UNIVERSITY OF CAPE COAST

INFLUENCE OF NUMBER LINE APPROACH ON LEARNING FRACTIONS: A CASE OF BASIC SIX LEARNERS IN THE CAPE COAST METROPOLIS

MAXWELL ADU SARPONG

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BY

MAXWELL ADU SARPONG

Thesis submitted to the Department of Basic Education of the Faculty of Educational Foundations, College of Education Studies, University of Cape Coast, in partial fulfilment of the requirements for the award of Master of Philosophy degree in Basic Education.

APRIL 2022

DECLARATION

Candidate's Declaration

I hereby declare that this is the result of my original research and that no part of it has been presented for another degree at this university or elsewhere.

Candidate's Signature ………………………….. Date …………….

Name:

Supervisor's Declaration

I hereby declare that the preparation and presentation of the thesis were supervised following the guidelines on supervision of thesis laid down by the University of Cape Coast.

Principal Supervisor's Signature …………………….. Date ………………..

Name:

ABSTRACT

While the number line approach is successful for fractional learning, there is a dearth of study on its usage in Ghanaian upper primary schools. The study explored the influence of the number line approach on learning fractions. Learners' achievements, attitudes, and learner's challenges when using the number line were explored. A quantitative research method embedded with a pretest-posttest non-equivalent design was used. The Fractions Achievement Test (FAT) and a structured questionnaire were the instruments used for data collection. Eighty-one basic six learners with two intact classes (39 in control and 42 experimental groups) were purposively selected within the Cape Coast Metropolis. The experimental group was exposed to learning fractions using the number line, while, in the control group, fraction lessons were carried out through the use of set models. An independent samples *t*-test was employed to analyse the differences between the pre-test and post-test of the control and experimental group. The findings revealed that there was a significant difference in performance between the post-test scores for the control group $(M = 7.36, SD = 1.78)$ and the experimental group $(M = 9.21, SD = 2.83)$; $t\ (69.77) = -3.55; p = 0.001$ respectively. This implies that the experimental group outperformed the control group due to the influence of the number line approach. Learners in the experimental group had better, understanding, application, and positive attitude toward learning fractions due to the use of number line. Finally, learners however had some challenges, such as the inability to recognise that zero is part of the number system, poor estimation, and counting the tick lines on the number line instead of the intervals between them.

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DEDICATION

To my father's memory, family and friends for the love and encouragement.

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CHAPTER ONE

INTRODUCTION

Background to the Study

The Institute for Statistics of the United Nations Educational, Scientific, and Cultural Organisation [UNESCO] (2017) has launched a community-wide advocacy campaign emphasising the relevance of obtaining a literacy education, particularly among teenagers and learners. Gravemeijer, Stephan, Julie, Lin, and Ohtani (2017), Chai (2019), Eismawati, Koeswanti, and Radia (2019) all conducted studies on the relevance of learners' mathematical skills and education worldwide. In a similar vein, Hasler and Akshoomoff (2019) asserted that both the International Mathematics Union (IMU) and the African Mathematical Union (AMU) support research into the importance of mathematical education, emphasising that every child, without exception, should possess fundamental mathematical skills. Additionally, because mathematics is a required core subject in primary education and frequently opens doors for those who possess it (Buenrostro & Radinsky, 2019), mathematics learners should have a thorough understanding of concepts, as the subject is a "calculation process" (Ghani & Mistima, 2018, p. 11). Charles-Ogan and Otikor (2016) believed that a significant amount of mathematics knowledge is requisite in the field of Science, Technology, and Engineering (STEM) for learners to relate what they have studied, which will then be used to solve any problems they encounter (Putra, Setiawan, Nurdianti, Retta & Desi, 2018).

Notwithstanding the importance of mathematics, most fourth-grade learners are not proficient in mathematics (National Assessment of Education Progress, 2019), most learners face learning difficulties, and teachers suffer in making learners understand the subject (Hidayat & Prabawanto, 2018; Copur-Gencturk, 2021).

In Ghana, the curriculum is concerned with all learners acquiring skills in the 4Rs: Reading, Writing, Arithmetic, and Creativity. All learners are to be provided with these essential skills and knowledge at any departure point from formal education, which seems a requirement to become a learning nation. Fully functioning citizens should become graduates of the 4Rs and lifelong learners (National Council for Curriculum and Assessment, Ministry of Education, 2019).

In the view of Wijaya (2017), fractional knowledge provides the basis for algebraic thinking and proportional justification and is a vital element for the development of mathematical understanding and a stepping stone to a variety of desirable careers (Hoon, Yaakob & Singh, 2016; Mousley & Kelly, 2018; Hughes, 2019; Teoh, Kor, Mohamed & Singh, 2020).

Carpentry, cooking, money management, and auto mechanics are just a few of the non-STEM middle-income jobs that require fractions (Handel, 2016; Bouck, Maher, Park, & Whorley, 2020) and are widely spread in everyday life and a necessity as early as the first grade (Dewi, Suratno, Suryadi, Mulyana, & Kurniawan, 2017). Fractions are among the most commonly studied concepts of mathematics. Despite the relevance of understanding fraction operations for learners, accurately representing and performing fraction operations is difficult in early and middle schools (e.g., Sidney & Alibali, 2017; Nasution & Putri, 2018; Provasnik, Dogan, Erberber, & Zheng, 2020). There are four procedures to represent fractions: symbolically, objectively, verbally, and in a model (Deringöl, 2019). All of these must be understood by the learners to relate each of these expressions.

Learners who struggle with elementary, middle, high school, and college never gain fractional arithmetic skills (Braithwaite, Pyke & Siegler, 2017; Bentley & Bossé, 2018). Braithwaite et al. (2017) posited that learners' inability to learn advanced mathematics hinders their employment success because, in a representative sample of U.S. workers, 68% reported using fractions at work (Handel, 2016). Researchers assessed Grade-eight learners in the USA on fraction addition and discovered that only 27% correctly identified the closest whole number to $\frac{12}{13} + \frac{7}{8}$ (the answer choices were 1; 2; 19; and "I do not know") (2). (Lortie, Forgues, Tian & Siegler, 2015). One possible reason for poor fractional understanding was that the mathematics textbook presented only one definition for fractions, i.e., fractions as only part of wholes. Also, the curriculum did not focus sufficiently on introducing the four operations of fractions until much later in the year (Wijaya, 2017).

If learners' conceptual understanding of fractions is weak, they will be unable to acquire a fundamental knowledge of fractions and be more prone to errors (Van Steenbrugge, Lesage, Valcke & Desoete, 2014). Given this, Wilkings and Norton (2018) contended that these challenges raise a critical challenge for learners. For example, textbook analyses reveal that learning opportunities frequently concentrate on procedural knowledge, contributing to learners' conceptual knowledge deficits (Lenz & Wittmann, 2020). In addition to general cognitive abilities and whole number skills, the knowledge of fractional magnitude predicts overall and more specific results for mathematics (Karamarkovich & Rutherford, 2019).

Many learners showed little or none of their accuracy between the fourth and sixth grades despite three years of fractional education in school (Firmender, Gavin & McCoach, 2014). Therefore, the National Governors Association Centre for Best Practices and Council of Chief State School Officers (2010) posited that addition and subtraction should usually be introduced in the fourth grade, and the main objective is fraction arithmetic up to sixth grade. In mathematics, integrating fractions into number lines becomes crucial (Slyke, 2019). Fazio, DeWolf, and Siegler (2016) discovered that the number line learning platform provides an appropriate context for learners to estimate fraction values.

Teachers must understand how their learners have demonstrated their fractional abilities with a number line since the number line is an excellent representation of increasing the fraction's ability (Hwang, Riccomini, Hwang, & Morano, 2019). In the views of Altıparmak and Palabıyık (2019), when learners are permitted to "understand" fractions using the number line, it assists them in "remembering," which enables them to "apply" according to the revised Bloom's taxonomy and can significantly promote meaningful learning. Barbieri, Rodrigues, Dyson, and Jordan (2020) recommended that using the number line approach increases learners' attention and attitude because it will allow more discussion and strategies to be demonstrated.

Similarly, number lines can help learners understand the relative magnitude of fractional mathematics (Resnick et al., 2017; Hamdan & Gunderson, 2017). Learners who can reliably locate fractions on number lines have a much higher chance of succeeding than those who cannot (Siegler & Pyke, 2013). While Yu (2018) opined that fractions on number lines could help learners overcome their whole number bias and teach fractional magnitudes. The Practice Handbook of the Institute of Education Sciences (IES) recommends using numerical line models as evidence for reasons and understanding fractional dimensions since it is an effective external visual representation. Yet, Mazana, Suero Montero and Olifage (2019) revealed that the learners attitudes may be related to their success in fractions using the number line approach.

Many interventions have been implemented throughout the years to help learners understand fractions (Jordan, Resnick, Rodrigues, Hansen & Dyson, 2017; Nasution & Putri, 2018), but most produced minor effects.

Statement of the Problem

Learning fractions demand not only the familiarity with fractional concepts, definitions, and properties but also the capacity to build cardinal and ordinal numbers and compare and organise fractions such as $\frac{1}{2}$, $\frac{1}{4}$ etc., and use them to find fractions of shapes and numbers. Upper Primary learners are to simplify fractions, utilise them as operators, and find fractions of integers and quantities (NaCCA, 2019). However, many learners struggle with fractions despite years of instruction (Hwang et al., 2019; Roesslein & Codding, 2019). The "whole numbers bias," as coined by Ni and Zhou (2005), is a significant contributor to the challenges of learning fractions. This prejudice makes it hard to think of whole numbers as units that can be subdivided. Learners face difficulties because there are so many different ways to interpret fractions.

The Trends in International Mathematics and Science Study [TIMSS, 2007, 2011], the (Program for International Student Assessment) [PISA], (2015), and the (Early Grade Mathematics Assessment) [EGMA] (2015) all revealed that Ghanaian learners could not perform above curriculum expectations (Mereku, 2016), highlighting the critical role of educators. A similar scenario exists in the Basic Education Certificate Examination (WAEC, 2015, 2016, and 2017), where the Chief Examiner has repeatedly reported learners' deficits in the fractional concept. Nevertheless, failure to progress from Basic Education Certificate Examination (BECE) and West African Senior Secondary Certificate Examination (WASSCE) will deny candidates access to the next level of education (Ntow, 2009).

A number line is suggested to represent part-whole, quotient, measure, ratio, and operate among a schemata (Morano, Riccomini & Lee, 2019; Barbieri et al., 2020), which comprises rote knowledge, relationship knowledge, and visualisation abilities. Despite its effectiveness, most teachers and researchers do not employ the number line approach, which serves as one of the common reasons for learners' difficulties (Gersten, Schumacher & Jordan, 2017; Dyson, Jordan, Barbieri, Rodrigues & Rinne, 2018). Anecdotal evidence from the researcher indicates that most Ghana Education Service (GES) approved textbooks do not consider number lines to solve fractions, even though the syllabus suggests number lines on fractional chart.

While the number line approach is successful for fractional learning (Gersten et al., 2017; Dyson et al., 2018), there is a dearth of research on its usage in Ghanaian upper primary mathematics. In numerous ways, a study on using number lines in learning fractions in primary school learners will be essential. However, interventions in Ghana have primarily focused on either area or set models, with linear models being neglected (number line) (Ametepeh, 2018; Amuah, Davis & Fletcher, 2019; Bernard, Golbert & Gabina, 2020). The current study employed a pretest-posttest non-equivalent quasi-experimental design embedded with a quantitative research approach to examine basic six learners' performance, attitude, and challenges when using the number line in learning fractions. In addition, differences between the learners' pre-test and post-test scores were examined to explore the influence of the number line approach on learners learning of fractions.

Purpose of the Study

The study examined the influence of the number line approach on learning fractions by comparing the achievement of the basic six learners taught using the number line and those taught without using the number line. Particularly, the study sought to:

1. examine basic six learners' level of performance through the use of the number line in learning fractions.
2. examine the basic six learners' attitudes towards the learning of fractions using the number line.

3. explore any possible challenges basic six learners face using the number line in learning fractions.

Research Questions

The following research questions guided the study:

1. What are basic six learners' level of performance through the use of the number line in learning fractions?
2. What are the basic six learners' attitudes toward the learning of fractions using the number line?
3. What are the basic six learners' challenges in using the number line in learning fractions?

Research hypotheses

The following hypotheses were formed to guide the study;

H_O1: there is no statistically significant difference in performance between the two groups on the pre-test scores.

H_A1: there is a statistical difference in performance between the two groups on the pre-test scores.

H_O2: there is no statistically significant difference between the two groups on the post-test scores.

H_A2: there is a statistical difference between the two groups on the post-test scores.

Significance of the Study

First, the results of the study would bring to bear the effects of the number line approach as an essential factor in improving learner's fractional magnitude and would provide relevant information to the schools under study, the Ministry of

Education or educational policymakers, and researchers in the field of education and mathematics on the state or extent of the use of number line and its effectiveness.

The study results will inform mathematics teachers in the Cape Coast North community's primary schools about factors that enhance or impede teaching fractions among their upper primary learners and what can be done to curb the situation.

Finally, the findings will serve as a source of empirical review on the state of the number line approach to learning fractions in Ghanaian primary schools.

Delimitation of the Study

The study of school fractions covers a wide range of this subject; the attention is on identifying fractions on the number line and explaining their thinking. The lessons on fractions were limited to the following content areas: identifying and locating fractions (MOE, 2012; NaCCA, 2019). This was relevant because if learners are permitted to identify fractions on the number line, it improves their attitude and, as a result, enhances their performance.

The study was also confined to two basic six intact classrooms, one each from the two upper primary schools involved. This was to prevent interaction between the learners from both schools. The use of the entire class ensured that everyone stood to benefit from the study. These classes were targeted because fraction starts to become a focus in primary four, and by primary six, learners should have foundational fraction knowledge. Finally, the study did not focus on the operations of fractions. The investigation was conducted in the Cape Coast

Metropolis. The area was chosen due to its familiarity with the researcher and convenient population accessibility for the study.

Limitations of the Study

According to Simon and Goes (2013), researchers cannot always prevent external factors from influencing their findings. These are issues with instruments, samples, analysis, self-report methodology, insufficient funding, and research design (Siddiqui, 2010). The primary flaw of the study was the questionnaire utilised to gather information. This included the possibility that respondents would interpret the items differently, inconsistency, and respondent unfairness. To address these concerns, the researcher explained the study's objectives and provided respondents with an interpretation of the questionnaire items. Again, because the study was limited to two different public upper primary schools in the Cape Coast North, the study conclusions may affect the generalisability of the findings.

Operational Definition of Terms

Fractions: Mathematically, a fraction is a representation of a subset of a set. The top number, called the numerator, stands for the fractional component, while the bottom number, called the denominator, stands for the entire.

Number Lines: In mathematics, number lines are helpful visual representations that can be utilised to better comprehend and solve problems. They are a straight line with marks or ticks at regular intervals along it. These notations stand in for numbers and facilitate operations like adding, subtracting, multiplying, and dividing, and provide a visual representation of quantities and comparisons of values. Some number lines only show whole numbers, whereas others can show decimals, fractions, and even negative numbers.

Behavioural Engagement: An individual's "behavioural engagement" level in a task is measured by how much effort they put into it. A student's involvement in their education is reflected in their level of behavioural engagement in the classroom, making it an important factor in learning and academic success. Learners that show behavioural engagement are more likely to learn and like their schoolwork.

Attitudes: A person's reaction or response to any given person, thing, or circumstance is based on their attitude towards them.

Interest: to be intensely interested in something one has never tried before. Feeling interested, enthusiastic, and driven to find out more about it. Pursuing or experiencing that thing might also bring about feelings of pleasure. Individual differences in taste, background, and education all shape what piques people's interest.

Confidence: Confidence is the conviction that one's evaluations of one's skills, attributes, judgements, and choices are accurate and worthy of trust.

Control Group: A control group is an experimental subset from which independent variable effects can be eliminated. In this case, it means the responders who were educated through the tried-and-true classroom approach.

Experimental Group: The Experimental Group is a study section where the actual experiment occurs. The effects of the experimental independent variable are recorded and tracked after being implemented in this group.

Organisation of the Study

The study was conducted in five chapters. Chapter one dealt with the introduction, background to the study, statement of the problem, the purpose of the study, research questions and hypotheses, the significance of the study, delimitations, and limitations. The second chapter examined the theoretical framework, conceptual and literature review highlighting research and other writers' perspectives. Chapter three covered the research methods, including design, population, sampling processes, data collection instrument, pre-testing, validity and reliability of the research instruments, data collection procedures, data processing and analysis, and ethical considerations. In chapter four, the results were presented, and the outcomes were examined. Chapter five focused on the study summary, major findings of the study results, conclusions, recommendations, and suggestions for inquiry.

CHAPTER TWO

LITERATURE REVIEW

Overview

The following subheadings were used to organise the literature review: fractions as a concept, the concept of the number line, misconception about fractions**,** the importance of the number line as an approach in learning fractions, conceptual and procedural knowledge, difficulties learners encounter when learning fractions with the number line approach. The theory underpinning this study, a review of related empirical studies, and the summary.

Theoretical Framework: Constructivism

According to Jacobs (2016), a theoretical framework can be viewed as an angle, a vantage point, or a set of lenses to conduct research. It is, therefore, a descriptive stage of the research procedure. According to Cline's (2011) assessment, this helps clarify the research problem by limiting the scope of the study. According to the hypothesis of Varpio, Paradis, Uijtdehaage, and Young (2020), a theoretical framework is a collection of related concepts (or variables) and the definitions used to create a preposition or hypothesis that describes the connection between the constructs. Fundamentally, a theoretical framework is the conceptual backbone of a study. The theory of constructivism, which states that individuals learn most effectively from materials they have actively participated in producing, provided the theoretical foundation of this study. The learner is considered to be the main concern during the teaching procedure. Prejudices, experiences, the period we live, and physical and mental maturity all influence how we learn. When a learner is motivated, he or she uses willingness, determination, and action to gather, convert, formulate hypotheses, use applications, interactions, or experiences to test these assumptions and draw accurate conclusions.

Two of the most influential figures in the development of constructivist theories are Jean Piaget and Lev Vygotsky. They both believe that classrooms should be constructivist environments, but their theories differ, and there are differences in how constructivism should be implemented in classrooms. Piaget (1980) believes that learners must be challenged to accept individual differences in a constructivist classroom, greatly enhance their commitment to study, develop new ideas and build their knowledge through different activities. When a person connects with an experience, condition, or idea, Piaget (1964) believes that one or two things can occur. Whether the current experience is connected to the existing pattern or is not (the assimilation process), the current pattern has been adjusted to suit the unique idea or experience (accommodation or adaptation process). The use of new understandings of an existing scheme is referred to as assimilation.

A learner's proficiency in recognising common characteristics between objects and connecting new ideas to ones they already know is the foundation of assimilation. While adapting available ways of looking at concepts that are not in line with existing schemes is also known as adaptation. Reflexive thinking facilitates accommodation and changes or changes in existing schemes.

Learners participate independently in constructive learning, work silently through specific tasks, allow their minds to screen through materials, and consolidate new ideas with old ones by the principle of constructivism. As constructivism implies, learners are not “blank slates” devoid of ideas, concepts, or brain structures. Furthermore, constructivist recognises that learners are not empty vessels or blank slates waiting for knowledge (Noureen, Bashhir & Arshad, 2020). Instead, learners build new knowledge from diverse past experiences, acquaintances, and beliefs (Noureen et al., 2020). Gupta and Gupta (2017) agreed that similar to how all cells develop from pre-existing cells in cell theory, information already exists in the human body, and all that is required is methods for investigation. This demonstrates that learners have a sense of self-awareness when they come to class. They do not absorb ideas presented by teachers but instead, create their knowledge.

This current study looked into the effectiveness of the number line approach in learning fractions. As a result, constructivism is relevant in this study because when learners construct knowledge independently, misconceptions may occur as they seek to form new ideas. Although misconceptions can never be avoided entirely, teachers can intervene before they become deeply rooted. Before addressing errors or developing interventions to promote understanding, teachers need to figure out why their learners make mistakes or how misunderstandings have developed (Harbour, Karp & Lingo, 2016).

Conceptual Review

In this sub-heading, the researcher reviewed concepts relating to fractions, misconceptions of fractions, the importance of the number line approach in learning fractions, and learners misconceptions in learning fractions using the number line.

The Concept of Fractions

Lamon (2020) stated that one aspect that impedes learning fractions is their different meanings. The word "fraction" emerges from the Latin word "fractio," derived from the word frangere, which means "to break." (Bennett, Burton, Ediger & Nelson, 2015, p. 151; Bassarear & Moss, 2016, p. 28). Fundamental fraction knowledge includes the understanding that fractions constitute a part of an object or a set of elements that fractional symbols can be used to represent them, and that numbers can represent numerical magnitudes (Jordan et al., 2013). To comprehend fractions, it is necessary to understand the difference between whole and equal fractional parts. Fractions, above all, are not self-contained units. As a result, fractions have meaning only when applied to the whole.

According to Pienaar (2014), a fraction is a number that represents a portion of a whole as an integer quotient (where the denominator is not zero). Van de Walle, Karp, and Bay-Williams (2016) define a fraction only as a relationship between the part and the whole. Additionally, Witherspoon (2019) posits that learners' overall knowledge of fractions must be exposed to various contexts. Fractions, for example, are defined as numbers that represent magnitudes (for example, $\frac{2}{5}$, $\frac{2}{4}$, and $\frac{2}{3}$) and can be ordered from smallest to highest. Pantziara and Philimppou (2012), on the other hand, suggest that the term "fraction" refers to the multidimensional ideas of a quotient, part-whole, operator, ratio, and measure. For example, the fraction $\frac{3}{5}$ can be thought of as a quotient (three divided by five), a component of a whole (three of five equal parts), an operator (three-quarters of a quantity), a ratio (three parts of five parts), and lastly as a measure (as a part on a number line). At this juncture, there is no commonly consented definition of fractions in the mathematics education literature. Even though most sources utilise a fraction note to emphasise that the numerator and denominator are integers and the denominator is non-zero, teachers may interpret fractions differently in different publications (Bennett et al., 2015; Morrison & Hamshaw, 2015).

The Concept of Number Line as Model for Fractions

Elementary and middle school learners employ the number line to help them understand both positive and negative whole numbers, decimals, fractions, and integers (Lahme, McLeman, Nakamaye & Umland, 2019). For the last decade, intervention research has used the number line to teach fractions and decimals concepts (e.g., Malone & Fuchs, 2017; Barbieri et al., 2020), most notably when teaching fractions magnitude. According to several studies, using a number line to estimate fraction magnitude helped in addition and subtraction (e.g., Fuchs et al., 2013; Tian & Siegler, 2017). Area model interventions such as cookies, pizza, and brownies are frequently utilised in the United States to teach fractions (Freeman & Jorgensen, 2015). Due to the Common Core State Standards, learners in the United States now view fractions as a number line. Similarly, elementary schools in nations such as Japan, China, and Korea emphasise the linear representation of fractions and numbers (Lewis & Perry, 2017).

Learners can be instructed to view $\frac{1}{5}$ of a pizza as one of five pizza slices (part-whole interpretation) or as $\frac{1}{5}$ of the distance between zero and one on a number line, as done in those mentioned above, highly successful countries in terms of literacy proficiency such as PISA, TIMSS and EGMA (Siegler, Thompson, & Schneider, 2011). When teaching and understanding fractions, the number line is a resource of considerable interest (Fisher & Dennis, 2023). According to the 2 016 Mathematics Standards for Elementary Schools Grade three (Permendikbud, 2016), two fundamental competencies exist: Explain whole numbers and simple fractions (such as $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$) addressed on the number line and using simple whole numbers and fractions (such as $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$) displayed on the number line; actually acknowledge that fractions can be used to portray parts of a whole, parts of a set, points on a number line, or distances on a number line. As a result, it is critical for instructors to (a) develop theories and applied handicrafts and understanding related to teaching and learning fractions on the number line, (b) comprehend at which point learners know when solving number line problems, and (c) acknowledge learners' misconceptions regarding fractions on the number line. Given the significance of this subject, the problem under study determined the sorts of errors learners make when dealing with fractions on the number line. Additionally, instructors can assist learners in comprehending fractions by explaining their numerical thinking and exhibiting their methodologies and viewpoints to their colleagues (Humphreys & Parker, 2015).

According to the Australian Curriculum Assessment and Reporting Authority [ACARA] (2012), learners are commonly taught fractions through the part-whole concept. Lamon (2007) asserts that teachers frequently employ the part-whole technique does not support the fraction being considered a number. Wu (2011) asserted that the actual number on a number line should be defined as a point. Even if this is the definition utilised later, kindergarten and grade one learners require a more explicit foundation to justify their decision. After learners were sufficiently knowledgeable about learning and explaining these representations, the diagrams of the number lines were recognised as the primary representation. Learners can naturally comprehend the entire numbering system without becoming mathematically confused by associating every concept of a number with the number line. A numerical line is a visual image of order and magnitude. The number line can teach different concepts connected with arithmetic learning and causally (Booth & Siegler, 2008). Learners need more outstanding practices integrating schemas of fractions to get a better understanding of fractions.

In accordance with the progression for numerical magnitudes described in the integrated theory of numerical development, the number line helps students better understand how fractions and whole numbers fit together (Siegler et al., 2011). Following this idea, numerical development enhances magnitude representations for an increasing number range, and learning fractions integrates fractions with previously acquired whole number knowledge. As a result, the number line appears to be more beneficial for learners' fraction learning than the area model. Number lines are tools that necessitate a grasp of numerical magnitude, as opined by (Namkung, Fuchs & Kozio, 2018). Because this form corresponds to how people think about numbers, using a number line can help learners make estimations and organise numbers (i.e., increasing from left to right; Hamdan & Gunderson, 2017). Even though the number line is most abstract than other physical and graphic representations, it is an important model.

The United States' [NMAP] (2008) promoted the number line as a model that bridges conceptual and procedural knowledge. According to the Common Core State Mathematical Standards, third-grade learners must demonstrate an understanding of the fraction line model (CCSMS, 2010). Learners are likely to comprehend the proportional relationships between a symbolic number and the linear size of the line when they place their numbers on a number line. A traditional number line task presents a horizontal line with labelled endpoints of zero 0 on the left and a base-ten number of 100 or 1,000 on the right. The participants are given an intended number, which they must place on the number line. Estimating numbers on a number line is associated with adults' and learners' mathematical abilities (Schneider et al., 2018). Performance on the fraction number line estimation tasks is also linked to mathematics achievement (Hamdan & Gunderson, 2017). However, there is significantly less research on fraction number lines and the strategies for arranging rational numbers on a number line than on traditional number lines (Zhang, Stecker & Beqiri, 2017).

According to one school of thought, learners begin with a logarithmic representation of numerical magnitude, exaggerating the interval in the middle of small numbers while reducing the distance between larger numbers (Libertus, Feigenson, Halberda & Landau, 2014). This logarithmic representation changes to a linear representation over time and education, where the distance between consecutive integers is equal (Namkung et al., 2018). Learners' estimates become more appropriate in Grade two, and their number line estimates show a linear representation of numerical magnitudes (Booth & Siegler, 2008). When this shift occurs, learners can extend their knowledge to include rational numbers, beginning with fractions (Siegler et al., 2011). To comprehend the ongoing character of rational numbers, it is necessary to reorganise numbers (Siegler & Pyke, 2013).

Another viewpoint contends that developmental progression is due to the importance of the number line in proportional reasoning. Participants make judgements about the relative magnitude of the numbers by comparing the goal number to the constrained endpoint on a number line task. This point of view is represented by a cyclical power function that shows how learners progress from estimating with only the left endpoint (i.e., open-ended judgment) to assessing with both endpoints (i.e., bounded judgement) (Slusser & Barth, 2017). Fractions can be included in the same way as the logarithmic to linear shift model once learners can make bounded judgements.

Importance of the Number Line as an Approach in Teaching Fractions

A numerical line is an essential tool to use regularly in fractions during the lesson. A number line is a helpful tool for seeing which fraction is smaller and which is bigger and reinforces that those two fractions always have another fraction (Van De Walle et al., 2013). A number line is a graph showing the range of infinity-to-infinity integers. The number line is a great pedagogic tool, according to

Skoumpourdi (2010), particularly as it allows learners to view mathematical concepts directly: the number line is utilised for counting, estimating, and time representation and for presenting distinct number sets. In addition, the number line can give geometric representation and measure and compare arithmetic processes.

According to Lamon (2012), there are three primary reasons why fractions must be taught:

- Fractions have a significant impact on learners' attitudes about mathematics.
- Fractions are a necessary part of school mathematics and daily life. Fractions are not just important in mathematics; they provide the basis of more advanced notions like ratios, rates, percent, proportions, proportionality, linearity, and slope. The ability to work with fractions is useful in many areas of life, including cooking, discount calculations, rate comparisons, unit conversions, reading maps, and financial planning.
- In order to be mathematically competent, a solid foundation in fractions is required. The final report, Foundations for Success, written by the National Mathematics Advisory Panel in 2008, concluded that algebra is the most important subject for students to learn in high school and college. The primary reason American learners struggle in algebra is a deficiency in fraction fluency. The challenging "algebra for everyone" task will remain unachievable until "fractions for everyone" is achieved.

Additionally, Gray (2018) lists the following benefits of utilising a number line to teach fractions:

- Number lines assist learners in visualising fractions as portions of a whole or a set and as a fraction of distance or time.
- Number lines aid in the comparison of fractions.
- Number lines are more successful than traditional visual models for teaching fractions.
- Number lines assist us in determining equal fractions.
- Number lines assist us in visualising a fraction as a number between two whole numbers.

According to Van de Walle, Karp and Bay-Williams (2013), number lines help learners compare numbers and acknowledge fractions as quantities rather than as "one number over another." Additionally, number lines can broaden learners' understanding of fractions by including negative fractions and fractions with values higher than 1, decimals, and per cent. Number lines are effective for illustrating the concept of fraction density.

Learners' Misconceptions in Learning Fractions

Numerous studies have been conducted on learners' mathematical misconceptions and errors (e.g., Mohyuddin & Khalil, 2016; Burgoon, Heddle & Duran, 2017; Aliustaoglu, Tuna & Biber, 2018). These misunderstandings and errors may be caused by a variety of factors, including student disposition toward mathematics (Kusmaryono, Suyitno, Dwijanto & Dwidayati, 2019), teaching framework (Skott, 2019), teaching skills (OECD, 2019), learners' preconceptions (Diyanahesa, Kusairi & Latifah, 2017), limited understanding (Saputri & Widyaningrum, 2016), and a lack of appropriate mathematical misunderstandings appear to be connected with incorrect concepts developed by learners in mathematics as a result of a lack of clarity in concept learning. Such misunderstandings may stem from their prior knowledge, which they improperly generalised (Im & Jitendra, 2020), and they believe either that what they are doing is correct or that they are unsure of what they are doing (Neidorf, Arora, Erberber, Tsokodayi & Mai, 2020). An error may occur due to incompetence or a lack of awareness regarding verifying the answers provided (Hansen et al., 2015). Persistent misconceptions can impair learners' comprehension of mathematical topics, resulting in frequent errors (Im & Jitendra, 2020). Such inaccuracy may result in poor performance, generating concern about the topic and resulting in unfavourable attitudes and a negative image of mathematics (Belbase, 2013).

Misconceptions are logical errors. At any level of fraction knowledge, there is the possibility of making a mistake. Makonye and Fakude (2016) define misconceptions as misguided beliefs and concepts that underpin a person's state of mind, resulting in a cascade of errors. Failure to recognise that components of the whole are of equal size is an example of a misperception in the early stages of fraction learning (i.e., $\frac{2}{3}$ would represent 2 of 3 equal parts). Siegler and Lortie-Forgues (2015) believed that misconceptions, such as learners not understanding an infinite number of fractions referring to the same magnitude, still support misconceptions. However, Fazio et al. (2011) posit those problem-solving errors are caused by inadequate confidence when dealing with fractions. Given this, Ramadianti, Priatna, and Kusnandi (2019) pointed out that this error occurred due to learners' lack of context for recognising fractions. Furthermore, learners sometimes avoid the fractional parts of operations when doing arithmetic with mixed numbers. Learners' misconceptions about fractions and the avoidance behaviours that result from them are common throughout their schooling. Fraction problems can last well into adolescence and adulthood (Siegler & Lortie-Forgues, 2015).

Fitri and Prahmana (2019) concluded from a sample of 30 seventh-grade learners from SMP Negeri 1 Piyungan that "learners continue to make mistakes when they recruit the unknown components of the problem and cannot use fractional concepts in counting and incorrectly convert mixtures into ordinary fractions." In addition, learners make mistakes when converting integers to fractions and are less careful when counting. Finally, learners sort fraction numbers incorrectly (p. 8).

Widodo and Ikhwanudin (2018) reached a similar conclusion after interviewing, recording, observing, and using paper and pencil measures on 31 grade six learners about the challenges they encounter when dealing with fractions on the number line; they described four common student blunders: misunderstanding units, misinterpreting tick marks, incorrectly partitioning, and guessing. When examining learners' answers, it was found that the answer was incorrect because of factors such as the ranking by the numerator and denominator proximity and the ranking of the minority or the majority, among other natural numbers. Thus, they proposed that, when teaching fractions, teachers should focus on unit understanding, clarify tick mark interpretation, remind learners of the need for partitioning and un-partitioning operations, and teach good estimate techniques.

Alkhateeb (2019) highlighted fifth-graders common mistakes in fractions and their associated thinking strategies in Zarqa (Jordan). Using a mixed-method approach with the diagnostic test and individual interview as an instrument, 240 learners were randomly selected, while 30 were interviewed. The outcome of the study showed various mistakes made by learners, which are as follows: the common mistakes were learners' relations with fractions as integers, errors about basic concepts of the fraction such as taking into account that the fractional number is always higher than the figure $\frac{A}{B}$ and that figure $\frac{A}{B}$ is always less than one; another misconception was that learners misinterpret the numerator and the denominator with the actual value of the fraction without paying much attention to the integer in the fractional number. The results further show that more than 50% of the learners made mistakes associated with finding solutions to fractions issues regarding learners' thinking and associated errors. The most apparent error was stating the fractions without prioritising the equal parts.

According to Siegler et al. (2011), learners can learn estimation fraction magnitudes between 0 and 1 ($\frac{1}{2}, \frac{1}{3}, \frac{2}{3}, \frac{1}{4}, \frac{3}{4}$) to help and support them in generalising their fraction magnitude knowledge. Learners will be able to reject irrational solutions if they have a sense of how near the answer might be, based on fraction magnitude. For instance, learners may refuse the approach that results in arithmetic errors of type $\frac{1}{2} + \frac{1}{3} = \frac{2}{5}$. This may prompt them to experiment with alternative approaches and see whether their response made sense. To support a more general knowledge of fraction magnitude, possessing a feeling of learners will be able to reject implausible answers by knowing what the answer might be near, depending on fraction magnitude (Siegler et al., 2011).

Trivena, Ningsih, and Jupri (2017) also observed: "how primary five learners understand fraction addition and subtraction." Both learners and teachers were subjected to a test that included the Certainty Response Index (CRI) and an interview. In analysing student responses, both the CRI and interviews with both learners and teachers were used. The findings revealed that learners' mastery of addition and subtraction concepts was dominated by the category "misconception." These data revealed that the mastery concept of fraction addition and subtraction in fifth grade remained low. The learners, in particular, are unaware that addition and subtraction operations must equalise the denominator.

Hıdıroğlu (2016) argued two reasons why targeted results in the fractional unit were of fragile accessibility: learners' misconceptions and the teachers' learning-teaching process, which does not consider the learners' prior knowledge. Learners' thinking is transformed when they learn about fractions. Learners experience difficulties moving from whole numbers to fractions because they do not focus on "numeric entities" (Siegler et al., 2011, p. 274). Even if fractional education starts at primary school, even secondary and school learners often confuse fractions and entire properties (Siegler et al., 2011; Vosniadou, 2014).

While Lewis, Matthews, and Hubbard (2016) confirm that this mistake is not unique to learners when university undergraduates were asked which sum of $\frac{12}{13}$ and $\frac{7}{8}$ was closest to 1, 2, 19, or 21, and 15% chose 19 or 21. This incorrect response indicates that learners focused on the fraction's components (numerator and denominator) rather than on its overall meaning and added the numerator (to get 19) or denominators (to arrive at 21). Due to their inability to process fractions holistically, individuals may wrongly apply their knowledge of whole number properties to fraction tasks, resulting in a "whole number bias" (Ni & Zhou, 2005; Siegler & Pyke, 2013; DeWolf & Vosniadou, 2015). For example, because the entire number 9 is higher in magnitude than the number 2, this prejudice may mislead someone to perceive the number 9 as more important than the number 2 and regard $\frac{1}{9}$ as larger than $\frac{1}{2}$.

Eroğlu (2012) discovered that Moss and Case conducted a study to determine whether prospective primary and secondary school mathematics teachers knew their learners' fractional errors. The future teachers were cognisant of their learners' errors but limited their explanations. They suggested using verbal descriptions, area models, real-world examples, preliminary knowledge replicas, standard teaching solutions, questions, simple examples, and exhibitions to assist learners in resolving their errors. They proposed verbal explanations, area models, real-world examples, standard teaching solutions, leading questions, straightforward examples, opposite examples, exercises, and practises to help learners recognise and correct their errors. These earlier syntheses' results helped develop a sense of useful teaching components for challenging learners in fractions.

However, these studies reviewed by Eroğlu (2012) and Zhang, Clements, and Ellerton (2015) focused on learners' conceptual misunderstanding from area models to multiple representations using 40 respondents. Fitri and Prahmana's (2019) study focused on the problems of learners in solving fractions using a descriptive research approach and a sample of 30 learners; learners' errors were detected without emphasis on how those learners overcame their issues. Cramer, Ahrendt, Monson, Wyberg, and Miller (2017) also looked at the challenges that third-grade learners encounter using number lines as a model for a fraction using interview and qualitative research design.

Mitchell and Horn (2008) conducted a study to discover learners' misconceptions regarding number lines. Twenty-nine grade six learners worked on eight number line tasks using an interview from two schools in metropolitan Melbourne. The fraction number line task was chosen to examine learners' reasoning comprehension to measure the fraction sub-construct. An interview from year five learner completed question 11 during his interview on how to place $\frac{1}{2}$ on the number line. "Put a cross where the number half would be on the number line," he read aloud. He drew a cross halfway between 2 and 3. "Half of it," he said when asked how he figured it out. Because zero is not a number in the middle [indicating the 1 and 4 on the number line]. This is the middle "[Counting in from both sides]" He did not count zero because he did not take into account the number, so $2\frac{1}{2}$ was the halfway point between 1 and 4. His answers during the interview and 24.4% of learners who completed the question on paper support the idea that it suggests procedure rather than a quantity or a distance from zero.

Causes of Misconceptions

In Prediger's (2006) view, learners' obstacles can be either didactical or epistemological. Prediger (2006) defines didactical difficulties as stumbling blocks evoked by a teacher's method of instruction and epistemological difficulties as those arising from the structure of mathematical content. The source of learners' misconceptions is that which misleads them, whether didactically or epistemologically, and this study is interested in both. There is a risk of errors if learners build knowledge by reconstructing and rearranging previous knowledge with new knowledge (Machaba, 2016). That means misunderstandings can result in an inadvertent transition to reorganise previous knowledge. Otherwise, a defective connection and connection between experience and the material can jeopardise conceptual development.

"Naive theories," according to Ojose (2015), hinder the rational reasoning of the learner and lead to misunderstandings. Unsupported theories that learners quickly deduce from their daily mathematics can be described as naïve theories. Changes to current knowledge posed by Ojose (2015), which will require prior learning material, can be required in new instruction. In this way, cognitive conflicts lead to misunderstandings during a learning-unlearning process. The nature of mathematics is tied to the expressions of misconceptions (Ojose, 2015). He further argues that learners change the rules between concepts. The alleged inconsistency of the rules probably leads to misunderstandings.

According to Sarwadi and Sharhill (2014), mathematical misunderstandings may begin in elementary school and worsen in secondary school due to inattention.

Teachers are accused of inattentiveness because they believe learners clearly understand previous grade subjects. Teachers are less concerned with monitoring learners' progress on earlier concepts and more about developing new education based on learners' unstable conceptual foundations.

Misconceptions can be generated by inaccurate prior information or an incorrectly created schema (Sarwadi & Sharhill, 2014). From an educational standpoint, teachers not only ignore these myths. They further opined those certain misunderstandings result from learners' incompatibility with instructions and specialised teaching strategies. This means that how teachers convey their lessons can result in learners having misconceptions. Lessons can demonstrate using words, instructional examples, allusions to previous topics, and misconceptions.

On the other hand, Li and Li (2008) believed that a deficiency in mathematics or education strategies that prevent learners from assimilating new concepts could lead to misunderstandings. Consequently, curriculum planners and curriculum implementers are involved in promoting or avoiding misunderstandings. Prediger (2006) believes that learners who only know a portion of a fraction model face a didactic impediment that can be avoided using different fraction interpretations. The didactic aspect requires education to be adequately empowered regarding fractional skills, dissemination, and alternative instructional methods.

In Makhubele's (2021) view, the main source of learners' errors in learning fractions was inadequate knowledge of the basic concepts, learners' previous knowledge, misconception, and misapplication of rules. In furtherance, Kori and

Sitio (2021) believed that using a single image in the representation caused learners to have trouble dealing with two image components, namely the difference in size and form, resulting in their limits in understanding the idea of fractions. Even though the two portions come from the same flat building, learners are nevertheless influenced by the fact that two pieces with distinct shapes have separate spaces.

Conceptual and Procedural Knowledge in Fractions

To comprehend mathematical concepts, learners must possess two distinct types of knowledge: conceptual and procedural knowledge. While "conceptual and procedural knowledge cannot be distinguished, the distinctions between the two are understandable" (Rittle-Johnson & Schneider, 2015, p. 588). Conceptual knowledge refers to an individual's grasp of core concepts and principles within a domain (Kilpatrick & Swafford, 2017), while knowledge of why a mathematical procedure works are referred to as procedural knowledge (Crooks & Alibali, 2014). According to Merriam-Collegiate Webster Dictionary (2012), conceptual knowledge is "an abstract or generic idea that has been generalised from specific instances." For example, understanding the unique properties of fractions (e.g., the relationship between the numerator and denominator, the density of fractions) and various aspects of fractions requires a conceptual understanding of fractions (Padberg & Wartha, 2017). In Wiest and Amankonah's (2019) views, conceptual understanding entails recognising relationships between concepts and procedures and applying mathematical principles in numerous circumstances.

However, mathematics researchers occasionally used a more specific definition. The term conceptual knowledge has been employed not only to attribute to what is known (conceptual knowledge) but also to describe concepts in the same way (for example, in-depth and with numerous connections), “relationally rich” knowledge is most clearly defined as conceptual knowledge (Star, 2005, p. 408).

It is comparable to an interconnected knowledge web, a network in which the links are just as important as individual data. For instance, individual facts and statements are interconnected with relationships, creating an information network. Learners with solid conceptual understanding can solve problems they have never seen before. Shade fraction figures to demonstrate quantity, compare fraction quantities, identify equivalent fractions, and locate fractions on a number line are all examples of conceptual fraction skills (Jordan et al., 2013; Bailey et al., 2015).

Procedural knowledge, however, is characterised as rules and algorithms for collection to solve mathematical issues (Ghazali & Zakaria, 2011). Procedural knowledge of fractions entails understanding how fractional arithmetic procedures, such as fraction addition, work, which entails understanding procedural actions performed within the context of specific measures or partial measures to accomplish specific goals (Rittle-Johnson & Schneider, 2015). Numerous research studies have demonstrated how difficult it is for learners to understand fractions (Simon, Placa, Avitzur & Kara, 2018; Lamon, 2020).

These issues are frequently the result of a lack of conceptual understanding, with many learners viewing fractions as meaningless symbols (Fazio et al., 2016). These findings imply that learners face a significant obstacle in developing a mental comprehension of fractions (Wilkins & Norton, 2018). Fraction arithmetic, equivalent fraction creation, and converting fractions to decimals and percentages are all examples of procedural skills (Hallett, Nunes, Bryant & Thorpe, 2012; Bailey et al., 2015). A fraction is taught procedurally; learners use the common denominator method to add fractions. Procedurally, based on Hiebert and Wearne, learners get procedural understanding or syntax thinking (Hiebert & Wearne, 2005). Multiplication of fractions, for example, allows for component-wise numerator and denominator processing, whereas addition and subtraction do not.

The key elements of the linkages between procedural and conceptual knowledge and the interdependence of the two have been acknowledged (Rittle-Johnson & Schneider, 2015). They believe that teachers and mathematical researchers have a legitimate interest in researching these knowledge components (e.g., Rittle-Johnson & Schneider, 2015). Mack (2001) argues that learners' representation and clarification of fractional issues involve applying these strands when teaching primary school fractions. Relative roles and relationships between the two knowledge domains should be recognised when decoding and addressing partial problems. The debate on this subject appears to be divided into three distinct camps. According to one perspective, learners develop conceptual awareness of fractions (Groth & Bergner, 2006). The second perspective posits that learners acquire procedural knowledge before conceptual understanding (Baroody, Feil & Johnson, 2007). Finally, it appears that young learners develop conceptual and procedural knowledge concurrently (Lenz, Dreher, Holzäpfel & Wittmann, 2020).

Karika (2020) addressed this issue by providing a test item that validly assesses learners' conceptual and procedural knowledge of fractions. Eighth and ninth-grade learners were used, constituting 235 learners across Germany.

Previous investigations were expanded and implemented on a conceptual or procedural scale with the assistance of professionals. The data were found to fit the theoretically assumed two-dimensional model the best. Significant differences existed in the correlations between the two forms of knowledge and overall cognitive performance. Additionally, by utilising their signs, pre-existing buildings can be confidently predicted. The findings revealed that it is possible to acquire conceptual and procedural knowledge.

Jordan et al. (2013) identified a comparable connection between two types of knowledge on a manifest level $(r = .62, p.001)$. In the United States $(n = 357)$, the study looks at the broad determinants of sixth-grade learners' conceptual and procedural fraction knowledge (attentive behaviour, language, and nonverbal reasoning), as well as traits related to numbers (number line estimation, calculating fluency). The data revealed that indicators aided conceptual and procedural knowledge differently, demonstrating separability. Aside from that, the study found only a little evidence that conceptual and procedural knowledge are distinct. In 47 prospective instructors from Taiwan and 49 from the United States of America, Lin, Becker, Byun, Yang and Huang (2013) discovered no link between procedural and conceptual knowledge in their study.

Furthermore, Özpınar and Arslan (2021) proved that conceptual learning could enhance procedural learning, but this is not always the case. Their research looked into which type of knowledge (conceptual or procedural) was more prevalent in elementary mathematics issues among pre-service primary mathematics instructors and the connection between these two types of knowledge.

Ninety-seven first-year learners were enrolled in the study at Turkey's Department of Primary Mathematics Teaching. A descriptive approach was combined with a case study technique.

Nahdi and Jatisunda (2020) performed a qualitative study using a case study approach in an elementary school in Majalengka (Indonesia) to demonstrate the connection between conceptual and procedural knowledge. They were interested in determining whether conceptual knowledge had a major effect on procedural knowledge and vice versa and could successfully mix conceptual and procedural knowledge. They stated that conceptual comprehension and procedural knowledge are crucial for learners to grasp since they will influence their mastery of subsequent mathematics topics. Since this knowledge cannot be achieved without teachers, Copur-Gencturk's (2021) concern was to examine how teachers' conceptual understanding affects the quality of their education. Using a sample of 303 elementary school teachers from around the United States of America, she tested primary school teachers' mastery of several fraction concepts. Teachers' explanations were coded according to their accuracy and the concepts and representations used. According to the findings, teachers lacked fraction arithmetic skills, especially fraction division. However, there was a moderate link between teachers' capacity to understand fraction addition and division and their teaching ability. Moreover, veteran educators better understood fractional mathematics than their special education counterparts.

Previous Research on Fraction Interventions

For learners in grades 3 through 12, two previous studies laid the ground for effective fraction intervention programmes. Misquitta (2011) looked at papers published between 1990 and 2008 concentrating on teaching fractions to struggling learners and the research quality. The author looked into whether effective techniques for teaching mathematics to struggling mathematics learners (Gersten et al., 2017) were equally beneficial for fraction learning and discovered that specific order, progressive pattern, and strategic teaching all improved fraction performance. Shin and Bryant (2015) extended Misquitta's investigation by looking at the years 1975–2014, reviewing studies associated with the CCSSM, and examining more relevant teaching aspects embedded in fraction treatments for underperforming mathematics learners. According to these researchers, specific, structured instruction combined with pictorial representations significantly improved fraction concepts and abilities.

Additionally, fraction outcomes improved when heuristic techniques and contextual challenges were used with explicit training. The findings from these prior syntheses have aided in understanding successful teaching approaches for difficult learners in fraction learning. However, both Misquitta (2011) and Shin and Bryant (2015) focused on secondary learners, leaving a research gap regarding which instructional components are most beneficial for primary learners. Given that learners' challenges with fractions begin early and continue throughout their education (Myers, Mazzocco, Hanich, Lewis & Murphy, 2013, Lortie-Forgues et al., 2015), it is necessary to discover common and successful strategies for learners' early fraction understanding.

Additionally, Shin and Bryant (2015) opined that most investigations studied a limited perspective of fraction learning that corresponded to the CCSSM, yet neither examined how to demonstrate fractions on a number line to assist learners in making the conceptual to procedural connection. Given the recent emphasis on conceptual understanding of fraction learning, additional research on this ability at the primary school level was considered convenient.

Studies on Fractions Using Number Line

Studies on fractions instructions with number lines have proven success in learners' conceptual and procedural skills. They have also been demonstrated to improve learners' learning and performance. Hoon, Narayanan, and Singh (2021) studied strategies primary five learners from Malaysia used when applying fractions on a number line. A total of eight learners using a task-based clinical interview from a qualitative study were applied. They found three types of fractional strategies. They are: (1) finding a fractional interval on a number line, (2) applying decimal and fraction interchange concepts, and (3) comparing fractional values. The study results showed that mastering fraction arithmetic is critical for learning fraction magnitude representations on the number line. They also speculate that those learners may have had the possibility of finding a difference in an arithmetic operation but that the concept of obtaining a difference using intervals was not well implemented in fractions (Hoon et al., 2021).

Jayanthi et al. (2021) investigated the effectiveness of fraction intervention on learners experiencing mathematical difficulties in grade five. With two groups, a pre-test-post-test quasi-experimental design was adopted. Cuisenaire Rods and a number line were employed. Number lines were employed in the intervention group (n = 186), and Cuisenaire rods were used in the comparison group (n = 99). On fraction proficiency and understanding tests $(g = 0.68 - 1.23)$, number line estimating tests $(g = 0.80 - 1.08)$, fraction procedures tests $(g = 1.07)$, and explanation tasks $(g = 0.68 - 1.23)$. The study's findings revealed that intervention learners significantly outscored learners in the control condition. They also claim that treatments such as explicit education can increase their fraction proficiency and understanding by using number lines frequently and having a chance to explain their reasons, allowing them to solve fraction problems successfully.

Tian, Bartek, Rahman, and Gunderson's (2021) investigation took place in the North-eastern U.S.: learning proper and improper fractions with the area model or the number line; 129 participants completed the study after each participant worked with a trained experimenter for 20-30 minutes. In part one, learners took a pre-test; in session two, they were randomly allocated to either the number line or the area model training condition. The results of the ANCOVA on PEA of number line estimation of trained fractions show that there was no influence of condition, $F\ (2.108) = 0.53, p = .593$, and $n^2{}_p = .01$. However, as in previous studies with similar training, learners in the number line training condition improved in proper fractions (Hamdan & Gunderson, 2017; Gunderson, Hamdan, Hildebrand & Bartek, 2019).

Even on proper fractions, the conclusions of a recent study found little evidence of improvement: According to the results of the ANCOVA on PEA of number line estimation, learners in the number line training performed similarly to those in the other two conditions at post-test, $F\ (2.108) = 1.45, p = .240$, n^2_p = .03. Finally, they discovered that learners find it more difficult to understand the number line procedure than the area model training method. An analysis of variance (ANOVA) as the interdisciplinary variable shows that learners with a condition in the numerical line $(M = 2.42, SD = 3.02)$ corrective input were given on a somewhat higher number of training trials than those in the area model $(M = 1.46, SD = 1.55), F\ (1.79) = 3.27, p = 076, n2p = .04)$.

In Southern California, Soni and Okamoto (2020) investigated improving learners' understanding using the number lines. They wanted to determine if the number line approaches are equally effective regardless of how they are implemented. In their study, 53 fourth-grade learners enlisted in three schools were taught differently. Learners from two schools using an iPad digital game had no fraction instruction before the study, whereas learners from the intervention group who used paper-and-pencil workbooks in another school had some fraction instruction.

The study confirms that numerical lines effectively improve fractional student knowledge when operated in a digital game or workbook. However, since technology costs could prevent digital games from being adopted, they recommended that paper-and-pencil booklet intervention be encouraged; however, this current study explored the effectiveness of number lines on learning fractions using two different schools in the Cape Coast Metropolis. A quasi-experimental pre-test-post-test score regarding basic six learners' achievements was used using a number line against the traditional classroom instructions between two different schools. The fraction Assessment Task and questionnaire on learners' attitudes and challenges using the number line were analysed.

The outcomes of Hamdan and Gunderson (2017) and Gunderson et al. (2019) suggest that learners in second and third grades may benefit from learning fractions using number lines. Hamdan and Gunderson (2017) divided second-and third-grade learners into three groups in the first study. Two groups received 15 minutes of instruction on fractions, while the third group completed a crossword puzzle (i.e., non-numerical control group). One group of learners learned about fractions using a number line, while the other used area models made of circles (Hamdan & Gunderson, 2017). Both groups learned that a fraction (e.g., $\frac{1}{4}$) has a number on top (the numerator; in this case, 1) and a number on the bottom (the denominator; in this case, 4).

However, depending on the group assignment, the visual representation of this fraction varied (i.e., dividing the number line vs dividing the area model). The experimenter demonstrated the fractions $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{1}{5}$ in both conditions and learners practised modelling them as well as the fractions $\frac{2}{4}$, $\frac{3}{4}$, $\frac{2}{5}$, $\frac{3}{5}$, and $\frac{4}{5}$. Learners assessed their fraction magnitude knowledge right after the training. Learners taught to use a number line improved their efficiency in showing fractions on a number line, while learners taught to use a circle area model improved their ability to show fractions on a circle. Learners who learned fractions through the number line were more adept at comparing fractions (which is greater: $\frac{2}{4}$ or $\frac{1}{5}$?) than learners who learned fractions through the area model. These results are astounding because none of the learners received direct instruction on fraction comparison. A subsequent study (Gunderson et al., 2019) replicated these findings and investigated why number line approaches outperformed area models. Even when a square was designed to resemble a number line, Gunderson et al. (2019) discovered that learners who learned fractions using a traditional number line outperformed those who learned fractions using area models.

In a brief, controlled experiment, second and fourth-graders who were taught to draw fractions on a number line outperformed learners who were taught to draw fractions on a circle on fraction magnitude and transfer tasks (Hamdan & Gunderson, 2017). According to the U.S. Department of Education's fraction practise guide, applying the numerical line as the primary tool for introducing fractions and introducing learners to a category of fractional concepts is recommended because "it is an effective way of developing student understanding of fractions as numbers with magnitude." (Siegler et al., 2011, p. 20). Beyond fundamental number skills and general competencies, fraction magnitude knowledge is a significant factor in mathematics proficiency (Resnick et al., 2016). Despite its importance, learners in the USA only obtain significant preparation in fractions at the starting point of second and third grade, with number lines coming later (Common Core State Standards Initiative, 2010).

According to Tunç-Pekkan (2015), the number line model facilitates learners learning of fractions more than the area model (part-whole) representations. Learners learn that fractions evolve in the same way as whole numbers, that there is an infinite number of fractions between two real numbers, and that an equivalent fraction can constitute the same position on a number line when presenting fractions (Gersten et al., 2017; Zhang et al., 2017).

When it comes to assisting struggling learners to better understand fraction magnitude, the number line is an extremely useful yet sometimes overlooked tool (Saxe, Diakow & Gearhart, 2013; Gersten et al., 2017; Dyson et al., 2018). The number line has been a traditional mathematical tool for representing numbers and developing number concepts. Its inherent property of magnitude and direction makes it very useful in comparing and ordering numbers (McNamara & Shaughnessy, 2011); it is also a good context for assessing learners' acquisition of mathematics knowledge, identifying their errors and misconceptions as well as influences learners' abilities to understand and operate on whole numbers. (Fuchs et al., 2013; Bailey et al., 2015).

Kara and Incikabi (2018) used a case study method to investigate 59 grade six learners from three middle schools in northern Turkey. Their study desired to explore which numerical model, number line, and verbal representations effectively solve the addition and subtraction of fractions. The Multiple Representations in Fraction Operations Test was utilised to collect data. The findings reveal that learners' achievement in employing alternative representations in fraction operations is higher in addition operations than in subtraction operations. Learners also improved performance on issues requiring numerical responses when the answers were presented in model representation (the shift from model to numerical model) (verbal to numerical transition). Yet, learners struggle with alternative representations like number lines and switching between verbal and visual representations.

Cramer et al. (2017) investigated how third-grade learners struggle with fraction representation using the number line. In a qualitative research design, an interview was used as an instrument. From what we can tell, students frequently misinterpret the entire segment of a particular number line as the unit, drawing on their familiarity with paper strips or fraction circles. When presented with a number line with multiple units, such as a 0 to 4 line, students often insist that the location of $\frac{3}{4}$, for example, is at the number 3.

Cramer et al. (2019) further investigated the topic: Reconstructing the unit on the number line using a unique number line instrument called Reconstructing-the-unit interview tasks. They wanted to see how fourth-graders work these distinct number line tasks and how they employed their past knowledge of essential fractional concepts to make sense of them. This study replicated these researchers using two basic six schools in the Ghanaian context. The experimental group was introduced to learning fractions using the number line, while the control group used the set models by their classroom teachers. Learners were given a FAT test to gather information on their performance and the challenges of using these tasks. In addition, learners were given a questionnaire on their attitude using the number line.

Zhang et al. (2015) examined 40 fifth-grade learners to determine their ability to represent unit fractions, $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$ using a variety of different models. Participants excelled at regional partitioning models but struggled with unit fractions in non-area-model contexts. Learners' test scores improved significantly after receiving an instructional intervention based on Dienes' (1960) dynamic principle. Following instruction, their conceptual understanding of unit fractions had advanced to the point where they could explain how they arrived at solutions.

Adom and Adu (2020) investigated paper folding on grade nine learners' performance in fractions in Chris Hani West District in South Africa. A pre-test and post-test quasi-experimental design were used. The study used various sampling techniques, including systematic simple random, convenience, purposive, and stratified sampling. The experimental group (n = 125) used paper folding, and the control group (n = 125) used the traditional method. The analysis results of paper folding revealed that the pre-test $(M = 8.372, SD = 1.770)$, posttest $(M = 11.792, SD = 4.256), t = 12.024 < 0.05$, which means the use of paper folding has a positive influence on learners' performance in fractions.

To assist JHS one learners to add unlike fractions using Cuisenaire rode and Paper folding in Chamba M/A basic schools was one of the action research designs employed by Bernard et al. (2020) using 45 learners randomly. Test (Pre-test-post-test) and observation were used as data collection instruments. The study's findings suggested that mathematics teachers teach mathematics abstractly with little interaction from learners. The researcher recommended prioritising thorough preparation and meaningful teaching approaches and activities to facilitate class delivery and student involvement. This study was embedded with tests (FAT) and a questionnaire with two different schools with intact classes.

Amuah et al. (2019) made an interesting contribution by investigating how junior high school learners understand some selected concepts of fractions using a sample of nine qualitatively in the Cape Coast Metropolis. The study revealed that learners better understood the part-whole concepts; the least understood concept was the equivalence concept. Wong (2009) evaluates how a primary school in Sydney learners understand the number line model for fractions, considering the learning demands and number line conventions learners must learn. The conventions of number lines were first presented, followed by assessing learners' understanding. Findings from the 297 learners investigated revealed that 20.2% (*n* = 60) could not answer. By iterating one-third three times, these learners could estimate the unit.

In comparison, 22.6% of learners (*n* = 67) ignored the scale and positioned 1 at the end of the number line. Pearn and Stephens (2007) report that these learners comprehend the quantitative features of fractions but not their proportionality. Because one is less than or equal to one-third, the results indicate that these learners do not perceive one-third as a quantity. According to studies, learners who marked one-third of a third may interpret fractions as an action, making it difficult to discover one-third of something. The additional 13.5% (*n* = 40) replies had no observable structure, while another 11.1% (*n* = 33) pointed 1 at the unit's halfway point. Thus 24.6% of learners are ignorant of their thoughts. The activity mentioned above gives teachers an idea of the challenges learners may face when learning to comprehend the concept of a fraction on the number line as a representation of a quantity (Wong, 2009).

These authors, Adom and Adu (2020), Subaar, Asechoma, Asigri, Alebna, and Adams (2010), offered some glimpses into the effectiveness of the area model (manipulatives), whiles Soni and Okamoto (2020) point out how iPad digital game can be used to illustrate fractions on a number line, it was recommended in Soni and Okamoto (2020) study that, this item comes with many costs; therefore, the paper-and-pencil book should be encouraged. Kara et al. (2018) looked into how multiple representations could be used to locate a fraction. Deringöl (2019) focused on pre-service and in-service teachers. Current studies from Tian et al. (2021) and Hoon et al. (2021) used a random sampling technique and eight learners, respectively, leaving a deficiency in the literature regarding the most effective instructional components for learners at the primary school level. This study used two intact classes to examine how number lines effectively improved learners' conceptual and procedural knowledge. Learners were asked to identify fractions; for example, $\frac{1}{2}$ and $\frac{1}{4}$ on a number line from 0 to 1, common errors were pinpointed, and the learners were allowed to explain their thinking on locating answers to identify their procedural skills on the Fraction Achievement Test.

Learners' Attitude Towards the use of Number Line Approach in Learning Fractions

Karika and Cskos (2022) used the number line approach to study how well learners can conceptualise fractions in their heads. High reliability (alpha=.95) was found for the test among a sample of 124 fifth graders. According to the results, the correlation coefficients between learners' overall performance and their attitude factors range from 0.21 (the usefulness of learning fractions) to 0.62 (the importance of studying fractions) (attitude towards fractions). Each of these R-values is statistically significant at the $p<0.05$ level. Each group of items investigated thus far showed correlation coefficients of around the same size and significance. Performance was significantly correlated with learners' attitudes toward learning fractions using a number line.

The fundamental objective of the research conducted by Govindarajan and Choo (2022) was to enhance the performance and attitudes of elementary school students in mathematics by introducing them to an effective learning approach (the number line). A quasi-experimental time-series design was used. Forty students were assigned to the treatment group, which used a blended learning platform (Moodle), whereas the same number of students in the control group traditionally received their education. Pre-Test, Tests 1 and 2, Post-Test, and Attitude Questionnaires and Interviews were utilised to collect data. One-way analysis of variance (ANOVA) was used to compare the experimental and control groups to assess the data analysis strategy. The data shows a noticeable distinction between the two approaches at the $p<0.05$ level. The study's results confirmed that blended learning effectively raises students' academic performance. It was also discovered that students' attitudes improved due to their time in blended learning.

Hensberry, Moore, and Perkins (2015) investigate how using a simulation to teach mathematics impacts students' motivation and performance. During four days, two groups of fourth graders used the simulation to practice basic fraction skills. Pre- and post-tests, a survey of students' attitudes, and in-depth interviews with a sample of students all contributed to the data collected. Both procedural and conceptual understanding of fractions improved significantly between the pre-and post-tests. The focus group interview data corroborated the survey results showing that most students felt the interactive simulation helped teach them about fractions. These findings show that interactive simulations can be powerful instruments for fostering procedural and conceptual comprehension when combined with good instruction.

This study was further concurred by Barbiere et al. (2020), whose preliminary analyses indicated a statistically significant interaction between classroom attentive behaviour and intervention group on fraction concepts on the posttest, implying that there was a moderating effect of the experimental intervention on the detrimental effect of low attentive behaviour on learning. Students who struggle with fractions benefit greatly from being taught using a number line-based method that combines research-based learning strategies.

Chapter Summary

This research is grounded in the theoretical framework of constructivism. The study of fractions, the concept of fractions, the concept of the number line, the importance of number lines, the various misconceptions students encounter when learning fractions with the number line, the causes of these errors, and the conceptual and procedural knowledge required to overcome them were all reviewed. In the end, the researcher also considered the results of empirical studies conducted by various authors.

CHAPTER THREE

RESEARCH METHODS

Introduction

This study examined the influence of the number line approach on learners learning of fractions, their performance in fractions, their attitudes when using the number line, and their challenges with the approach. This chapter discussed the research strategies and procedures that were put into place to collect the information needed to answer the research questions. As a result, the study concentrated on the research design, population, sample size and sampling procedure, research instrument, data collection and data processing and analysis, and ethical considerations.

Research Design

According to Rahi (2017), a research design is a pattern or detailed plan for conducting a study; it directs the logical data collection and analysis framework to arrive at conclusions. This research used a quasi-experimental design. A pre-test-post-test non-equivalent group design was used. Quasi-experiments are common in the social sciences, psychology, and education since respondents are not subjected to a completely arbitrary treatment allocation (LaCaille, LaCaille, Damsgard, & Maslowski, 2019; Tuti & Liebe, 2021). Experiments are commonly conducted in classrooms, where the experimental and control groups are made up of natural groups like whole classes. This also allowed for studies on mathematics instruction in its natural habitat (Hanfy, Daleure, Abuquad & Al Hosani, 2022).

The study comprised an experimental and a control group. The experimental group was exposed to using the number line to learn fractions. Two schools with an intact class of 42 (Experimental) and 39 (Control) were selected purposively for the study. A pre-test was administered to the two schools during the first week; the mean scores were established. With the mean scores, the learners from the low mean school were placed in the experimental group using the number line model. The learners from the school with a high mean were placed in the control group, who experienced fractions using the set and area models. The teacher introduced the topic according to the guidelines used by the basic six curricula. Shaughnessy, Zechmeister, and Zechmeister (2014) posited that this design is appropriate if the purpose is to analyse the pre-test mean scores between the experimental and control groups and the post-test mean scores of the group. In the pre-test-post-test control group, one of the groups formed previously for some other purpose was randomly selected as the experimental group.

Population

The term “population” is used by Enos, Yensu, and Obeng (2020) to refer to a group of cases that share common characteristics and are extrapolated from in order to draw broader conclusions about the phenomenon under study. Others choose a place they believe will produce comparable or unique outcomes to address the study questions. For this reason, the selection of these schools from the two towns was influenced by geographical proximity, time constraints, and the desire to avoid treatment contamination. The two groups were decided after the mean scores were established. With the means scores, the learners from the school with low mean scores were placed in the experimental group to experience teaching and learning fractions using the number line model. The learners from the school with a high mean were placed in the control group, who experienced fractions using the set and area model. Table 1 shows the demographic characteristics of the learners who took part in the study:

Table 1: Demographic Characteristics of Learners

	Experimental Group		Control Group	
Gender	N	%	N	%
Male	22	52.4	20	51.3
Female	20	47.6	19	48.7
Total	42	100	39	100

Source: Field Data (2021)

Sample and Sampling Procedures

Sampling refers to selecting a sample from a larger population with the same characteristics as the remaining units (Saunders, Lewis, & Thornhill, 2019). The units are assumed to exhibit the same unit attribute due to the population. Participants for this study comprised basic six learners within the Cape Coast Metropolis. These learners were considered ideal for the study. These classes were targeted because fraction starts to become a focus in primary four, and by primary six, learners should have foundational fraction knowledge.

Data Collection Instruments

Two instruments, Fraction Achievement Test (FAT) and a structured questionnaire were used. The Fraction Achievement Test (FAT), a number line consisting of 10 items, was used for the study. The test allows learners to identify fractions on the number line and explain their thinking. The tests were adopted, and the item construction used the recommended textbooks and the Basic six syllabus. The Fraction Achievement Test (FAT) test instruments consist of ten subjective and nine explanations of learners' thinking on how the number line approach is being used, and one question gives room for learners to apply their understanding of how to draw a number line and show the following fractions $\frac{2}{3}$ and $\frac{4}{5}$. However, only the positions of the post-test items were changed compared to the pre-test. There were ten items on the post-test. Under each item, eight items allow learners to explain their thinking; one item asks learners to draw a number line and show $\frac{2}{3}$ while the remaining questions ask learners to locate a point on the number line. Each test was graded on a scale of one to twenty (Barbieri et al., 2020).

Questionnaire

A questionnaire is a type of research instrument that consists of questions designed to elicit information from respondents (Kim, McLeod & Kiss, 2018). In this case, a self-made survey was employed. The purpose of the questionnaire was to collect data on learners' attitudes before and after the number line instructions. Pozzo, Borgobello, and Pierella (2019) outline some benefits of using a questionnaire. The first benefit is that it guarantees that a large crowd will show up. The second advantage is that it can be distributed to a non-selective group of people. The questionnaire was used to elicit information on learners' attitudes and whether they embrace fraction lessons using the number line. It was divided into three sub-construct, "Behavioural Engagement," "Interests," and "Confidence." Questions like, "I can locate fraction problems faster when I use the number line," I now see the number line as part of a fraction. The reliability of the findings and the amount of information gleaned from the survey increased using a 4-point Likert scale (Swan, 2006). There were 15 questionnaires on learners' attitudes towards learning fractions with the number line.

Validity and Reliability of the Instruments

This section paid attention to how the instruments reliability and validity were tested. The Cronbach alpha and the (KR20) were the main reliability procedures, while the face and content validity were also assured.

Validity of the Instruments

According to Révész (2012) and Gravetter & Forzano (2018), validity is the extent to which a study's findings address the question for which they were intended, or the extent to which they provide an answer to the problem that they were intended to solve. The validity of a study is determined by how well its findings fit with prior knowledge and how well they fit the needs of the researcher (Sileyew, 2019). Face and content validity analyses were performed on the Fraction Achievement Test and questionnaire.

First, students at the University of Cape Coast enrolled in the same programme were asked for their thoughts on a questionnaire. Colleague students reviewed the questions to ensure that the wording offered no room for interpretation. Next, the supervisor checked the content validity of the Fraction Achievement Test and questionnaire concerning learners' attitudes using the number line approach.

The research supervisor also examined the research questions alongside each instrument's item to determine whether they measured what they were supposed to have measured. A mathematics teacher, an expert in mathematics instruction, measurement and evaluation, and the classroom teacher also checked the validity of the test.

Reliability of the Instruments

The reliability of a measuring device is defined as the degree to which multiple readings taken from the same subject under identical conditions produce the same result (William, 2006). The researcher adopted Fraction Achievement Test (FAT) on fractions given to Basic 6 learners of similar characteristics but in different Municipalities in the Central Region of Ghana. A questionnaire on learners' attitudes toward learning fractions with the number line was scored immediately. Those items which were ambiguous were taken out. Based on the data analysis, it was established that the Cronbach's alpha reliability coefficient for the instrument was = 0.898. Values of 0.70 or greater for Cronbach's Alpha reliability coefficient are deemed dependable, as stated by Creswell (2007). After that, the reliability of the achievement test was established by applying the Kuder-Richardson formula 20 (KR20). The test item's quality was analysed, and the reliability coefficient was calculated using the modified version of Kuder and Richardson formula 20 (KR20) by Brennan, Lee, and Kolen (2006).

$$KR_{20}=\frac{K}{K-1}\left(1-\frac{\Sigma pq}{\sigma^2 X}\right)$$

$$KR_{20}=\frac{15}{(15)-1}\left(1-\frac{3.5069}{35.74306}\right)$$

$$KR_{20}= [1.0714]\ [0.9019] = 0.96$$

The KR_{20} value was 0. 96. Therefore, the FAT assessment adopted in the Ghanaian context was reliable.

Table 2: Reliability coefficients on attitude sub-scales

Variables	No. of items	Actual study
Behavioural engagement	5	0.76
Interest	5	0.68
Confidence	5	0.77

Sources: Field Data (2021)

Data Collection Procedures

In order to have a complete understanding of a topic, data collecting is used to compile and analyse information from various sources (Kachikis et al., 2019). Permission from the school administration and participants was sought using an introductory letter from the Institutional Review Board (IRB) of the University of Cape Coast. The day, time, and venue for the data collection were agreed upon by the Headmistresses, the classroom teachers of the two schools, and the researcher. After securing the head’s and teachers’ consent, preparations were made to gather data on the agreed dates, times, and venue. Teachers and learners were assured of confidentiality and anonymity. Thus, it was explained that the data collected were purely for academic purposes. The number line approach is used to identify cardinal and ordinal numbers, help learners compare and organise fractions, and use them to find fractions of shapes and numbers (NaCCA, 2019). It also offers learners the opportunity to justify their reasons. During the third term of the 2021 academic year, comprehensive data gathering and instruction were done in the two schools.

Learners consented to be part of the study, following which the researcher administered a questionnaire and Fraction Achievement Test. Both teachers from the two schools taught fractions for two weeks following instruction. A written test was taken by both groups at the end of the two weeks lesson, marked and scored out of 20 marks.

The pre-test was given to the basic six learners at both schools during the first week of the study. Learners were taught the concept of fractions for two weeks by their teacher. In the third week of the study, a post-test was administered to the experimental and control group. The mean scores of the learners in the post-test were compared to the mean scores in the pre-test to establish the influence of the number line approach on learners' performance on fractions with the conventional approach.

Data Processing and Analysis

Data analysis is making the idea of data by combining, reducing, and interpreting the information. The method you will use to respond to your research question(s) (Merriam & Grenier, 2019). The learners' responses to the after-instruction questionnaire and the lesson test scores were entered into Microsoft Excel and then transferred to SPSS (version 26). Marks from the FAT test pre-test-post-test score were coded, and descriptive statistics such as mean, standard deviations, and percentages were computed to examine the level of performance between the two groups. The quantitative data was analysed using descriptive and inferential statistics.

To answer *research question One: What are the basic six* learners' *attitudes toward learning fractions with the number line?* Data were analysed using a paired samples t-test, percentages, and mean to explore their attitude before and after the intervention.

Research question Two: What are the basic six learners' challenges in using the number line in learning fractions? Percentages, excerpts, and snapshots of both groups' challenges were identified, and explanations were provided.

Hypothesis One: There is no significant difference between the two groups on the pre-test scores. An independent samples t-test was used to test if there was a significant difference between the pre-test scores at the 5% significance level between the two groups.

Hypothesis Two: There is no significant difference between the two groups on the post-test scores. An independent t-test, mean and standard deviation were utilised at the 5% significance level between the two groups on the post-test scores.

Ethical Considerations

Research must be conducted ethically if it is to meet the validity and trustworthiness of the research process (Kyngäs, Kääriäinen & Elo, 2020). Challenges in quantitative research include gaining approval, protecting the privacy of respondents, minimising disruptions at research sites, and defining the study's goals.

I began by visiting the schools to become acquainted with the atmosphere in which teachings were delivered. I explained the research goal to all individuals involved in the two selected schools using the introductory letter. I met with the learners and informed them of the study aim, meeting dates, and session times. They were required to sign a consent document. Finally, I assigned each participant a unique serial number. As a result, the data for this study were coded anonymously, and so cannot be linked to specific individuals.

Chapter Summary

This chapter mentioned that a quantitative approach was utilised. The chapter justified the study replication in the two intact classes from the two schools. The chapter also examined the population, participants, research instruments, quasi-experimental design, and data collection procedures. In addition, questions about the instrument and study validity and reliability were highlighted. Finally, the chapter discussed the ethical implications and the steps that should be taken to mitigate and address them.

CHAPTER FOUR

RESULTS AND DISCUSSION

Introduction

The study investigated the influence of the number line approach in learning fractions on Basic six learners in Cape Coast Metropolis. The (FAT) test results written by learners and their explanations of how they understood and responded to the questionnaire were used as the instruments. A questionnaire on learners' attitudes which were sub-categorised into three, "behavioural engagement," "confidence," and "interest," were gathered before and after the intervention. A pretest-posttest non-equivalent design under a quantitative research approach was used. Two intact classes were used with eighty-one learners from two different schools. Research questions on learners' attitudes and challenges were analysed. Hypotheses were tested at a 5% significance level to determine significant differences between the groups on pre-test and post-test scores. In addition, the study findings are also discussed in relation to the literature reviewed.

The findings pertinent to the first hypothesis are presented in the following subheading.

Before the intervention, a pre-test was conducted between the two groups. The research hypothesis was formulated as follows.

***H_O*1:** *There is no statistically significant difference between the two groups on the pre-test scores.*

An independent samples t-test was conducted. At an alpha α level of 0.05. A Levene's test of equal variance was also performed. The results show that the sig. value for Levene's test was below 0.05 (0.091), indicating that the assumption has been violated, and the results on the second row were reported. Table 3 shows the outcome of the pre-test scores.

Table 3: Difference in pre-test scores

Schools Categories	*N*	*M*	*SD*	***t***	df	Sig.
				-0.11	63.29	0.91
Control	39	4.97	2.413			
Experimental	42	5.02	1.522			

Source: Field Data (2021)

From Table 3, the Control group had a mean score of $(M = 4.97, SD = 2.41)$. The maximum score was 10, with the minimum score being 2 compared to the Experimental group $(M = 5.02, SD = 1.52)$ where learners had a maximum score of 9, with 2 being the minimum score. The results from the t-test, as shown in Table 2, indicated that there was no statistically significant difference in the pre-test scores for the Control $(M = 4.97, SD = 2.41)$ and Experimental group $(M = 5.02, SD = 1.52)$; $t\ (63.29) = -0.11; p = 0.91$. This implies that the two groups were performing almost equally at the start of the intervention. Therefore, I failed to reject the null hypothesis, which states that "There is no significant difference between the control and the experimental group on the pre-test."

After the intervention, a post-test was conducted. The research hypothesis was formulated as follows.

***H*$_O$2:** *There is no significant difference between the two groups on the post-test scores.*

An independent samples t-test was conducted at an alpha α level of 0.05. A Levene's test of equal variance was also conducted. Since the sig. value for Levene's test was below 0.05 (0.001), indicating that the assumption has been violated, and the second row (equal variances not assumed) was reported. Table 4 shows the outcome of the post-test scores.

Table 4: Difference in post-test scores

Schools Categories	*N*	*M*	*SD*	*t*	df	Sig.
				-3.55	69.77	0.001
Control	39	7.36	1.78			
Experimental	42	9.21	2.83			

Source: Field Data (2021) *Significant, p < .05

From Table 4, the Control group had a mean score of $(M = 7.36, SD = 1.78)$. The maximum score was 13, with the minimum score being 4, compared to the Experimental group $(M = 9.21, SD = 2.83)$ where learners had a maximum score of 16, with 5 being the minimum score. The results from the t-test, as shown in Table 3, indicated that there was a statistically significant difference in the post-test scores for the Control $(M = 7.36, SD = 1.78)$ and the Experimental group $(M = 9.21, SD = 2.83)$; $t\ (69.77) = -3.55; p = 0.001$. This indicates that the experimental group outperformed the control group following the intervention.

Noticing a difference in performance, the researcher went further to find out where these differences lie according to lower-order taxonomy measured at the Knowledge, Understanding, and Application levels. Four test items measured the Knowledge level, five measured the Understanding level, and one measured the Application. It was observed that the Control ($M = 4.85, SD = .630$) and the Experimental group ($M = 4.71, SD = .970$) performed almost similarly at the knowledge level. But when it comes to understanding and application levels, the Experimental group outperformed the Control group by a significant difference, the control group ($M = 4.85, SD = .630$) and experimental group ($M = 4.71, SD = .970$) $t\,(70.91) = .731; p = 0.001$.

Here are excerpts of learners' work for the various taxonomies.

Excerpts of questions classified under Knowledge.

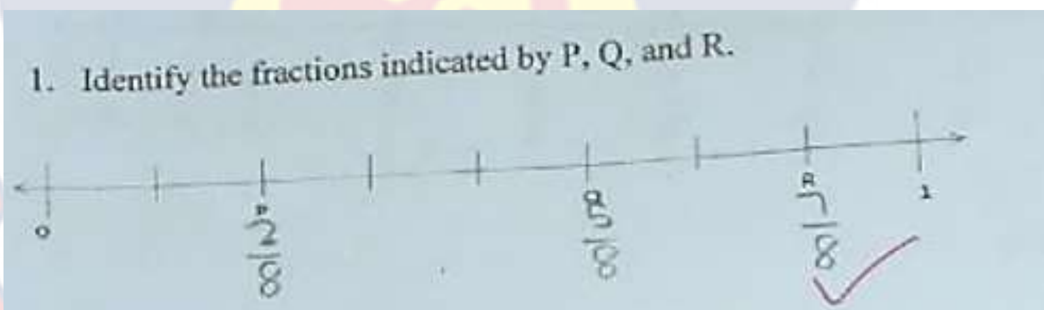

Figure 1: "Cont.30" Knowledge level on item 1

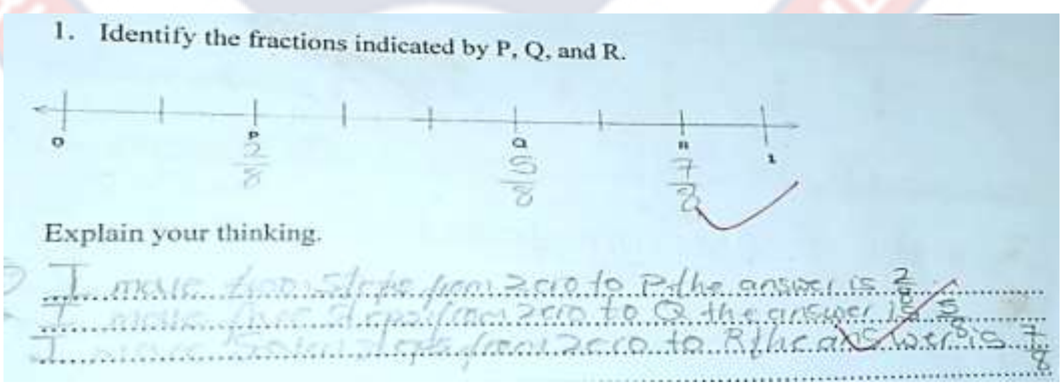

Figure 2: "Exp.13" Knowledge level on item 1

Excerpts of questions classified under Understanding.

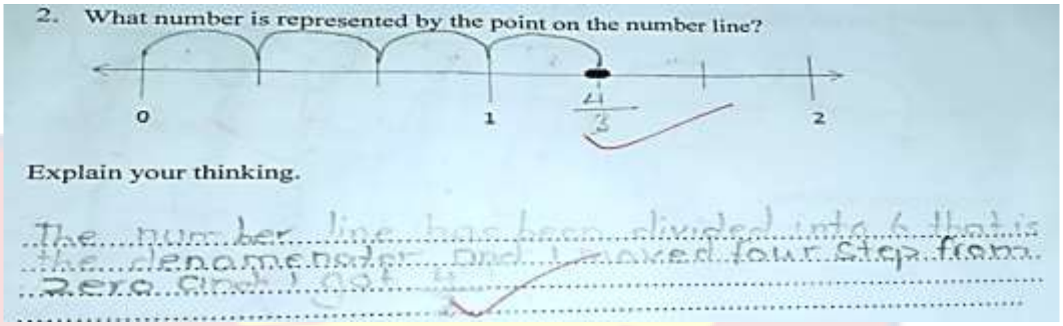

Figure 3: "Exp.31" Understanding level on item 2

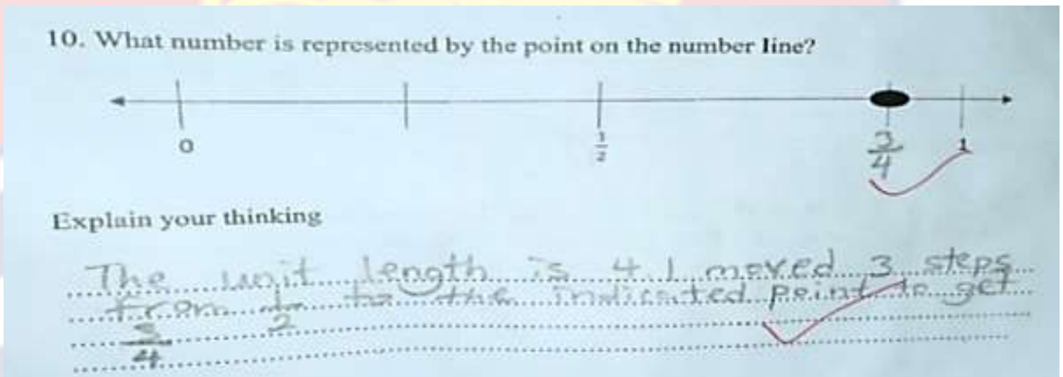

Figure 4: "Exp.29" Understanding level on item 10

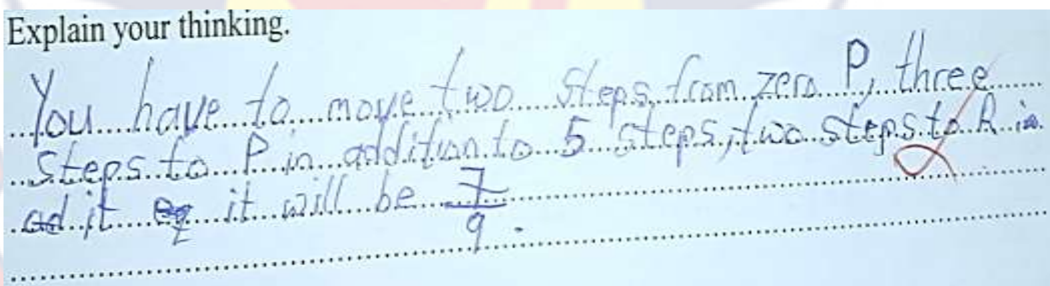

Figure 5: "Cont.12" Understanding level on item 2

Excerpts of questions classified under Application

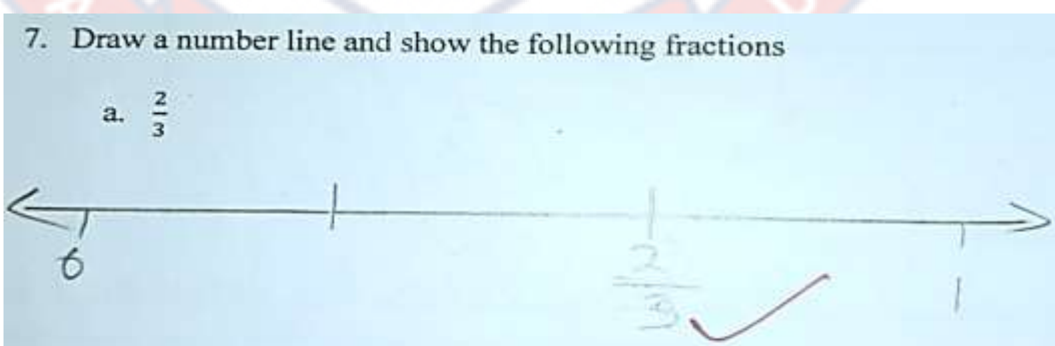

Figure 6: "Exp.11" Application on item 7

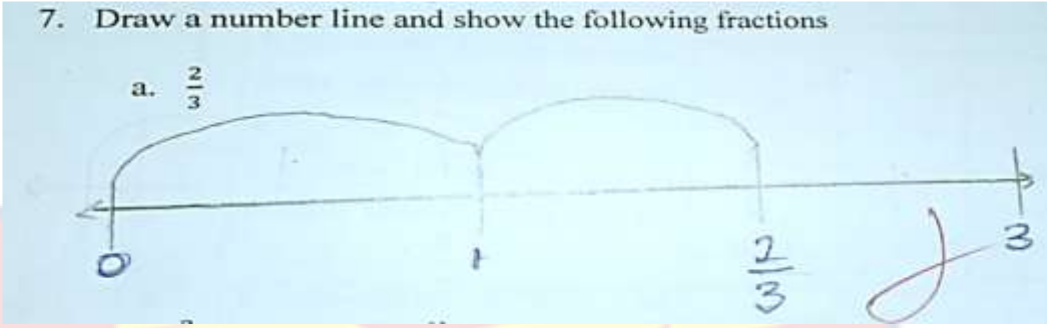

Figure 7: "Cont.10" Application on item 7

This study confirms with Jayanthi et al. (2021), whose studies investigated the effectiveness of fraction intervention on learners experiencing mathematical difficulties in grade five. The results of their ANCOVA revealed that intervention learners outscored learners in the control condition by a significant margin.

This study contradicts Tian et al. (2021), who discovered that learners find it more challenging to understand the number line procedure than the area model training method. Using traditional number lines, as in this study, the findings support Gunderson et al. (2019), who also discovered that learners who learned fractions using a traditional number line outperformed those who learned fractions using area models. This finding also reveals that learners could recognise equivalent fractions, supporting earlier findings (Zhang et al., 2017; Gersten et al., 2017).

This study confirmed Soni et al.'s (2021) findings which support that applying the number line improved learners' fraction-related concepts as measured using knowledge, understanding, and application. The findings also support Hamdam et al. (2017), who revealed that learners who learned fractions through the number line were more adept at performance level than those using area models. Finally, the findings relate to Kara and Incikabi's (2018) study, who believed that number line enhances learners' performance and their numerical expression of verbal questions improves. This study concurs with Altıparmak and Palabıyık (2019), who opined that when learners are permitted to "understand" fractions using the number line, it assists them to "remember," which enables them to "apply" according to the revised Bloom's taxonomy and can significantly promote meaningful learning.

Research Question One: *What are basic six learners' attitudes toward learning fractions with the number line?*

A four-Likert scale was used to examine the experimental learners' attitudes toward learning fractions with the number line. Fifteen questions were subdivided into three sub-constructs, with five statements each. The first construct, "*Behavioural Engagement,"* meant how learners acted when learning fractions with the number line. The second factor was "*Interest*" regarding how much they enjoyed the lesson and worked extensively on the tasks. "*Confidence*" looked at learners' confidence in learning and understanding fractions with the number line. This was based on learners' ability to explain their thinking and justify their answers on FAT tasks. The result is shown in Table 5.

Table 5: Experimental Learners' Attitude toward Learning Fractions with the Number Line

Items	Statement	Mean	S.A %	Agree %	Disagree %	SD %
	Behavioural Engagement					
1	I can locate fraction problems faster when I use the number line	1.64	42.9	50.0	7.1	0
2	For some reason, the use of number lines is really hard for me	2.50	19.0	28.6	35.7	16.7
3	It was easier to learn and understand fraction concepts using the number line	1.40	71.4	16.7	11.9	0
4	I can now understand how to locate $\frac{1}{2}$ with the number line	1.29	81.0	9.5	9.5	0

5	I now see number line as part of a fraction	1.64	59.5	23.8	9.5	7.1
	Confidence					
6	I have much self-confidence when using the number line in solving fractions	1.43	64.3	28.6	7.1	0
7	I can now apply and explain my thinking on how to locate fractions using the number line	1.62	45.2	47.6	7.1	0
8	Once I start working with the number line, it is hard to stop	1.86	50.0	23.8	16.7	9.5
9	The individual tests helped me to check my understanding of the lessons	1.50	66.7	19.0	11.9	2.4
10	I have a few problems on how to explain how to locate $\frac{1}{2}$ using the number line	2.40	19.0	33.3	35.7	11.9
	Interest					
11	The examples and exercises given in the lessons enhanced my understanding of fraction concepts	1.48	59.5	35.7	2.4	2.4
12	The activities in the number line helped me to follow the lessons and think critically in understanding the concept	1.62	57.1	28.6	9.5	4.8
13	From the teacher's examples during the lesson, I can relate	1.43	69.0	21.4	7.1	2.4

	the concepts learned to real-life situations.					
14	The use of the number line helped to identify patterns, compare and relate ideas to make generalisations	1.45	66.5	21.4	11.9	0
15	I prefer that my teacher use this approach to teach us different concepts of fractions.	1.19	81.0	19.0	0	0

Source: Field Data (2021)

From Table 5, it was indicated that twenty-one learners representing (50%) agreed with the statement that they could locate fraction problems faster on the number line, with 18 (42.9%) strongly agreeing, while 3 learners (7.1%) disagreed, indicating a mean of 1.64. fifteen learners (35.7%) disagreed with the statement that "for some reason, the use of number lines is hard for them," with 12 learners (28.6%) agreeing, while the least strongly disagreed (16.7%), given a mean of 2.50. Again, 30 (71.4%) strongly agreed that using the number line made learning and understanding fraction concepts convenient, with 16.7% representing 7 learners agreeing and only 5 (11.9%) disagreeing.

Despite item number 4 being one of the most strongly agreed (81%) statements, "I can now understand how to locate $\frac{1}{2}$ with the number line." 4 (9.5%) support the assertion, and 4 (9.5%) disagree. Twenty-five learners (59.5%) with a mean of 1.64 (*SD*=.932) strongly agreed that they can now see the number line as part of a fraction, while (23.8%) agreed on the motion, with 9.5% and (7.1%) disagreed and strongly disagreed respectively. Also, 47.6% of learners agreed that they can now apply and explain their thinking on locating fractions using the number line, with 45.2% strongly agreeing, with the least being 7.1%, representing a mean of 1.62.

Most learners support the statement, "Once I start working with the number line, it is hard to stop," with 50% strongly agreed and 23.8% agreed. While a minimum of 16.7 disagreed, providing a mean of 1.86. Most learners, 66.7%, strongly agree that the tests helped them check their understanding, whereas only 11.9% disagree. Furthermore, the majority, 35.7%, disagreed that they have few problems how to locate $\frac{1}{2}$ using the number line, with 33.3% agreed, given a mean of 2.40. Most learners (59.5%) thought that the examples and exercises provided in the lessons helped them better understand fraction concepts. Most learners (57.1%) strongly agreed that the number line exercises assisted them in following the lessons and thinking critically about the concept. Most learners (69%) strongly agreed that the teacher's examples helped them apply the principles learnt in class to real-life circumstances. Last but not least, 66.5 percent of learners strongly believed that using the number line assisted them in identifying patterns, comparing and relating ideas, and making predictions. Lastly, it was observed that most learners representing 81%, strongly agreed that they prefer their teacher to use this approach to teach them different concepts of fractions.

In addition to research question two, a paired-samples t-test was performed to compare experimental group learners' attitudes towards learning fractions with the number line before and after the intervention. The data is presented in Table 6.

Table 6: Paired Samples t-test on Experimental Learners' Attitude toward Learning Fractions with Number Line (N=42)

	Before Intervention		After Intervention		*p* *value*	Effect Size(d)
Attitude	*M*	*SD*	*M*	*SD*		
Behavioural Engagement	1.13	0.227	3.22	0.612	0.0001	3.26
Interest	1.36	0.350	3.56	0.305	0.0001	5.13
Confidence	1.42	0.363	1.43	0.244	0.0001	4.82
Overall attitude	1.303	0.223	3.424	0.247	0.0001	6.08

Source: Field Data (2021)

From Table 6, it appeared that the learners' overall mean attitude on learning fractions using the number line improved significantly after the intervention on all the sub-constructs. The overall mean on attitude before the intervention for the experimental group ($M = 1.303, SD = 0.223$) and after the intervention ($M = 3.736, SD = 0.247$), $t\,(41) = 31.57; p < 0.001$ indicated a significant difference. Cohen's *d* was estimated at 6.08, which is a large effect. This suggests that the experimental group of learners acknowledged that they developed a positive attitude toward learning fractions with the number line.

This study's findings on learners' attitudes toward learning fractions with the number line support earlier findings from Erlinda and Surya (2017), who revealed that learners find the number line intervention more interesting. This finding aligns with Wall, Thompson, and Morris (2015), who revealed that learners' confidence before the study trial was the lowest compared to after employing number line instructions. This study supports Karika and Csíko's (2022) findings, revealing that learners have a positive attitude toward learning fractions with a linear model. In addition, this study supports Govindarajan and Choo's (2022) findings, which indicated that learners' attitudes regarding fractions improved due to the number line. Hensberry, Moore, and Perkins (2015) found that the experimental group's attitude during the questionnaire corroborated the findings that students had a favourable attitude toward learning fractions with a number line. Barbieri et al. (2020) concur that using the number line approach increases learners' attention and attitude since it allows more discussion and strategies. Finally, Lamon (2012) also believes fractions on the number line significantly influence learners' attitudes about mathematics.

It was observed that learners encountered some challenges when using the number line. I present to you the last research question below.

Research Question Two: *What are the basic six learners' challenges in using the number line in learning fractions*?

The last research question explored learners' various challenges in interpreting and responding to fraction-related items. Table 7 presents the experimental group's results of incorrect and correct answers.

Table 7: Incorrect and correct responses from the Experimental group

Item No.	Incorrect Answers (%)	Correct Answers (%)
1	9(21.4)	33(78.6)
2	12(28.6)	30(71.4)
3	7(16.7)	35(83.3)
4	14(33.3)	28(66.7)
5	5(11.90)	37(88.1)
6	9(21.4)	33(78.6)
7	18(42.9)	24(57.1)
8	15(35.7)	27(64.3)
9	33(78.6)	9(21.4)
10	33(78.6)	9(21.4)
Total	42	100

Source: Field Data (2021)

Results from Table 7 indicate that 9(21.4%) learners incorrectly answered, whiles in question item two, 12(28.6%) learners had it wrong from the group. It was also observed that only seven learners representing 16.7%, incorrectly answered the question. From item four, 33.3% of learners wrongly answered the question. Only 5(11.9%) from the experimental group incorrectly answered item five.

From item 7 it was observed that nine 9(21.4%) learners from the experimental group incorrectly answered it. Results from item eight indicate that 15(35.7%) incorrectly answered the question from the experimental group. 33 (78.6%) learners incorrectly answered questions 9 and 10, respectively.

Six challenges were identified, which are explained and shown below.

1. **Counting of tick marks instead of the interval between them**

It was observed that 19 learners' faced the challenge of counting the tick marks instead of the interval between them. A learner, as indicated in Figures 8 and 9, wrote $\frac{4}{8}$. The learner believed that since the first indicated point, "*P*" is $\frac{2}{8}$, the next indications should be $\frac{3}{8}$ and $\frac{4}{8}$ respectively for both *Q* and *R*.

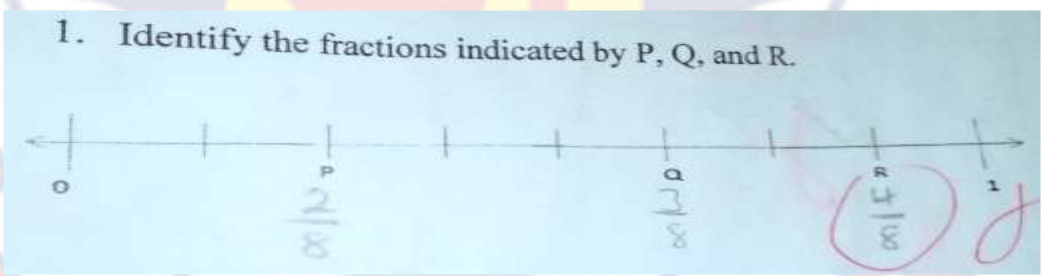

***Figure 8*: incorrect sample response for item 1 "*Exp. 17*"**

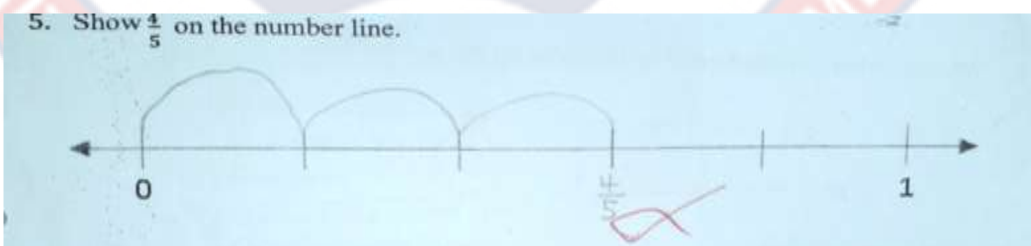

***Figure 9*: incorrect sample response for item 5 "*Exp. 14*"**

2. Difficulty differentiating fractions from the whole number

It was observed that 18 learners had difficulty differentiating fractions from the whole. An example of an incorrect response was that some learners failed to realise that the point was between one and two on the number line. Given less regard for the unit in which the point was situated.

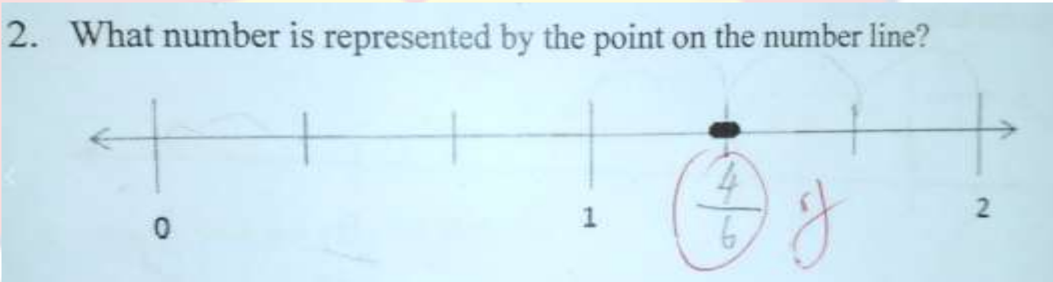

***Figure 10*: incorrect sample response for item 2 "*Exp. 15*"**

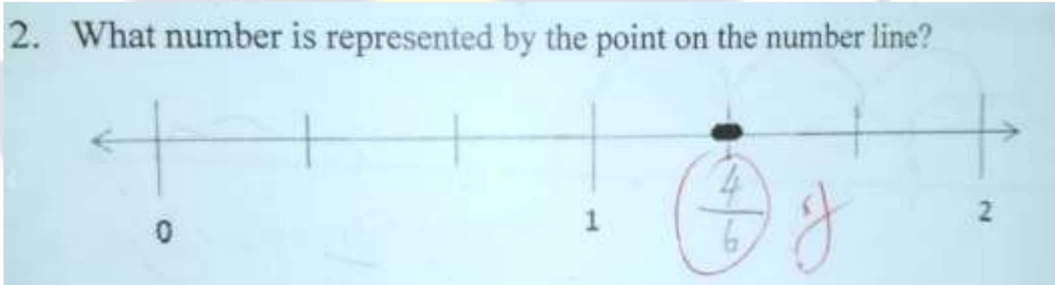

***Figure 11*: incorrect sample response for item 2 "*Exp. 19*"**

3. Considering only the numerator in locating fractions with less regard to the number of partitions.

It was observed that nine learners' faced this challenge. For example, a learner believed that since the denominator of $\frac{3}{4}$ was not noted within the fractions on the number line; it was ignored, thus locating the fraction with only the numerator, as shown in Figures 12 and 13.

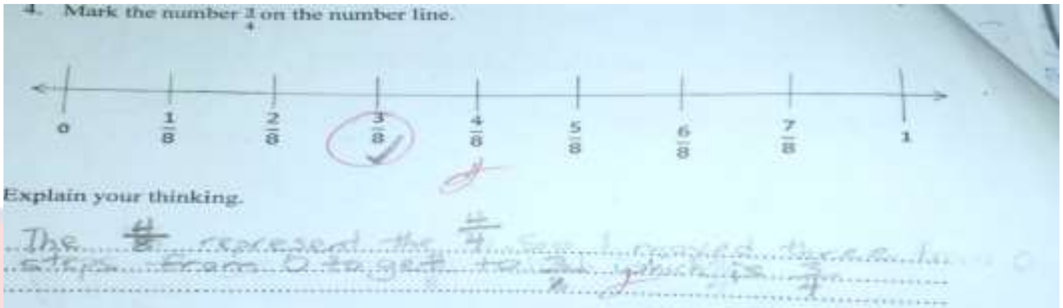

Figure 12: **incorrect sample response for item 4 “*Exp. 18*”**

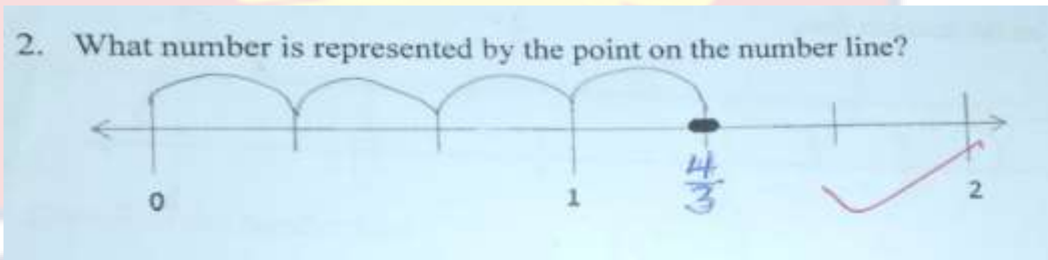

Figure 13: **incorrect sample response for item 2 “*Exp. 21*”**

4. **Difficulty naming or locating fractions in unit lengths other than 0 – 1**

More than half (33) of the learners find it difficult to locate fractions in unit lengths other than 0 – 1. An excerpt is shown in Figures 14 and 15.

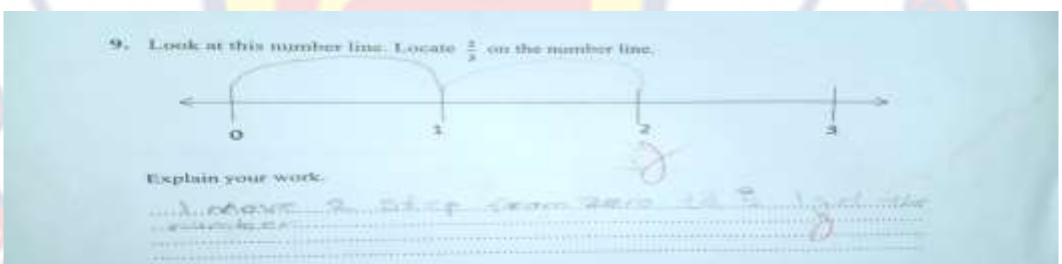

Figure 14: sample of incorrect answers for test item 9 *“Exp.13”*

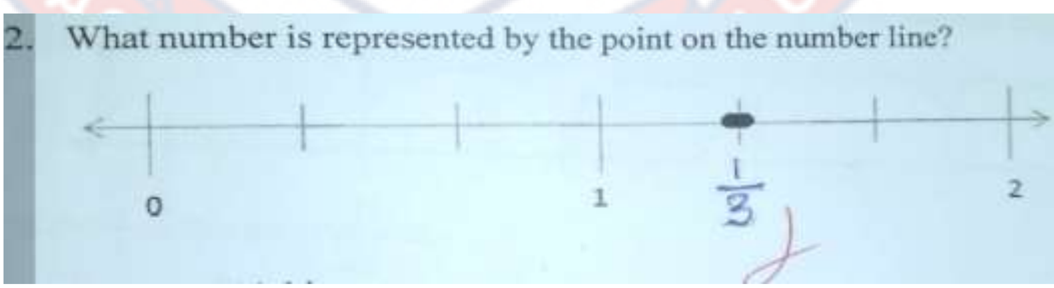

Figure 15: sample of incorrect answers for test item 9 *“Exp.11”*

5. **Poor estimation:**

The result showed that thirty-three learners failed to recognise that $\frac{1}{3}$ cannot be located between one and two, neither can $\frac{2}{3}$ be located between two and three on the number line. The result is shown in Figures 16 and 17.

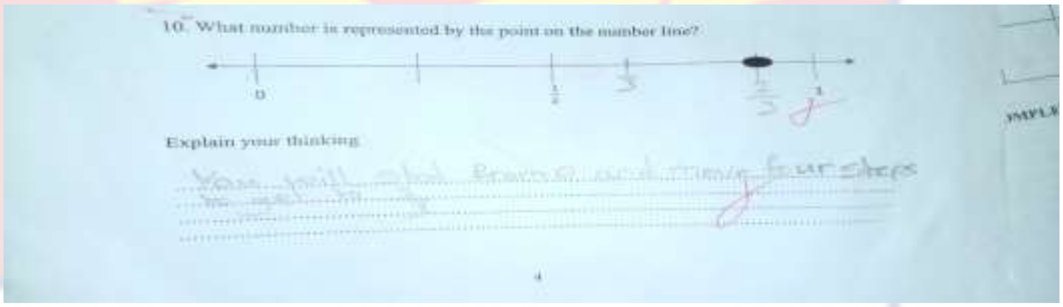

Figure 16: sample of incorrect answers for test item 10 "*Exp.35*"

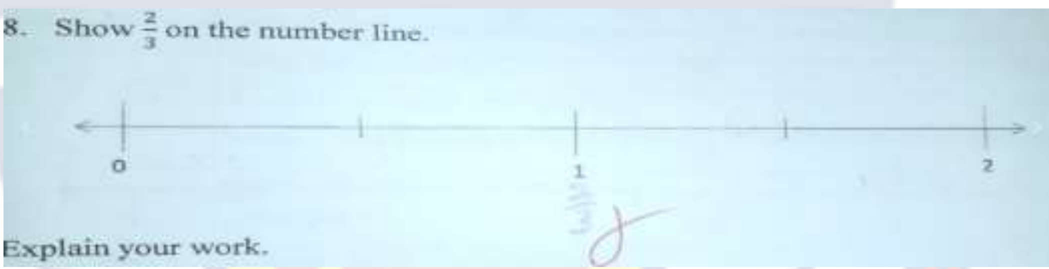

***Figure 17*: incorrect sample response for item 8 "*Exp 41*"**

6. **Unable to recognise that 0 is part of the number system.**

It was observed that 19 learners failed to recognise 0 as a number during estimation, as shown in Figures 18 and 19.

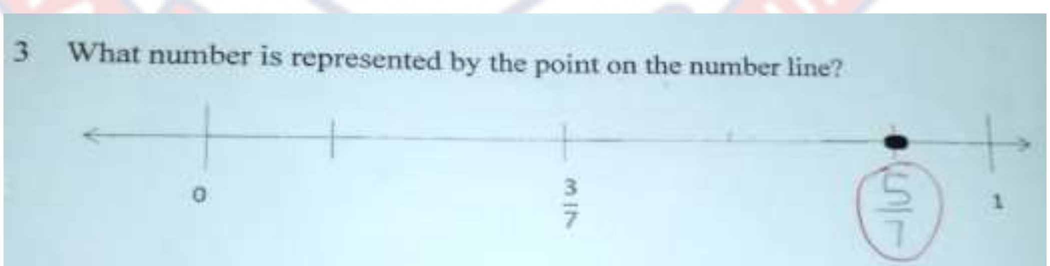

***Figure 18*: incorrect sample response for item 3 "*Exp 41*"**

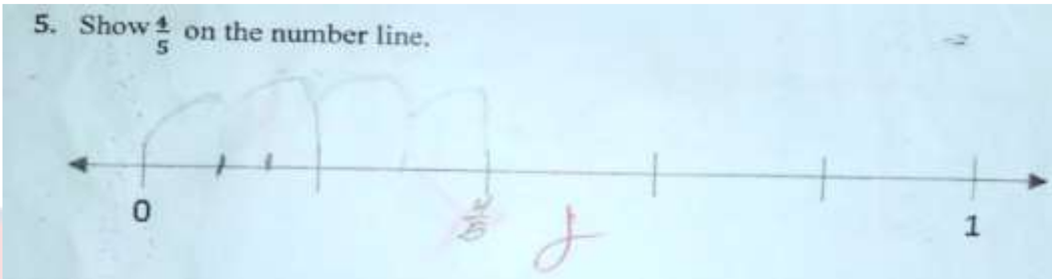

***Figure 19*: incorrect sample response for item 5 "*Exp 11*"**

The literature showed that learners faced challenges when using the number line in learning fractions. This study's findings are consistent with Widodo and Ikhwanudin (2018), who posited that learners have difficulty interpreting tick marks. When a number line contains multiple units, such as 0 to 3, learners find locating it on the number line challenging, as confirmed by Alkhateeb (2019). The challenges learners face align with Aliustaoğlu et al. (2018), who opined that learners encounter difficulties concerning part-whole, determining fraction units incorrectly, and misinterpreting fractions as whole numbers.

This study aligns with Mitchell et al. (2008) after interviewing five basic six learners. Their result was that 24.4% of learners do not consider zero when working with the number line. This misconception in this study could account for what Sarwadi et al. (2014) posited as "inaccurate prior knowledge." While Makhubele (2021) also sees it as inadequate knowledge of the basic concepts.

Chapter Summary

In this study, learning fractions with the number line approach was beneficial because learners' performance, knowledge, understanding, application, and attitude improved after the intervention. Although there were challenges such as poor estimation, counting tick marks, unable to recognise 0 as part of the number system, learners built the following ideas and connections using the number line: (a) how to locate and identify a fraction on the number line, (b) how to express what you have learned, (c) how to draw a number line and where fractions are located.

In summary, the findings imply that the number line strategy significantly aided learners learning of fractions. Attitudes by learners and the challenges faced when learning fractions on the number lines were discussed.

CHAPTER FIVE

SUMMARY, CONCLUSION AND RECOMMENDATIONS

Overview

This chapter provided conclusions based on the research findings on data collected on the influence of the number line approach in learning fractions and a summary and recommendations for future research.

Summary

The study aimed to examine the influence of the number line approach on learning fractions. The study was guided by two research questions and two research hypotheses. A pretest-posttest non-equivalent quasi-experimental design under a quantitative research approach was used. Thus, the two distinct Basic six classes consisting of eighty-one learners were purposively selected within the Cape Coast Metropolis. One school was exposed to the number line intervention, whereas the other received set and area model instruction. Descriptive (means, standard deviations, percentages, and snapshots of learners' tests) and inferential statistics (independent and paired-samples t-test) were used to analyse the research questions and hypotheses.

Findings

The study revealed the following findings:

1. It was revealed that, after the instruction, the experimental group performed higher than the control group due to the intervention. It was observed that learners' performances were significant as measured at the Knowledge, Understanding, and Application.

2. It was revealed that learners' attitudes regarding number lines in learning fractions were positive.
3. Experimental learners encountered challenges such as the following.
   i. Counting the tick marks instead of the intervals between them.
   ii. Difficulty in differentiating fractions from whole numbers.
   iii. Consideration of only the numerator in locating fractions with less regard for the number of equal partitions.
   iv. The misconception of the unit length: thinking of all unit lengths as 0 – 1, thus difficulty naming or locating fractions in unit lengths other than 0 – 1.
   v. Poor estimation: failure to recognise that $\frac{1}{3}$ cannot be located between one and two, neither can $\frac{2}{3}$ be located between two and three on the number line.
   vi. Unable to recognise that 0 is part of the number system

Conclusions

From the findings, it was noted that learners' level of performance was high due to the number line intervention. Most of the learners understood the unit length 0 – 1, but differentiating that from other units was challenging as they found it difficult to name fractions located in other unit lengths aside from 0 – 1. The number line shows an infinite number of unit lengths. Unlike other models, the number line model does not always show a clear unit, and choosing a unit length to work with may sometimes be challenging for learners. Real-life activities during the lesson made learners recognise and connect fraction lessons to everyday life. Learners found the number line lessons enjoyable; their attitude and interest aroused; they became familiar with the instruction, making them less anxious. Learners gained confidence in responding to questions and engaging in conversations.

Though the denominator sometimes tells the number of equal partitions of the unit, it cannot always be the case, as some fractions may be renamed with their equivalents. The fact that you want to model $\frac{4}{8}$ does not necessarily mean the unit should be partitioned into eighths. It can be sixths or quarters or even tenths, yet $\frac{4}{8}$ can be modelled by using the idea of equivalents. An inadequate understanding of this probably made some learners locate fractions using only the numerator.

The number line model can incorporate the idea of fractions into the whole number system, making it a good model for teaching distinction between these classes of numbers and developing the idea of those number concepts simultaneously. However, learners' conception of this distinction was limited as they exhibited difficulties in distinguishing $\frac{2}{3}$ from 2. Partly, poor estimation strategies culminated in this failure, as good estimation techniques could have given them the idea that $\frac{2}{3}$ is not the same as 2.

Recommendations

Based on the significant findings of the study and its conclusions, the following recommendations are given:

1. The number line has proven to be a key model in developing knowledge, understanding, and application when learning fractions. Learners develop a positive attitude toward learning fractions with the number line.

Stakeholders should organise in-service training for mathematics teachers to make use of the number line in addition to other models for teaching fractions.

2. Mathematics teachers should take time to explain the various terms in fractions to learners.
3. The Ghana Education Service-approved textbooks should consider the number line to solve fractions, and other stakeholders, such as mathematics teachers, should also give attention to the tick marks and the spaces between them to help learners relate to the tick marks on the number line.

Suggestions for Further Research

The following recommendations for future research are given based on the findings and conclusions of the study.

1. Only one municipality, two schools, and one intact class from each were included in this study. The study also looked into the location and explanation of learners' thinking regarding fractions; nevertheless, replicating the study in other locations and focusing on fraction operations would add to the study's conclusions.
2. The study used a quantitative approach with eighty-one respondents in one circuit out of the six circuits in the Cape Coast Metropolis. In essence, further studies should employ a mixed method.
3. Finally, further studies should focus on teachers' understanding of using the number line in teaching fractions at the upper primary school level.

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APPENDICES

APPENDIX 'A': ETHICAL CLEARANCE FORM

UNIVERSITY OF CAPE COAST
COLLEGE OF EDUCATION STUDIES
ETHICAL REVIEW BOARD

UNIVERSITY POST OFFICE
CAPE COAST, GHANA

Our Ref: CES-ERB/ucc.edu/vs/21-91

Your Ref: ……………………

Date: 3rd November, 202

Chairman, CES-ERB
Prof. J. A. Omotosho
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0243784739

Vice-Chairman, CES-ERB
Prof. K. Edjah
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Secretary, CES-ERB
Prof. Linda Dzama Forde
lforde@ucc.edu.gh
0244786680

Dear Sir/Madam,

ETHICAL REQUIREMENTS CLEARANCE FOR RESEARCH STUDY

The bearer, Maxwel Sarpong Adu Reg. No. …………………… is an M.Phil. / ~~PhD.~~ student in the Department of Basic Education …………………… in the College of Education Studies, University of Cape Coast, Cape Coast, Ghana. He / She wishes to undertake a research study on the topic:

The influence of number line approach in learning fractions: A case of basic six pupils in Cape Coast Metropolis.

The Ethical Review Board (ERB) of the College of Education Studies (CES) has assessed his/her proposal and confirm that the proposal satisfies the College's ethical requirements for the conduct of the study.

In view of the above, the researcher has been cleared and given approval to commence his/her study. The ERB would be grateful if you would give him/her the necessary assistance to facilitate the conduct of the said research.

Thank you.
Yours faithfully,

Prof. Linda Dzama Forde
(Secretary, CES-ERB)

APPENDIX 'B': INTRODUCTORY LETTER

UNIVERSITY OF CAPE COAST
COLLEGE OF EDUCATION STUDIES
FACULTY OF EDUCATIONAL FOUNDATIONS
DEPARTMENT OF BASIC EDUCATION

Telephone: +233 - (0)3321 33379
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UNIVERSITY POST OFFICE
CAPE COAST, GHANA

Our Ref: DBE/32/V.3

Your Ref:

16th September, 2021

Dear Sir/Madam,

LETTER OF INTRODUCTION

This is to inform you that Maxwell Adu Sarpong **(EF/BEP/19/0014)** is a second year M.Phil student at the Department of Basic Education, University of Cape Coast.

He is undertaking a study on **"The Influence of Number Line Approach in Learning Fractions: A case of Basic Six Pupils in Cape Coast Metropolis".** In connection with this, he needs to collect data.

The study is academic in purpose and data collected will be treated as confidential. We would therefore be grateful if you could give him the necessary assistance.

Thank you.

Yours faithfully,

Dr. Mumuni Thompson
HEAD OF DEPARTMENT

DEPARTMENT OF BASIC EDUCATION
UNIVERSITY OF CAPE COAST
CAPE COAST

APPENDIX 'C'

UNIVERSITY OF CAPE COAST

COLLEGE OF EDUCATION STUDIES

DEPARTMENT OF BASIC EDUCATION

LEARNERS' CONSENT FORM

Title of Project: Influence of number line approach on learning fractions: A case of basic six learners in the Cape Coast Metropolis.

Name of Researcher: Maxwell Adu Sarpong

Dear Learners,

Would you consider taking part in some research?

I am planning to do some research to investigate how using the number line approach can improve Basic six learners learning of fractions. I would like you to take some time to read the following information to understand why the research is being done and decide if you would take part or not.

What is the Purpose of the Study?

This research aims to examine the influence of the number line approach on the learning of fractions. The study also wishes to examine the influence of number lines on learners' conceptual and procedural knowledge toward fraction learning.

Who is asked to take part, and why have I been chosen?

All Basic six learners in the Cape Coast North are invited to participate in the study. You have been chosen to participate because of your school; likewise, your class has been selected for the study.

What will happen if I take part?

If you choose to be included, you will be asked to participate in the lesson on fractions using the number line approach, complete some questionnaires and interviews and take part in a test if you agree.

What is the benefit of taking part?

After completing the study, I will provide you with a summary of the results if you wish to. The information you provide will also help educational authorities improve the quality of teaching and learning fractions in your school through the number line approach.

Confidentiality- what will happen to the information I provide?

When I write up the study, students' names and schools will be changed so that no one can be identified. Also, any information that may identify the schools or the students will be removed from the final write-up. If you have any questions, do not hesitate to ask. If you would like to participate, please complete the next section.

Learners' Consent

	………………	15-11-2021
(Name of Participant)	(Signature)	(Date)
EDNA SOFU	………………	15-11-2021
(Name of Witness)	(Signature)	(Date)
Adu Sarpong Maxwell	………………	15-11-2021
(Name of Researcher)	(Signature)	(Date)

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APPENDIX 'D'

EXPERIMENTAL SCHOOL LESSON PLAN

Week ending		Class size:
Subject: Mathematics	Date:	
Class: Basic Six	Day	
Topic: Fractions	Time:	
Ref: Mathematics curriculum (NaCCA, 2019)		

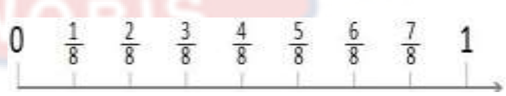
2. Instructional Objectives
By the end of the lesson, the student will be able to: **a.** explain fraction **b.** show the difference between a numerator and a denominator (e.g., $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$); **c.** generate unit fractions and locate a unit fraction, e.g., one-eighth, on a number line by defining the interval from 0 to 1 as the whole and partitioning it into 8 equal parts and that each part has size $\frac{1}{8}$ **d.** Estimate location and name points on the number line; **e.** Locate, compare and order sets of integers using the number line **f.** Identify simple unit fractions such as 'a halves' or 'a quarter.'
Learners Expectation: pupils will use the number line to examine relationships between groups of numbers (e.g., greater/less than, relative distances from one another, etc.)
3. **Content** a. Meaning of fraction b. Classify this into numerators and denominators, e.g., $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$) c. Meaning of the number line d. Locate, compare and order sets of integers using the number line and symbols e. Estimate location and name point on the number line
4. Learning Environment

a. Pupils will be seated in small groups and collaboratively work together to complete assigned tasks. Pupils will be made up of 10 groups with 4 to 5 members in each group.

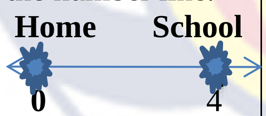
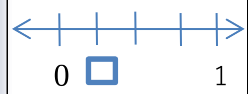
b. Pupils will be in their seats to take an independent test

5. GENERAL OBJECTIVES

This lesson will allow pupils to explore the relative positions of numbers. Pupils will be encouraged to think about the distances separating numbers and how the number line can help determine their differences. As pupils work through this lesson, they will have opportunities, though perhaps implicit, to think about number combinations and locate numbers in distances. This lesson will be done with rulers and with pencil and paper.

DAYS	**PHASE 1: STARTER** 1**0** MINS (Preparing the Brain for Learning)	**PHASE 2: MAIN** 30MINS (New Learning Including Assessment)	**PHASE 3: REFLECTION** 10MINS (Learner and Teacher)
Week one Lesson 1	Engage learners to sing fraction songs to get them ready for the class Put learners groups. Call out a number between 1 and 6. E.g., 3. Demonstrate how to understand fraction languages eg., $\frac{3}{7}$ (three-seventh)	Pupils sit attentively and observe the teacher Define what fractions is with examples, and exposed them to part-whole, set, and ratio. Introduce how to locate one eight by defining the interval from 0 to 1 as the whole and partitioning it into 8 equal parts, as shown below: 	**Try** List five examples of fractions. Express fraction in the form of ratio and part-whole Assist learners to practice with more examples In groups, let pupils summarise what they learned.

Lesson 2	Review what was learned in the previous Guide pupils to differentiate between numerator and denominators Suppose a number has to be divided into four parts, then it is represented as $\frac{1}{4}$. So, the fraction here, $\frac{1}{4}$, defines $\frac{1}{4}$th of the number 1. Hence, $\frac{1}{4}$ is the fraction here.	**PHASE 2: MAIN** 30MINS (New Learning Including Assessment) Ask pupils to explain how to share 7¢ among 3 friends in the form of $\frac{3}{7}$ on the number line Show which number is the whole $\frac{2}{5}, \frac{5}{7}$ and which is the dividends	Review lessons with learners by giving them a task to solve in their workbooks. Review the lesson with learners
Lesson 3	**PHASE 1: STARTER** 1**0** MINS (Preparing the Brain for Learning). Monitor and guide pupil on how to draw a straight line using a pencil Ask pupils to estimate $\frac{3}{5}, \frac{4}{9}$ using a ruler and a pencil on the number line	**PHASE 2: MAIN** 30MINS (New Learning Including Assessment) Ask pupils to draw several lines, 10 units each, and mark the ends 0 and 1. For each line, ask them to partition the interval from 0 to 1 into each of the following unit fractions $\frac{1}{2}, \frac{1}{3}; \frac{1}{5}; \frac{1}{6}$ and $\frac{1}{10}$ Ask pupils to explain how to locate one-third, one-sixth, and two-fourth on the number line	Review lesson with learners by giving them a task to solve in their workbooks.

Week 2 Lesson 1	**PHASE 1: STARTER** 10 MINS	**PHASE 2: MAIN** 30MINS (New Learning Including Assessment)	
	Use the everyday activity to demonstrate and present fractions on the number line: **E.g.** Ama's distance from her house to school is 4 miles. She walked toward the school for about $\frac{3}{4}$ of a mile before her friend (Joyce) father picked her up in his car to school. Show where $\frac{3}{4}$ of a mile would be on the number line. 	Put pupils in groups of 5 each to brainstorm and explain their understanding Assist pupils to estimate location and name points from their class to the school canteen on the number line. E.g., Mark and label $\frac{1}{3}$ and $\frac{1}{5}$ on the number line. Guide pupils to understand points on the number line and; Explain how to move from 0 to 1 and within intervals	Ask pupils to Locate $\frac{1}{4}$ on the number line 

Lesson 2	You walked from your house toward the market. This is 2 miles. You walked about $\frac{1}{4}$ a mile when you met your friend. Show on the number line exactly where you met your friend. **House Market**  0 1 2mls	Guide pupils on how to position fractions on the number line.	Guide and revise the previous lesson with pupils
35 mins	**CONCLUSION** Direct pupils to sit in their position	Summarise the previous lesson. Arrange pupils to write FAT independently and hand over test sheet for scoring.	30 minutes Answer the questions by providing the needed information
REMARKS			

APPENDIX 'E'

EXPERIMENTAL LEARNERS' ATTITUDE TOWARD LEARNING FRACTION WITH THE NUMBER LINE

Complete the statements below on how you think your attitudes towards learning fraction concepts through the number line instruction will be. Please tick [✓] in ***only one*** of the appropriate columns for your response to the following statements.

N$^{\underline{O}}$	**Statement**	**SA**	**Agree**	**Disagree**	**SD**
1	I can locate fraction problems faster when I use the number line				
2	For some reason, the use of number lines is really hard for me				
3	It was easier to learn and understand fraction concepts using the number line				
4	I can now understand how to locate $\frac{1}{2}$ with the number line				
5	I now see number line as part of a fraction				
6	I have much self-confidence when using the number line in solving fractions				
7	I can now apply and explain my thinking on how to locate fractions using the number line				

8	Once I start working with the number line, it is hard to stop.				
9	The individual tests helped me to check my understanding in the lessons				
10	I have a few problems on how to explain how to locate $\frac{1}{2}$ using the number line				
11	The examples and exercises given in the lessons enhanced my understanding of fraction concepts				
12	The activities in the number line helped me to follow the lessons and think critically in understanding the concept				
13	From the teacher's examples during the lesson, I can relate the concepts learned to real-life situations.				
14	The use of the number line helped to identify patterns, compare and relate ideas to make generalisations				
15	I prefer that my teacher use this approach to teach us different concepts of fractions.				

APPENDIX 'F'

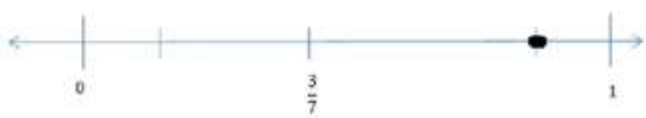
FRACTIONS TASKS

ATTEMPT ALL QUESTIONS

1. Identify the fractions indicated by P, Q, and R.

Explain your thinking.

2. What number is represented by the point on the number line?

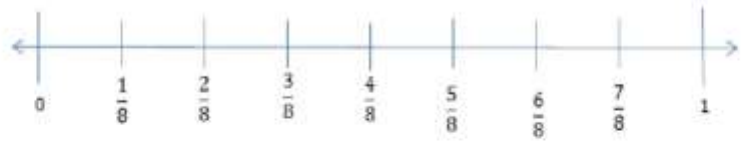


3. What number is represented by the point on the number line?

Explain your thinking.

1

4. Mark the number $\frac{3}{4}$ on the number line.



Explain your thinking.

5. Show $\frac{4}{5}$ on the number line.

Explain your thinking.

6. Show $\frac{4}{8}$ on the number line

Explain your thinking.

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....

7. Show $\frac{2}{3}$ on the number line.

Explain your thinking.

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8. Draw a number line and show the following fractions

a) $\frac{2}{3}$

Explain how you constructed the number line and how you identified the position for the fractions

....
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....

9. Look at this number line. Locate $\frac{2}{3}$ on the number line.

Explain your thinking.

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10. What number is represented by the point on the number line?

Explain your thinking

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....

APPENDIX 'G'

SCORING RUBRICS FOR FRACTION ASSESSMENT TASK

Ans.1: The unit was partitioned into eighths. To locate *P*, you count two steps from 0 to reach $\frac{2}{8}$, five steps to reach $Q\ \frac{5}{8}$ and seven steps to reach $R\ \frac{7}{8}$.

Ans 2: Each unit was partitioned into thirds, and the indicated point happens to be the fourth tick mark from zero; the point was represented as $\frac{4}{3}$ or $1\frac{1}{3}$.

Ans 3: The unit was first partitioned into sevenths. The indicated point was noted as $\frac{6}{7}$.

Ans 4: By dividing the numerator and denominator into two each, $\frac{6}{8}$ is equivalent to $\frac{3}{4}$. Hence $\frac{6}{8}$ was marked as $\frac{3}{4}$.

Ans 5: The unit length was partitioned into fifths. $\frac{4}{5}$ corresponded to the fourth tick mark.

Ans 6: The unit length was partitioned into sixths. To locate $\frac{4}{8}$, the fourth tick mark was considered because it corresponded to the numerator value.

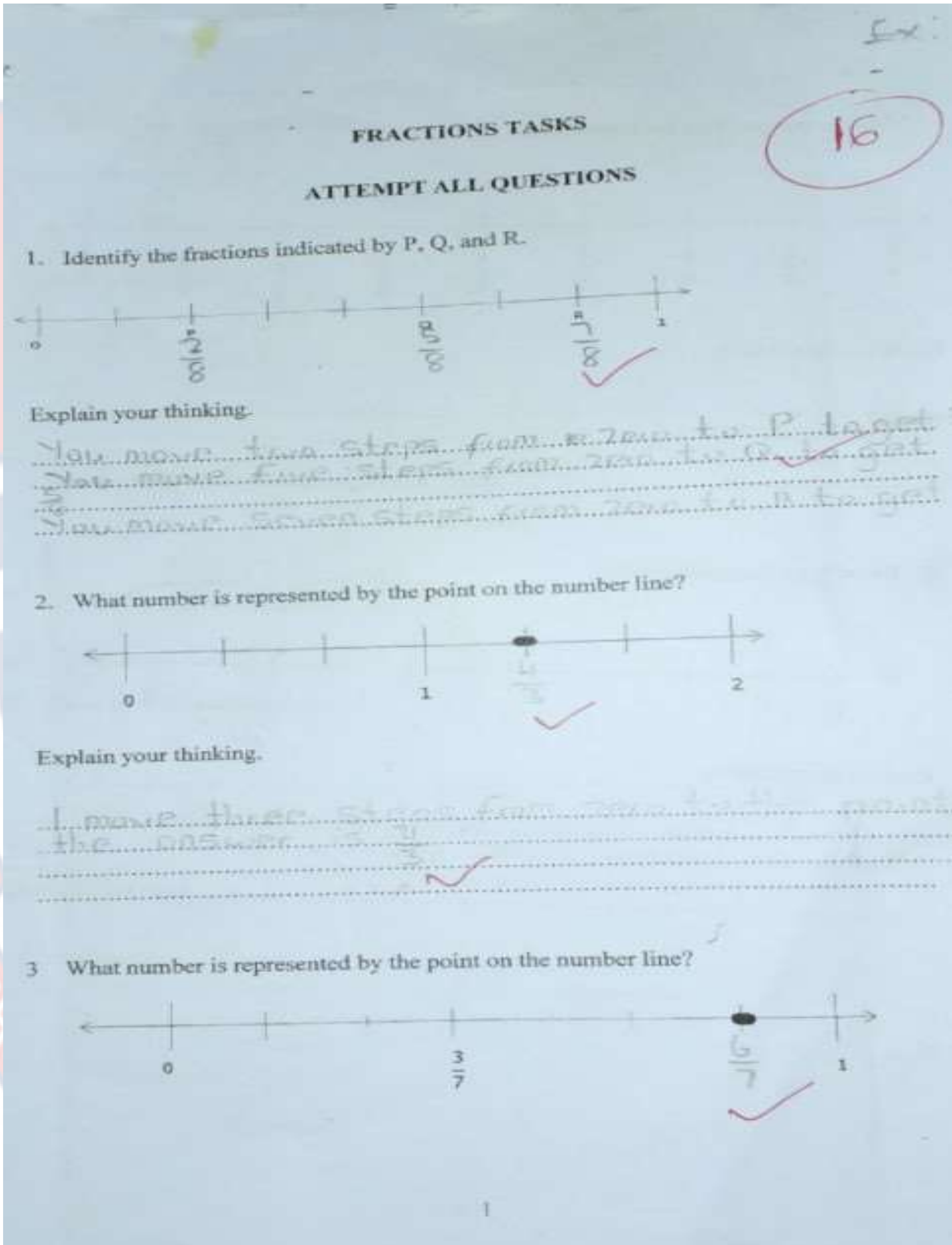
Ans 7: The unit length (0 – 1) was repartitioned into thirds before locating two-thirds.

Ans 8: A number line was drawn to show a unit partitioned into thirds. Two-thirds were subsequently marked as the second tick mark.

Ans 9: The unit length (0 – 1) was repartitioned into thirds before locating two-thirds.

Ans 10: The unit length was partitioned into fourths. The indicated point was the third tick mark hence named $\frac{3}{4}$.

APPENDIX 'H': SNAPSHOT OF "EXP.31" FAT RESULTS



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FRACTIONS TASKS

ATTEMPT ALL QUESTIONS

1. Identify the fractions indicated by P, Q, and R.

Explain your thinking.

You move two steps from zero to P to get

You move five steps from zero to Q to get

You move seven steps from zero to R to get

2. What number is represented by the point on the number line?

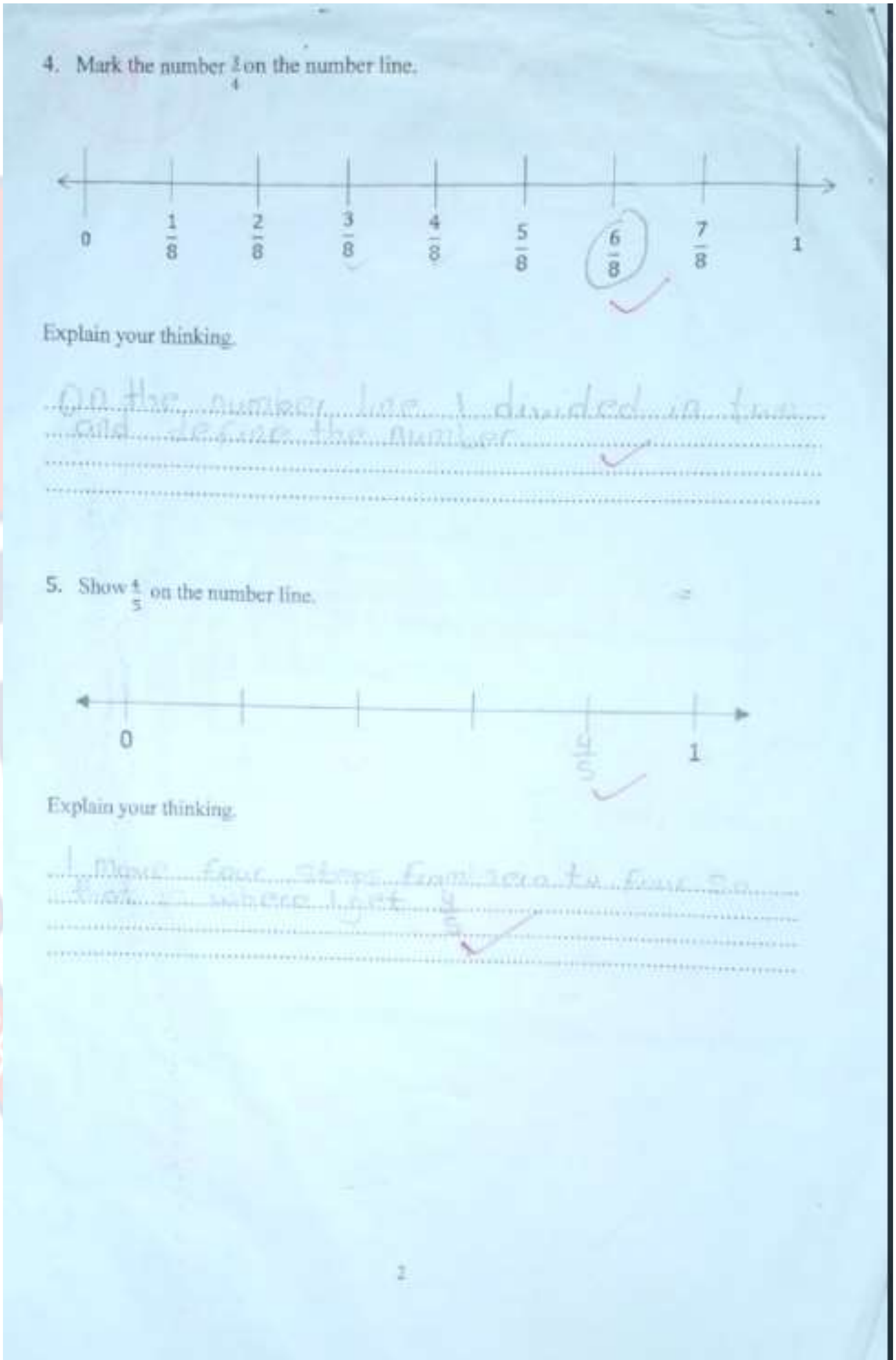
Explain your thinking.

I move three steps from zero to the point

the answer is 4/3

3 What number is represented by the point on the number line?

1



4. Mark the number $\frac{3}{4}$ on the number line.

$0 \quad \frac{1}{8} \quad \frac{2}{8} \quad \frac{3}{8} \quad \frac{4}{8} \quad \frac{5}{8} \quad \frac{6}{8} \quad \frac{7}{8} \quad 1$

Explain your thinking.

On the number line I divided in two and desire the number

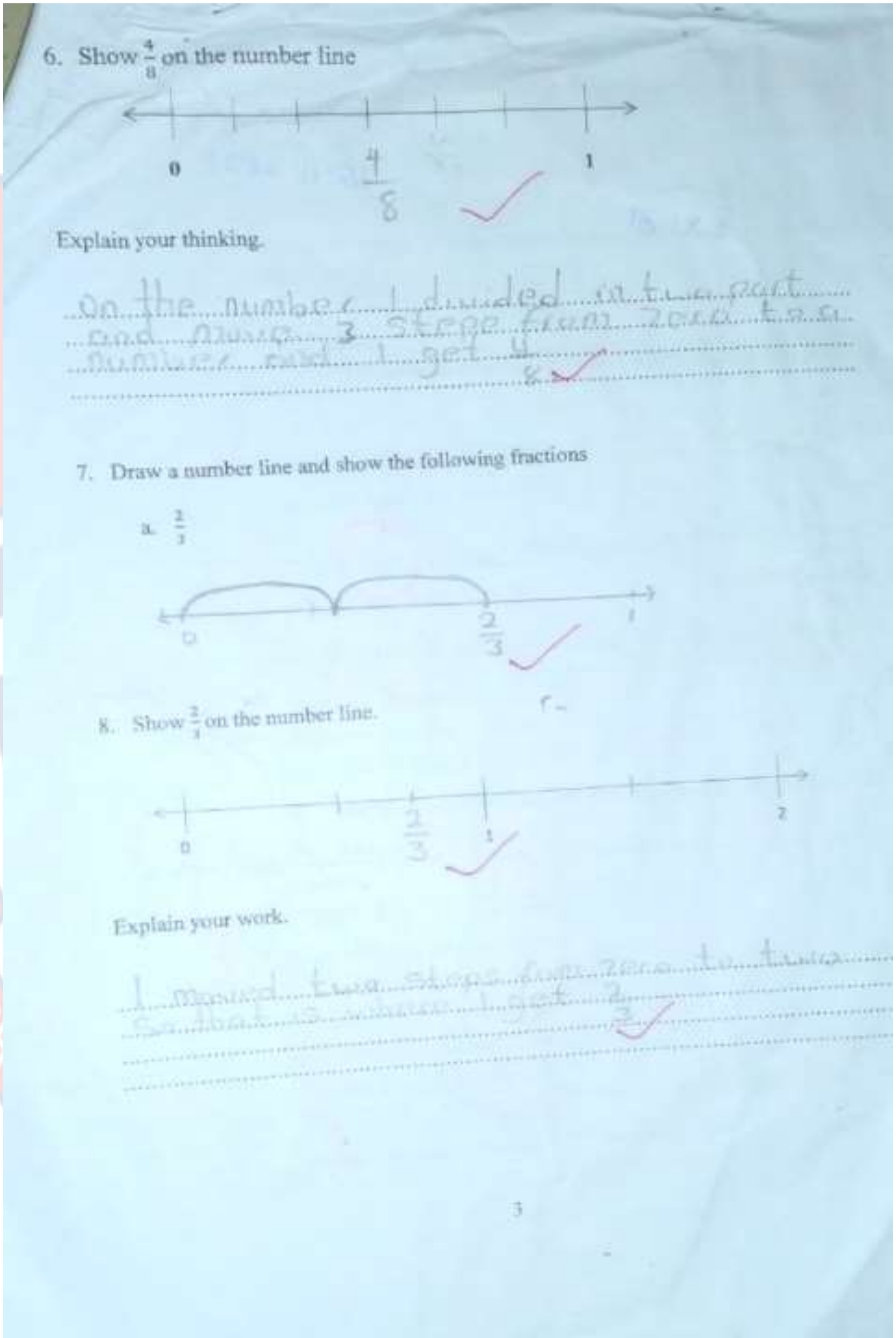
5. Show $\frac{4}{5}$ on the number line.

0 $\frac{4}{5}$ 1

Explain your thinking.

I move four steps from zero to four so that is where I get $\frac{4}{5}$

2



6. Show $\frac{4}{8}$ on the number line

0 $\frac{4}{8}$ 1

Explain your thinking.

On the number l divided in two part and move 3 steps from zero to a number and I get $\frac{4}{8}$

7. Draw a number line and show the following fractions

a. $\frac{2}{3}$

0 $\frac{2}{3}$ 1

8. Show $\frac{2}{3}$ on the number line.

0 $\frac{2}{3}$ 1 2

Explain your work.

I moved two steps from zero to two so that is where I get $\frac{2}{3}$

3

Explain how you constructed the number line and how you identified the position for the fractions

....

9. Look at this number line. Locate $\frac{2}{3}$ on the number line.

Explain your work.

You moved two steps from zero to two. that is where I get $\frac{2}{3}$

10. What number is represented by the point on the number line?

Explain your thinking

On the number line I divided in two and define the number and move four steps from zero to four number and I got $\frac{2}{3}$

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