes) that outline the information required by Trigonometry students.					
5	Determine learning objectives (i.e., outcomes) that outline the attitudes that students must adopt regarding trigonometry.					

6	Determine learning objectives (i.e., outcomes) that outline the abilities that students must gain in Trigonometry.					
7	Determine acceptable learning objectives (i.e., outcomes) for encouraging multicultural approaches of learning in Trigonometry.					
8	Create lesson plans (i.e., learning activities) that include the history of trigonometry.					
9	Create lesson plans (learning activities) that integrate Trigonometry with other subjects.					
10	Choose commercially produced instructional resources for Trigonometry (e.g., textbooks, charts, models, etc.).					
11	Create lesson plans (i.e., learning activities) for Trigonometry subtopics.					
12	Encourage students to study trigonometry.					
13	In Trigonometry, use an inquiry/discovery teaching strategy (i.e., method).					
14	In the Trigonometry educational lessons, use hands-on teaching approaches.					
15	In Trigonometry, demonstrate process abilities (e.g., generalizing, defining, etc.).					
16	In Trigonometry, demonstrate manipulative abilities (e.g., measuring).					
17	Apply Trigonometry ideas to learners' daily lives (i.e., to real-life situations)					

18	Conduct a field excursion to help students study Trigonometry more effectively.					
19	In Trigonometry, use teaching approaches (i.e., procedures) that allow you to focus on educating individuals rather than the entire class.					
20	For teaching big courses in Trigonometry, use instructional methodologies (i.e., methods).					
21	In Trigonometry, use instructional methods that require students to teach each other (i.e., peer tutoring).					
22	To enhance Trigonometry instruction, use audio-visual technology (e.g., overhead projector, cassette or video recorder, etc.).					
23	Trigonometry may be taught using computers.					
24	In your Trigonometry lesson, maintain learner discipline.					
25	Evaluate your own teaching efficiency as a Trigonometry instructor.					
26	Maintain student discipline in your classroom.					
27	Assess your own teaching efficacy as a teacher.					
28	In the Trigonometry classroom, organize and manage physical space (e.g., arrangement of learners' desks, etc.).					
29	In trigonometry, use a computer to aid organize teaching (e.g., storing student records).					

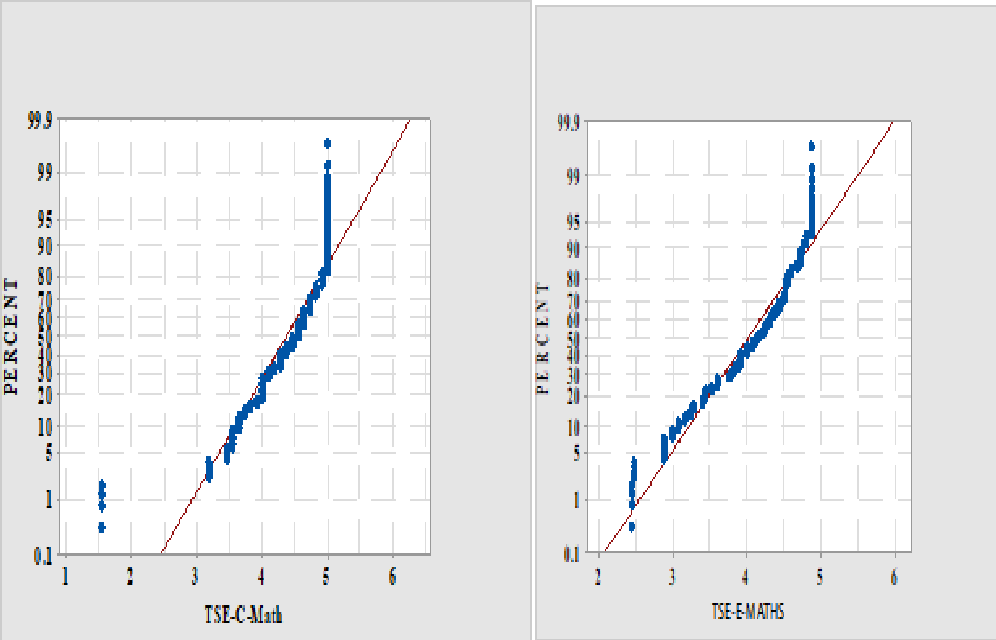
30	Update your understanding about Trigonometry-related job options for students.					
31	In Teaching Trigonometry, refresh your understanding of successful teaching techniques (i.e., strategies).					
32	Update your understanding on Trigonometry-related societal issues (economics, electrification, etc.)					
33	Update your understanding of how students learn trigonometry in a global culture.					
34	Update your learning expertise to include a constructivist approach to learning. Trigonometry					
35	Update your understanding of how Trigonometry is applied in society.					
36	Improve your ability to identify and rectify frequent trigonometry misunderstandings and mistakes.					
37	Improve your Trigonometry content understanding.					
38	Update your Trigonometry knowledge and skills.					
39	Refresh your understanding of Trigonometry's history.					
40	To enhance Trigonometry instruction, use audio-visual technology (e.g., overhead projector, cassette or video recorder, etc.).					
41	Computers are used to teach Trigonometry.					
42	In Trigonometry, use a computer to aid organize teaching (e.g., keeping student records).					

43	Choose supplementary materials (such as library and reference books, films, and so on) for teaching trigonometry.					
44	Internet use (choosing appropriate websites, participating in user groups/discussions, etc.)					
45	Powerpoint and other presentation software					
46	Spreadsheet programs such as Microsoft Excel are used to create statistical graphs.					
47	Use of a graphical calculator					
48	Using a Scientific Calculator					
49	Multimedia operation training (using digital video and/or audio devices in trigonometry)					
50	Subject-specific training using learning software to achieve particular subject maths objectives (e.g. tutorials, simulations, etc.)					
51	Course on pedagogical concerns with the incorporation of ICT into teaching and learning.					
52	Develop own teaching materials/aids for Teaching Trigonometry					
53	In the Trigonometry educational lessons, use hands-on teaching approaches.					
54	In Trigonometry, demonstrate manipulative abilities (e.g., utilize trigonometric models).					
55	Identify free and locally available Trigonometry teaching materials.					

56	Choose supplementary materials (such as library and reference books, films, and so on) for teaching trigonometry.					
57	To detect trigonometric learning, use performance records.					
58	Interpret performance data to assess whether pupils are ready for trigonometry education.					
59	Create assessment items that accurately assess trigonometric instructions.					
60	In trigonometry instruction, set up informal evaluation settings.					

THANK YOU FOR YOUR ASSISTANCE– IT IS GREATLY APPRECIATED

Appendix E: Normality Tests



Normality Test for Trigonometry Self-Efficacy (TSE)

Appendix F: Overall Mean Score on Trigonometry Content Knowledge

Variable	Mean	SD
Questions 1-Question 10	40.64	24.19
Tasks 1&2	34.80	35.02

Appendix G: Overall Mean and SD of Least and Most Confident Items

Overall Level of Confidence	Mean	SD
Overall Least Confidence	3.826	0.719
Overall Most Confidence	4.306	0.596

Appendix H: Categorization of Errors in Hypothesized Student Solutions

Task 1: Given that $\sin(\theta+1)=0.5$, find the value of θ if $0^0<\theta<90^0$.

STUDENT A	Errors and Misconceptions	Error Categories	STUDENT B	Errors and Misconceptions	Error categories
$\frac{\sin(\theta+1)}{\sin}=\frac{0.5}{\sin}$ $\theta+1=\frac{0.5}{\sin}$	Yes, there are errors Dividing through by 'sin' as a common factor instead of a function.	ERROR 1	$\sin\theta+\sin(1)=0.5$	Yes, there are errors Opening the bracket at the LHS i.e. $\sin(\theta+1)\neq\sin\theta+\sin(1)$	ERROR 1
$\theta+1=\frac{1}{\sin}(0.5)$ $\theta+1=\sin^{-1}(0.5)$ $\theta+1=30$ $\therefore\theta=29^0$	$\frac{1}{\sin}\neq\sin^{-1}$ This is because sine inverse is a function These errors and misconceptions renders the final answer $\therefore\theta=29^0$ Not Judicious (NJ)	ERROR 2	$\sin\theta+0.0175=0.5$ $\sin\theta=0.5-0.0175$ $\sin\theta=0.4825$ $\theta=\sin^{-1}(0.4825)$ $\theta=0.00842^0$	This expansion messed up the whole calculations Also, $\sin^{-1}(0.4825)\neq0.00842$ Therefore $\theta=0.00842$ is not a solution	ERROR 2
				Also, $\theta=0.00842$ has no unit of measurement (degrees)	ERROR 3

Task 2: Solve $\cos\left(\theta+30^0\right)+\cos\left(\theta-30^0\right)=\cos 30^0$ for values of θ, if $0^0<\theta<360^0$.

STUDENT A	Errors and Misconceptions	Error categories	STUDENT B	Errors and Misconceptions	Error Categories
	Yes. There is something wrong		$\cos\left(\theta+30^0\right)+\cos\left(\theta-30^0\right)$ $=\cos 30^0$	Yes. There is something wrong	
$\cos\theta+\cos 30^0+\cos\theta$ $-\cos 30^0=\cos 30^0$	There is error in the expansion of the cosine of compound angles identity (incorrect application of distributive property)	ERROR 1	$\cos\theta+\cos 30^0+\cos\theta-\cos 30^0$ $=\cos 30^0$	There is error in the expansion of the cosine of compound angles identity	ERROR 1
$\cos 2\theta=\cos 30^0$	$\cos\theta+\cos\theta$ $\neq\cos 2\theta$	ERROR 2	$2\cos\theta=\cos 30^0$ $but\ \cos 30^0=\dfrac{\sqrt{3}}{2}$		
$\dfrac{\cos 2\theta}{2}=\dfrac{\cos 30^0}{2}$ $but\ \cos 30^0=\dfrac{\sqrt{3}}{2}$ $\cos\theta=\dfrac{\sqrt{3}}{4}$ $\theta=\cos^{-1}\dfrac{\sqrt{3}}{4}$ $\theta=64.34^0$	Dividing both sides of the equation using "2" as a factor instead of an angle of a function in the cosine double angle ($cos2\theta$). These errors and misconceptions rendered the final answer $\theta=64.34^0$ incorrect Therefore $\theta=64.34^0$ is not a solution	ERROR 3	$2\cos\theta=\dfrac{\sqrt{3}}{2}$ $\cos\theta=\dfrac{\sqrt{3}}{4}$ $\theta=\cos^{-1}\dfrac{\sqrt{3}}{4}$ $\theta=64.34^0$	These errors and misconceptions rendered the final answer $\theta=64.34^0$ incorrect. Therefore $\theta=64.34^0$ is not a solution.	ERROR 2