UNIVERSITY OF CAPE COAST

VALUING IN MATHEMATICS TEACHING AND LEARNING AMONGST TEACHERS AND STUDENTS: WHAT DOES IT LOOK LIKE ACROSS GRADE LEVELS?

BRIGHT DAPAAH ADDAE

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BY

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Thesis submitted to the Department of Mathematics and ICT Education of the Faculty of Science and Technology Education, College of Education Studies, University of Cape Coast, in partial fulfilment of the requirements for the award of Doctor of Philosophy degree in Mathematics Education

APRIL 2024

DECLARATION

Candidate's Declaration

I hereby declare that this thesis is the result of my own original research and that no part of it has been presented for another degree in this university or elsewhere.

Candidate's Signature: .... Date:..........................

Name: Bright Dapaah Addae

Supervisors' Declaration

We hereby declare that the preparation and presentation of the thesis were supervised in accordance with the guidelines on supervision of thesis laid down by the University of Cape Coast.

Principal Supervisor's Signature:.................................. Date: ....

Name: Prof. Ernest Kofi Davis

Co-Supervisor's Signature: ………………………… Date: ………………

Name: Prof. Douglas Darko Agyei

ABSTRACT

This study was set out to investigate what teachers and students value in mathematics learning at the pre-tertiary level of education in a metropolis in the southern part of Ghana. The study employed positivist research paradigm. A descriptive cross sectional survey research design was used in which a quantitative data was collected using "What I Find Important (in my mathematics learning)" [WIFI] questionnaire for students and a modified version of the students' questionnaire for teachers. The study adopted a multistage sampling procedure to select participants. First, a stratified random sampling technique was used to select Primary schools and JHS based on urban and rural locations of schools and SHS according to school type (co-educational, single sex female and single sex male schools). Second, simple random sampling (SRS) technique with proportional allocation of samples was used to select Primary, JHS and SHS students from each stratum to take part in the study. Also, all the teachers who teach mathematics in the sampled schools were purposefully selected as participants. A total of 1263 students and 177 teachers from 34 public pre-tertiary schools in the metropolis were selected to participate in the study. Principal Component Analysis (PCA), specifically Exploratory Factor Analysis (EFA) and one-way Multivariate Analysis of Variance (MANOVA) were used to explore what the teachers and their students value in mathematics learning. The study revealed that the teachers valued the attributes: *Understanding*, *Versatility* and *Achievement* in students' mathematics learning. The students on the other hand, valued the attributes: *Fluency*, *Understanding*, *Instructional Materials/Activities*, *Connections*, *ICT*, *Feedback and Learning Strategies* in their mathematics learning. The study pointed out that teachers who hold Master degree (Math) with no teacher training valued *Versatility* most. However, teachers who hold Bachelor degree (Math) with teacher training valued *Understanding* the least. In this regard, the study recommends that Colleges of Education and Universities that train mathematics teachers should reform their curricula in mathematics education to include education on values to enable mathematics teacher trainees to be conscious of their valuing stances and how it impacts on their classroom instructions. Also, Curriculum developers at the pre tertiary level of education in Ghana should ensure that the values adopted by the pre tertiary mathematics teachers and their students are in line with those of the mathematics curriculum.

KEY WORDS

Pre-tertiary mathematics teachers

Pre-tertiary students

Mathematics

Mathematics educational values

Mathematical values

Values

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DEDICATION

To my beloved wife and children

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

Figure **Page**

CHAPTER ONE

INTRODUCTION

Teachers have been trained to give the needed direction in the classroom in the realisation of the objectives that have been spelt out in the curriculum. The mathematics curriculum has been designed to promote relational understanding in learners (Skemp 1979). Learners who value relational understanding of mathematics topics as important ‘recognise the relationships between concepts in mathematics’, ‘relate mathematics to other school subjects’ and can ‘connect mathematics to everyday life’ (Singapore Ministry of Education, 2019).

Values are critical components of mathematics learning (Seah, 2019). Tang, Seah, Zhang and Zhang (2021) opined that values are great motivators that influence how we think, feel and study or impart mathematics to others. Unfortunately, Clarkson and Bishop (1999) found that mathematics educators and their learners do not understand the role of values. Recent literature by Seah in 2018 continued to point out the contribution of values in providing support for affective state and cognitive development of students in mathematics though significant, still remain unnoticed. Mathematical connections are developed, cultural identities of students are validated, active participation, significant improvement in mathematics learning (Hill, Hunter & Hunter, 2019) when values are recognised. Thus, value research continues to be of high priority.

Background to the Study

Among the many subjects taught in school, mathematics remains unique. In school, mathematics occupies an enviable position as a "mother subject" (Kafata & Mbetwa, 2016). This is because the success of students in most school subjects and in most cases depends to a very large extent on their success in mathematics. For mathematics related school subjects, transfer of sound mathematical knowledge is implicitly or explicitly a necessity. Also, the usefulness of mathematics is still evident in other non-mathematics school subjects such Languages, History, etc. because of the logic that embodies mathematical ideas.

Mathematical knowledge permeates all cultures and societies making it a prerequisite for proper functioning of a person in every society (Martin, 1997). Bishop (1988) reported that all people across the world in the past, now and in the future go through the 'six universal mathematical activities' that are fundamentally mathematical: measuring, designing, counting, playing, locating and explaining. Every cultural group does them knowingly or unknowingly. Mathematics in needed in all spheres of human endeavour. This has made what teachers and students find important in mathematics learning a growing issue of interest to education stakeholders all over the world. Values play a significant role in creating a sense of self as well as social uniqueness for learners (Le Métais, 1997) and pedagogical identity for teachers (Chin, Leu & Lin, 2001). Values represent affective variable such as beliefs, attitudes and motivation (Bishop, 2008). Values impact on the cognitive makeup of students and teachers. What is this cognitive supporting, conative, affective and motivational construct called "values"?. Several definitions have been adduced in literature for values in mathematics education (Bishop, 2016; Bishop, FitzSimons, Seah, & Clarkson, 1999; Seah & Wong, 2012). Seah (2018) definition of values/valuing in mathematics learning as individual's convictions, which are considered to be of importance and worthwhile will be the focus of this study.

Values and valuing have been categorised into three: mathematical values (Bishop, 1988), mathematics education values, and general educational values (Bishop, 1996). Mathematical values are beliefs in mathematics that are taught in contemporary schooling. According to Bishop (2008), mathematical values are those that Western Mathematics has adopted as its knowledge and ideas have developed within 'Westernised cultures'. They comprise values showing mathematical knowledge, its scientific nature, produced by the influences of mathematicians from various traditional backgrounds (Bishop, FitzSimons, Seah, & Clarkson, 1999). Also, mathematics educational values are the instructional strategies of school mathematics which teachers take learners through. These values include but are unlimited to ICT, understanding, group work, versatility, and effort. In the perspective of Bishop (2008), mathematics educational values are those found in the syllabus, textbooks and instructors' professional practices. Again, general educational values are values the system of education expect to instil in learners under the auspices of any of the subjects in school. Examples include but not limited to honesty, punctuality, and creativity. General educational values do not have direct impact on students' mathematics achievement (Barkatsas, Law, Seah & Wong, 2012) and therefore, was not the focus of this study. However, the interest of this research work was on mathematics educational values and mathematical values (Bishop, et al.). According to Seah, Baba and Zhang (2017), these two value concepts perform a critical function in the quality of learning experiences of learners and outcomes in mathematics in general.

During mathematics classroom discourse, there are values embedded in the discussion of meanings between the learners and the teacher. The communication among teachers and learners provides an avenue for differences in what each of them values. Most often than not, these differences result in disagreement which can potentially lead to conflicts until a consensus is reached. Thus, purposeful learning takes place when teachers and their students in the mathematics classrooms agree on some fundamental values in mathematics and mathematics education (Anderson & Österling, 2019). Unfortunately, in most cases in the mathematics classroom, mismatch occurs between what school instructors and learners find important in the learning of mathematics (Pampaka & William, 2016). For instance, whilst the students want instrumental understanding, the teacher teaches relationally. Here, the students are only interested in some kind of rules for getting the right answers and once these rules are obtained, they keep them and forget about all the details that led to the generation of these rules. Other mismatch occurs between the learner and school instructors. Here, the learners want to understand relationally but the teacher teaches instrumentally. The school instructor's teaching approach makes the students' relational understanding impossible. Thus, such students often make attempts to understand the teacher's rules (instrumental activities) and it appears such students may not be able to easily recall them for use.

Students whose values in mathematics learning are compatible with what teachers value in mathematics learning tend to apply what they have learnt more effectively, retain information longer. These learners continue to experience positive disposition after going through academic course compared to their contemporaries who experience values mismatch (Felder, 1993). Purposeful mathematics learning takes place when teachers and students have in common some fundamental values of mathematics and mathematics education (Andersson & Österling, 2019). According to Borg and Stranahan (2002), if what mathematics teachers preferred in mathematics learning matches with that of students', learners' academic achievements increase considerably. They further argued that the more mathematics teachers' value in mathematics learning align with that of learners' value in mathematics learning, students' academic achievements also go up.

Studies by Wei and Eisenhart (2011) and Byun and Park (2012) have reported in wonder disparities in achievement in mathematics among East Asian learners and their non-East Asian ethnic group counterparts always go in favour of the Eastern Asians. They further reported that these East Asian learners went to the same educational institution as their peers, were taught by the same teachers, went through the same centralized mathematics curriculum with the same or similar teaching activities during lesson presentations, they go through the same mathematics assignments and home works as well as mathematics exercises, problems and investigations, they would have encountered similar classroom learning conditions and environment. Similar observations have been reported in the United States in mathematics achievement of migrant school children (Lee & Zhou, 2015). Questions will be asked with regard to why academic achievement gap still exist among school children who share similar characteristics in different facets of life. Research suggests that East Asian female parents trust that hardwork is an essential requirement for academic attainment and American mothers also believe that children's mathematics achievement is an innate construct and that a child needs to have a mathematics gene to perform well in mathematics (Stevenson, 1992; Tsao, 2004). Wei and Eisenhart have argued that unique values in people's culture trigger variations in mathematics instructional and learning strategies by mathematics instructors and learners respectively. Aside achievement differences reported among students who go through common learning experiences from different countries, ethnicity and sociocultural backgrounds, gender difference in mathematics learning among learners is popular in research (Leder, Forgasz, & Solar, 1996). Differences in mathematics achievement between female and male learners in PISA, TIMSS, BECE and WASSCE have inspired gender research in mathematics education in recent times (Else-Quest, Hyde & Linn, 2010; Hanna, 2000). Variables such as culture, biology and society have featured prominently in studies in mathematics learning involving learners of different sex. Gender differences in mathematics achievement and learning though continue to exist, the extent of the difference keeps on narrowing (Leder, 1992). For example, no sex difference in achievement in mathematics was observed in the meta–analysis of the 2003 TIMSS data (Else-Quest, Hyde & Linn, 2010). This study finds out where the narrowing occurs as the researcher explores what school instructors and learners value in the study of mathematics across grade levels. As gender roles and stereotyping intensifies among students and their ramification on values and valuing as they move from one grade level to the other, could the same be said of what mathematics teachers' value in mathematics learning as learners advance in grade levels? Values and valuing are motivational concepts that inform choices and engagements in the mathematics classroom. They explain why male and female students in the same classroom behave differently when learning mathematics (Barkatsas, Law, Seah & Wong, 2019). This study has enabled the researcher to use values and valuing to justify why male and female students and teachers value the things they value in mathematics learning which a grey area in research in mathematics education is.

Students value mathematics when they know that what they are learning meets their individual needs, being able to think critically for themselves and will help them in their future aspirations. Davis, Seah, Howard and Wilmot's (2021) study on the attributes of mathematics learning which Ghanaian SHS learners' find important, argued strongly that valuing represents an essential feature of teachers' mathematics pedagogy and therefore outcomes of students learning. Recently, there has been an increase in research studies on values in mathematics and mathematics education. In the year 2008, Bishop made a proposal that to investigate reasons underpinning the things teachers do in the classroom, it is germane to find out what they see as important as teachers. Prior to this proposal, Bishop (1988) had suggested that culture and society have a major impact on individual school instructor's actions and choices in the classroom during instructions. Again, what mathematics instructors value is influenced by the culture of the school, curriculum expectations, the instructor's knowledge of mathematics and for teaching mathematics including their attitudes and beliefs (Bishop & Whitfield, 1972; Jacobs, Lamb, & Philipp, 2010). It can be inferred from the above statements that there seems to be a host of culturally and societally compelled values reinforced by institutional environment, institutions including the school and the society. Instructors of mathematics show what they value in mathematics, its instruction and learning through method selection and sequencing for teaching mathematics concepts (Seah, 2002) and achievement of learners. For instance, if what a mathematics teacher values in mathematics instruction is for learners to get higher marks in mathematics then he/she will teach for them to memorise mathematical formulae and ideas to get instrumental understanding of mathematical concepts (Skemp, 1976). Again, if what a student values in mathematics learning is to get a higher score in mathematics, such a student will memorize mathematical formulae and facts just to achieve his/her aim. According to Felder (1993), no two students learn and assess their knowledge, skills and understanding the same way. He acknowledges that not only heredity factors but the environment the learner lives contribute significantly to the way the student learns.

Teachers' academic qualification and teaching experience are two important variables that promote higher academic achievement and academic work of students (Bonney, Amoah, Micah, Ahiamenyo, & Lemaire, 2015). This is due to the fact that these variables are critical in the teaching and learning process.

Research shows that people develop their values generally and values in mathematics learning specifically through the educational system they go through (Bishop, 2016). For Ghanaian teachers and students, the pre-tertiary educational system they have passed through has shaped their values and valuing in mathematics learning. Ghana's pre-tertiary educational system last for 14 years. This system of education is made up of two years of pre-school referred to us Kindergarten education, six years of primary school education, three years of JHS education and three years of SHS education. As a result of parents going to work to overcome the increasing socio-economic challenges of the present life, children are made to begin Nursery School at the early stage of their lives. The youngsters at this level are taken through education to help form their personal identities, learning to take care of their bodies and learning how to study. It is aimed at preparing them adequately for pre-school education. All Nursery schools in Ghana are privately owned. The youngsters are expected to begin pre-tertiary education at age four for kindergarten education popularly referred to as pre-school. The youngsters are expected to begin primary school at age six after having spent two years at the pre-school level. The primary school education lasts for six years. Children start the first phase of high school education which is the Junior High School (JHS) at the age of twelve. They are slated to go through education at that level for three years. Learners at the JHS level are aged between 12 and 15 years inclusive. Learners at the JHS level are supposed to write the Basic Education Certificate Examination (BECE), a national examination as a requirement to enter SHS. The last stage of the pre-tertiary education is the SHS education. Students complete this level of education by going through, WASSCE, which is the requirement to enter a tertiary institution in Ghana. All things being equal, learners at the SHS level are anticipated to complete at the age of 18 years.

In 1995, Ghana government began the execution of Free Compulsory Universal Basic Education (FCUBE). The programme was aimed at increasing learners' access to basic education. The policy was characterised by the implementation of social intervention programmes such as school feeding, capitation grant, provision of school uniforms and exercise books. Ghana's pre-tertiary education system is both compulsory and free.

Ghana, in 2017, began the execution of Free Senior High School (FSHS) education programe to increase access to senior high school education. The FSHS Education policy meaningfully adds to Ghana's FCUBE programme and has ensured access to senior high school education for all Ghanaian children. The FSHS education policy has resulted in all-time record of SHS student enrolment in Ghana's history. This policy is aligned with achievement of Sustainable Development Goal 4 (SDG 4) which addresses in part access to quality education for all by the year 2030. The implementation of FSHS education policy did not come without challenges. Its implementation met infrastructural deficit across all SHSs. In order to overcome this obstacle to the policy, the government of Ghana introduced a double track system. Double track is a system of education where the total number of staff and student population is divided into two to go through academic work in an alternation form (Mensah, 2019). The double track system has enabled the SHSs to double their students' intake whilst maintaining the existing school infrastructure. The schools run a green and gold tracks shift system in a non-overlapping manner.

Mathematics is mandatory at pre-school, primary and high school stages in Ghana. Mathematics is used as one of the critical filters for learners to gain admission to SHS and tertiary institution in Ghana (Addae & Agyei, 2018). In Ghana, mathematics curriculum and textbooks are written in English, the country's official language. At the pre-school and lower primary school (primary 1-3) levels, learners are expected to be taught mathematics and the other school subjects in their local language. According to the World Bank (2005), using learners' indigenous language as the language for teaching and learning (LFTL) ensures authentic education, increased access to education, reduced possibility of learners' academic retrogression as well as improved learning outcomes. From the upper primary up to the SHS, all mathematics instructions are done in English Language. At the SHS level, learners are obliged to study additional mathematics course (Elective mathematics) depending upon the programme they are studying. Academic programmes of study at the SHS level range from technical, general science, agricultural science, general arts to business. Students are placed into these programmes based on interest and performance on BECE.

The mathematics syllabus at the pre-tertiary level stresses the use of Information Communication Technology (ICT) in mathematics teaching and learning process (Agyei, 2012). Notwithstanding the importance of ICT integration in mathematics education at the pre-tertiary level in Ghana, its usage is limited. Calculators provide opportunity to integrate ICT in the teaching and learning of mathematics at a cheaper cost (Clark, 2011). Using calculator for classroom activities in mathematics is cheaper than a desktop computer or a laptop. Calculator usage is not encouraged at primary and the JHS though the mathematics syllabus of the latter advocates for its usage as a learning tool. Perhaps, not only do teachers want the students to pass well in their internal and external examinations where calculator usage is not allowed but acquire basic mathematics computational skills necessary for a successful SHS education. Learners at that age needed to develop an understanding of mathematical concepts rather exposing them to calculators with its attendance over-reliance on them. SHS students are permitted to use calculators in the classroom, and during internal and external examinations. It is believed that mathematics computations at the SHS level are complex and that learners are expected to use calculators to solve them.

Statement of the Problem

Learners' poor achievement in mathematics across the various levels of education has been recognised across the world (Chand, Chaudhary, Prasad & Chand, 2021) and Ghana is not an exception (TIMSS, 2003, 2007, 2011 & WAEC, 2014, 2015, 2016). There have been series of efforts to address the challenges learners face with mathematics learning. For instance, in 2014, the World Bank commissioned a project to enhance mathematics teaching and learning in Ghana (World Bank, 2018). Also, Science, Technology and Mathematics Education (STME) Innovation Camps have been instituted and implemented annually to expose pre-tertiary education students to the wonders and joy of science and the mysteries, beauty and utility of mathematics. Furthermore, GES through the supported of Japan International Cooperation Agency (JICA) has developed training manuals to implement the Pre-tertiary Teacher Professional Development and Management (PTPDM) blue print.

In this regard, mathematics teachers were taken through in-service training programme with the aim of equipping them to enhance learners' performance on perceived difficult topics in mathematics with focus on teachers' content and pedagogy (Ministry of Education, 2014). These interventions notwithstanding, reports have shown that not much progress in the performance of students in mathematics has been realized (Frimpong, 2017; Ghana Star News, 2016; Ghana News Agency, 2015). Several years of rigorous research initiatives and interventions in mathematics education across the globe have been unsuccessful to ensure momentous improvements in mathematics learning by students. It is possible that adequate attention has not been paid to the role of conation in facilitating mathematics teaching and learning, specifically to the variable of values (Davis, Carr & Ampadu, 2019; Seah, 2019). Thus, learners' learning results are not just connected to other affective and cognitive processes but include how essential these processes have been considered important and therefore valued by students and teachers.

Durmus and Bicak (2006) opined that it only when values are taken into account that improvement in quality of mathematics education will be realized. In a review of at least 500 research studies by Nuffield Foundation, Askew, Hodgen, Hossain and Bretscher (2010) found that high achievement in mathematics may be much more closely connected to values than to particular mathematics teaching practices. Bishop (1999) argued in affirmative that adequate consideration should be given to values in mathematics education if success will be achieved by teachers and learners in the learning of mathematics. What an educator values is reflected in his/her classroom practices openly or secretly (Bishop & Clarkson, 1998). Bishop (2008a) stated that mathematical and mathematics educational values of teachers are seen in the selection of pedagogy and content of mathematics. Classroom practices such as discussions and various problem-solving methods, as well as small group work, are important practices that reflect the values that teachers hold.

Teachers' preference to pedagogy and content and how their students can learn best is informed by their values. For effective mathematics teaching and learning, it is critical to pay adequate attention to values of teachers and students (Bishop & Clarkson, 1998). According to the literature, "values" is an important variable in the implementation of mathematics curriculum in schools (Bishop, 2008). He acknowledged that in mathematics instruction, values remain a critical part of affective classroom environment. It affects how students choose to involve or not to involve in mathematics. The need for values transmission and acquisition by teachers and students respectively is re-echoed in the main rationale for teaching SHS core mathematics in Ghana. It focuses on attaining one crucial goal: to enable all Ghanaian young persons to acquire the mathematics skills, insights, attitudes and values that they will need to be successful in their chosen careers and daily lives (Ministry of Education, Science and Sport, 2007).

Notwithstanding the significance of values in mathematics and mathematics instruction, its role has sadly been neglected (Bishop, 2008). In the past, research studies on affect have focused on attitudes, beliefs and motivation with values not given adequate attention in literature. However, there has been a rise in global attention on the importance of values in mathematics education in the past 15 years. Research studies in mathematics education on values is largely categorised into three broad areas: teachers' values in mathematics (Bishop, Clarkson, FitzSimons & Seah, 2001; Dede, 2015), students' values in mathematics learning (Davis, Carr & Ampadu, 2019; Davis, Seah, Howard & Wilmot, 2021; Seah, Davis & Carr 2017; Seah & Wong, 2012; Zhang, 2019) and values in mathematics textbooks or curriculum (Dede, 2006; Seah, Anderson, Bishop & Clarkson, 2016). Initially, small-scale studies of teacher valuing in mathematics education were conducted through the lens of values and valuing (Chin & Lin, 2000). Some studies have also explored values and valuing in mathematics among primary and secondary school students (Dede, 2015; Seah, Baba & Zhang, 2017a; Zhang, 2019).

Interestingly, these studies into values and valuing in mathematics learning have focused on developed countries whose students' performance on Trends in International Mathematics and Science Study (TIMSS) and Programme for International Student Assessment (PISA) has been relatively better. As a way of taking values research up in Sub-Saharan Africa because of the role it plays in enhancing mathematics learning in schools, Davis, Seah, Howard and Wilmot (2021) have explored the attributes of mathematics learning which Ghanaian SHS learners' value. Earlier, Davis, Carr and Ampadu (2019) expanded the scope of their study and examined values in mathematics learning among Ghanaian learners across grade levels in the Cape Coast Metropolis. Even though the scope for the former study by Davis et al. was extended across all grade levels at the pre-tertiary level of education, the two studies were still limited to students valuing in mathematics learning. Although Abass (2021) looked at what mathematics teachers and their students value in mathematics teaching and learning, his study was limited to only the SHS level. Going through literature, no study had looked at what teachers and students value in mathematics learning across pre-tertiary education level (primary, JHS and SHS levels) in a single study. In her broad assessment of research studies on values and valuing in mathematics, Carr (2019) brought attention to this research gap.

Purpose of the Study

The study explored what Ghanaian educators and their learners find important in mathematics learning. It found out if there is an alignment between what mathematics instructors value in their students' mathematics learning and what the students also value in their own mathematics learning.

Research Objectives

The objectives of this study were:

1. To identify what primary, Junior High School (JHS) and Senior High School (SHS) teachers' and their students' find important in mathematics learning.
2. To find out the similarities and differences in the values of teachers and students in mathematics learning.
3. To explore the effects of grade levels, sex, academic qualification and teaching experience on valuing in mathematics learning.

Research Questions

1a. What do primary, Junior High School (JHS) and Senior High School (SHS) teachers' value in their students' mathematics learning?

1b. What do primary, Junior High School (JHS) and Senior High School (SHS) students' value in their mathematics learning?

2. How similar or different are values of teachers and their students in mathematics learning?

3a. What is the effect of grade level teachers teach (primary, JHS and SHS) on the attributes they value in students' mathematics learning?

3b. What is the effect of grade level of students (primary, JHS and SHS) on the attributes they value in their mathematics learning?

4a. What is the effect of sex on valuing in students' mathematics learning among mathematics teachers?

4b. What is the effect of sex of students on the attributes they value in their mathematics learning?

5. What is the effect of teachers' academic qualification on the attributes they value in students' mathematics learning?

6. What is the effect of teachers' teaching experience on the attributes they value in students' mathematics learning?

Significance of the Study

The study aimed to reshape curriculum design and teaching activities, as well as improve understanding of the role of values in mathematics learning at the pre-tertiary level of education in Ghana. It describes mathematical and mathematics educational values that mathematics instructors and learners should pay attention to in the mathematics classroom. The findings of the study which outline the seven attributes students across grade levels at the pre-tertiary level of education value in mathematics learning will enable mathematics curriculum designers to suggest relevant pedagogies that will support students' affective and cognitive development in the mathematics classroom.

Since values and valuing are societally and culturally laden constructs, the gathering and analysis of a Ghanaian data on what mathematics instructors and learners value in mathematics learning at a metropolis in the Western region of Ghana will enable teachers and other educational authorities to inculcate in students the right values that will motivate them to develop good mathematics learning attitudes, behaviours, beliefs and habits to enhance the teaching and learning of mathematics across educational levels.

The findings of this study could be used by mathematics teachers, school administrators, and curriculum planners to improve their understanding of what pre-tertiary students value in mathematics learning and to better plan and deliver mathematics teaching experiences in school.

To researchers in Ghana and elsewhere, the researcher hopes that the outcome of this research will augment the emerging knowledge and literature on values and valuing in mathematics amongst mathematics teachers and learners. It sought to expand our knowledge about values and valuing from teacher or learner specific to a combination of teachers and students valuing in mathematics learning.

The study provided useful information on the values of pre-tertiary students in the learning of mathematics across grade levels. It provided insight into value changes in mathematics learning as students' progress from one level of education to the other on the academic ladder. Students' grade level transitional movement from primary school through JHS to SHS may come along with its new focus in their valuing in mathematics learning. This will enable teachers to instil in their students the right values needed for success in mathematics and mathematics learning at that level.

Delimitations

Since the researcher explored valuing in mathematics amongst Ghanaian teachers and students across educational levels, the research was limited to SHS, JHS and Primary levels. This is because they constitute the pre-tertiary level of education in Ghana where mathematics is mandatory unlike at the tertiary level where mathematics is offered by those who offer mathematics as a programme of study or its related courses. At the pre-tertiary level of the educational system of Ghana, the lower Primary school consists of Primary One to Primary Three including Kindergarten. Although at that level mathematics is a mandatory subject, the learners at the lower primary level were not included in this study. The exemption of the lower primary school pupils from the scope of the study was based on the fact that the researcher adopted the original "What I Find Important (in my mathematics learning)" [WIFI] questionnaire which was written in English Language. Administering this kind of questionnaire at the lower primary school level was likely to pose comprehension problems to the learners on the items. English Language is used as the Medium of Instruction (MoI) in upper primary and above in the country. As a result of this, the data collected was limited to Upper Primary teachers and students (specifically primary six pupils), JHS mathematics teachers and students (specifically JHS 1 and 2 students) as well as SHS mathematics teachers and students (specifically SHS 1 & 2 students).

Limitations

Researching into values is thought-provoking because values of people are not easily expressed. One has to go the extra mile to discover the values that are concealed in the brain of the person. That notwithstanding, one cannot conclude with surety of someone's values. Also, mathematics educators and their learners of a metropolis in the Western region of Ghana participated in the study. The geographical area of this metropolis is small relative to the geographical coverage of Ghana. It is impossible to survey the values of all the teachers and students in the country at all levels of education. The data from 177 - teacher and 1263 - student participants only painted a picture of mathematical values and mathematics educational values of instructors and learners in the Primary schools, JHS and SHS in the country. Thus, the generalisability of the outcomes of this research on the entire population of Ghanaian educators who teach mathematics and students at the pre-tertiary level of education is limited by this small sample.

Definition of Terms

To ensure easy understanding in the context of the study, it is germane to provide operational definitions of key variables and terms used in the study.

Values: Refer to what is important to us in the teaching and learning of mathematics.

Understanding: Refers to identifying connections among mathematical concepts, owning mathematical knowledge and applying it to solve problems.

Fluency: Refers to having command over mathematical knowledge.

Learning Strategies: Refer to knowledge construction involving a wide range of cognitive activities such as obtaining information, establishing connections among concepts, scrutinizing and elaborating mathematics content and evaluating the truth and consistency of information.

Organisation of the Study

The study is made up of five Chapters numbered chapter one through to chapter five. The first Chapter has been outlined as indicated above.

Chapter Two highlights on the literature review of the study. It reviews the literature under the following broad areas: conceptual reviews, theories on values, theoretical framework, and empirical review. This Chapter ended with the summary of the salient points that have emerged from the literature review and their implications as far as the study is concerned.

The third Chapter focused on the research methods for the study. It discusses the research design, the study area, population of the study, sampling procedure, data collection instruments, the data collection procedures, data processing and analysis and finally the statistical tools used for the analysis of the data gathered.

Chapter Four presents the results and discussion of the findings from the study.

Finally, Chapter Five highlighted the summary of this study, conclusions that were drawn and recommendations made based on the study's key findings and suggestions for further research in the area.

CHAPTER TWO

LITERATURE REVIEW

Overview

The study explored what teachers and students in Ghana find important in mathematics learning. The chapter focuses largely on the various perspectives or what other authors have written on what mathematics teachers and students value in mathematics learning across grade levels at the pre-tertiary level of education. In this context, the general meaning of the concept of 'values' and 'valuing' and in particular 'values and valuing in mathematics' were examined. This study organises the literature review into four main components as follows;

1. Conceptual Reviews
2. Theories on Values
3. Theoretical Framework
4. Empirical Review

Conceptual Reviews

Values and Mathematics

The term "values" is a relatively new area of study in mathematics education research (Seah, Bishop, FitzSimons, & Clarkson, 2001). The origin of 'values' research could be traced from ethnomathematics research about culture and studies (Bishop, 1988). It has different meanings in different contexts. It could represent a variable in an equation in the context of school mathematics. When used in general context, it can also refer to the value of something. According to DeBellis and Goldin (2006), values, which include ethics and morals, are "deep, personal truths" or commitments that individuals cherish.

According to Philipp (2007), 'values' is a deeply held belief that one cherishes and acts on. Thus, values and beliefs are linked, however, beliefs are subsumed in values. Although values and beliefs are different related constructs, they can be used interchangeably in every day conversations. Seah in 2007 attempted to draw a distinction between the two variables by indicating that beliefs are context specific but values are not. Krathwohl, Bloom and Masia (1964) argued that values emanate from beliefs. Also, Clarkson, Bishop, FitsSimons and Seah's (2000) definition of values as beliefs in action put values in the domain of beliefs. In his chapter, McLeod (1992) stated, "we think of beliefs, attitudes, and emotional states as depicting growing degree of affective engagement, reducing levels of cognitive engagement, increasing levels of response intensity, and decreasing levels of response stability" (p. 578-579). According to Raths, Harmin and Simon (1987), belief as a value indicator has been put into some criteria. These involve a person selecting and prioritizing certain values over others with a high level of cognitive processing. Situating values and values research in the context of affective domain does not promote understanding of how values are taught in school mathematics to enhance effective pedagogy in mathematics.

In this study, the researcher operationalised ‘‘values' by looking at it from Bardi and Schwartz (2003) perspective, that is, something that one considers to be worthwhile and important and Mathews (2001), perspective, as mediators of learning behaviours. These perspectives enabled the researcher to situate what teachers and learners find important in mathematics learning as something that mediates their behaviours in mathematics learning and mathematics classroom. In the study of culture, society, and personality, values have been described as the main dependent variable, while in the study of social attitudes and behavior, values have been described as the main independent variable (Rokeach, 1973). Value is a generic concept which is experienced each day in our cultures. Values can also be seen as discipline specific. Values in mathematics education have been defined as important and valuable characteristics that an individual internalizes, providing him or her with the will and determination to pursue any course of action selected in the teaching and learning of mathematics (Seah & Andersson, 2015).

Bishop (1999) has indicated that "values in mathematics education are the deep affective qualities which education fosters through the school subject of mathematics" (p. 2). This is in line with the affective educational objective in the taxonomy that sees value construct as an affective variable (Krathwohl, et. al, 1964). These are contrary to peoples' misconception that mathematics is a value – free discipline. This perception emanates from the assumption that mathematics as a subject perhaps has one correct solution and one best method/procedure for finding an answer to a routine mathematical task (FitzSimons, Seah, Bishop, & Clarkson, 2000ab). There is another school of thought that believes that mathematics learning is value – laden. They argued further that notwithstanding the value – laden nature of mathematics, there are no clearly defined set of mathematical values that all mathematics instructors and students must adhere to (Liman, Salleh, & Abdullahi 2013).

Theories on Values

Values as a conative variable

Values are the socially mediated attributes that are regarded as essential (Lewin, 1938). Atkinson (1957), Higgins (2007), Torelli and Kaikati (2009), added to this understanding to include valuing as a driving force in obtaining what one desires. Values and beliefs are constructs that are at times used interchangeably. They do, however, refer to different qualities. Leder, Pehkonen, and Törner (2003) stressed on Bar (1990)'s explanation of beliefs in the first chapter of their book 'Beliefs: A Hidden Variable in Mathematics Education' to mean "what people consider as facts, opinions, hypotheses, as well as faith" (p. 12). On the contrary, when we think of our values, we think of what is important to us (Schwartz, 2012). Other definitions of beliefs and values share resemblance with these. Beliefs reveal what are true or untrue in social and cultural contexts. However, values represent what are individually important or unimportant in a non – contextual fashion. Values produce a strong motivating force for a person, culture or society in achieving a goal. This explains in part why values usually linked with traits of individuals or groups. "To talk about culture is to talk about values" (Frade & Machado, 2008, p. 34). Values ensure that a person becomes diligent and resolute in his actions in pressing on towards a goal as well as continuously sustaining hard and effort in attaining the goal which is a characteristic of conation. Values inform one's intellectual and emotional disposition. Therefore, a person valuing creativity and strategies will inform his approach to problem-solving, such as numerous efforts at looking for alternatives, 'appropriate' heuristic for problem-solving. Also, valuing of creativity can control a person's affective wellbeing. This includes experiencing happiness, approval, and accomplishment which are end products of problem-solving. Attaching confidence to actions and ways of doing things ensures self-efficacy. These come about when one is knowledgeable in the various problem solving heuristic models and can use what fit a particular problem to solve them. Vallerand et al. (1992) conducted a structural equation model to demonstrate how social norms and behavioural beliefs contribute to a person's attitudes.

In the Theory of Reasoned Action, Fishbein and Ajzen (1975), proposed a set of beliefs which underlined them. "This type of beliefs, namely personal normative beliefs, refers to one's beliefs about what should be or ought to be done" (Vallerand et al., 1992, p. 106). Thus, what one values contribute to the formation of the person's cognition and behavioural beliefs and this influence attitudes, behaviour and intentions underlying those behaviours. The researcher believes that behaviours do not always reflect the relevant valuing.

Values as regulator of Cognition

Higher order cognition in mathematics is an endeavor of humans. In an empirical review of research articles on human behaviour, the influence of affective variables on meta-cognitive process remains a critical issue of concern. Cognition cannot be separated from the system of affect particularly values (Blanchette & Richards, 2010). Seah (2019) conceptualised that the human brain is made up of interactions between cognition and the affective variable of values which guide behaviour, actions and decision making. The 'values' one holds tend to either promotes or impedes cognition and thus values have consequences for cognitive activities. Blanco, Guerrero and

Caballero (2013) explained why curriculum designers should come out with educational programme that integrate affective and mental strategies to mathematics education. On the contrary, DeBellis and Goldin (2006) have opined that teaching and learning of mathematics is a rigorous cognitive activity that is devoid of emotions.

Constructivism learning theory

Two opposing theoretical perspectives have characterised mathematics education across the globe. Behaviourist perspective and constructivist perspective have dominated mathematics education from ancient times up to date. According to Mereku (2003), behaviourist orientation to instruction and learning impacted on the teaching and learning of mathematics in Ghana throughout the 1970s and early part of 1980s. The many disadvantages of behaviourist perspective ranging from pedagogy, knowledge acquisition, assessment, to the teacher's and learner's roles have been identified (Hofstetter, 1997). It is against this background that constructivist approach has overshadowed mathematics education for more than three decades (Boaler, 2009). In the context of the role of the teacher, Proulx (2009) has argued that teachers have shifted their teaching from the behaviourist perspective of the teacher being the one who gives or tells information to the passive learners to a more participatory approach of constructivism in which the teacher serves as a guide or a facilitator to active learners who are seen as unique with rich ideas that can be used to construct new concepts and procedures.

Learners' active involvement in the learning and instruction of mathematics stimulates procedural and conceptual understanding. Learners' comprehension of mathematical concepts and their application in resolving problems at hand and future problems they may encounter is enhanced when the teacher links mathematical theory to practice. Thus, linking everyday mathematics to school mathematics ensure smooth transfer of sound mathematical knowledge. One fundamental ideology underlying constructivism is the philosophy that the learner uses previous relevant knowledge and experiences to construct new knowledge and understanding. Modern pedagogies ascribe to constructivism as far as mathematics education is concerned. Constructivism ensures learner's control over their own learning and independence.

Constructivist learning theory has been the focus of modernist mathematics curriculum development across the world and Ghana is not an exception. The philosophies underpinning the new basic school mathematics curriculum in Ghana emphasize on the adoption of constructivism by instructors and learners in mathematics teaching and learning. Notwithstanding the importance of constructivist approach to teaching and learning, Westwood (1999) has argued that it has failed to guarantee students fluency and automaticity with computation and basic number.

Values as a sociocultural variable

Values are socioculturally relevant because valuing is done by people. Sociocultural viewpoint on "values" is critical in appreciating their significance in mathematics education. Mathematical symbols, ideas, practices and products do not have any inherent values. However, it is the people and the institutions who make use of them place some kind of worth on them (Davis & Hersh, 1981; Joseph, 1991; Wilson, 1986). In a relatively current study, Barkatsas and Seah (2015), argued that values are extremely sensitive to sociocultural influences and depend on the society and culture of the person. Later on, Seah (2019) reported that this sociocultural construct is environmental in nature and therefore depends on the setting one finds him or herself. In the mathematics classrooms, learners who come from diverse sociocultural backgrounds and environments may value the same attributes in their mathematics learning but disparities may exist in how they attach importance to each value attribute.

Thus, comparing research study by Davis, Carr and Ampadu (2019) in Ghana to Swedish study by Andersson and Oüsterling (2019) portrayed value differences among learners in mathematics in the two countries. Whilst Swedish study reported that learners in Sweden value activities such as explanation by the teacher, knowing the times tables, and rightness, their counterpart in Ghana valued characteristics such as strategy, achievement, fluency, authority, the use of ICT, relevance, and versatility.

Researchers see general educational values, mathematics educational values and mathematical values as different entities (Bishop 1988, 2008; Dede 2011). In mathematics learning and instruction, the focus should not be restricted to general educational values (respect for peoples' opinions, honesty, candidness, promptness, good behaviour etc.), sociocultural values, educational values or religious values a lone. This is because these values are not subject specific and so come naturally along with the learning of mathematics and other school subjects (Davis, Seah, Howard & Wilmot, 2021). Values are acquired (UNESCO, 1991) from different instructional strategies in any subject area. The instructional techniques include simulation, drama, educational games, debates, discussions, role-play projects, group work, educational visits, interviews, brainstorming, and utilising the resource materials using poems, stories, songs, photographs, art work, and mantras. Other instructional methods include evaluation of projects, assessment of group work, observation techniques, interviews, pre-test, post-test, anecdotal records, and audio-visual evaluations (Churchill et al., 2013).

General educational values are connected to the customs of a specific people and of a specific educational body. Such values are ingrained in our lives and therefore cannot be detached from society and culture (Corrigan, Dillon & Gunstone, 2007; Kang & Glassman, 2010). Based on Dede (2015) clarification of values, general educational values are viewed as implicit valuing in the planned mathematics curriculum and invariably have the tendency of impacting on what teachers and learners deem essential as they participate in mathematical skills and concepts (Bishop, 1988). Dede (2009) had earlier linked general educational values to communal or societal values.

Theoretical Framework

What educators and learners value in mathematics learning are revealed through their classroom interactions and learning practices (Seah, 2019). The theoretical framework adopted for this research study combined White (1959) and Bishop (1988, 1991, & 1999) philosophies of mathematical values because they portray the classroom actions taken by mathematics instructors and their learners. A blend of philosophies from interconnected viewpoints is considered to be more efficient than any single perspective (Westwood, 1999).

White (1959) Dimensions of Values

White (1959) proposed three dimensions of values to explain growth in culture. These dimensions include ideological, sentimental (or attitudinal) and sociological values. The value dimensions by White (1959) is presented in Table 1.

Table 1: Mathematical Value Dimensions

Value Dimension	Description
Ideological Values	Epistemology of the mathematics knowledge
Sentimental or Attitudinal Values	Individual's attitudes toward mathematics knowledge
Sociological Values	Mathematical knowledge and society

Source: Bishop (2008)

Bishop (2008) provided a lens through which White (1959) mathematical value dimensions could be viewed with their respective descriptions. Teachers and students relate to these value dimensions to identify which one(s) they deem important.

White (1959) outlined three component analysis of culture:

1. Ideological component: made up of ideologies that rely on symbols, and philosophies.
2. Sentimental (attitudinal) component: attitudes, feelings about people, and behaviour.
3. Sociological component: interpersonal relationship, patterns, guidelines, behaviours, traditions and institutions.

According to Bishop (1988), mathematical values are values which characterise the nature of mathematics as it is presented in the classroom. Bishop (1988 & 1999) expanded the work of White (1959) to include complementary pairs under each component. Each component has two complementary pairs of value attributes such as rationalism and objectism, control and progress and openness and mystery.

The Ideological component of Mathematical values

With regard to this component of the Mathematical culture, it is argued that the essential aspect of mathematical values are rationalism and objectism (Bishop, 1988, 1991). Valuing rationalism means emphasising argument, reasoning, logical analysis, and interpretations, arguably the most relevant value in mathematics education. Valuing Objectism entails emphasising the objectification, concretization, symbolization, and application of mathematical ideas.

Rationalism - Objectism

To rationalise means to form a sensible connection between two or more ideas that are not related or related in absurdity. According to Andersson and Osterling (2019), rationalism as an ideological phenomenon enables learners to write or verbalize their thoughts and argue for correct response or line of reasoning in mathematics. This reason comes with the concept of sharing and comprehending the arguments of a person and the idea of participation as an agentic activity. Rationalism is the life line of mathematics. It is perhaps the most significant value in mathematics education (Bishop, 2008). It is concern with sense making and logic which are common traits of mathematics. Rationalism has made mathematics what it is today.

It is regarded as the anchor of mathematical growth. According to Kline (1972), mathematics has rationality spirit that motivates and pushes the human brains to function to its full potential. Out of the six mathematical value systems identified by Bishop (1988), it is the only mathematical value that is guaranteed with mathematical authority and power. Rationalism involves both inductive and deductive reasoning. Rationalism transcends beyond mathematics and are applied in all aspects of human endeavour. This argument is supported in the words of Weizenbaum (1984) that "the introduction of computers into our highly technological society has merely reinforced and amplified those antecedent pressures that have driven man to an ever more highly rationalistic view of his society" (p. 11). Doing mathematics involves formulating, conjecturing, proving and disproving formulae and hypotheses, contradictions, giving examples and counter-examples which are all tenets of rationalism.

According to Andersson et al. (2019), objectism is the use of pre-determined formulae and the practice of symbolizing. Objectism involves transforming an abstract idea in mathematics into real object or practical forms that make sense to an objective mind. It is a pedagogical idea that concretizing mathematical ideas and concepts provide a scaffold for students' understanding of mathematics (Tang, Seah, Zhang & Zhang, 2021). The use of symbols in mathematics help to transform mathematical knowledge or idea into forms that aid easy understanding and communication. Symbolical representations are human creation and yet have powerful connections with the mathematical concepts they convey.

A symbol is a sound or something visible, mentally connected with an idea. It is a sign that reminds us something apart from the sign itself. It calls to attention a certain imagery which accompanies the class of things it symbolizes. Thus, symbols are embodiment of the ideas they put across. This has made symbols as system of convention in the world of mathematics. Mathematical symbols are used in different subjects such as physical science, computer science, commerce, statistics etc. because of transferability in the ideas of mathematics. Mathematical ideas serve as the foundation of all ideas. The evolutional emergence of mathematical ideas and concepts have come along with historical generation of mathematical symbols and representations.

Today, technological development such as computers have brought new paradigms to the transformation and pedagogy in the area of mathematics teaching and learning. The application of ICT in the learning of mathematics by students could be done in groups. Gender related collaborative pairings (male – male, female - female or male – female pairs) put female – female pairs as very reliable in applying computer software in solving mathematical problems (Yelland, 2001). Yelland supported the argument by reporting that girl pairs demonstrated higher order reasoning and critical thinking in the use of technology in solving mathematical problems. According to Forgasz (2006), teachers and their students trust that applying computers enable them to comprehend mathematical concepts better. Irrespective of the outcome of using computer to learn mathematics, boys value the chance of improving their computer skills and knowledge (Vale, 2003b)

The Sentimental (Attitudinal) component of Mathematical values

The sentimental component of mathematical values according to White (1959) is concerned primarily with attitudes and feelings. The essential values in this element are Control and Progress. Control is valued by prioritising the authority of mathematical knowledge through competence in rules, facts, procedures, and predetermined criteria. Valuing Progress entails highlighting how mathematical ideas evolve and grow through alternative theories, the creation of innovative methodologies, and challenging of existing ideas.

Control - Progress

The value of “control” is concerned with how mathematical knowledge and ideas are used appropriately in the classroom and outside the classroom ((Bishop, 1988). It signifies one’s ability to use the ideas of mathematics to solve problems he/she faces in the classroom or real life. It also involves explaining, predicting, describing and applying mathematical ideas. The teacher having control over mathematical concepts / subject matter is a major requirement for their students’ success in mathematics. This is because the teacher is able to bring into being different perspective of a mathematics idea. It is in line with this that Stigler and Hiebert (1999) argued that many answers to a mathematics problem enhance relational comprehension among teachers and students in that mathematics concept. In support of this, one of the best methods of developing a deeper understanding among teachers and students is to solve a problem in diverse ways (Leikin & Lavan – Waynberg, 2007). There is also flexibility in the choice of appropriate method for teaching. The teacher with control over mathematical content is able to link abstract mathematical ideas to reality.

This enables the students to appreciate and practicalise the mathematics they learn at school. Students’ questions emanating from their misconceptions are also addressed to their satisfaction because the teacher has control over the content and methodology. When mathematics is mastered and understood by teachers and students, there is a feeling of security of knowledge. Davis et al. (2019) reported that students’ sentimental value of control increases as one climbs higher the academic ladder. In the world of mathematical values, there are interconnections with different complementary pairs of values. Thus control is linked to its complementary pair of progress and ideological component of objectism. The demand for people to show command in every endeavour has been the way to go in the past and present. It is in line with this that Schaaf (1963) says "the spirit of the nineteenth and twentieth centuries is typified by man's increasing mastery over his physical environment" (p. 15). Control has helped to predict natural disasters in our societies and the necessary precautions taken to reduce the effects on humanity. Mathematical ideas are applied in the area of Science to help man to have total dominance over his physical and social environments.

Progress on the other hand, refers to valuing of alternate approaches, initiating innovative ideas and interrogating and critiquing prevailing ones or status quo with the aim of making it better (Bishop, 2008). In mathematics, knowledge and ideas develop, grow and change with time. Students' misconceptions are corrected as students' progress from one academic level to the other. For instance, at the basic school level, pupils and students think that adding and multiplying numbers always make the result greater. Subtracting and dividing numbers make the result smaller. When these students have progressed on the academic ladder to high school, they are exposed to more fractions and negative numbers which enable them erase this misconception and make their knowledge more refined.

The progressive nature of mathematical knowledge is exemplified in the mathematics curriculum across all levels of education. In school mathematics, some mathematics topics are taught at different levels. The same topic taught at different levels differ in scope, content and complexities as students' progress from one stage to the other. The spiral nature of the mathematics curriculum at all levels of education does not only provide growth in mathematics knowledge of students but also provides high level of control, security and progress of mathematical knowledge. In the contexts of the above, the researcher believes that progress in the learning of mathematics is both assimilation and accommodation. How mathematical knowledge has evolved in history has impacted on mathematics education (Tozluyurt, 2008). The history of mathematics provides a natural flow of cognitive ideas which is in line with Piaget's theory of intellectual development which sees mathematics learning as an evolving phenomenon occurring in identifiable stages.

A qualitative study by Goodwin (2007) showed that teachers who are more knowledgeable in the history of mathematics help their learners to achieve better in mathematics. Thus, mathematics teachers' integration of the history of mathematics in lessons enable their students to appreciate and recognise the evolving and changing nature of mathematics (Karakus, 2009). It makes mathematics teaching and learning more meaningful and inspiring. The history of mathematics is topic specific. Topics and mathematical symbols such as rational numbers, money, probability, measurements, pyramids, geometry, fractions, volume, algebra, counting systems, and the square root symbol, pi, etc. have their emergence traced from history.

The Sociological component of Mathematical values

The vital values in this element are openness and mystery. By emphasising the democratic process of knowledge through examples, evidence, and unique justifications, openness is valued. Putting premium on the value of mystery means emphasising the amazement, fascination, and mystique of mathematical concepts. These are what Bishop (1988) believes to be the fundamental values that support the growth of mathematical thinking in the classroom and thus underlie the development of mathematics. If more research studies are conducted to explore how to instill these values in learners and instructors, we will make better strides toward solving most of our educational problems.

Mystery - Openness

Bishop (1988) indicated that mystery and its complementary pair openness aid in the comprehension of mathematics to a very large extent. Valuing mystery involves the nature of knowledge and its abstraction and reflecting about the genesis of the mathematical creation (Bishop, 2016). Mystery is key in sustaining the interest and appreciation of teachers and students for mathematics. According to Mason (2015), mystery makes teachers and students more curious, wonder and excited about mathematics. This is seen in mathematical computations such as multiplying 111 111 111 by 111 111 111 to give 12345678987654321; dividing the circumference of any circular object by its diameter irrespective of its size and dimension gives a constant value; Pythagorean triple that have 3, 4, 5 cm or 5, 12, 13 cm edge lengths always give a multiple of 60 when they are multiplied together. Pythagoras' philosophy about mathematics resonates with that of Ernest (2015) who sees the beauty of mathematics as embedded in its creativity, cleverness and wonder which speak of the mystery of mathematics that educators and learners experience in the classroom.

Openness is a value attribute that holds the view that mathematical facts, axioms, assumptions, ideas, truths and principles are open to everyone for scrutiny. Mathematical ideas and facts can be verified several times within the globe. It involves the teacher’s way of making mathematical ideas known to everyone in the classroom. It can be in the form of questions and answers, classroom discourse be it monologic or dialogic. It is the method the teacher uses to bring out mathematical ideas to students. Openness makes mathematical ideas known to the public. This can occur in conferences, workshops, viva, etc.

The WIFI instrument has related outlines that show how the questionnaire's items are shared among mathematical and mathematics educational values (Anderson & Osterling, 2019). In the guidelines, some of the items connected to the three complementary pairs of value dimensions are shown in Table 2.

Table 2: Mathematical Values and their Corresponding Value Items on WIFI Questionnaire

Mathematical Value	Value Item
Rationalism	
	Learning the proofs
	Abstract or theoretical aspects of maths
	Verifying theorems or hypotheses
Objectism	
	Using concrete materials to understand mathematics
	Using diagrams to understand mathematics
	Connecting mathematics to real life
	Looking out for mathematics in real life
	Hands-on activities
	Outdoor mathematics activities
Control	
	Students posing mathematics problems
	Making up my own mathematics questions
	Looking for different possible answers
Progress	
	Stories about recent developments in maths
	Relating maths to other subjects
	Relationships between concepts
Mystery	
	Mystery of mathematics
Openness	
	Debates
	Small-group discussions
	Whole-class discussion
	Explaining where formulae or rules came from

Source: Anderson & Osterling (2019)

The overlap of the six mathematical values by Bishop (1988)

There are several interconnections among the six mathematical values by Bishop (1988). Some authors believe that complementary pairs of mathematical values exist independent of each. This demonstrates that an ideological value is clearly an objectism value if it is not rationalism value. In a similar perspective, if a sentimental value is control, then it is not a value of progress and the reverse is also true. This analogy is extended to the sociological value of mystery and openness. Others believe that they have some common intersections (Bishop, 1988 & Osterling & Andersson, 2013).

The researcher is of the view that these intersections do not occur within the same pairs but across different complementary pairs. For example, in the activities leading to the proving or finding the generalization of the sum of inside angles of a triangle which constitutes mathematical value of rationalism, one needs several cut out shapes of different triangles of varying dimensions. One has to apply the idea of the properties of triangles in this generalization which is an idea of mathematical value of objectism. At this point, it is worthy of note that these two values are not complementary but overlapping. For instance, progress and openness overlap. Mathematics teachers make use of logical relationship if they are justifying or generalising. There is also an overlap between rationalism and mystery. Davis et al. (2019) indicated that mystery and openness as sociological values of mathematics decrease at the SHS. Thus, Primary and JHS students value mystery and openness more than SHS students.

Bishop (1988) indicated the mystery and rationalism of Pythagorean triples and supported the claim that relational understanding of mathematics deepens its mystery. Thus, mathematics instructors should be motivated to draw a connection among mathematical values in their teaching to aid understanding and appreciation of mathematical ideas. It is in the light of this that mathematics teachers are urged to reason comprehensively on the mathematical values that form the foundational frame for the understanding of school mathematics.

Mathematics Educational Values

In Third Wave Project Studies 1 and 2, Bishop (2008) investigated mathematics educational values of learners. Mathematics educational values can also be located in mathematics syllabus, mathematics educator's classroom instructional methods and other official mathematics text materials. Values of this nature are presented less in explicit forms than in implicit forms. They are elements of mathematics teaching and learning instructors would stress and believe are relevant to learner's mathematics accomplishment. According to Bishop (1996), mathematics educational values connect particularly to methodology and learning of the subject. Mathematics instruction and learning involves arranging authentic contexts of activities designed to enable learners learn.

The teacher's decision making in which pedagogy will best address the learning difficulties of learners is paramount. As students are continually exposed to the very methodology the teacher finds important in helping learners to learn, they become use to it. These values may affect students' choices for the pedagogy and learning activities used in the classroom. Some of the examples of mathematics educational values include consistency, clarity, creativity, open mindedness, accuracy, connection, understanding, ICT integration, etc.(Durmus & Bicak, 2006).

Connections

The utilitarian value of mathematics is seen in making mathematics practical, being relevant in everyday life of students or relevant in the future success and opportunities for students. Learning mathematics becomes authentic when teachers create opportunity for learners to experience mathematical concepts and skill by applying real life scenarios (Osterling & Anderson, 2013). To enable learners appreciate the value of the relevance of mathematics, questions must be cast in real life context during mathematics teaching and learning. According to Young-Loveridge, Taylor, Sharma, and Hawera (2006), irrespective of the culturally diverse backgrounds of learners, they hold the belief that mathematics is useful for their later endeavours and academic success.

Mathematics teachers and mathematics textbook authors should begin mathematics from real life problem, scaffold by thinking through the problem with the students on mathematical concepts and ideas in the mathematics curriculum essential for solving the problem. Mathematics teaching should go through the three transition stages of concrete phase through semi-concrete phase to abstract phase. The use of real objects in teaching mathematics usually happens at the early part of school life. At this level, teachers are able to connect school mathematics with everyday mathematics. A study by Davis et al. (2021) found among other things, that valuing of connection by learners appeared as one of the highly valued attributes at the SHS level in Ghana. At SHS, the focus of the students is on high stake international examinations where their success in these examinations will guarantee them education at the next level. This research confirms with the study by Zhang et al. (2016) that learners' values can and do change as they move from one educational level to another.

In the opinion of Seal et al. (2017b), Ghanaian students' valuing in mathematics learning is not intrinsic in nature. To them, the reason why Ghanaian students like to engage in mathematics and its learning appeared more with what could be done with the mathematics knowledge they acquire through instruction but not what goes into mathematics as a subject. Based on the findings of the study above, students across grade levels in Ghana value connection and utilities.

Understanding

The resolve to explore what educators and their learners value in mathematics learning should take into cognizance the clarity and understanding they get in mathematics concepts. Different researchers have categorised different forms of understanding (Herscovics & Bergeron, 1988; Franke, Kazemi & Battey, 2007; Skemp 1976, 2006). According to Pirie and Kieren (1994), understanding consists of relational and instrumental, intuitive and formal and concrete and symbolic.

Relational understanding is derived from knowledge of concept. It comprises comprehension, making networks and managing relationships. When learners understand concepts in mathematics relationally, they become problem solvers and independent thinkers and they retain knowledge better and longer. On the other hand, procedural knowledge leads to instrumental understanding among learners. Instrumental understanding takes place when people apply memorised mathematical formulae and rules in solving mathematics problems (Skemp, 2006). It confirms a formula-base understanding without recourse to explanation, textbook-methods and generalization. According to Perkins and Unger (1999), understanding is learning flexibly, implemented through inference, interpretation, application and confirmation. It includes having conventional knowledge and skills. That is, "learning with understanding can be expected to yield higher engagement, more active use and transfer of knowledge than learning with rote emphasis" (p. 95). Relational understanding (Skemp, 1976) of mathematics is the ultimate aim of instruction and learning in Ghana. Once this goal is achieved, learners can discover mathematical ideas by themselves without the help of the teacher. This is one of the requirements of becoming a global learner. It is characterised by knowing what to do at every point in time during mathematical engagements. Developing deep mathematical understanding in students is difficult because learners are part of the enactment process of teaching and learning.

ICT

In Ghana, the application of technology in mathematics education involves internet, computer and calculator usage. However, there are perceive obstacles in the schools which suppress technology integration in mathematics instruction and learning. They consist of teachers' limited understanding and expertise on how to incorporate technology into mathematics instructions, limited access to technologies by educators as well as learners and lack of opportunities for teachers to study and practice technology integration. Most public elementary schools do not have computers, no internet connectivity and learners are not allowed to use calculators. However, public SHSs in Ghana have computer laboratories but most of these laboratories are not connected to the internet. Incorporating ICT into mathematics instruction and learning cannot take place successfully without classrooms adequately prepared to suit its implementation. Bariham, Ondigi and Kiio (2020) have reported that majority of classrooms in SHS in Northern Ghana were unconnected to electricity to enhance academic work with computers. SHS in Ghana lack mathematical software to facilitate teaching and learning (Agyei, 2012). SHS learners are obliged to make use of calculators in the classroom and as such almost all students have calculators.

Empirical Review

Values of Mathematics Teachers and Students in Ghana

Teachers all over the world do not operate from the same society and culture. Thus, their societal, cultural and educational backgrounds are different. Following the new educational reforms in 2018, teachers who teach at the Basic Education level (for now, it consists of kindergarten, primary and JHS) are required to possess at least a bachelor's degree. Majority of these educators possess Diploma in Basic Education from Colleges of Education, College of Distance Education, University of Cape Coast (UCC) and Institute of Distance Learning, University of Education, Winneba (UEW) with few of them from Department of Basic Education of these two universities. All teachers at the basic schools and the senior high schools are grouped into two: professional and non-professional educators (MoE, 2012).

The qualified educators are the ones who possess either Diploma, Bachelor's or Master's degree in Education from officially recognised higher educational institutions for training teachers and those who possess Diploma, Bachelor's or Master's degree in non-education discipline and have done Post Graduate Diploma in Education (PGDE). Unprofessional educators are grouped into three: university graduates with no certificate in education, graduates with diploma from accredited Polytechnics (currently Technical Universities) with no PGDE certificate and those with SHS certificate with passes in English and Mathematics (MoE, 2012). Most of the instructors teaching at the elementary school particularly Kindergarten and Primary schools are class teachers who are suppose to teach mathematics together with all the other subjects prescribed by the curriculum at that level. At the JHS and SHS levels however, teachers are specialists who teach specific subjects. At the SHS level, mathematics teachers are required to hold a minimum certificate of B.Ed (Mathematics) or BSc (Mathematics Education) or BSc. (Mathematics) with PGDE certificate or its equivalent from an accredited university.

At UCC, the B.Ed (Mathematics) programme has four components: general education, mathematics education, mathematics content and teaching practice. The general education courses are taught in the Department of Educational Foundations. The mathematics education courses are taught in the Department of Mathematics and ICT Education. Similarly, the mathematics content courses are taught in the Department of Mathematics and Department of Statistics. The organization and placement of pre–service mathematics educators for the University on-campus and off-campus Teaching Practice programme is coordinated by the Centre for Teacher Professional Development (CTPD), UCC. The preparation of the prospective mathematics instructors enables them to possess both the pedagogical side and content side of the subject (Dede, 2009). Because of inadequate number of qualified mathematics teachers, graduates who do not have any of these qualifications are in the classroom teaching mathematics (Buabeng, Ntow & Otami, 2020).

This is likely to bring about variations in what they find important in mathematics teaching and learning since in the opinion of Seah, Baba and Zhang (2017a) people from varying educational backgrounds have different mathematical values. Mathematics teachers from different backgrounds are likely to instill different distinct values in their students even when the teachers are teaching the same concept in the mathematics curriculum (Bishop, Clarkson, FitzSimons & Seah, 2000). This perhaps may impact on what mathematics teachers see as crucial for ensuring learners develop mathematical thinking (Bishop, 2008). Values have been connected to the idea of 'worth' or important / unimportant (Begg, 2001; Swadener & Soedjadi, 1988). Teachers who believe in value items such as 'investigations', 'problem-solving', 'connecting mathematics to real life', etc are more likely to regard them as essential characteristics in their mathematics teaching and learning repertoire. Most often than not, what teachers perceive as 'worthy' or 'important' become part of them thereby stimulating their actions in the classroom.

In Ghana, the problem of shortage of teachers, particularly qualified mathematics teachers has been a recurrent phenomenon (Cobbold, 2015). The causes of this perennial shortages range from absence of interest in the study of the subject up to the tertiary level, government policy on education, educational expansion to high attrition rate of qualified mathematics teachers.

The canker of shortage of qualified teachers in schools is not peculiar to Ghana. Eurydice (2002) reported that United Kingdom faced an arduous task of filling classrooms with qualified teachers. Nations that belong to the Organization for Economic Cooperation and Development (OECD) have instructor shortfall.

Mathematics educators' conceptualisation of mathematics and the strategies they use in teaching their students have an impact on the things they value in mathematics teaching and learning (Wong, Ding & Zhang, 2016; Wong, Marfo, Wong & Lam, 2002). Teachers' control over teaching methods, teaching resources, curriculum scope selection and assessment practices have an influence on what learners also see as useful in their learning. Studies in recent times have shown the benefits of modernist teacher preparation programmes far outweigh those who were prepared in the traditional 4-year teacher training programmes (Denton & Peters, 1988; Dyal 1993; Shin, 1994). Most school districts and teacher education institutions have had to create educational programmes and opportunities outside the conventional 4-year teacher preparation programmes. This allows for a more extensive study of the mathematics discipline coupled with research to guide their practice on the field. In this regard, such programme is synonymous to that of teaching hospitals in the study of medicine.

A purposefully structured programme designed to support the training of novice professional teachers. The programme must be in every district across the nation. Such programmes are subject specific to help newly trained teachers to deliver effectively in the classroom. The services of highly experienced veteran teachers from the teacher training institutions and classroom could be sought. Similar programme is being done in Belgium, Luxembourg, Germany and France where prospective teachers are required to go through 2 to 3 years advance level mentorship programme to augment an undergraduate training and a strictly monitored mentorship only in the schools affiliated to the university. Pre-service teachers who go through such a training programme are seen by their colleagues as more experience, better prepared and effective with students. Also, teachers trained in these programmes are open minded and view the world from different perspectives. They see beyond their own world and view situations not only from their cultural perspective. They tend to switch to comprehend the behaviours, encounters, viewpoints, knowledge as well as misconceptions of students from several sociocultural and cognitive backgrounds. There is high probability for such educators to enter the education field and stay compared to their counterparts who were trained in the conventional 4-year programmes only (Andrew, 1990; Andrew & Schwab, 1995; Arch, 1989).

Teachers exhibition of their values are restrained by the influence of classroom ecology. An instructor can portray dissimilar values in diverse classrooms based upon the nature of students and the concept that is being taught. At times, the teacher has to shelve his values to be able to go through an overloaded syllabus within the school structured timetable. However, some teachers maintain the values they portray irrespective of the prevailing situation in the classroom. Such teachers may change their teaching method but maintain their mathematical values they believe in. There are research studies that show a mismatch between mathematics teachers' values and the kind of values exhibited in the classroom (Sosniak, Ethingston & Varelas, 1991; Thompson, 1992; Tirta, 1999).

Students take decisions and make choices which affect them in the classroom. The responsibility of taking decisions in the classroom has been with students since they began doing mathematics. It is worth emphasising that students can pretend on the values they espouse because of the presence of teachers. This shows that students' mathematical values and mathematics educational values are influenced by their teachers. In support of this, Wang (2012) reported that classroom experiences of students impacted strongly on the development of their values. Bissell-Havran and Loken (2009) also indicated the role of students' peers in the progress of student valuing in mathematics.

Furthermore, some mathematics educational values and mathematical values are learnt at home through the activities children undertake before they begin school. The household activities are carried out with direct involvement and supervision of parents. It is in the light of this argument that Gniewosz and Noack (2012) reported that student's value in the development of mathematics is a function of parental values. Values students learn at school are further reinforce by what happens around them in their immediate environment. According to Rivera (2010), students' involvement in activities that are not academic in nature at school and outside school leads to improved grades at school since through that they imbibe the value of focus. A student who views mathematics as a collection of facts, rules as well as guidelines will value instrumental comprehension or learning. Students in a constructivist classroom, however, might have a broad - based understanding of mathematics and exhibit more advanced learning strategies. According to Ding and Wong (2012), these learners put premium on mathematics, are less nervous about learning it, and have good educational behaviours.

What students see as valuable in mathematics learning impact on the effort they put into their mathematical reasoning, their selection of mental strategies and their decision making (Seal et al., 2017a). It is a common knowledge that students from varied school contexts being it rural or urban may value things differently. This is because the school culture and climate, its setting, predominate occupation of the people, parental educational background may affect what students consider imperative in the learning of mathematics (Bishop, 2008). Educators perform a significant role in transmitting mathematics concepts and values to students. According to Kluckhohn (1962), peoples' actions and inactions are directed by their values. When teachers became aware and comprehend their individual values, they consciously or unconsciously transfer these values to their learners.

Valuing in Mathematics across Educational Levels

Gniewosz and Noack (2012) compared learners' values in mathematics learning across school context – Primary, JHS and SHS. Later on, Hsiang-Wei (2017) narrowed the scope relative of the former study and conducted a study across both Grade 4 and Grade 8 in America, Singapore and Taipei. The study found that students' competence beliefs as well as their values reduce as they progress on the academic ladder. Also, Dede (2015) discovered that the school levels of the teachers within and between Germany and Turkey had significant impact on their values for teaching mathematics in primary and secondary schools. Differences among Learners is possible because differences among teachers is recognised. In Psychology, grade level research on learners valuing have been reported (Gniewosz et al., 2012). Dietz, Hofer and Fries (2007) discovered, along with other things, that "learners in 8th grade valued achievement less than learners in 6th grade" (p. 10).

This was observed when routine and academic procrastination of learners in Germany were explored. Research work on values at different levels of schooling help determine the impact of context of school on value in mathematics education. This is because mathematics curriculum and its implementation vary widely across many nations in terms of school context. For instance, the level of qualification one requires to be a mathematics educator, teachers' training in mathematics, and the level of difficulty of mathematics learners are required to learn may vary among nations. It is important to note that grade level tends to influence what learners value in the learning of the subject.

Sex and Mathematical Values

In recent times across the world, premium has been placed on how mathematics could be taught and learned well especially at the beginning years of school life. This is to ensure sex equity as students progress to the higher level on the academic ladder. According to Watt (2004), female students need to be convinced to value studying mathematics at the pre-tertiary and university levels. Mathematics classroom lessons with ICT integration tend to favour boys than girls (Vale, Forgasz & Horne, 2004). In the beginning years of school life, females performed significantly lower than their male counterpart in subtraction algorithms and solving of addition problems (Horne, 2002 & 2003). Horne further reported that girls outperformed their boys counterpart in properties of shapes. Later on, with the same students the performance trend reverse in favour of boys in number concept. Zhang (2019) reported in the study of values in mathematics learning among Chinese Mainland primary and secondary school learners that gender difference exist with respect to their values. In this regard, males tend to place a higher value on capacity, logical comprehension, and innovation than females, whilst females place a higher value on investigations in mathematics.

Mathematics Teachers' Academic Qualifications and their Mathematical Values

Students learning and academic performance are influenced by teacher qualification (Fallon, 1999; Metzler & Woessmann, 2010). Iheanachor's (2007) study on effect of educators' instructional behaviour, academic proficiency, and career development on learners' achievement in mathematics further supported the claim by the earlier and later researchers. The study revealed among others that instructor's academic credentials, subject specialization and number of years in teaching are the main forecasters of learners' achievement in mathematics. Evaluation of Texas school districts showed that instructors' licensure examination marks, teaching experience and higher academic qualification resulted in the differences among grade 1 to grade 11 students' mathematics achievement across districts (Ferguson, 1991). In the same research, differences in mathematics attainment between white and black learners were predicted highly by difference in the academic qualification of their teachers.

According to Fuller (1999), districts dominated by fully licensed teachers are more likely to have their students passed the Texas State achievement tests irrespective of teacher experience, educational level of parents, school status and background of students. In the same way, a research in North Carolina (Strauss & Sawyer, 1986) reported that instructors' mean marks on National Teacher Examination (NTE) impacted significantly on learners' pass rates on State Competency Examinations (SCE). The NTE tests content knowledge and pedagogical knowledge of instructors. A 3% to 5% decrease in the proportion of pupils failing the SCE was correlated with a 1% rise in teacher effectiveness as measured by NTE scores.

It appears that an instructor's pre-service preparation programme has an impact on what they find important in their mathematics instructional practices (Clarkson, Bishop, & Seah, 2010). There are two forms of qualifications for mathematics teachers: content-area qualifications and pedagogy qualifications. In Ghana, some teachers in the classroom only possess qualification in only the mathematics content with no qualification in mathematics pedagogy whilst other possess both. The latter is recognised as a professional teacher. It is believed that to be able to teach well, one needs to have some skills and desirable knowledge for teaching mathematics while other think that mathematics teaching is best learnt on the job. This is supported strongly with an argument that all that a mathematics teacher needs to succeed in the classroom is the knowledge of the content of mathematics and the rest is learnt on the job by trial-and-error basis.

However, evidence available from research suggests otherwise (Hill, Rowan & Ball, 2005). Reflective summaries of studies in teacher education indicate that notwithstanding the weaknesses of teacher preparation and teacher licensing, teachers who have all the two forms of teacher qualifications help their students to succeed and are rated in the education sector better than their counterpart who have one (Ashton & Crocker, 1986; Evertson, Hawley & Zlotnik, 1985; Haberman, 1984; Greenberg, 1983; Olsen, 1985). Research also shows that having qualification in mathematics pedagogy such as knowledge of psychology of mathematics, curriculum issues, mathematics history, mathematics methods, classroom management as well as good communication skills and mathematics content impact on student achievement and tend to exert better effects on students achievement than when one possesses only content knowledge (Addae & Agyei, 2018; Ashton & Crocker, 1987; Begle & Geeslin, 1972; Ferguson & Womack, 1993; Guyton & Farokhi, 1987; Monk, 1994; Perkes, 1967-68). People who are highly intelligent and have strong desire to teach cannot do so effectively when they are to teach learners with special needs without training. Qualifications in pedagogical and subject matter knowledge are what prospective instructors need to succeed in the classroom. These are evident in the programme instituted in United State of America known as Teach for America (TFA).

This is an academic intervention programme designed to recruit brilliant university graduates to teach in low performing schools. Notwithstanding the passion and the high Intelligent Quotient (IQ) levels of the graduates, they were unable to prepare their students adequately for their examination (Grady & Grady, 1991; Roth, 1993; Popkewitz, 1995; Texas Education Agency, 1993). Most of these graduates were disappointed for not being successful teachers. There was high attrition rate for the programme in subsequent years and those of the graduates who had the intention of having teaching as a career dropped out of the programme because the outcome was not the best. One of the major reasons why these graduates could not perform to satisfaction was because probably they were not trained on pedagogy and andragogy of teaching. These graduates found it difficult to transmit concepts they imbed painlessly to their students. The developers of this teaching programme unknowingly implemented the philosophy of 'bright person myth' of instruction, which holds that anybody can impart their knowledge to others without training. According to Goldhaber and Brewer (1997), mathematics teachers' professional mathematics training has significant effect on mathematics achievement scores of students. These trainings enable mathematics teachers to acquire new knowledge and skills in mathematics content and methodology that are implemented in the classroom by mathematics teachers (Borger & Tilleman, 1993; Cohen & Hill, 2000).

It is in line with this that Hedge, Laine, and Greenwald (1994) reported in their meta-analysis research that, there exist a direct relationship between learner learning outcomes, teacher qualifications and practices. "Teacher academic qualifications" are the taught courses, academic authorizations, expertise and academic exposures an instructor brings to the classroom. In this research study, teacher academic qualification is operationalised to include knowledge in mathematics content, mathematics pedagogy, teacher professional development and level of preparation. Holding all variables constant, mathematics teachers with higher academic qualification should have their students achieve better in mathematics.

According to Tchoshanov (2010), mathematics teachers with inadequate mathematical knowledge teach mathematics concepts procedurally whilst those with higher mathematical knowledge tend to teach conceptually.

Notwithstanding the above, Wenglinsky (2000) reported that there is no discernable variation between graduate mathematics teachers and those with higher degrees with respect to their effectiveness in the classroom. Mathematics teachers with higher qualification in mathematics were found be less effective than mathematics teachers with lower levels of qualification (Askew, Brown, Rhodes, Johnson & William, 1997). Research by Darling-Hammond and Ball (1998) and Monk (1994) indicated that there is a minimum qualification in mathematics below which mathematics teacher will not be effective. Teacher academic qualification differ between low-achieving countries and high-achieving countries.

Teachers' Experience in Mathematics Teaching and their Mathematical Values

Ghana has instituted a formal educational policy on the induction into the teaching profession for beginning teachers after their teacher education training known as Ghana National Teachers' licensing examination (GNTLE). Ghana National Teachers' licensing examination is an event which does not adequately give the needed support to new teachers on the field. It is in the light of this that Keengwe and Boateng (2012) indicated that new educators begin real classroom instruction without receiving any kind of initiation. Newly trained educators are made to figure out how to succeed in their novel career by themselves (Mereku, 1998). As a result, young educators who had previously begun their careers with vigor and optimism must now deal with challenges in their classrooms and the schools as a whole (Manuel, 2003), with others returning to the instructional practices they had previously learnt.

Studies show that there is a direct relationship between mathematics teachers' instructional exposure and student academic attainment in mathematics (Buddin & Zamarro, 2010; Toropova, Johansson & Myrberg, 2019). Experienced mathematics teachers have developed network of mathematics knowledge schemas (Jacobs, Lamb & Philipp, 2010). According to Hill, Rowan and Ball (2005), mathematics teachers with more years of teaching utilize their expertise in mathematics to influence learners' mathematics achievement. They argued that mathematics teachers with advanced expertise in mathematics tend to achieve better with their students than their counterparts with less advanced expertise.

The years of experience of mathematics teachers remain a critical element in identity formation of learners' values. In contrast, other researchers have reported that mathematics teachers' classroom teaching effectiveness should not be premised on their teaching experience (Martin, Mullis, Gregory, Hoyle & Shen, 2000; Wenglinsky, 2000). Abbot-Chapman, Hughes, Holloway and Wyld (1990) have reported that experienced mathematics teachers tend to be promoted to administrative and managerial levels and therefore their experience is likely to be felt in those positions in the school but not in mathematics teaching.

Chapter Summary

The literatures which have been reviewed above show that 'values' and 'valuing' studies in mathematics learning is a novel field of study. Meanings of 'values' are diverse and ranges from general through subject specific values to values in mathematics education (see for e.g. Bishop, 1999; DeBellis & Goldin, 2006; FitzSimons, Seah, Bishop & Clarkson, 2000b; Seah &

Anderson, 2015). Values emanate from our culture and society and that general educational values are the ones acquired through our day-to-day interactions within the environment. From the literature above, general educational values are the values society hands over from one generation to the other and are applied in everyday life including school subjects. However, mathematical and mathematics educational values are those that apply directly to mathematics as a discipline. Even though literature has it that values are important in students' mathematics learning and school mathematics curriculum delivery (Bishop, 2008), it still remains a grey area in mathematics education research particularly what mathematics educators value in their students' mathematics learning and what their learners' themselves value in their own mathematics learning especially in a single study. In this regard, there is a pressing need to shed some light on what mathematics teachers at the pre-tertiary level of education in Ghana and their students' value in mathematics learning. It is this gap in literature that the study sought to address.

CHAPTER THREE

RESEARCH METHODS

Overview

The study explored what teachers and students in a metropolis in the southern part of Ghana value in mathematics learning. It found out if there is value alignment between mathematics instructors and their learners as far as mathematics learning is concerned.

This chapter focuses on how the study was set up and executed. It specifies the research design used and the population from which the sample was taken. Additionally, it details the procedures used to select the study's sample from the population as well as the sample size. The chapter also describes the type of research instrument used. It states how reliable the instrument was. It details the location and processes of pilot test. The chapter concluded by discussing the procedures used for data collection, data processing, and data analysis. The chapter ended with the summary of the major issues that have emerged.

Research Design

The decision to choose a research design and methodology for a research study is underpinned by the type of paradigm selected for the research. In broad terms, paradigms are point of views or philosophies (Perera, 2018). Contradictory value systems or belief systems that influence and direct the choices that researchers make are known as paradigms in research (Tashakkori & Teddlie, 1998). Perera opined that research paradigms are a collection of shared views and agreements among scientists about how to understand and handle challenges. Bryman (2012) identifies research paradigm as a belief system.

It establishes and directs what needs to be researched, how the study ought to be carried out, and how the findings ought to be analysed in a particular discipline. Research paradigms can be grouped into interpretivism, positivism, post positivism, constructivism, critical theory and pragmatism. This study adopted positivism research paradigm. Hirschheim (1985) has argued that knowledge claims that are not grounded in positivist thought are not valid and not scientific. Positivist research paradigm relies on quantitative methods (Park, Konge, Artino, 2020). The research designs or methodologies that lend themselves to positivist research paradigm include survey (longitudinal, cross-sectional and trend) research methodology, experimental methodology, quasi-experimental methodology, randomized control trials methodology, correlational methodology and causal comparative methodology (Kivunja & Kuyini, 2017).

Based on the nature of the research questions the researcher aimed to address, a descriptive cross-sectional survey research design was adopted. A cross sectional design is used when a researcher makes use of different categories of people (Enon, 1998). Thus, the researcher surveyed what teachers and students at different educational levels (Primary, JHS and SHS) value in mathematics learning. Descriptive cross-sectional survey study offers a numerical or quantitative description or exploration of patterns, behaviours, or viewpoints of a group of people by looking at a section of them. It entails using questionnaires to gather data with the intention of extrapolating from a sample to the general population. It enabled the researcher to have access to large volumes of data from a large demographic sources in comparatively limited period of time. Its versatile nature with respect to non-technological (face-to-face) and technological modes (on your website, email it to individuals, or on social media platforms) in which data is collected from respondents makes it an attractive option for researchers.

The choice of descriptive cross-sectional survey was also occasioned by the fact that this design allows for data to be collected from a sizable samples and generalization made over the entire population. Thus, a sample of 177 mathematics educators and 1263 learners from the population of instructors and learners in the selected metropolis is quite large relative to teacher and student population. Once more, data was gathered from mathematics instructors and learners of different grade levels. (Primary school, JHS and SHS) at a particular period of time. Usually, questionnaires are used to determine and report on people's behaviours, ideologies, thoughts, and/or attitudes (Vogt, Vogt, Gardner & Haeffele, 2014). The researcher intended to explore other affective constructs such as values and valuing in exploring what educators and their learners value in mathematics learning.

Study Area

According to Yin (1989), the study area selected by the researcher should contribute and provide relevant data for the study. Other researchers select a study area based on some considerations. For instance, studies by Audet and D'Amboise (2001) and Yin (2009) showed that accessibility, geographical proximity and convenience account for a choice of a study area of research. Whilst some value studies have been conducted in some parts of Ghana, Western region was randomly selected among the regions that value studies have not been carried out. for the study. A district in the region was randomly selected for the study using computer generated numbers. Figure 2 below shows the map of the study area.

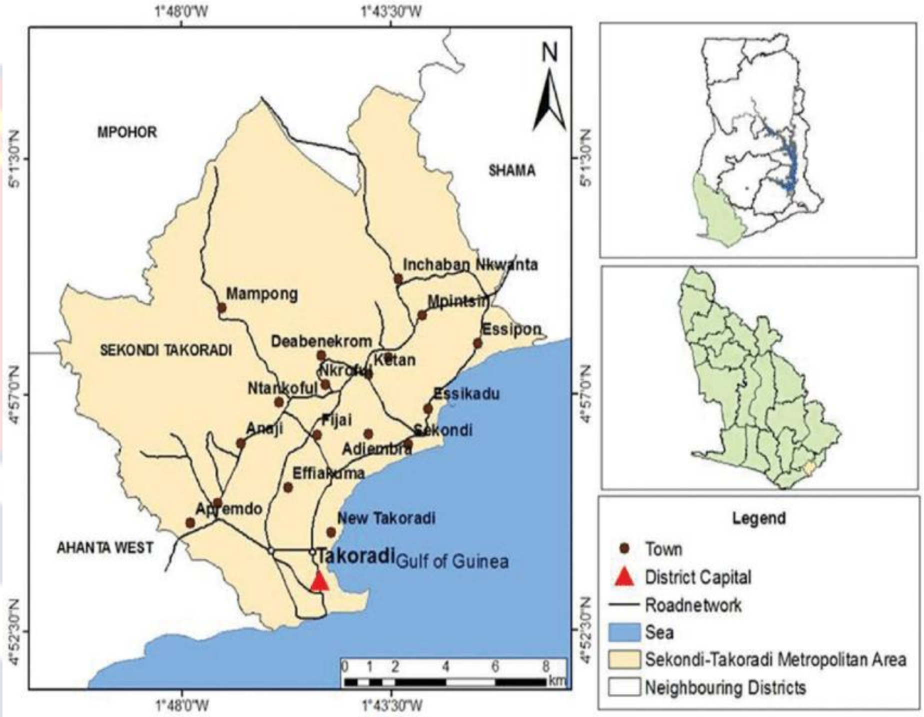

***Figure 1*: Map of the study area**

The metropolis is made up of rural and urban communities where the schools are located. Almost all the SHSs are located in the urban communities of the metropolis whist Primary and JHSs are scattered across rural and urban communities of the metropolis. Notwithstanding varying school contexts (rural/urban) at the basic schools in the metropolis, SHSs in the metropolis are also a mixture of varied school type (single sex male, single sex female and co-educational schools) and school category (category A – C). The school categories are determined by Ghana Education Service based on infrastructure and past academic performance records of the schools on WASSCE. The varied school context, school type and school category are likely to go a long way to provide the researcher with divergent and heterogeneous results and ensure a better understanding and perspective to the problem at hand. Also, the researcher selected this metropolis out of convenience because that is where the researcher lives and works.

Schools in remote communities of Ghana lack good facilities and infrastructure. As a result, they have inadequate qualified teachers, less school enrollment, lack resource materials for learning and instruction and textbooks. The reverse of this is witnessed in educational institutions in the urban areas (Siaw, 2009). Siaw further argued that governments approach to bridge the gap in achievement between rural and urban schools has still gone in favour of the schools in the urban areas.

Population

The population for the study comprised all the upper primary, JHS and SHS students in a metropolis in the southern part of Ghana as well as their mathematics teachers. The metropolis has 1,215 teachers and 55,346 students at the pre–tertiary public schools.

Table 3 shows the distribution of enrolment of students and staff strength of mathematics teachers in the sampled schools in the selected metropolis in the 2021/2022 academic year.

Table 3: Enrolment of Students and Staff Strength of Mathematics Teachers in the Sampled Schools in the Selected Metropolis - 2021/2022

Level	Number of Schools	Student Enrolment	Number of Teachers
Primary	12	874	36
JHS	15	816	34
SHS	7	12,562	107
Total	34	14,252	177

Source: Ghana Education Service (GES), 2021.

Sampling Procedure

A sample of 1263 students and 177 teachers were selected for the study. It comprised 336 upper primary pupils and 36 teachers, 325 JHS students and 34 teachers and 602 SHS students and 107 teachers. The teachers and the students were selected from 34 out of the 114 public basic and SHSs in the metropolis. The number of teachers and students involved in the study were selected from more than 20% of the total number of schools in the metropolis selected for the study. This proportion is within the acceptable proportion of a given population that can be sampled for a survey (Krejcie & Morgan, 1970).

The Primary schools and JHSs were grouped based on urban and rural setting of schools. The grouping of the schools into urban and rural context is in line with GES approved classification of basic schools in Western Region (GES, 2010). The study adopted a multistage sampling procedure to select participants. The Primary schools and JHSs in the metropolis selected for the study were put into four strata. Urban Primary schools represented one stratum, Urban JHSs constituted another stratum, Rural Primary schools formed a stratum and Rural JHSs formed the last stratum. A stratified random sampling technique was used to select schools from each stratum. Table 4 shows the distribution of number of Primary schools and JHSs with their corresponding sampled schools according to urban and rural location of schools in the selected metropolis.

Table 4: Distribution of Number of Primary and JHS with their Corresponding Sampled Schools According to School Setting in the Selected Metropolis

School Setting	Number of schools	Number of schools sampled
Urban Primary	30	7
Urban JHS	35	9
Total	65	16
Rural Primary	17	5
Rural JHS	21	6
Total	38	11

Simple random sampling, specifically, computer generated numbers with proportional allocation of samples was employed to select schools and students from each stratum. To ensure representativeness, the researcher employed proportional allocation of samples such that strata with higher number of schools and students had more representation than those with fewer numbers. 200 and 136 primary school pupils were randomly selected from 7 urban and 5 rural schools respectively in the metropolis to participate in the study. 21 and 15 of the teachers from the urban and rural schools respectively who teach mathematics in these schools were purposively selected to partake in the study. On the other hand, 180 and 145 JHS students were randomly selected from 9 urban and 6 rural schools respectively in the metropolis to take part in the study. 20 and 14 of the teachers from the urban and rural schools respectively who teach mathematics in these schools were purposively selected to partake in the study. All the SHSs are located in the urban settings of the metropolis. In this regard, SHSs were put into strata according to school type (co-educational, single sex female and single sex male schools). Co-educational schools were put in one stratum, single sex female schools constituted a stratum and single sex male schools formed another stratum. By putting the first cycle schools into strata based on urban and rural context and the second cycle schools based on school type ensured diverse representation of participants for the study (Opoku-Asare & Siaw, 2015). Table 5 shows the number of SHS and those randomly selected in a metropolis of the Western region of Ghana according to school type.

Table 5: Distribution of Number of SHS and Number of SHS Sampled According to School Type in the Selected Metropolis

School Type	Number of schools	Number of schools sampled
Co-educational	7	5
Single sex female	2	1
Single sex male	2	1
Total	11	7

Simple random sampling technique with proportional allocation of samples was used to select schools and students from each stratum. Monette, Sullivan and DeJong (2002) stated that "the easiest method for selecting samples by chance is simple random sampling (SRS), whereby every person in the group has the same likelihood of being chosen" (p. 136-137). SRS handles the target population as a single entity.

271 students from co-educational, 158 students from single sex girl and 173 students from single sex boy schools were randomly selected. The corresponding number of their teachers (45 from co-educational schools, 26

from single sex female schools and 36 from single sex male schools) were purposively selected to take part in the study from the seven selected SHSs.

With regards to SHS, all the mathematics instructors in the sampled schools participated in the research study. For the JHS mathematics teachers, all the mathematics teachers from the sampled schools took part in the study. Also, for the Primary school teachers, all Primary 4, 5 and 6 mathematics teachers in the sampled schools took part in the study. The researcher used purposive sampling technique to select all the mathematics teachers in the sampled schools (Primary, JHSs and SHSs). This was because the researcher intended to find out if what teachers value align with what students' value in mathematics learning.

Upper primary pupils, particularly primary 6 pupils in the sampled schools were the partakers for the study at the primary school level. However, primary 4 and 5 pupils were not included in the study because they may not be ready for responding to all English Language questionnaire. Although the language policy of schools in Ghana requires the use of English Language as a medium of Teaching (MoT) from upper primary school onwards, Davis (2010) has argued that primary 4 and 5 pupils in schools in Ghana may not have good mastery of technical language of mathematics. Unfortunately, the WIFI questionnaire contains a lot of mathematical terminologies which the respondents should understand before they could respond to them well. For primary 6 pupils, all the items in the students questionnaire that contained technical mathematics terms were explained to them by the researcher. On the part of the JHS students, only JHS 1 and 2 learners took part in the study. This was because JHS 3 learners were candidates who were busily making preparations toward their 2021 national examinations (Basic Education Certificate Examination [BECE]) and so letting them be part of the study would have bothered them in one way or the other. In the same way, the SHS 3 students did not partake in the research. This was because during the period of data gathering by the researcher, they had commenced their WASSCE and so using them in the study was inappropriate. In all the schools selected for the study, the student participants were selected from intact classes by the use of simple random sampling. In all, 177 mathematics teachers (124 males and 53 females) and 1263 students (682 males and 581 females) across the three pre-tertiary levels of education answered the questionnaires.

Data Collection Instrument

The study's data was gathered using quantitative instruments (questionnaires). Questionnaires helped the researcher to carry out the study with large samples, statistically analyse and interprete the findings so that meaningful conclusions could be drawn. The research instruments were adopted and adapted based mainly on the research questions and the purpose of the research. 1263 students from the selected pre-tertiary state-owned schools in a metropolis in the southern part of Ghana completed the "What I Find Important" (WIFI) in my mathematics learning questionnaire. Also, 177 teaching staff from the selected schools also completed the modified version of the students' questionnaire.

The original WIFI project is a multi-national research study with participating continents including Africa, Europe, Asia, Australia and America (Seah, Davis & Carr, 2017b). Out of the five continents that took part in the study, 20 countries participated and Ghana was one of them. The WIFI questionnaire was initially developed in English Language. Other researchers have translated it to other languages such as Japanese, Turkish and Chinese to suit the language policy of teaching in their respective countries. In line with the language policy for teaching in Ghana, the researcher wrote the questionnaires for teachers and students in English language (Obeng, 1997). The researcher designed the questionnaires in two different forms namely Teachers' Questionnaire (TQ) and Students' Questionnaire (SQ) (see Appendices A and B respectively). Questionnaires are, without a doubt, a major source of getting data for a research project. The use of questionnaires for collecting quantitative data helps to reach out to many respondents and with little difficulty. Thus, survey instrument typically, though by no means exclusive, rely on large scale sampling which aligns with the sample size of the present study (Seliger & Shohamy, 1989).

However, according to Richards and Schmidt (2002), when drafting a questionnaire, the researcher must make sure it is dependable, unambiguous and valid. For greater preferred sides of the aforementioned cautioned qualities, closed-ended questionnaire was used. Closed-ended questionnaire provides the researcher with numerical data and are more efficient thereby making analysis easier (Seliger & Shohamy, 1989). Gillham (2000) supported the idea of using closed-ended questionnaires in collecting quantitative data by admitting that "data from open-ended questionnaires are not easy to analyse and report the findings because of their qualitative nature" (p. 5). It is safe to say that the advantages of appropriately developed questionnaires have been outlined in literature (Fraenke & Wallen, 2003; Gillham, 2000; Muijs, 2004; Nunan, 1999). The use of a questionnaire saves time and guarantees the privacy and confidentiality of participants (ibid.). In comparison to interviews, questionnaires can be distributed to a broad spectrum of individuals. One drawback of questionnaires is that, unlike other data collection tools, they cannot be linked to specific individuals.

Teachers' Questionnaire (TQ)

The researcher modified the Students' Questionnaire (SQ) to obtain the Teachers' Questionnaire (TQ). TQ is divided into four sections: A, B, C, and D. In all, the TQ is semi-structured with 90 items. Section A elicited demographic information about the instructors which comprises their sex, teaching experience, the grade level they teach and the school type and category, professional training and qualification, rank in GES and why they are teaching mathematics.

The section B comprises 64 items requiring respondents to select one out of the 5 – point Likert – scale item options ranging from Absolutely Unimportant = 1 to Absolutely Important = 5. The items were made up of mathematical and mathematics educational values. The respondents were requested to specify how useful mathematics instructional actions which include *whole – class discussions* (see item 7), *doing a lot of mathematics work* (see item 37), *hands – on activities* (see item 52) etc. are to them. A five-point Likert scale response format (1= Absolutely Unimportant, 2 = Unimportant, 3 = Neither Important nor Unimportant, 4 = Important, 5 = Absolutely Important) was utilized to analyse all the questionnaire items. The five-point Likert scale response provides the opportunity for the participants to have choices that go beyond "important" or "unimportant" to a statement. It provides the opportunity for respondents to select neutral responses (Nworgu, 1991). On the contrary, the neutral option rated as 3 in the five-point Likert scale response format does not provide logical and efficient data analysis (Swan, 2006).

Section C consists of 10 paired items of phrases (which are not opposite in meaning) where the participants were requested to specify the degree to which their valuing in their mathematics learning tilts towards one of the bi-polar items.

Section D is made up of four items. It looks at what the teacher sees as the most important things arranged in descending order, what their students need to do at school in order to do well in mathematics and the justification for the order (see appendix A).

Students' Questionnaire (SQ)

The researcher adopted the original WIFI questionnaire. The SQ consists of four sections: A, B, C and D. In all, the SQ is semi-structured with 84 items. Section A solicited general information about the learners. They included their sex, age, current level of education and their school type. The section B comprises 64 items requiring respondents to select one out of the 5 – point Likert – scale item options ranging from Absolutely Unimportant = 1 to Absolutely Important = 5. The items were made up of mathematical values and mathematics educational values. The respondents were asked to indicate how important the value items *investigations* (see item 1), *problem - solving* (see item 2), *connecting mathematics to real life* (see item 12), *teacher helping me individually* (see item 41) etc. are to them in the learning of mathematics.

A five-point Likert scale response format (1 = Absolutely Unimportant, 2 = Unimportant, 3 = Neither Important nor Unimportant, 4 =

Important, 5 = Absolutely Important) was utilized to analyse all the questionnaire items .

Section C consists of 10 paired items of phrases exploring students' views on how their valuing in mathematics learning inclines towards one of the bi-polar pronouncements.

Section D is made up of four items. They consist of contextualised questions which aim at arousing values-related responses from the students (see appendix B).

Because the study was interested in exploring what educators and learners value in mathematics learning, only the items in Sections A and B of the adapted and adopted WIFI questionnaire were used for both the TQ and SQ respectively to gather data for this study. A widely accepted valid practice and pragmatic research methodology is to shorten an existing questionnaire by choosing sample items devoid of sacrificing the overall scope and content of the questionnaire. The technique is often utilized in research on values, when "measures are all lengthy and require relatively long time to complete" (Roccas, Sagiv, & Navon, 2017, p. 27).

Validity and reliability of the instruments

Bogdan and Biklen (1992) defined reliability as a match between data obtained by an inquirer and what actually occurs on the ground. Reliability also refers to the degree upon which data collected from a respondent is constant and stable over time (Creswell, 2012). Such respondent provides the same answer to the same questions asked at different times. In the context of assessment, reliability refers to the extent upon which learners' assessment outcomes are consistent when: a) they complete different but equivalent or alternative tasks on the same or different occasions, b) they complete the same task(s) on two different occasions, c) two or more assessors score their performance on the same task(s) (Amedahe & Asamoah-Gyimah, 2005). Reliability and validity are two concepts that go hand in hand and need each other. However, it is essential to point out that although reliability is important, it is insufficient condition for validity. The validity of an instrument for a study evaluates the degree to which it measures what it meant to measure (Robson, 2011; Thatcher, 2010). It ensures that the results of a study are truthful (Mohajan, 2017) and that the concepts in the study are properly measured (Pallant, 2011). Mohajan argues that validity plays two essential roles namely credibility and transferability of research instrument. The credibility of the instrument ensures legitimacy of the results of the study and this is observed in sampling, data gathering as well as analysis of data. The transferability of the instrument on the other hand determines the degree to which the findings of the research is extended to other groups of interest.

The instruments for the data collection were scrutinised by two Professors who are experts in mathematics education in the Department of Mathematics and ICT Education, University of Cape Coast to ensure content and construct validity of the instruments. The researcher also requested the services of his PhD course mates who critiqued and scrutinised the instruments to ensure face and content validity. The work done by the Professors enabled the researcher to modify the SQ to suit the teachers who teach mathematics at the pre-tertiary level of education in Ghana. Based upon these, the researcher made few amendments into the two questionnaire instruments (TQ and SQ). The updated questionnaires were piloted tested across the pre-tertiary educational levels namely primary school, JHS and SHS.

Bless, Higson-Smith and Kagee (2006) defined pilot study as mini research done before a large-scale study to find out whether or not the research methods, sampling technique and process, instruments for gathering data and processing and analysis of data are suitable and sufficient. Doody and Doody (2015) shared in the idea of Bless et al. but focused on the participants of the pilot study. They proposed that "a pilot study is a small-scale version of a planned study conducted with a small group of participants similar to those to be recruited later in the larger scale study" (p. 1074). Piloting the instrument enables the researcher to correct mistakes and rewrite items whose errors and ambiguity were not noticed in the course of the questionnaire design. It is an important mechanism to check the reliability and validity of the instruments and to find out if the data gathered for the pilot research can help answer the research questions (Feng & Yamat, 2019). According to Luiz (2017), pilot research enables the researcher to predict the practicability or otherwise of the actual research. This helps to avoid wastage of time and resources.

The pilot study was carried out in a metropolis in the southern part of Ghana. The metropolis was chosen for the pilot study because it shares resemblance with the metropolis selected for the main study on two grounds. First, the basic schools in the metropolis for the pilot study have rural and urban contextual characteristics. Second, the SHSs in the metropolis for the pilot study have varied school type (co-educational, single sex female and single sex male schools). These go a long way to bring into perspective, varied characteristics in the data for the pilot research as envisaged in the main research. Fifty (50) pre-tertiary instructors who teach mathematics comprising six (6) primary school teachers, four (4) JHS teachers and forty (40) SHS teachers from the metropolis participated in the pilot study.

Also, 150 of the learners who are taught by the selected mathematics educators consisting of forty (40) primary, fifty (50) JHS and sixty (60) SHS students were selected randomly to partake in the pilot research. The pilot study provided the opportunity for the researcher to modify ambiguous items, delete errors in the questionnaires and eliminate unclear items. This ensured that the actual data collection had responses that were valid and reliable.

A Cronbach alpha reliability coefficient of 0.947 was obtained for the pilot study for the items in the TQ and that of the SQ was 0.820. In the actual data collection, Cronbach alpha reliability coefficient for the instruments were obtained as 0.914 for TQ and 0.922 for SQ. According to Tavakol and Dennick (2011), if the relationship among the items in the questionnaire is strong, the Cronbach apha reliability coefficient will be close to one (1) and close to zero (0) if it is low. More specifically, a Cronbach alpha value exceeding 0.90 is considered as exceptionally good, above 0.80 is considered as good and above 0.70 is considered generally as satisfactory and acceptable (Nunnally, 1978).

Data Collection Procedures

Prior to the start of the data gathering, the researcher applied for and obtained ethical approval from the University of Cape Coast's Institutional Review Board (IRB) (see Appendix D). Permission was subsequently sought from the Metropolitan Education Directorate to collect data from instructors and learners in the public pre-tertiary educational institutions in the metropolis through an open introductory letter from the Department of Mathematics and ICT Education, University of Cape Coast (see Appendix C). After permission was granted, the researcher continued to seek permission from the Heads of the various schools whose mathematics teachers and students participated in the study through the same introductory letter from the Department of Mathematics and ICT Education, University of Cape Coast. The researcher also requested for the consents of the teachers and students before participating in the study. The partakers of the study were given assurance of confidentiality. No name was required on the questionnaires. They were informed of the study's goals, potential advantages of the study to the respondents in particular, and the benefits for mathematics teaching and learning in general.

Generally, there are several methods for distributing questionnaires. Fink (2013) outlined the methods to include; over the internet, by post, face-to-face, and through the telephone. For this study, the researcher used the face-to-face approach of questionnaire distribution. Though, each method has its own limitations, some are identified to have more flaws than the others. The face-to-face method of administering questionnaire was used because; (a) the rate of return is high, (b) the researcher is personally involve to monitor and address concerns, (c) unclear information can be explained instantly by the researcher, and (d) the researcher is cognizant of the conditions under which the questionnaires were completed.

The researcher administered the questionnaires himself. The data collection took place from 22nd October, 2021 to 9th December, 2021.

Data Processing and Analysis

The data obtained from the answers to the items in the questionnaires by the study participants were coded and analysed by the help of Statistical Package for Social Sciences (SPSS) software version 23. As a means of reducing data entry errors, the researcher diligently entered the data himself. The researcher further run frequency distribution to check if each variable has value range within the Likert scale range of 1-5. Those that were not within the stipulated range were re-entered correctly.

Descriptive and inferential statistics were used to analyse the data for the study. Specifically, rank means and standard deviations were used to describe what pre-tertiary mathematics teachers find important in their students' learning of mathematics and what the students also find important in their own mathematics learning. These descriptive statistics were further used to determine how the components in the Principal Component Analysis (PCA) were valued by mathematics teachers and their student as far as their mathematics learning is concerned. PCA, specifically Exploratory Factor Analysis (EFA) and one-way Multivariate Analysis of Variance (MANOVA) were the inferential statistical techniques used to analyse the data collected for the study. PCA was used to put the value items into variable components for analysis. EFA was adopted for the study because exploring values of teachers and students in mathematics across grade levels at the pre-tertiary space is a gray area in values research. Thus, in an area of research that has not been thoroughly investigated, EFA is the more appropriate technique to employ. Also, MANOVA was used to find out if statistically significant differences exist in the various variable components across the educational levels (Primary school, JHS and SHS) and sex (male and female) of instructors and learners.

To enable the researcher find answers to research questions one, two and three: “What do primary, Junior High School (JHS) and Senior High School (SHS) teachers’ value in their students’ mathematics learning?”, “What do primary, Junior High School (JHS) and Senior High School (SHS) students’ value in their mathematics learning?” and “How similar or different are values of teachers and their students in mathematics learning?”, a PCA was used for the analysis. The questionnaire items were examined using PCA with a varimax rotation and Kaiser normalization. Varimax rotation provides a clearer separation of factors and it is the most widely used orthogonal rotation method (Hair, Black, Babin & Anderson, 2019). A cut-off criterion was set at 0.45 at 5% level of significance for the PCA.

Also, to find answers to research questions four to nine: “What is the effect of grade level teachers teach (primary, JHS and SHS) on the attributes they value in students’ mathematics learning?”, “What is the effect of grade level of students (primary, JHS and SHS) on the attributes they value in their mathematics learning?”, “What is the effect of sex (male and female) on valuing in students’ mathematics learning among mathematics teachers?”, “What is the effect of sex (male and female) of students’ on the attributes they value in their mathematics learning?”, “What is the effect of teachers’ academic qualification on the attributes they value in students’ mathematics learning?”, “What is the effect of teachers’ teaching experience on the attributes they value in students’ mathematics learning”, a one-way MANOVA was carried out.

Suitability of the data for PCA

Prior to the data analysis, the data collected from the adopted and adapted WIFI questionnaires were checked. According to Kaiser (1970), factorability of correlation matrix is assumed if the Kaiser–Meyer–Olkin (KMO) measure of sampling adequacy is greater than 0.6 and the Bartlett's (1950) test of sphericity (BTS) is significant. Results from the data gathered showed KMO measures of sampling adequacy of 0.720 and 0.926 for Teacher Questionnaire (TQ) and Student Questionnaire (SQ) respectively. Bartlett's (1950) test of sphericity in each case was significant at the .000 level. Based on these, factorability of the correlation matrix was assumed. This confirms that the identity matrix of the questionnaire was valid and reliable and affirmed the suitability of PCA for the data.

Suitability of the data for MANOVA

For any statistical test, there are assumptions that need to be satisfied in order for the statistical test to be valid. Before MANOVA was carried out, the following tests were conducted and met the criteria for its assumptions: Sample size, test for normality, homogeneity of variance-covariance matrices (Box's M test) and independence of observations. Sample size assumption is satisfied by using the number of cells generated by combining the independent and dependent variables and the number of participants for the research. In this study, the three pre-tertiary educational levels (Primary school, JHS and SHS) represented the independent variables and three dependent variables for teacher valuing (Understanding - C1, Versatility - C2 and Achievement - C3) produced nine cells. Also, the cell sample sizes for the independent variables included 36 Primary school teachers, 34 JHS teachers and 107 SHS teachers who teach mathematics. The sample sizes are in agreement with the recommendation by Tabachnick and Fidell (2013) who opined that a sample size of at least 20 observations in each cell should ensure robustness. Also, on the part of the students, the three independent variables represent grade levels of students (Primary school, JHS and SHS) and seven dependent variables (Fluency - C1, Understanding - C2, Teaching materials/activities - C3, Connections - C4, ICT- C5, Feedback - C6 and Learning strategies - C7) produced twenty-one cells. The cell sample sizes for the independent variables of 336, 325 and 602 for Primary, JHS and SHS learners respectively are far more than the threshold sample size of 20 recommended by Tabachnick and Fidell. The large sample sizes of mathematics educators and students for the research as justified in literature make sample size assumption for data suitability for MANOVA satisfied.

Normality of the data can be tested using the statistical approach mainly skewness, kurtosis, Kolmogorov-Smirnov and Shapiro-Wilk. Ghasemi and Zahediasl (2012) indicated that normality of a data is determined by both pictorial, that is, graphical method (Histogram, Box and whisker plot and Normal Q-Q Plot) and non - pictorial forms, that is, statistical method (Skewness and kurtosis, Kolmogorov-Smirnov and Shapiro-Wilk). Notwithstanding Ghasemi and Zahediasl's approach of assessing normality, visual interpretation of using the graphical method to check normality is not reliable and does not guarantee a proof of normality of a data (Altman & Bland, 1995; Field, 2009; Oztuna, Elhan & Tuccar, 2006). They believe that using the visual method is the layman's way of assessing normality of a distribution. It is further argued that even within the statistical method of assessing normality, Kolmogorov- Smirnov (K-S) has lower power compared to Shapiro – Wilk (S-W) and that S-W is the finest alternative for testing the normality of a data (Thode, 2002). S-W was performed on the data and did not show evidence of non-normality ($W = 0.97,\ p = 0.20$). Box's M Test of Equality of Covariance Matrices was used to determine the assumption of homogeneity of variance- covariance matrices. The results of Box's M Test of Equality of Covariance Matrices show that $[Box's\ M = 27.371,\ F = 4.453,$ $df1 = 6,\ df2 = 62333.894,\ p = 0.14]$ the assumption for MANOVA has not been violated. Grade levels (Primary school, JHS and SHS) as well as the values teachers and their students hold in mathematics learning are observations that are independent and do not violate the assumption of independence of observations.

Scoring of the instruments

A Likert scale with five options format (Absolutely Unimportant, Unimportant, Neither Important nor Unimportant, Important, Absolutely Important) was used to measure what teachers who teach mathematics at the pre-tertiary level in Ghana find important in their students' mathematics learning and what the learners also find important in their own mathematics learning. The items (value attributes) in the questionnaires were scored as shown below:

Response Options	Score
Absolutely Unimportant	1
Unimportant	2
Neither Important nor Unimportant	3
Important	4
Absolutely Important	5

Using mathematical approximation as a bench mark for the interpretation of the responses on the Likert scale for this study, a mean score of 1.00 - 1.49 indicates absolutely unimportant statement, an average score of 1.50 – 2.49 reports an unimportant statement, a mean score of 2.50 – 3.49 shows neither the statement is important nor unimportant in the learning of mathematics, a mean score of 3.50 – 4.49 is seen as important and lastly, a mean score of 4.50 – 5.00 reveals an absolutely important statement with regards to teachers' and students' valuing in mathematics learning.

Chapter Summary

The study employed a descriptive cross-sectional survey research design with positivist philosophical research paradigm. The study adopted a multistage sampling technique to select participants. Stratified, simple random and purposive sampling techniques were employed to select mathematics educators and their students across primary, JHS and SHS according to school context (rural and urban) and school type (co-educational, single sex girl and single sex boy schools) respectively. Primary schools and JHSs were put into school context and SHSs were also put into school type for the purposes of stratification. Seven SHSs from three school type (co-educational, single sex girl and single sex boy schools) and twenty-seven basic schools from two school contexts (rural and urban) in a selected metropolis in the southern part of Ghana were used for the study. One thousand, two hundred and sixty-three pre-tertiary students and their teachers (177) participated in the research. Specifically, 107 SHS mathematics teachers and 602 students across school type, 34 teachers who teach mathematics at the JHS and 325 of their students and 36 primary school teachers and 336 of their students from urban and rural schools participated in the study.

A WIFI questionnaire was used to collect data from students. A modified version of students' questionnaire was also used to collect data from mathematics teachers. Rank means and standard deviations were used to describe what mathematics teachers find important in students' mathematics learning and what the learners also find important in their own mathematics learning. Descriptive statistics (mean and standard deviation) were used to determine how the components in the PCA were valued by mathematics teachers and their students. PCA and MANOVA were the inferential statistical techniques used for the study. PCA was used to put the value items into components for analysis. MANOVA was used to determine whether or not statistically significant differences exist in the various components across the grade levels (Primary school, JHS and SHS), sex (male and female), academic qualification and teaching experience.

A study of this nature could not have been undertaken without the researcher revising what he intended to achieve as the study progressed. Notwithstanding the fact that the study's sample size was quite large (177 mathematics teachers and 1263 students) to support generalization of the findings, it is germane to say that the research was carried out in only one metropolis out of one hundred and sixty-one Metropolitan, Municipal and District Assemblies (MMDAs) across the sixteen regions of Ghana. Therefore, the findings of the study may not portray the actual situation in the country with respect to values of instructors and learners in mathematics learning.

CHAPTER FOUR

RESULTS AND DISCUSSION

Overview

The study explored what teachers who teach mathematics at the pre-tertiary level in Ghana value in their students' mathematics learning and what the students' value in their own mathematics learning. The study also explored what pre-tertiary students' value in their mathematics learning across grade levels and sex and what mathematics teachers who teach at the pre-tertiary level of education value in mathematics learning across grade levels, sex, academic qualification and teaching experience.

In this chapter, the results of the study are presented and discussed in relation to the nine research questions outlined in Chapter one. The presentation of the results of the study starts with the demographic characteristics of the respondents since the study explores values across grade levels, sex, academic qualification and teaching experience.

Demographic Characteristics of the Research participants

The demographic characteristics of pre-tertiary mathematics teachers who were sampled for the study are presented in Table 6. This includes their sex, age, grade level they teach, the type of school they teach, school category, number of years of teaching mathematics, their qualification as well as their rank in GES.

Table 6: Demographic Characteristics of Teacher Participants

Variable	Category	Frequency	Percentages (%)
Sex			
	Male	124	70.06
	Female	53	29.94
	Total	**177**	**100.00**
Age (in years)			
	33 and below	82	46.33
	34-44	72	40.68
	45-55	20	11.30
	Above 55	3	1.69
	Total	**177**	**100.00**
Grade Level			
	Primary	36	20.33
	JHS	34	19.21
	SHS	107	60.45
	Total	**177**	**100.00**
School Type			
	Co-educational	115	64.97
	Single sex female	26	14.69
	Single sex male	36	20.34
	Total	**177**	**100.00**
School Category			
	Category A	53	47.32
	Category B	35	31.25
	Category C	24	21.43
	Total	**112**	**100.00**
Years of teaching mathematics			
	5 years and below	84	47.46
	6 – 10 years	41	23.16
	11 – 15 years	32	18.08
	16 – 20 years	13	7.34
	Above 20 years	7	3.95
	Total	**177**	**100.00**
Academic Qualification			
	Teachers' Certificate A	2	1.13
	Diploma in Basic Education	71	40.11
	Bachelor Degree in Basic Education	28	15.82

Table 6:cont'd

	Bachelor Degree (Math) with teacher training	51	28.81
	Bachelor Degree (Math) with no teacher training	8	4.52
	Master Degree (Math) with teacher training	14	7.91
	Master Degree (Math) with no teacher training	3	1.69
	Total	**177**	**100.00**
Rank in GES			
	Senior Superintendent	68	38.42
	Principal Superintendent	48	27.12
	Assistant Director	56	31.64
	Director	5	2.82
Total		**177**	**100.00**

The results from Table 6 show that out of the 177 pre-tertiary teachers who teach mathematics and participated in the study, $70.06\%\ (n = 124)$ were males and $29.94\%\ (n = 53)$ were females. The number of male mathematics teachers who participated in the study were more than twice the number of female teachers who took part in the study. This confirms the old Adamic reality that females are underrepresented in Science, Technology, Engineering and Mathematics (STEM) related programmes as they climb up the educational ladder (Hanson, 1996). Also, Joensen and Nielsen (2013) opined that the more advanced the mathematics course becomes, the greater the proportion of men enrolled. The reduction is as a result of female students switching to other academic programmes or dropping out of school entirely. The age range of the pre-tertiary mathematics teachers who participated in the study was between 33 years and below and 60 years inclusive. This suggests an active working age period for workers in Ghana. The percentage of SHS mathematics teachers who responded to the questionnaire was 60.45% $(n = 107)$. A high proportion of mathematics teachers at the pre–tertiary educational level (64.97%) who responded to the questionnaire teach mathematics at co-educational schools. This is because in the metropolis, there are fewer number of single sex male and female schools relative to co-educational schools. The few single sex schools in the metropolis are SHSs. This implies that all the primary and JHS selected were co-educational institutions which accounted for high number of respondents from co-educational schools. Also, for the teachers who teach mathematics at the various categories of SHS and participated in the study, majority of them 47.32% $(n = 53)$ teach in category A schools. This is not surprising because Category A schools run a lot of academic programmes and have infrastructure to admit more students with their corresponding number of teachers compared to category B and C schools. This phenomenon is affirmed by the teacher-student ratio policy of 1 teacher to 35-40 students by GES. Again, 88.70% $(n = 157)$ and 11.29% $(n = 20)$ of the teachers who participated in the study have taught mathematics for 15 years or below and above 15 years respectively. Promotions in GES are based on long-service. It is possible that majority of these teachers who have taught mathematics for more than 15 years have been promoted to administrative positions as School Heads or Assistant School Heads with few of them directly involved in the teaching of mathematics.

It is also possible that many of those who have taught mathematics for more than 15 years have left GES to teach at the tertiary level of education or have left the teaching profession entirely (Cobbold, 2010). The occurrence of this phenomenon is parallel to what Smith (2014) reported that the graph of teacher attrition is U-shaped with the maximum points occurring at teachers who have been in the teaching profession for less than six years.

Majority of the teachers 40.11% $(n = 71)$ who participated in the study hold Diploma in Basic Education (DBE) Certificate. This is because at the time they were trained as teachers, Diploma was the minimum qualification one needed to teach at the basic level of education in Ghana. Also, the rank of majority of teachers in GES who participated in the study was Senior Superintendent 38.42% $(n = 68)$. This is to be expected because Senior Superintendent is the starting rank for Diploma certificate holders in GES.

Table 7 shows the demographic characteristics of students. This includes their sex, age, grade level, school type, and school category.

Table 7: Demographic Characteristics of Student Participants

Variable	Category	Frequency	Percentages (%)
Sex			
	Male	682	54.00
	Female	581	46.00
	Total	**1263**	**100.00**
Age (in years)			
	10-12	197	15.60
	13-15	432	34.20
	16-18	584	46.24
	19-21	46	3.64
	Above 21	4	0.32
	Total	**1263**	**100.00**
Grade Level			
	Primary	336	26.60
	JHS	325	25.73
	SHS	602	47.67
	Total	**1263**	**100.00**
School Type			
	Co-educational	932	73.79
	Single sex female	158	12.51

Table 7: Cont'd

	Single sex male	173	13.70
	Total	**1263**	**100.00**
School Category			
	Category A	319	52.99
	Category B	137	22.76
	Category C	146	24.25
	Total	**602**	**100.00**

Out of the 1263 learner participants for the study, 54.00% $(n = 682)$ were males and 46.00% $(n = 581)$ were females. 96.04% $(n = 1213)$ of student respondents aged between 10 – 18years inclusive shows that the respondents' ages were within the school going age of Ghanaian learners from grade 6 to grade 12 (see Table 7). As a breakdown, Ghanaian children are expected to begin pre-school at age four, spend two years at pre-school level and start primary school at age six. The duration for primary school education is 6 years before they begin the first part of the high school education which is a junior high school lasting for three years. SHS which is the last stage of the pre-tertiary education in Ghana lasts for a period of three years. Majority of the respondents 47.67% $(n = 602)$ were SHS students. The students in the SHSs in the metropolis are aggregation of students from almost all the JHSs in the metropolis as well as JHSs in the region and across the country which make the students population in the SHSs quite huge. This is coupled with the fact that mathematics is a core subject which is offered by all SHS students irrespective of the academic programme they offer.

Single sex female and single sex male schools have 12.51% $(n = 158)$ and 13.70% $(n = 173)$ of the respondents respectively compared to the respondents from the co-educational schools, 73.79% $(n = 932)$. The results in Table 7 also show that there are more learner participants in the study from category A schools, 52.99% $(n = 319)$, in the metropolis than categories B, 22.76% $(n = 137)$, and C schools, 24.25% $(n = 146)$. The high number of student respondents from category A schools could partly be attributed to the fact that these schools offer more academic programmes than categories B and C schools and therefore offer admission opportunities to more students.

Also, category A schools are first class schools with better school facilities and infrastructure which aid effective teaching and learning. Parents are motivated by this and guide their children who are prospective SHS students to select those schools as their first-choice schools. JHS 3 students who are seeking admission into SHS learn so hard to gain admission to the category A schools of their choice.

What Primary, Junior High School (JHS) and Senior High School (SHS) Teachers' Value in their Students' Mathematics Learning

In order to answer research question 1a, the researcher found the attributes mathematics teachers at the pre-tertiary level value in their students' mathematics learning using the modified version of the students' questionnaire. The results are presented in Table 8. The results indicate that none of the value items on the questionnaire was valued as unimportant or absolutely unimportant. However, 12.50% $(n = 8)$ of the value items were valued as absolutely important, 82.81% $(n = 53)$ were valued as important whereas 4.69% $(n = 3)$ were valued as neither important nor unimportant. Table 8 shows the descriptive statistics (means and standard deviations) for all the 64 value items on the WIFI questionnaire arranged from most valued item to the least valued item. The means were ranked in order to find out the value items that were highly valued and least valued by the teachers.

Table 8: Ranked Means with their Standard Deviations of the Valued Items on what Teachers Find Important in Students' Learning of Mathematics on Modified Version of WIFI Questionnaire

	Mean	Std. Deviation
Connecting maths to real life	4.74	.441
Working step-by-step	4.64	.505
Using concrete materials to understand mathematics	4.59	.578
Teacher giving students feedback	4.59	.597
Examples to help students understand	4.58	.506
Problem-solving	4.53	.622
Using diagrams to understand maths	4.51	.565
Practicing with lots of questions	4.50	.596
Doing a lot of mathematics work	4.49	.575
Teacher asking students questions	4.49	.565
Appreciating the beauty of maths	4.48	.575
Mathematics homework	4.46	.522
Mathematics tests / examinations	4.46	.564
Small-group discussions	4.45	.592
Writing the solutions step-by-step	4.44	.572
Understanding why my students solution is incorrect or correct	4.41	.537
Explaining by the teacher	4.41	.677
Teacher asking students questions	4.40	.557
Looking out for maths in real life	4.40	.595
Students explaining their solutions to the class	4.37	.619
Hands-on activities	4.36	.679
Students working out maths by themselves	4.36	.633
Understanding concepts / processes	4.34	.746
Knowing the steps of the solution	4.33	.619
Looking for different ways to find the answer	4.32	.836
Investigations	4.30	.787
Knowing which formula to use	4.30	.645
Teacher helping students individually	4.30	.636
Teacher use of keywords (e.g 'share' to signal division; contrasting 'solve' and simplify)	4.28	.682
Completing mathematics work	4.28	.602
Relating mathematics to other subjects in school	4.27	.719
Mathematics games	4.27	.579
Mathematics puzzles	4.23	.664
Remembering the work we have done	4.23	.801
Using mathematical words (e.g angle)	4.23	.687
Practising how to use maths formulae	4.21	.618

Table 8: Cont'd

Relationships between maths concepts	4.19	.711
Students posing maths problems	4.17	.849
Alternative solutions	4.15	.702
Getting the right answer	4.14	.741
Looking for different possible answers	4.14	.749
Whole – class discussions	4.10	.771
Learning maths with the internet	4.10	.810
Outdoor mathematics activities	4.08	.714
Learning through mistakes	4.08	.678
Students making up their own maths questions	4.05	.737
Learning maths with the computer	4.03	.845
Feedback from my friends	4.00	.842
Knowing the theoretical aspects of mathematics (e.g proof, definition of triangles)	3.99	.932
Mathematics debates	3.96	.903
Learning the proofs	3.95	.952
Verifying theorems / hypotheses	3.95	.946
Explaining where rules/formulae came from	3.95	1.049
Knowing the times tables	3.94	.860
Stories about mathematicians	3.93	.830
Stories about recent developments in mathematics	3.88	.931
Given a formula to use	3.81	.838
Stories about mathematics	3.80	.911
Mystery of mathematics (e.g 111 111 111 × 111 111 111 = 12345678987654321)	3.63	1.053
Memorising facts (e.g Area of a rectangle = length × breadth)	3.57	1.048
Using calculator to check the answer	3.55	1.158
Being lucky at getting the correct answer	3.32	1.080
Using the calculator to calculate	3.25	1.379
Shortcuts to solving a problem	3.17	1.203

Source: Field survey (2021)

As displayed in Table 8, it is observed from the means that most of the items were valued as important by the teachers as far as their students' mathematics learning is concerned. The item, *Connecting maths to real life* has the highest ranked mean score *(mean = 4.74)* whilst the item, *Shortcuts to solving a problem* is least ranked with the lowest mean score *(mean = 3.17).*

Also, the standard deviations revealed a small spread around the means of the most valued items, *Connecting maths to real life (mean = 4.74, SD = 0.441), Working step-by-step (mean = 4.64, SD = 0.505), Using concrete materials to understand mathematics (mean = 4.59, SD = 0.578), Teacher giving students feedback (mean = 4.59, SD = 0.597), Examples to help students understand (mean = 4.58, SD = 0.506), Problem- solving (mean = 4.53, SD = 0.622)*,etc Larger spread around the means were observed among the responses of the least valued items such as *Shortcuts to solving a problem, (mean = 3.17, SD = 1.203), Using the calculator to calculate (mean = 3.25, SD = 1.379), Being lucky at getting the correct answer(mean = 3.32, SD = 1.080), Using calculator to check the answer (mean = 3.55, SD = 1.158), Memorising facts (mean = 3.57, SD = 1.048), Mystery of mathematics (mean = 3.63, SD = 1.053)*, etc. Higher deviations for lower ranked value items show that what pre-tertiary mathematics teachers value in students' mathematics learning deviate to both sides of the Likert scale options (Andersson & Österling, 2019). The nature of the items ranked highly on the TQ as absolutely important by the teachers suggest that they value Control in their students' mathematics learning.

Principal Component Analysis of What Pre-Tertiary Mathematics Teachers Value in their Students' Mathematics Learning

In order to find out the nature of what mathematics teachers at the pre-tertiary level of education in Ghana value in mathematics as far as their students' mathematics learning is concerned, a Principal Component Analysis (PCA) with Kaiser–Meyer–Olkin (KMO) measure of sampling adequacy and the Bartlett's (1950) test of sphericity (BTS) as well as varimax rotation were carried out to put the value questionnaire items into factors with the level of significance set at 5%. The outputs from KMO gave 0.720 which is greater than 0.6 and BTS showed a p-value of 0.000 which is less than 0.05. These indicate that the interrelation of variables assumption is satisfied.

Table 9 shows output of KMO and BTS on the data of teachers at the pre-tertiary level.

Table 9: KMO and Bartlett's Test - Teachers

Kaiser-Meyer-Olkin Measure of Sampling Adequacy.		.720
Bartlett's Test of Sphericity	Approx. Chi-Square	6168.570
	Df	2016
	Sig.	.000

Source: Field survey (2021)

A robust cut-off criterion within acceptable margin was set at 0.45 for the data output (Comrey & Lee, 1992). All value items that failed to meet the cut-off criterion were discarded. In line with the criteria set above, 8 value items were eliminated and 56 items out of the 64 items were retained. The PCA resulted in 18 components with eigenvalues greater than one accounting for about 72% of the total variance were retained (see Appendix E). According to Streiner (1994), factors retained in PCA should be explained by not less than 50% of the total variance. As a follow up to the use of eigenvalues approach to determine the number of retained components in PCA, Scree Plot (SP) was explored to identify the actual number of components to be retained. SP is a graph that connects the total eigenvalues with their corresponding components. SP breaks to separate components that have large values of total eigenvalues greater than 1 from smaller total eigenvalues of 1 or less. However, Zwick and Velicer (1986) have reported that SP has a general tendency to overrate the number of factors to retain. It does not offer an objective way of selecting the factors to be retained in PCA and its interpretation is largely vague (Hayton, Allen & Scarpello, 2004). Figure 2 shows the SP indicating the components to be retained and those to be discarded.

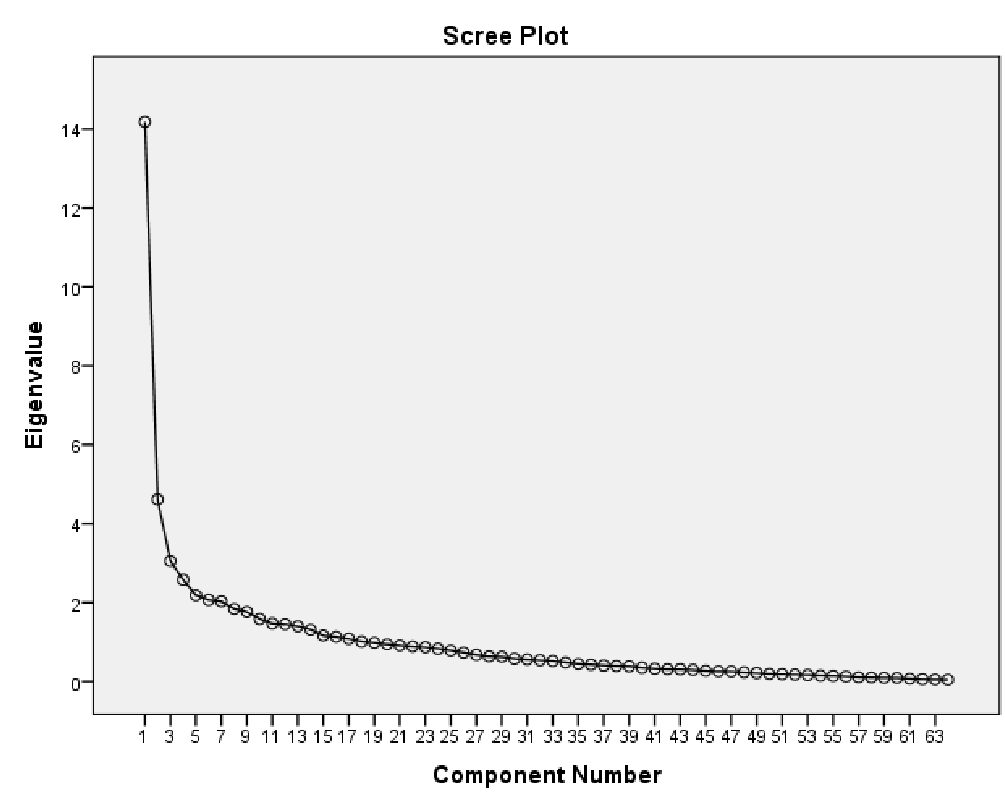

Figure 2: Scree plot showing the components to retain and those to discard - Teachers

From the SP above, it is not clear where the breaking point occurs or the point the eigenvalues start to level off. There is a disagreement as to whether the point that divides the 'major components' and the 'minor component' occurs at component 3, 4 or 5 (see Figure 2). Parallel Analysis (PA) was used as a method to overcome the 'number – of – components problem' that characterizes the SP (Humphreys & Montanelli, 1975; Zwick & Velicer, 1986). The PA for the study was carried out using Monte Carlo PCA for Parallel Analysis (Watkins, 2000). The first three actual eigenvalues from PCA (that is 14.180, 4.613 and 3.052) (see Appendix E) were larger than the criterion value from parallel analysis (2.9314, 2.7015 and 2.6957) (see Appendix G). Three components were retained in the final analysis based on the PA carried out. According to Hayton, Allen and Scarpello (2004), factors corresponding to actual eigenvalues that are greater than the criterion value from the PA should be retained. From the parallel analysis, thirty-five of the value items on the WIFI questionnaire loaded under component 1 (C1), eight items loaded under component 2 (C2) and three of the items loaded under component 3 (C3). The results of the PCA indicated that teachers who teach mathematics at the pre-tertiary level of education value *Understanding-C1* (Alternative solutions, understanding concepts/processes, writing the solutions step by step, knowing which formula to use, knowing the steps of the solution, using mathematical words e.g angle, teacher asking students questions, completing mathematics work,…, relating mathematics to other subjects in school), *Versatility-C2* (Using calculator to check the answer, using calculator to calculate, mystery of mathematics e.g 111 111 111 ×111 111 111 = 12345678987654321, verifying theorems/hypotheses,…, learning the proofs) and *Achievement-C3* (Being lucky at getting the correct answer, memorizing facts e.g Area of a rectangle = length X breadth) in their students' mathematics learning. The output from the PCA for the final components maintained is as shown in Table 10.

Table 10: Principal Component Analysis: Rotated Component Matrix - Teachers

C1 – UNDERSTANDING	1	2	3
30: Alternative solutions	0.673		
54: Understanding concepts/processes	0.658		
33: Writing the solutions step-by-step	0.642		
58: Knowing which formula to use	0.641		
56: Knowing the steps of the solution	0.585		
32: Using mathematical words (e.g angle)	0.680		
35: Teacher asking students questions	0.576		
62: Completing mathematics work	0.574		
53: Teacher use of keywords (e.g 'share' to signal division; contrasting 'solve' and simplify)	0.571		
20: Mathematics puzzles	0.570		
59: Knowing the theoretical aspects of mathematics (e.g proof, definition of triangles	0.569		
29: Students making up their own maths questions	0.569		
13: Practising how to use maths formulae	0.565		
21: Students posing maths problems	0.561		
16: Looking for different ways to find the answer	0.559		
50: Getting the right answer	0.551		
49: Examples to help students understand	0.551		
39: Looking out for maths in real life	0.544		
48: Using concrete materials to understand mathematics	0.538		
19: Students explaining their solutions to the class	0.537		
42: Students working out maths by themselves	0.535		
11: Appreciating the beauty of maths	0.526		
26: Relationships between maths concepts	0.526		
36: Practicing with lots of questions	0.524		
43: Mathematics tests / examinations	0.518		
63: Understanding why my students solution is incorrect or correct	0.515		
52: Hands-on activities	0.507		
61: Stories about mathematicians	0.483		
23: Learning maths with the computer	0.479		
64: Remembering the work we have done	0.477		

Table 10: Cont'd

3: Small-group discussions	0.468		
47: Using diagrams to understand maths	0.465		
16 Looking for different possible answers:	0.456		
12: Connecting maths to real life	0.451		
10: Relating mathematics to other subjects in school	0.450		
C2 – VERSATILITY			
22: Using calculator to check the answer		0.685	
4: Using calculator to calculate		0.550	
60: Mystery of mathematics (e.g 111 111 111 ×111 111 111 = 12345678987654321)		0.523	
31: Verifying theorems/hypotheses		0.509	
18: Stories about recent developments in mathematics		0.489	
40: Explaining where rules/formulae came from		0.468	
24: Learning maths with the internet		0.455	
8: Learning the proofs		0.451	
C3 – ACHIEVEMENT			
27: Being lucky at getting the correct answer			0.566
14: Memorizing facts (eg Area of a rectangle = length X breadth)			0.520
17: Stories about mathematics			0.459

Source: Field survey (2021)

Principal Component Analysis (PCA) is used when the objective of the study is to summarize most of the original information (variance) in a minimum number of factors for prediction purposes (Hair, Black, Babin, & Anderson, 2019). Thus, the use of PCA enabled the researcher to achieve what the study was set out to do. That is; to determine what teachers and students value in mathematics learning across grade levels.

Loadings for a variable must be above the cut-off criterion of 0.45 for significance. Decision to retain an item under a particular factor as a result of cross loadings is based on the following:

a. Between 1.0 and 1.5—problematic cross-loading and the variable with smaller loading a strong candidate for elimination to achieve simple structure.
b. Between 1.5 and 2.0—potential cross-loading, with deletion of a variable based on interpretability of resulting factors.
c. Greater than 2.0—ignorable cross-loading, where smaller loading, while significant, can be ignored for purposes of interpretation (see Hair, Black, Babin, & Anderson, 2019).

Table 11 shows the value items for which cross loadings occurred and how these items were maintained under a particular factor.

Table 11: Cross - Loadings – Teachers

	Factor Loadings Matrix			Squared Loadings			Ratio	Cross loading Classification
	Factor 1	Factor 2	Factor 3	Factor 1	Factor 2	Factor 3		
Practicing with lots of questions	**0.50**	**0.46**	0.35	**0.25**	**0.21**	0.12	1.19	Problematic
Mystery of mathematics (e.g 111 111 111 × 111 111 111 = 12345678987654321)	**0.46**	**0.48**	0.27	**0.21**	**0.23**	0.10	1.10	Problematic
Memorizing facts e.g Area of a rectangle = length × breadth)	**0.49**	0.26	**0.50**	0.24	0.07	**0.25**	1.04	Problematic
Looking for different ways to find the answer	**-0.56**	-0.27	**0.53**	**0.31**	0.07	**0.28**	1.11	Problematic

Results of Table 11 showed that items with the highest value was maintained under its corresponding factor. Thus, the value item, *Practicing with lots of questions* was maintained under factor 1; *Mystery of mathematics (e.g 111 111 111 × 111 111 111 = 12345678987654321)* was maintained under factor 2, *Memorizing facts e.g Area of a rectangle = length × breadth)* was maintained under factor 3 and *Looking for different ways to find the answer* was maintained under factor 1.

The results from Table 12 provide Cronbach alpha reliability coefficients for the three components using each teacher respondent as a unit of analysis. The Cronbach alpha values indicate the extent to which the value items on the WIFI questionnaire agree with the component under which they are loaded. Pallant (2011) has reported that a Cronbach alpha reliability coefficient of above 0.70 shows that there is high internal consistency within the items measuring the construct. The Cronbach alpha reliability values of 0.93 and 0.80 for *Understanding–C1* and *Versatility-C2* respectively affirm that the individual items that loaded under these components 'hang together' well. Even with the Cronbach alpha value of 0.63 for *Achievement – C3* does not mean that the items that loaded under *Achievement – C3* do not agree with the construct. Pallant has argued that fewer items, usually less than ten items produce lower Cronbach alpha value and does not mean that there is no internal consistency in the items. The overall Cronbach alpha value of 0.92 shows an excellent internal consistency (George & Mallery, 2003).

Table 12: Cronbach Alpha Coefficient for each of the Component Subscale for Teachers

Component	Alpha reliability
Understanding-C1	0.93
Versatility - C2	0.80
Achievement - C3	0.63
Overall	0.92

Source: Field survey (2021)

An examination of the descriptive statistics for the three components show higher mean scores for *Understanding - C1* and *Versatility – C2* with smaller deviation around the mean for *C1* compared to *C2*. That notwithstanding, the mean values for the three learning attributes for mathematics show that mathematics teachers at the pre-tertiary level of education value *Understanding*, *Versatility* and *Achievement* as important to students mathematics learning. Table 13 shows the descriptive statistics (means and standard deviations) of what teachers who teach mathematics at the pre-tertiary level value in their students' mathematics learning.

Table 13: Means and Standard Deviations of what Mathematics Teachers Value in their Students' Mathematics Learning

Value	Mean	Standard deviation
Understanding – C1	4.30	0.673
Versatility – C2	3.78	1.035
Achievement – C3	3.56	1.013

Source: Field survey (2021)

What Primary, Junior High School (JHS) and Senior High School (SHS) Students' Value in their Mathematics Learning

In order to address research question 1b, the researcher examined how pre-tertiary students value the individual value items on the WIFI questionnaire as far as their mathematics learning was concerned. The results in Table 13 indicate that none of the value items on the WIFI questionnaire was valued by the pre-tertiary students as either unimportant or absolutely unimportant. Thus, for the 64 items on the WIFI questionnaire that were valued by the pre-tertiary students, 15.63% $(n = 10)$ of the items were valued as absolutely important, 82.81% $(n = 53)$ were valued as important whilst 1.56% $(n = 1)$ was valued as neither important nor unimportant. The results from Table 14 present the means and standard deviations for all the 64 value items on the WIFI questionnaire arranged from the most valued item to the least valued item.

Table 14: Ranked Means and Standard Deviations of the Valued Items on What Students Find Important in their Mathematics Learning on the WIFI Questionnaire

	Mean	Std. Deviation
Remembering the work we have done	4.58	.741
Knowing the steps of the solution	4.58	.746
Examples to help students understand	4.57	.697
Knowing which formula to use	4.55	.721
Explaining by the teacher	4.55	.744
Getting the right answer	4.55	.771
Working step-by-step	4.53	.739
Mathematics tests / examinations	4.52	.828
Practicing with lots of questions	4.50	.742
Teacher asking students questions	4.50	.773
Mathematics homework	4.48	.877
Teacher asking students questions	4.47	.758
Problem-solving	4.43	.756
Writing the solutions step-by-step	4.43	.829
Doing a lot of mathematics work	4.42	.897
Understanding concepts / processes	4.41	.862

Table 14: Cont'd

Looking for different ways to find the answer	4.40	.811
Completing mathematics work	4.39	.792
Students explaining their solutions to the class	4.37	.772
Students working out maths by themselves	4.36	.829
Knowing the times tables	4.34	.941
Practising how to use maths formulae	4.34	.951
Connecting maths to real life	4.30	.959
Using diagrams to understand maths	4.28	.860
Learning through mistakes	4.28	.985
Understanding why my students solution is incorrect or correct	4.27	.935
Investigations	4.23	.933
Teacher helping students individually	4.22	.965
Teacher giving students feedback	4.22	.917
Teacher use of keywords (e.g 'share' to signal division; contrasting 'solve' and simplify)	4.19	.971
Given a formula to use	4.18	.983
Memorising facts (e.g Area of a rectangle = length × breadth)	4.16	1.012
Mathematics puzzles	4.14	.945
Whole – class discussions	4.14	.938
Students posing maths problems	4.14	.971
Hands-on activities	4.13	.964
Knowing the theoretical aspects of mathematics (e.g proof, definition of triangles)	4.13	1.070
Small-group discussions	4.12	.989
Using mathematical words (e.g angle)	4.12	.959
Alternative solutions	4.09	.951
Students making up their own maths questions	4.08	.974
Mathematics games	4.07	.952
Using concrete materials to understand mathematics	4.06	1.035
Learning maths with the internet	4.02	1.051
Looking out for maths in real life	4.02	1.023
Looking for different possible answers	4.00	1.038
Learning the proofs	3.98	1.139
Verifying theorems / hypotheses	3.94	1.118
Feedback from my friends	3.92	1.011
Outdoor mathematics activities	3.88	1.039
Relating mathematics to other subjects in school	3.86	1.198
Explaining where rules/formulae came from	3.84	1.178
Learning maths with the computer	3.83	1.145
Using calculator to check the answer	3.83	1.241

Table 14: Cont'd

Shortcuts to solving a problem	3.81	1.272
Mystery of mathematics (e.g 111 111 111 × 111 111 111 = 12345678987654321)	3.81	1.263
Relationships between maths concepts	3.80	1.078
Appreciating the beauty of maths	3.77	1.184
Stories about mathematicians	3.75	1.258
Mathematics debates	3.75	1.142
Stories about recent developments in mathematics	3.65	1.130
Being lucky at getting the correct answer	3.64	1.266
Stories about mathematics	3.59	1.252
Using the calculator to calculate	3.42	1.505

Source: Field survey (2021)

The results from Table 14 indicate that most of the value attributes were valued as important by the students in their mathematics learning. *Remembering the work we have done (mean = 4.58)* and *Knowing the steps of the solution (mean = 4.58)* were co-ranked as the most valued items with the highest mean score. However, *Using calculator to calculate was least ranked by the students with the mean score of 3.42*. Again, the most valued attributes show small spread around the mean, *Remembering the work we have done (mean = 4.58, SD = 0.741), Knowing the steps of the solution (mean = 4.58, SD = 0.746),* and *Examples to help students understand (mean = 4.57, SD = 0.697).* However, items with higher spread around the mean were the least valued, *Using the calculator to calculate (mean = 3.42, SD = 1.505), Stories about mathematics (mean = 3.59, SD = 1.252), Being lucky at getting the correct answer (mean = 3.64, SD = 1.266) etc.*

Principal Component Analysis of What Pre-Tertiary Students Value in their Mathematics Learning.

In order to find out the nature of what pre-tertiary students value in the learning of mathematics, a PCA with a KMO and BTS as well as varimax rotation were carried out to examine the value items with significance level of

0.05. The outputs from KMO gave 0.926 and BTS showed a p-value of 0.000 which is less than 0.05. These indicate that the interrelation of variables assumption is satisfied. Table 15 shows output of KMO and BTS on the data of the students at the pre-tertiary level.

Table 15: KMO and Bartlett's Test - Students

Kaiser-Meyer-Olkin Measure of Sampling Adequacy.		.926
Bartlett's Test of Sphericity	Approx. Chi-Square	26224.714
	Df	2016
	Sig.	.000

Source: Field survey (2021)

A cut–off criterion within acceptable margin was set at 0.45 for the data output. All the value items that failed to meet the criteria were discarded. In line with the criteria set above, 32 value items were eliminated, remaining 32 items out of the 64 value items on the WIFI questionnaire. The PCA resulted in 16 components with eigenvalues greater than one accounting for about 59% of the total variance were retained (see Appendix F). Figure 3 shows the SP indicating the components to be retained and those to be discarded.

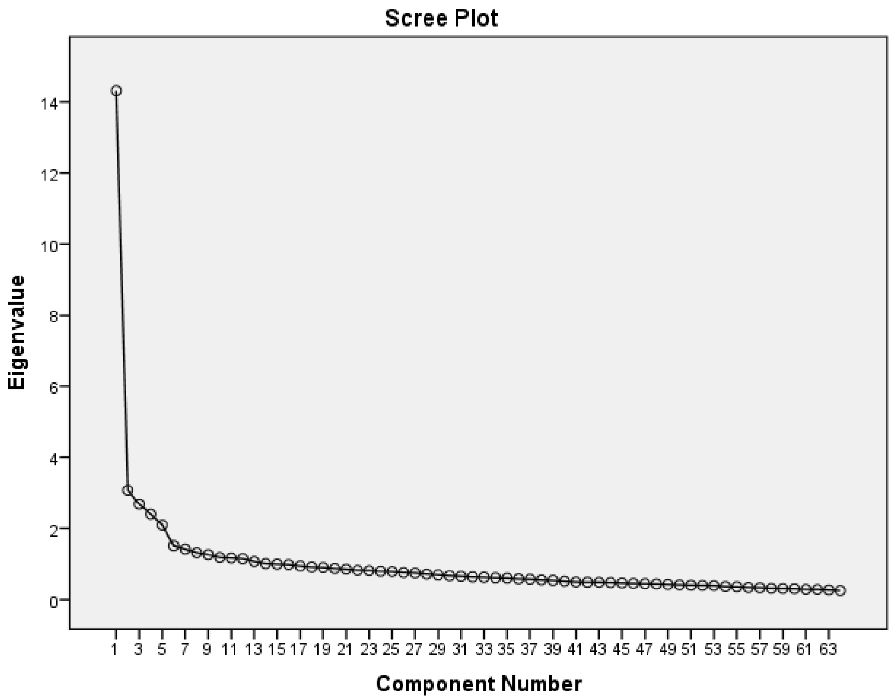

Figure 3: Scree plot showing the components to retain and those to discard - Students

The PA for the study was carried out using Monte Carlo PCA for Parallel Analysis (Watkins, 2000). The first seven actual eigenvalues from PCA (that is 14.529, 3.031, 2.633, 2.446, 2.130, 1.528 and 1.402) (see Appendix F) were larger than the criterion value from parallel analysis (1.5696, 1.5324, 1.5041, 1.4792, 1.4583, 1.4390 and 1.4013) (see Appendix H). Seven components were retained in the final analysis based on the PA carried out. According to Hayton, Allen and Scarpello (2004), factors corresponding to actual eigenvalues that are greater than the criterion value from the PA should be retained. The seven factors (Components) retained included: *Fluency-C1*, *Understanding-C2*, *Teaching materials/activities-C3*, *Connections- C4*, *ICT-C5*, *Feedback-C6*, and *Learning Strategies-C7*. Thus, seven items loaded on *C1*, eight items loaded on *C2*, three items loaded on *C3*, three items loaded on *C4*, four items loaded on *C5*, four items loaded on *C6*, and three items loaded on *C7*.

The output from the PCA for the 7 components produced for what students' value in mathematics learning is as shown in Table 16.

Table 16: Principal Component Analysis: Rotated Component Matrix – Students

	Component						
	1	2	3	4	5	6	7
C1-FLUENCY							
40: Explaining where rules /formulae came from	0.661						
8: Learning the proofs	0.654						
60: Mystery of mathematics (e.g 111 111 111 ×111 111 111 = 12345678987654321)	0.647						
59: Knowing the theoretical aspects of mathematics (e.g proof, definitions of triangles)	0.599						
31: Verifying theorems / hypotheses	0.580						
38: Given a formula to use	0.574						
13: Practising how to use maths formulae	0.540						
C2 – UNDERSTANDING							
50: Getting the right answer		0.661					
56: Knowing the steps of the solution		0.629					
64: Remembering the work we have done		0.592					
54: Understanding concepts / processes		0.540					
35: Teacher asking students questions		0.524					
58: Knowing which formula to use		0.520					
43: Mathematics tests / examinations		0.512					

Table 16: Cont'd

49: Examples to help students understand	0.503					
C3 – INSTRUCTIONAL MATERIALS/ACTIVITIES						
47: Using diagrams to understand maths		0.641				
48: Using concrete materials to understand mathematics		0.609				
34: Outdoor mathematics activities		0.474				
C4 – CONNECTIONS						
11: Appreciating the beauty of maths			0.613			
10: Relating mathematics to other subjects in school			0.606			
18: Stories about recent development in mathematics			0.482			
C5 –ICT						
4: Using the calculator to calculate				0.738		
22: Using the calculator to check the answer				0.712		
23: Learning maths with the computer				0.682		
24: Learning maths with the internet				0.582		
C6 – FEEDBACK						
45: Feedback from my friends					0.698	
44: Teacher giving students feedback					0.598	
51: Learning through mistake					0.523	
30: Alternative solution					0.467	
C7 – LEARNING STRATEGIES						
3: Small-group discussions						0.672
7: Whole-class discussions						0.541
5: Explaining by the teacher						0.471

Source: Field survey (2021)

Table 17 presents Cronbach alpha reliability coefficients for the seven components using each student's response as a unit of analysis.

Table 17: Cronbach Alpha Coefficient for each of the Component Subscales for Students

Component	Alpha reliability
Fluency -C1	0.83
Understanding-C2	0.65
Instructional materials/activities -C3	0.62
Connections- C4	0.61
ICT- C5	0.70
Feedback-C6	0.68
Learning Strategies -C7	0.65
Total	**0.88**

Source: Field survey (2021)

The Cronbach alpha values for each component show a range from moderate to high levels of internal consistency within the value items forming those components. The range of the Cronbach alpha reliability coefficients were 0.73 - 0.83 and 0.65 - 0.84 for teachers and their students respectively (see Tables 12 & 17). The alpha coefficients were within the acceptable limit of 0.60 for purposes of research (Nunnally cited in Henderson, Fisher and Fraser, 1998).

The results from Table 18 show the means and standard deviations of what pre-tertiary learners' value in their mathematics learning. An analysis of the descriptive statistics for the seven components (*Fluency – C1, Understanding – C2, Instructional materials/activities – C3, Connections – C4, ICT – C5, Feedback – C6* and *Learning Strategy – C7*) of the value attributes particularly the means indicate that pre-tertiary students either value those seven value attributes as "absolutely important" or "important" in their mathematics learning. Understanding–C2 remains the most valued attribute with the highest mean score (mean = 4.53) and smallest deviation in the responses around the mean. This is indicative of the fact pre-tertiary students see *Understanding – C2 (getting the right answer, knowing the steps of the solution, remembering the work we have done, understanding concepts/processes, teacher asking students questions, knowing which formula to use, mathematics tests/examinations and examples to help students understand)* as absolutely important in their mathematics learning. With the least mean score (mean = 3.76) for *Connections – C4 (appreciating the beauty of mathematics, relating mathematics to other subjects in school, stories about recent development in mathematics),* it still remains as an attribute pre-tertiary students value as "important" in their mathematics learning.

Table 18: Means and Standard Deviations of what Students' Value in their Mathematics Learning

Value	Mean	Standard Deviation
Fluency – C1	4.00	1.120
Understanding – C2	4.53	0.767
Instructional materials/activities – C3	4.07	0.978
Connections – C4	3.76	1.171
ICT – C5	3.78	1.235
Feedback – C6	4.13	0.966
Learning Strategies – C7	4.27	0.891

Source: Field survey (2021)

Similarities or Differences Between What Teachers and their Students Value in Mathematics Learning

To assess the similarities or differences between what mathematics teachers value in their students' mathematics learning and what the students themselves also value in their own mathematics learning, the components on the PCA for teachers and students were compared. Analysis using PCA identified 3 value attributes which teachers at the pre-tertiary level value in students' mathematics learning. They included *Understanding – C1, Versatility – C2* and *Achievement – C3*. On the other hand, PCA also provided 7 attributes that pre-tertiary students value greatly in their mathematics learning. They included *Fluency – C1, Understanding – C2, Instructional materials/activities – C3, Connections – C4, ICT – C5, Feedback – C6* and *Learning Strategies – C7*. It is inferred from the value attributes that teachers' valuing aligns with students' valuing for the component labelled *Understanding*. Although *Versatility – C2* and *Achievement – C3* were statistically significant, they were valued only by mathematics teachers (not co-valued by teachers and students). *Fluency – C1, Instructional materials/activities – C3, Connections – C4, ICT – C5, Feedback – C5* and *Learning strategies – C7* on the other hand, were singly valued by students (not co-valued by both students and teachers).

Effect of Grade Level Teachers Teach (Primary, JHS and SHS) on the Attributes they Value in Students' Mathematics Learning

In order to explore the effect of grade level teachers teach on the attributes they value in students' mathematics learning, MANOVA was performed. A one-way MANOVA was conducted with the independent variables having three educational levels: (1) Primary school, (2) JHS and (3) SHS. The dependent variables consisted of three value attributes held by pre-tertiary mathematics teachers as follows: *Understanding - C1, Versatility - C2* and *Achievement - C3.*

There are range of statistics to select from in determining if there is statistically significant effect of independent variable (educational levels teachers teach) on the combined dependent variables (C1, C2 and C3). These include Pillai's Trace, Wilks' Lambda, Hotelling's Trace and Roy's Largest Root. Notwithstanding the suggestion by Fabachnick and Fidell (2014) that Wilks' Lambda is used generally, for unequal sample size (36 Primary teachers, 34 JHS teachers and 107 SHS teachers), Pillai's Trace is more robust. The results from Table 19 indicated that with Pillai's Trace having significant level of 0.000 reveals that statistically significant differences exist among pre-tertiary mathematics teachers values in mathematics put together across grade levels. The results $[Pilai's\, Trace = 0.992,\ F(2,174) = 7335.481,\ p = 0.000,\ \eta^2 = 0.992]$ suggest a very high effect size ($\eta^2 = 0.992$) indicating that there is a strong relationship in teacher valuing across grade levels (see Table 19).

Table 19: Summary of Multivariate Analysis of Variance (MANOVA) for Grade Level's Groups Variable –Teachers

Effect		Value	F	Hypothesis df	Error df	Sig.	Partial Eta Squared (η^2)
Grade level	Pillai's Trace	0.992	73335.481	3.000	172.000	0.000	0.992

Source: Field survey (2021)

The results from Table 20 revealed significant tests of between subjects effects for all the three components:

Understanding –C1 $[F(2,174) = 2.017,\ p = 0.001,\ \eta^2 = 0.023]$;

Versatility – C2 $[F(2,174) = 7.951,\ p = 0.000,\ \eta^2 = 0.084]$ and

Achievement – C3) $[F(2,174) = 0.045,\ p = 0.000,\ \eta^2 = 0.001]$.

Table 20: Tests of Between-Subjects Effects of (MANOVA) for the Independent Variable and the Dependent Variables – Teachers

Source	Dependent variable	Type III Sum of Squares	Df	Mean Square	F	Sig	Partial Eta Square (η^2)
	Understanding – CI	596.043^a	2	298.021	2.017	0.001	0.023
Level	Versatility – C2	425.784^b	2	212.892	7.951	0.000	0.084
	Achievement – C3	0.372^c	2	0.186	0.045	0.000	0.001

Source: Field survey (2021)

a. R Squared = 0.023 (Adjusted R Squared = 0.011); b. R Squared = 0.084 (Adjusted R Squared = 0.073) c. R Squared = 0.001 (Adjusted R Squared = -0.011)

To determine where the significant differences were among the three dependent variables (C1, C2, and C3) across the three grade levels the teachers teach (Primary school, JHS and SHS), Tukey's Honestly Significant Difference (HSD) Post Hoc multiple comparison test was carried out. This test was performed to find out which difference resulted in the significant F- ratio obtained from the overall MANOVA. The use of Tukey's Honestly Significant Difference (HSD) Post Hoc test was informed by the fact that the number of mathematics teachers selected across the different grade levels (36 Primary, 34 JHS and 107 SHS teachers) for the study were not equal (Pallant, 2011). This is because the test makes use of the harmonic mean sample size for unequal group sizes.

Statistically significant pairwise differences were obtained in all the three significant components (C1, C2 and C3) as against grade levels the teachers teach mathematics: Primary/JHS teachers, Primary/SHS teachers and JHS/SHS teachers. The results in Table 21 indicated that statistically significant differences exist in two out of the three components (C1 and C3) between the independent variables Primary and JHS mathematics teachers. The results further showed that statistically significant differences exist in all the three components (C1, C2 and C3) between the independent variables, Primary and SHS mathematics teachers. For JHS and SHS mathematics teachers, statistically significant differences also existed between the two groups of mathematics teachers in all the three components (C1, C2 and C3).

Table 21: Post Hoc Tests (Tukey HSD) - Teachers

Dependent variable	(I) Grade Level	(J) Grade level	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
Understanding – C1	PRI.	JHS	3.65*	2.907	0.000	-3. 22	10.53
		SHS	4.70*	2.342	0.000	-0.84	10.24
	JHS	PRI.	-3.65*	2.907.	0.000	-10.54	3.22
		SHS	0.05*	2.393	0.001	-4.61	6.71
	SHS	Pri	- 4.70*	2.342	0.000	-10.24	0.84
		JHS	-1.05*	2.393	0.000	-6.71	4.61
Versatility – C2	PRI.	JHS	1.10	1.237	0.649	-1.83.	4.02
		SHS	-2.56*	0.997	0.021	-4.92	-0.20
	JHS	PRI.	-1.10	1.237	0.649	-4.02	1.83
		SHS	-3.66*	1.019	0.001	-6.07	-1.25
	SHS	Pri	2.56*	0.997	0.021	0.20	4.92
		JHS	3.66*	1.019	0.001	1.25	6.07
Achievement - C3	PRI	JHS	-0.10*	0.487	0.000	-1.25	1.05
		SHS	- 0.12*	0.393	0.000	-0.91	0.95
	JHS	PRI.	0.10*	0.487	0.000	-1.05	1.25
		SHS	- 0.02*	0.401	0.000	-0.83	1.07
	SHS	Pri	0.12*	0.393	0.000	-0.95	0.91
		JHS	0.02*	0.401	0.000	-1.07	0.83

Source: Field survey (2021)
Based on observed means

The error term is Mean Square (Error) = 4.153
*The mean difference is significant at 0.05 level.

The results from Table 22 show the marginal means estimated from the highest to the lowest for the three statistically significant components across the three pre- tertiary levels of education are as follows:

1. The Primary school teachers who teach mathematics had the highest mean followed by JHS and SHS mathematics teachers for component C1. This means that Primary school teachers valued *Understanding – C1* in mathematics more than JHS and SHS mathematics teachers.
1. For component C2, SHS mathematics teachers had the highest mean followed by Primary and JHS teachers who teach mathematics.
2. The teachers who teach mathematics at the JHS level had the highest mean for C3 followed by teachers who teach mathematics at the Primary and SHS. This implies that JHS mathematics teachers valued *Achievement – C3* more than their colleagues who teach the subject in Primary and SHS.

Table 22: Estimated Marginal Means for Teachers' Valuing Across Grade Levels

Dependent Variable	Educational level you teach	Mean	Std. Error	95% Confidence Interval Lower Bound	95% Confidence Interval Upper Bound
Understanding	Primary Sch.	149.889	2.026	145.890	153.888
	JHS	146.235	2.085	142.120	150.350
	SHS	145.187	1.175	142.867	147.506
Versatility	Primary Sch.	28.833	.862	27.131	30.536
	JHS	27.735	.887	25.984	29.487
	SHS	31.393	.500	30.405	32.380
Achievement	Primary Sch.	10.667	.340	9.996	11.337
	JHS	10.765	.349	10.075	11.455
	SHS	10.645	.197	10.256	11.034

Source: Field survey (2021)

Effect of Grade Level of Students (Primary, JHS and SHS) on the Attributes they Value in their Mathematics Learning

As with the teachers, the effect of grade level on valuing in mathematics learning among students was explored. A one-way MANOVA was performed. MANOVA offered the opportunity for the researcher to examine the effects of educational levels on students' valuing in mathematics learning. MANOVA was conducted with the independent variable having three educational levels: (1) Primary school, (2) JHS and (3) SHS. The dependent variables consisted of seven value attributes held by pre-tertiary students as follows: Fluency-C1, *Understanding – C2, Instructional materials/activities – C3, Connections – C4, ICT – C5, Feedback – C6* and *Learning Strategies – C7.* Pillai's Trace confirms a statistically significant differences within the three educational levels in terms of the combined value attributes $[Pillai's\,Trace = 0.989,\ F(7, 1253) = 14509.592,\ p = 0.000,\ \eta^2 = 0.989]$ (see Table 23).

Table 23: Summary of Multivariate Analysis of Variance (MANOVA) for Grade Level's Groups Variable

Effect		Value	F	Hypothesis df	Error df	Sig.	Partial Eta Squared (η^2)
Grade level	Pillai's Trace	0.989	14509.592	7.000	1253.000	0.000	0.989

Source: Field survey (2021)

Tests of between - subjects effects revealed significant univariate main effects for five out of the seven components: *Instructional materials/activities – C3* $[F(2, 1140) = 31.868,\ p = 0.000,\ \eta^2 = 0.053]$;

Connections – C4 $[F(2,1260) = 7.213,\ p = 0.001,\ \eta^2 = 0.012]$; *ICT – C5)* $[F(2,1260) = 201.576,\ p = 0.000,\ \eta^2 = 0.261]$; *Feedback – C6* $[F(2,1260) = 20.126,\ p = 0.000,\ \eta^2 = 0.034]$; Learning strategies – C7$[F(2,1260) = 5.767,\ p = 0.003,\ \eta^2 = 0.010]$.

However, there were no statistically significant univariate main effects for *Fluency– C1* $[F(2,1260) = 2.416,\ p = 0.090,\ \eta^2 = 0.004]$ and *Understanding – C2*$[F(2,1260) = 1.370,\ p = 0.254,\ \eta^2 = 0.002]$ (see Table 24).

Table 24: Tests of Between-Subjects Effects of (MANOVA) for the Independent Variable and the Dependent Variables – Students

Source	Dependent variable	Type III Sum of Squares	Df	Mean Square	F	Sig	Partial Eta Square (η^2)
	Fluency – C1	183.277[a]	2	91.638	2.416	0.090	0.004
Level	Understanding – C2	46.610[b]	2	23.305	1.370	0.254	0.002
	Instructional materials/activities – C3	302.700[c]	2	151.350	31.868	0.000	0.053
	Connections – C4	100.890[d]	2	50.445	7.213	0.001	0.012
	ICT – C5	2940.006[e]	2	1970.003	201.576	0.000	0.261
	Feedback – C6	303.038[f]	2	151.519	20.126	0.000	0.034
	Learning strategies – C7	43.327[g]	2	21.663	5.767	0.003	0.010

Source: Field survey (2021)

a. R Squared = 0.004 (Adjusted R Squared = 0.002);

b. R Squared = .002 (Adjusted R Squared = 0.001);

c. R Squared = 0.053 (Adjusted R Squared = 0.051);

d. R Squared = .012 (Adjusted R Squared = 0.011);

e. R Squared = 0.261 (Adjusted R Squared = 0.260);

f. R Squared = 0.034 (Adjusted R Squared = 0.032);

g. R Squared = 0.010 (Adjusted R Squared = 0.008);

In order to find out where the significant differences were among the five significant dependent variables (C3, C4, C5, C6 and C7) across three grade levels (Primary school, JHS and SHS), Tukey's Honestly Significant Difference (HSD) Post Hoc multiple comparison test was performed. These tests were performed to find out which difference resulted in the significant F-ratio obtained from the overall MANOVA. The use of Tukey's (HSD) Post Honestly Significant Difference (HSD) Post Hoc test was informed by the fact that the number of students sampled across the different grade levels for the study were not equal (Pallant, 2011). This is because the test makes use of the harmonic mean sample size for groups that do not have equal sizes.

Statistically significant pairwise differences were observed in five components (C3, C4, C5, C6 and C7) for Primary and JHS learners, Primary and SHS students as well as JHS and SHS students. The results from Table 25 indicated that statistically significant differences existed in three out of the five components (C3, C5 and C6) between the independent variables Primary and JHS students. The results also indicated that statistically significant differences exist in three out of the five components (C3, C4 and C5) between the independent variables Primary and SHS students. For JHS and SHS students, statistically significant differences also existed between the two groups of learners in four out of the five components (C4, C5, C6 and C7).

For the five statistically significant components, the marginal means estimated in descending order across the three levels of education are indicated as follows:

1. For *Instructional materials/activities* - *C3*, primary school learners had the highest mean score followed by SHS and JHS learners.
2. For *Connections* - *C4* and *Learning Strategies* - *C7*, JHS learners had the highest mean score followed by primary and SHS learners. This means that JHS students valued *Connections* and *Learning Strategies* more than their counterparts in primary and SHS.
3. For *ICT* - *C5*, SHS students had the highest mean score followed by JHS and primary school learners.
4. For *Feedback* - *C6*, SHS students had the highest mean score followed by primary and JHS students.

Table 25: Post Hoc Tests (Tukey HSD) - Students

Dependent variable	(I) Grade Level	(J) Grade level	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
Instructional Materials/activities – C3	PRI.	JHS	1.24*	0.170	0.000	0.85	1.64
		SHS	1.02*	0.155	0.000	0.66	1.39
	JHS	PRI.	-1.24*	0.170	0.000	-1.64	-0.85
		SHS	- 0.22*	0.156	0.334	-0.59	0.15
	SHS	PRI	- 1.02*	0.155	0.000	-1.39	-0.66
		JHS	0.22*	0.156	0.334	-0.15	-0.59
Connections – C4	PRI.	JHS	- 0.17	0.206	0.675	-0.66	0.31
		SHS	0.50*	0.188	0.021	0.06	0.94
	JHS	PRI.	0.17	0.206	0.675	-0.31	0.66
		SHS	0.67*	0.190	0.001	0.23	1.12
	SHS	PRI	-0.50*	0.188	0.021	-0.94	-0.06
		JHS	- 0.67*	0.190	0.001	-1.12	-0.23
ICT – C5	PRI	JHS	-1.58*	0.243	0.000	-2.15	-1.01
		SHS	-4.33*	0.222	0.000	-4.86	-3.81
	JHS	PRI.	1.58*	0.243	0.000	1.01	2.15
		SHS	-2.76*	0.224	0.000	-3.28	-2.23
	SHS	PRI	4.33*	0.222	0.000	3.81	4.86
		JHS	2.76*	0.224	0.000	2.23	3.28

Table 25 cont'd

Dependent variable	(I) Grade Level	(J) Grade level	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
Feedback – C6	PRI.	JHS	1.07	0.213	0.000	0.57	1.57
		SHS	- 0.11	0.195	0.832	-0.57	0.35
	JHS	PRI.	-1.07*	0.213	0.000	-1.57	-0.57
		SHS	- 1.18*	0.197	0.000	-1.65	-0.72
	SHS	PRI	0.11	0.195	0.832	-0.35	0.57
		JHS	1.18*	0.197	0.000	0.72	1.65
Learning strategies	PRI.	JHS	- 0.35	0.151	0.057	-0.70	0.01
		SHS	0.12	0.138	0.656	-0.20	0.44
	JHS	PRI.	0.35	0.151	0.057	-0.01	0.70
		SHS	0.47	0.139	0.002	0.14	0.79
	SHS	PRI	-0.12	0.138	0.656	-0.44	0.20
		JHS	- 0.47	0.139	0.002	-0.79	-0.14

Source: Field survey (2021)

Based on observed means.

The error term is Mean Square (Error) = 3.757.

* The mean difference is significant at the 0.05 level.

Effect of Sex on Valuing in Students' Mathematics Learning among Mathematics Teachers

In determining the effect of sex on valuing in students' mathematics learning among mathematics teachers, a one-way MANOVA was conducted with sex as the independent categorical variable. The dependent variables were made up of three subscales; *Understanding – C1*, *Versatility – C2* and *Achievement – C3.* MANOVA was chosen because it has the capacity to draw a comparison between the groups and determines if the differences in means between the groups on the combination of the dependent variables are due to probability (Pallant, 2011).

Results from Table 26 show that the Pillai's Trace value was significant at 0.000 indicating that there are statistically significant differences among male and female pre-tertiary level teachers who teach mathematics in terms of their valuing in students' mathematics learning.

This result revealed that pre- tertiary mathematics teacher's sex [$\text{Pillai's Trace} = 0.899,\ F(3,172) = 511.512,\ p = 0.000,\ \eta^2 = 0.899$] significantly affect teachers' valuing in students' mathematics learning (see Table 26.

Table 26: Summary of Multivariate Analysis of Variance (MANOVA) for Sex Variable - Teachers

Effect		Value	F	Hypothesis df	Error df	Sig.	Partial Eta Squared (η^2)
Sex	Pillai's Trace	0.899	511.512[b]	3.000	172.000	0.000	0.899

Source: Field survey (2021)
a. Design: Intercept + A1
b. Exact statistic
c. The statistic is an upper bound on F that yields a lower bound on the significance level (0.05)

Tests of between - subjects effects as displayed in Table 27 show that statistically significant differences between male and female teachers who teach mathematics were observed in only one out of the three components: *Versatility – C2* $[F(2,174) = 6.075,\ p = 0.003,\ \eta^2 = 0.065]$ with its estimated marginal mean higher in males. However, the other two components *Understanding – C1* $[F(2,174) = 1.593,\ p = 0.206,\ \eta^2 = 0.018]$ and *Achievement – C3* $[F(2,174) = 1.021,\ p = 0.362,\ \eta^2 = 0.012]$ did not show a significant difference between pre-tertiary mathematics teacher's sex (see Table 27). Thus, sex difference in mathematics learning is a sociocultural bias phenomenon but not biological (Ghasemi, Burley & Safadel, 2019).

Table 27: Tests of Between-Subjects Effects of (MANOVA) for the Independent Variable and the Dependent Variables - Teachers

Source	Dependent variable	Type III Sum of Squares	Df	Mean Square	F	Sig	Partial Eta Square (η^2)
	Understanding C1	473.139 [a]	2	236.569	1.593	0.206	0.018
Sex	Versatility – C2	331.895 [b]	2	165.948	6.075	0.003	0.065
	Achievement – C3	8.388 [c]	2	4.194	1.021	0.362	0.012

Source: Field survey (2021)
a. R Squared = 0.018 (Adjusted R Squared = 0.007);
b. R Squared = 0.065 (Adjusted R Squared = 0.055)

c. R Squared = .012 (Adjusted R Squared = 0.000)

Effect of Sex of Students on the Attributes they Value in their Mathematics Learning

Similar to the teachers, in exploring the effect of sex of students on valuing in mathematics learning, a one-way MANOVA was ran with the independent variable (sex) and the dependent variables (C1, C2, C3, C4, C5, C6 and C7). Pillai's Trace criterion was utilized to test if significant group (sex) differences existed on a linear combination of the dependent variables. The results indicated that there is statistically significant differences between male and female learners in the seven value subscales put together[$Pillai's\ Trace = 0.989,\ \ F(7, 1253) = 14822.108,\ p = 0.000,$ $\eta^2 = 0.989$] (see Table 28). This study agrees with several literature on gender differences among primary and SHS students that have been reported in mathematics education (see Lachance & Mazzocco, 2006; Hyde, Fennema & Lamom, 1990 Zhang, 2019). This result of the study also agrees in part with a study by Barkatsas, Law, Seah and Wong (2018) that statistically significant sex differences exist for secondary and primary school learners' values in the study of mathematics. However, they established among other things that there are no statistically significant sex differences in mathematics learning among primary school students.

Table 28: Summary of Multivariate Analysis of Variance (MANOVA) for Sex Variable - Students

Effect		Value	F	Hypothesis df	Error df	Sig.	Partial Eta Squared (η^2)
Sex	Pillai's Trace	0.989	14822.108[b]	7.000	1253.000	0.000	0.989

Source: Field survey (2021)
a. The statistic is an upper bound on F that yields a lower bound on the significance level (0.05)

Tests of between - subjects effects portrayed significant difference between female and male learners in two out of the seven value components: *Fluency – C1* $[F(1, 1260) = 5.792,\ p = 0.016,\ \eta^2 = 0.005]$ and *Learning strategies – C7* $[F(1, 1260) = 10.829,\ p = 0.001,\ \eta^2 = 0.009]$. However, there was no significant difference between female and male learners for the value components: *Understanding – C2* $[F(1, 1260) = 0.119,\ p = 0.730,$ $\eta^2 = 0.000]$; *Instructional materials/activities – C3* $[F(1, 1260) = 0.284,$ $p = 0.0.594,\ \eta^2 = 0.000]$; *Connections – C4*$[F(1, 1260) = 0.384,\ p = 0.536,\ \eta^2 = 0.000]$. *ICT – C5*$[F(1, 1260) = 0.043,\ p = 0.836,\ \eta^2 = 0.000]$ and *Feedback – C6*$[F(1, 1260) = 1.403,\ p = 0.236,\ \eta^2 = 0.001]$ (see Table 29).

Table 29: Tests of Between-Subjects Effects of (MANOVA) for the Independent Variable and the Dependent Variables - Students

Source	Dependent variable	Type III Sum of Squares	Df	Mean Square	F	Sig.	Partial Eta Square (η^2)
Sex	Fluency – C1	219.445a	1	219.445	5.792	0.016	0.005
	Understanding – C2	2.031b	1	2.031	0.119	0.730	0.000
	Instructional materials/activities – C3	1.424c	1	1.424	0.284	0.594	0.000
	Connections – C4	2.716d	1	2.716	0.384	0.536	0.000
	ICT – C5	0.569e	1	0.569	0.043	0.836	0.000
	Feedback – C6	10.921f	1	10.921	1.403	0.236	0.001
	Learning strategies – C7	40.676g	1	40.676	10.829	0.001	0.009

Source: Field survey (2021)

a. R Squared = 0.005; (Adjusted R Squared = 0.004); b. R Squared = 0.000 (Adjusted R Squared = -0.001)

c. R Squared = 0.000 (Adjusted R Squared = -0.001); d. R Squared = 0.000 (Adjusted R Squared = -0.001)

e. R Squared = 0.000 (Adjusted R Squared = -0.001); f. R Squared = 0.001(Adjusted R Squared = 0.000)

g. R Squared = 0.009 (Adjusted R Squared = 0.009)

Estimated marginal means were observed for *C1* in favour of males and *C7* in favour of females (see Table 30). Estimated marginal means were used to establish variations in what students' value in mathematics learning across sex of students.

Table 30: Estimated Marginal Means for Male and Female Students and their Values in Mathematics Learning

Dependent Variable	Sex	Mean	Std. Error	95% Confidence Interval	
				Lower Bound	Upper Bound
Fluency – C1	Male	32.295	0.247	31.810	32.779
	Female	31.415	0.270	30.885	31.944
Learning strategies – C7	Male	12.644	0.078	12.492	12.797
	Female	13.023	0.085	12.856	13.190

Effect of Teachers' Academic Qualification on the Attributes they Value in Students' Mathematics Learning

To explore the effect of academic qualification of teachers on attributes they value in their students' mathematics learning, a one -way MANOVA was conducted with academic qualifications as the independent variable and mathematics teachers' values in mathematics as the dependent variables: *Understanding – C1, Versatility – C2* and *Achievement – C3.*

Results from Table 31 show Pillai's Trace value was significant at 0.000 indicating that there are significant differences among mathematics teachers with different academic qualifications in terms of what they value in mathematics[Pillai's Trace $= 0.970$, $F(3, 164) = 1766.187$, $p = 0.000$, $\eta^2 = 0.970$].

Table 31: Summary of Multivariate Analysis of Variance (MANOVA) for Academic Qualification Variable

Effect		Value	F	Hypothesis df	Error df	Sig.	Partial Eta Squared (η^2)
Academic Qualification	Pillai's Trace	0.970	1766.187[b]	3.000	164.000	0.000	0.970

Source: Field survey (2021)

a. The statistic is an upper bound on F that yields a lower bound on the significance level (0.05)

Tests of between - subjects effects as displayed in Table 32 show that statistically significant differences exist among pre-tertiary teachers who teach mathematics with varied academic qualifications in terms of what they value in mathematics for two out of the three values of mathematics attributes: *Understanding – C1*[$F(7, 166) = 3.439$, $p = 0.002$, $\eta^2 = 0.127$] and *Versatility – C2* [$F(7, 166) = 3.405$, $p = 0.002$, $\eta^2 = 0.0126$]. However, mathematics teachers' value in students' mathematics learning subscale labelled *Achievement – C3* [$F(7, 166) = 1.324$, $p = 0.242$, $\eta^2 = 0.053$] did not show a significant difference among mathematics teachers with different academic qualifications (see Table 32).

Table 32: Tests of Between-Subjects Effects of (MANOVA) for the Independent Variable and the Dependent Variables

Source	Dependent variable	Type III Sum of Squares	Df	Mean Square	F	Sig.	Partial Eta Square (η^2)
Academic qualification	Understanding C1	33325.191 [a]	7	475.027	3.439	0.002	0.127
	Versatility – C2	608.232 [b]	7	86.890	3.405	0.002	0.126
	Achievement– C3	37.644 [c]	7	5.378	1.324	0.242	0.053

Source: Field survey (2021)

a. R Squared = 0.127 (Adjusted R Squared = 0.090);
b. R Squared = 0.126 (Adjusted R Squared = 0.089)
c. R Squared = 0.053 (Adjusted R Squared = 0.013)

Tukey's Honestly Significant Difference (HSD) Post Hoc multiple comparisons test was performed to determine the mean difference among the teachers' academic qualifications that contributed to the overall significant F-ratio value. Thus, to find out which of the seven different academic qualifications (independent variable) mathematics teachers at the pre-tertiary level (Teacher's Certificate 'A', Diploma in Basic Education, Bachelor degree in Basic Education, Bachelor degree (Math) with teacher training, Bachelor degree (Math) with no teacher training, Master degree (Math) with teacher training, Master degree (Math) with no teacher training) and the dependent variables involving the two teacher valuing in students' mathematics learning (C1 and C2) where the significant differences were located. The results of the Post Hoc test revealed significant pairwise differences between mathematics teachers who possess Bachelor degree in Basic Education and Bachelor degree (Math) with teacher training for the component labelled *Understanding – C1*. The analysis revealed significant pairwise differences between mathematics teachers who hold Bachelor degree (Math) with teacher training and their counterparts who hold Master degree (Math) with no teacher training for C1. The analysis further showed significant academic qualifications pairwise differences between teachers who hold Diploma in Basic Education and those who hold Bachelor degree (Math) with and without teacher training for *Versatility – C2* (see Table 33).

Table 33: Post Hoc Tests (Tukey HSD)

Dependent variable	(I) academic qualification	(J) academic qualification	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
UnderstandingC1	Teacher's Cert. 'A'						
		DBE	3.05	8.402	1.000	-22. 03	28.13
		BBE	-4.57	8.577	0.998	-30.17	21.03
		B.ed(Math)	5.88	8.457	0.993	-19.37	31.12
		Bsc.(Math)	-2.38	9.264	1.000	-30.02	25.27
		Mphil/Med (Math Edu)	5.86	8.858	0.994	-20.58	32.30
		Mphil/Msc (Math)	-15.83	10.697	0.756	-47.76	16.09
	DBE						
		Teacher's Cert. 'A'	-3.05	8.402	1.000	-28.13	22.03
		BBE	-7.62	2.615	0.061	-15.43	0.18
		B.ed (Math)	2.83	2.190	0.856	-3.71	9.36
		BSc (Math)	-5.42	4.370	0.877	-18.47	7.62
		Mphil/Med (Math Edu.)	2.81	3.427	0.983	-7.41	13.04
		Mphil/MSc (Math)	-18.88	6.907	0.096	-39.50	1.73
	BBE						
		Teacher's Cert. 'A'	4.57	8.58	0.998	-21.03	30.17
		DBE	7.62	2.62	0.061	-0.18	15.43
		B.ed (Math)	10.45*	2.79	0.004	2.13	18.76
		BSc (Math)	2.20	4.70	0.999	-11.82	16.22
		Mphil/Med (Math Edu.)	10.43	3.84	0.100	-1.02	21.88
		Mphil/MSc (Math)	-11.26	7.12	0.694	-32.51	9.98

Table 33 Cont'd

B.ed (Math)						
	Teacher's Cert. 'A'	-5.88	8.46	0.993	-31.12	19.37
	DBE	-2.83	2.19	0.856	-9.36	3.71
	BBE	-10.45*	2.79	0.004	-18.76	-2.13
	BSc. (Math)	-8.25	4.47	0.521	-21.61	5.11
	Mphil/Med (Math Edu.)	-0.02	3.56	1.000	-10.64	10.61
	Mphil/MSc. (Math)	-21.71*	6.97	0.035	-42.52	0.89
BSc. (Math)						
	Teacher's Cert. 'A'	2.38	9.26	1.000	-25.27	30.02
	DBE	5.42	4.37	0.877	-7.62	18.47
	BBE	-2.20	4.70	0.999	-16.22	11.82
	B.ed (Math)	8.25	4.47	0.521	-5.12	21.61
	Mphil/Med (Math Edu.)	8.23	5.19	0.692	-7.27	23.73
	Mphil/MSc (Math)	-13.46	7.93	0.619	-37.14	10.22
Mphil/Med (Math Edu.)						
	Teacher's Cert. 'A'	-5.86	8.86	0.994	-32.30	20.58
	DBE	-2.81	3.43	0.983	-13.04	7.42
	BBE	-10.43	3.84	0.100	-21.88	1.02
	B.ed (Math)	0.02	3.56	1.000	-10.61	10.64
	BSc. (Math)	-8.23	5.19	0.692	-23.73	7.27
	Mphil/MSc (Math)	-21.69	7.46	0.061	-43.94	0.56
Mphil/MSc (Math)						
	Teacher's Cert. 'A'	15.83	10.70	0.756	-16.09	47.76
	DBE	18.88	6.91	0.096	-1.73	39.50
	BBE	11.26	7.12	0.694	-9.98	32.51
	B.ed (Math)	21.71*	6.97	0.035	0.89	42.52
	BSc. (Math)	13.46	7.93	0.619	-10.22	37.14
	Mphil/Med (Math Edu.)	21.69	7.46	0.061	-0.56	43.94

Table 33 Cont'd

Versatility – C2	Teacher's Cert. 'A'	DBE	-3.63	3.62	0.952	-14.43	7.16
		BBE	-4.57	3.69	0.878	-15.59	6.45
		B.ed (Math)	-6.75	3.63	0.513	-17.61	4.11
		BSc. (Math)	-9.50	3.97	0.212	-21.40	2.40
		Mphil/Med (Math Edu.)	-6.79	3.81	0.563	-18.16	4.59
		Mphil/MSc (Math)	-9.67	4.60	0.358	-23.41	4.07
	DBE						
		Teacher's Cert. 'A'	3.63	3.61	0.952	-7.16	14.43
		BBE	-0.94	1.13	0.981	-4.30	2.42
		B.ed (Math)	-3.12*	0.94	0.019	-5.93	-0.30
		BSc. (Math)	-5.87*	1.88	0.034	-11.48	-0.25
		Mphil/Med (Math Edu.)	-3.15	1.47	0.336	-7.55	1.25
		Mphil/MSc (Math)	-6.03	2.97	0.400	-14.90	2.84
	BBE						
		Teacher's Cert. 'A'	4.57	3.69	0.878	-6.45	15.59
		DBE	0.94	1.13	0.981	-2.42	4.30
		B.ed (Math)	-2.18	1.20	0.539	-5.76	1.40
		BSc. (Math)	-4.93	2.02	0.190	-10.96	1.11
		Mphil/Med (Math Edu.)	-2.21	1.65	0.831	-7.14	2.71
		Mphil/MSc (Math)	-5.10	3.06	0.641	-14.24	4.05
	B.ed (Math)						
		Teacher's Cert. 'A'	6.75	3.64	0.513	-4.11	17.61
		DBE	3.12*	0.94	0.019	0.30	5.93
		BBE	2.18	1.20	0.539	-1.40	5.76
		BSc. (Math)	-2.75	1.93	0.786	-8.50	3.00
		Mphil/Med (Math Edu.)	-0.04	1.53	1.000	-4.61	4.54
		Mphil/MSc (Math)	-2.92	3.00	0.959	-11.87	6.04

Table 33 Cont'd

BSc. (Math)						
	Teacher's Cert. 'A'	9.50	3.97	0.212	-2.40	21.40
	DBE	5.87*	1.88	0.034	0.25	11.48
	BBE	4.93	2.02	0.190	-1.11	10.96
	B.ed (Math)	2.75	1.93	0.786	-3.00	8.50
	Mphil/Med (Math Edu.)	2.71	2.24	0.888	-3.96	9.39
	Mphil/MSc (Math)	-0.17	3.41	1.000	-10.36	10.02
Mphil/Med (Math Edu.)						
	Teacher's Cert. 'A'	6.79	3.81	0.563	-4.59	18.16
	DBE	3.15	1.47	0.336	-1.25	7.55
	BBE	2.21	1.65	0.831	-2.71	7.14
	B.ed (Math)	0.04	1.53	1.000	-4.54	4.61
	BSc. (Math)	-2.71	2.23	0.888	-9.39	3.96
	Mphil/MSc (Math)	-2.88	3.21	0.972	-12.46	6.70
Mphil/MSc (Math)						
	Teacher's Cert. 'A'	9.67	4.60	0.358	-4.07	23.41
	DBE	6.03	2.97	0.400	-2.84	14.90
	BBE	5.10	3.06	0.641	-4.05	14.24
	B.ed (Math)	2.92	3.00	0.959	-6.04	11.87
	BSc. (Math)	0.17	3.41	1.000	-10.02	10.36
	Mphil/Med (Math Edu.)	2.88	3.21	0.972	-6.70	12.46

Source: Field survey (2021)
*The mean difference is significant at 0.05 level.

The estimated marginal means of the teachers' academic qualifications show that teachers who hold Master degree (Math) with no teacher training value *Understanding – C1* most in mathematics with teachers who hold Bachelor degree (Math) with teacher training valuing *Understanding – C1* the least. Again, teachers who hold Master degree (Math) with no teacher training had the highest mean with teachers who hold Teacher's Certificate 'A' having least mean in the value attribute labelled *Versatility– C2*. This implies that teachers who hold Master degree (Math) with no teacher training value *Versatility – C2* the most in mathematics with teachers who hold Teacher's Certificate 'A' valuing *Versatility– C2* the least (see Table 34).

Table 34: Estimated Marginal Means for Academic Qualifications of Teachers and their Valuing in Students' mathematics Learning

				95% Confidence Interval	
Dependent Variable	Academic Qualification	Mean	Std. Error	Lower Bound	Upper Bound
Understanding	Teacher's Certificate A	148.500	8.311	132.092	164.908
	Diploma in Basic Education	145.451	1.395	142.697	148.205
	Bachelor degree in Basic Education	153.071	2.221	148.686	157.457
	Bachelor degree (Math) with teacher training	142.596	1.714	139.211	145.980
	Bachelor degree (Math) with no teacher training	150.875	4.155	142.671	159.079
	Master degree (Math) with teacher training	142.643	3.141	136.441	148.845

Table 34: Cont'd

	Master degree (Math) with no teacher training	164.333	6.786	150.936	177.731
Versatility	Teacher's Certificate A	25.000	3.572	17.947	32.053
	Diploma in Basic Education	28.634	.600	27.450	29.818
	Bachelor degree in Basic Education	29.571	.955	27.686	31.456
	Bachelor degree (Math) with teacher training	31.681	.737	30.226	33.136
	Bachelor degree (Math) with no teacher training	34.500	1.786	30.974	38.026
	Master degree (Math) with teacher training	31.786	1.350	29.120	34.451
	Master degree (Math) with no teacher training	34.667	2.917	28.908	40.425

Effect of Teachers' Teaching Experience on the Attributes they Value in Students' Mathematics Learning

In order to explore the effect of teaching experience of teachers on attributes they value in their students' mathematics learning, a one-way MANOVA was conducted with teachers' teaching experience as the independent variable and mathematics teachers' valuing in students' mathematics learning as the dependent variables: *Understanding – C1*, *Versatility – C2* and *Achievement – C3*.

Results from Table 34 showed that Pillai's Trace value was significant at 0.000 with effect size (partial eta squared) of 0.986 indicating that 98.6 percent of the variance in what mathematics teachers value in students' mathematics learning is explained by teachers' experience in teaching mathematics. The results also show that there are significant differences among mathematics teachers with diverse teaching experiences relative to what they value in students' mathematics learning[Pillai's Trace $= 0.986,\ F(3, 168) = 3985.703,\ p = 0.000,\ \eta^2 = 0.986$].

Table 35: Summary of Multivariate Analysis of Variance (MANOVA) for Teachers' Teaching Experience Variable

Effect		Value	F	Hypothesis df	Error df	Sig.	Partial Eta Squared (η^2)
Teaching Experience	Pillai's Trace	0.986	3985.703[b]	3.000	168.000	0.000	0.986

Source: Field survey (2021)

a. The statistic is an upper bound on F that yields a lower bound on the significance level (0.05)

Tests of between - subjects effects as displayed in Table 35 show that statistically significant differences exist among pre-tertiary teachers who teach mathematics with different teaching experiences in terms of what they value in students' mathematics learning for only one out of the three values of mathematics attributes: *Versatility – C2* $[F(4, 166) = 4.761,\ p = 0.001,\ \eta^2 = 0.101]$. However, mathematics teachers' value in mathematics subscale labelled *Understanding – C1*$[F(4, 166) = 2.286,\ p = 0.062,\ \eta^2 = 0.051]$ and

Achievement – C3 $[F(4, 166) = 0.955,\ p = 0.434,\ \eta^2 = 0.022]$ did not show a significant difference among mathematics teachers with diverse teaching experiences (see Table 36).

Table 36: Tests of Between-Subjects Effects of (MANOVA) for the Independent Variable and the Dependent Variables

Source	Dependent variable	Type III Sum of Squares	Df	Mean Square	F	Sig	Partial Eta Square (η^2)
	Understanding C1	1312.247 [a]	4	328.062	2.286	0.062	0.051
Teaching Experience	Versatility – C2	506,079 [b]	4	126.520	4.761	0.001	0.101
	Achievement – C3	15.842 [c]	4	3.961	0.955	0.434	0.022

Source: Field survey (2021)

a. R Squared = 0.051 (Adjusted R Squared = 0.029);
b. R Squared = 0.101 (Adjusted R Squared = 0.080)
c. R Squared = 0.022 (Adjusted R Squared = - 0.001)

Discussion of Results

The study's findings are discussed in accordance with the nine research questions. What primary, Junior High School (JHS) and Senior High School (SHS) teachers' value in their students' mathematics learning was explored in research question one. The study revealed that the seven most rated items on the WIFI questionnaire by the pre-tertiary mathematics teachers include: *connecting mathematics to real life, working step-by-step, using concrete materials to understand mathematics, teacher giving feedback, examples to help students understand, problem solving* and *using diagrams to understand mathematics*. These value characteristics that were rated higher by the mathematics teachers will ensure student-centred teaching in the mathematics classroom. The teachers rating these value items higher than the rest of the items show that pre–tertiary mathematics teachers' conception of mathematics teaching is geared towards students' conceptual and procedural knowledge as well as having control over content and subject matter.

Conversely, the study revealed seven least rated value items by the pre-tertiary mathematics teachers: *shortcuts to solving a problem, using the calculator to calculate, being lucky at getting the correct answer, using calculator to check the answer, memorizing facts, mystery of mathematics* and *stories about mathematics*. It is obvious that mathematics teachers will rate *shortcuts to solving a problem* as least valued because mathematics teachers believe that solving of problems in mathematics is a process but not a product (Giannakopoulos, 2012). Mathematics teachers most often will like their students to go through the rigorous step by step way of solving mathematics problems. They believe that there should be coherence between the answers to mathematics problems and the processes leading to those answers. Thus, the processes a person goes through when solving mathematics problems are as important as the answers. Also, *using the calculator to calculate* and *using calculator to check the answer* being part of the seven least valued attributes by the pre-tertiary mathematics teachers is to be expected. This is because two-thirds of the teacher cohort (primary school and JHS teachers) who responded to the TQ do not use calculators when teaching their students mathematics and therefore will not value them highly in their students' mathematics learning. Again, the value item *being lucky at getting the correct answer* being among the least valued attributes is a demonstration that mathematics teachers at the pre-tertiary level of education recognise that obtaining correct answers in mathematics is not by mere luck but it is about having relational understanding of the concepts and spending adequate time practicing with varied problems in mathematics.

The large variance observed in the teachers' responses to these seven value items which lie at the base of the list of the valued attributes give an indication that the respondents failed to reach a consensus in their response on how important these items are to their learners' learning of mathematics. It is important to emphasize that the highly rated value items on the modified version of the WIFI questionnaire by the pre-tertiary mathematics teachers are enshrined in the core competencies which the new Ghanaian mathematics curriculum want teachers to inculcate in students compared to the least valued items.

Further analysis using PCA confirmed 3 value attributes which teachers who teach mathematics at the pre-tertiary level value in their students' mathematics learning. These subscales in descending order of contribution to the total variance explained were *Understanding – C1, Versatility – C2* and *Achievement – C3*. Teachers are trained to focus on the important aspect of mathematical knowledge which will guarantee students' academic advancement.

Thus, teachers who exhibit longevity and resilience in learning demonstrate efficient teaching techniques, high content knowledge, deeper comprehension of utility of mathematical concepts, conversant with theories underlying instructional choices and self-confidence in making decision (Ma, 1999). Versatile teachers are highly qualified teachers who are well versed in mathematics content knowledge and pedagogical skills and are more effective. Highly qualified teachers with the purpose of improving students mathematics achievement is basically a goal which teachers cannot afford to ignore (Bransford, Brown & Cocking, 1999).

What primary, Junior High School (JHS) and Senior High School (SHS) students' value in their mathematics learning was explored in research question two. The results of the study revealed that *Remembering the work we have done, Knowing the steps of the solution*, *Examples to help students understand, Knowing which formula to use, Explaining by the teacher, Getting the right answer and Working step-by-step* were the seven most valued items by the pre-tertiary students on the WIFI questionnaire. Pre-tertiary students valuing these items as their topmost priority in their mathematics learning show that they are focused on achieving success in mathematics through instrumental and relational understanding of mathematical concepts. These value items being the highest valued attributes by the students at the pre-tertiary level affirm that they are mostly concern with experiencing success in their high stake national and international final examinations.

On the contrary, *Using the calculator to calculate*, *Stories about mathematics, Being lucky at getting the correct answer, Stories about recent developments in mathematics, Mathematics debates, Stories about mathematicians* and *Appreciating the beauty of mathematics* were the seven lowest ranked value items by the pre-tertiary students. The use of calculator is allowed in examinations in Ghanaian schools from SHS upwards. This implies that only one-third of the student cohort (SHS students) who responded to the WIFI questionnaire are allowed to use calculator to do mathematics. That explains in part why *Using the calculator to calculate* was the least ranked value attribute on the WIFI questionnaire by the learners. Also, *Stories about recent developments in mathematics, Mathematics debates, Stories about mathematicians* and *Appreciating the beauty of mathematics* might be new to the students. Perhaps, it was the first time the student came across such terminologies in mathematics. These attributes that have been ranked low by the pre-tertiary students in terms of their valuing in their mathematics learning seem not to be directly related to the Ghanaian pre-tertiary mathematics curricula.

PCA also identified seven attributes that pre-tertiary students valued greatly in their mathematics learning. They included: *Fluency – C1, Understanding – C2, Instructional Materials/Activities – C3, Connections – C4, ICT – C5, Feedback – C6 and Learning Strategies – C7*. These attributes which have been arranged according to their individual proportional contribution to the total variance altogether explains 43.00% of the total variance. The seven attributes (*Fluency, Understanding, Instructional Materials/Activities, Connections, ICT, Feedback and Learning Strategies)* valued in the learning of mathematics by the students participants in this study compared favourably with that of a study in the year 2017 involving public primary and high school students in the Cape Coast metropolis of Ghana by Seah, Davis and Carr (2017). At the SHS level, the students valuing similar attributes in their mathematics learning may probably be that some of the learners who were partakers in the study by Seah et al. who were then in JHS and Primary school might have gained admission in some of the boarding SHS in the metropolis selected for the study.

Duplantier, Ksoll, Lehrer and Seitz (2017) have reported that intranational mobility particularly migration to access senior high school education is highest across nearby regions in Ghana. They further reported that the Western region is one of the regions in the country that attract JHS graduates from the other regions to SHS in the region.

Ghana's education system runs boarding school system where students from far and near localities are housed and fed in the schools. Based on this, Ghanaian students can choose to attend SHS in any place of their choice across the nation to access education. It is possible that some of the students who took part in the Cape Coast study though were staying there during the period of their JHS and Primary school education had migrated to study in some of the boarding SHSs in the metropolis selected for this study and coincidentally participated in this study. It is possible that the learners whose values in their mathematics learning were explored during their JHS and Primary school days in the Cape Coast study had held on to their values up till now. Seah, Pan and Zhong (2022) have reported that values remain the most stable affective and conative variable in learners as they progress through pre-tertiary educational levels.

The attributes established in this study also demonstrate more of extrinsic valuing (*instructional materials/activities, connections, feedback and learning strategies*) by Ghanaian students which Seal et al. (2017) earlier argued that perhaps may explained in part why Ghanaian students over the years have performed poorly in mathematics on international examinations such as TIMSS. This assumption is further anchored by intrinsic valuing of Ghanaian students' counterpart in countries in Eastern Asia (Korea, Shanghai, Hong Kong, and Singapore). Intentionally or coincidentally, the students' intrinsic valuing in the learning of mathematics result in high performance in mathematics on PISA and TIMSS.

How similar or different are values of teachers and their students in mathematics learning were explored in research question three. While the teachers value three attributes (*Understanding*, *Versatility* and *Achievement*) in students' mathematics learning, their students value seven attributes (*Fluency, Understanding, Instructional Materials/Activities, Connections, ICT, Feedback, and Learning Strategies*) in their mathematics learning. What pre-tertiary students value in their mathematics learning was more than twice what their teachers value in mathematics learning. Studies involving students valuing in mathematics learning (Davis, Carr & Ampadu, 2019; Davis, Seah, Howard & Wilmot, 2021; Hill, Hunter & Hunter, 2019; Seah, Davis & Carr, 2017; Yankson, 2020; Zhang, 2019) provide a more significant number of value attributes portrayed by students compared to the number of value attributes exhibited by teachers in research studies involving both teachers and students valuing or teacher valuing alone (Abass, 2021; Dede, 2019; Dede, 2015). Among the three value attributes identified by teachers as important to students mathematics learning and that of the seven value attributes portrayed by students as important to their mathematics learning, only the component labelled *Understanding* (understanding concepts / processes; examples to help students understand; students posing mathematics problems; remembering the work we have done; knowing which formula to use;

getting the right answer and knowing the steps of the solution) was co-valued by both teachers and learners. This outcome of the study is consistent with Frade and Machado's (2008) finding that teachers' values in mathematics education impact strongly on learners' values, attitudes, beliefs as well as their feelings. The finding with respect to co-valuing of *Understanding* by teachers and students was very much expected. Firstly, mathematics teachers valuing *Understanding* is perceived as a requirement to be a successful mathematics teacher in particular and excelling in the teaching profession in general (Yankson, 2020). Secondly, primary school pupils and JHS students valuing *Understanding* suggest that getting it right at the foundational levels of education propels them to excel at the JHS and SHS levels respectively where the mathematics content is perceived to be quite challenging. This same component being valued by SHS students perhaps suggest that they will want to be well grounded in the SHS mathematics to overcome the ever demanding nature of mathematical thinking needed to succeed at the tertiary level and the world of work.

Also, pre-tertiary mathematics curriculum advocates for a little below one-third of the total percentage weightings for instruction, learning and evaluation in mathematics should be on knowledge and understanding. Teachers and students put emphasis on acquisition of this profile dimension when teaching and learning mathematics to ensure attainment of the planned mathematics curriculum. *Understanding*, though constitutes one of the lower order dimensions within the spectrum of the profile of learning behaviour dimensions, still relevant in the scheme of mathematics instruction, learning and assessment. *Versatility* and

Achievement were valued singly by only teachers. On the other hand, the six remaining components (*Fluency*, *Teaching materials/activities*, *Connections*, *ICT*, *Feedback* and *Learning strategies*) were independently valued by only students. Notwithstanding the non-alignment between learners' and teachers' valuing in mathematics learning in certain value attributes, there are some value items that loaded under both *Versatility* (using calculator to check the answer, using calculator to calculate, learning with the internet, verifying theorems/hypotheses, learning the proofs, stories about recent developments in mathematics, explaining where rules/formulae came from; and mystery of mathematics e.g 111 111 111 × 111 111 111 = 12345678987654321) for teacher valuing and *ICT* (using the calculator to calculate, using the calculator to check the answer, learning maths with the computer and learning maths with the internet) for students valuing in mathematics learning. The differences in label names emanate from additional value items that loaded under *Versatility* which resulted in labelling it as *Versatility* but not *ICT*.

The effect of grade level teachers teach (primary, JHS and SHS) on the attributes they value in students' mathematics learning was explored using research question four. According to the findings of the study, there were statistically significant differences among pre-tertiary mathematics teachers' values in students' mathematics learning across grade levels. There was a significant difference in all the three mathematics teachers' values in students' mathematics learning (C1, C2 and C3) across grade levels (primary school, JHS and SHS). This is partly due to the fact that as learners go through the educational system from one grade level to the next, they are likely to engage in more complicated mathematics which are more likely to bring forth distinct values (Davis, Carr & Ampadu, 2019). Teachers who teach primary school pupils mathematics value *Understanding – C1* in their students' mathematics learning more than their colleagues who are teaching mathematics in JHS and SHS. At the primary school level, the concentration of mathematics teachers is to help their learners to develop high interest in studying the subject at the early stage of their academic life.

Mathematics teachers at this level teach their learners to have deeper (relational) understanding of the concepts in the mathematics syllabus. The foundation blocks of learners' mathematical proficiency for higher level mathematics are laid at the Primary school. Primary school teachers should create zone of proximal development for their learners, encourage co-construction of meanings and ideas with the learner based on the understanding and awareness of the child's perspective with the aim of making the learner a critical thinker and academically independent (Bruner, 1986). SHS mathematics teachers valuing *Versatility – C2* in students' mathematics learning more than teachers who teach mathematics at both Primary school and JHS was expected. Mathematics teachers at SHS level need to expose their students to a myriad of mathematics questions (mathematics tasks, mathematical problems and mathematical investigations) and mathematics problem solving strategies (mathematical heuristics) to make them flexible in the study of mathematics (Graham & Thomas, 2005). Teachers in JHS valued *Achievement – C3* higher than their counterparts in Primary and SHS.

Probably, the focus of the teachers is for their students to perform well on their BECE in order to gain admission into their first choice SHS which is highly competitive and also meet the pass rate target the metropolitan/municipal/district directorate of education has set for core subject teachers in the BECE.

The effect of grade level of students (primary, JHS and SHS) on the attributes they value in their mathematics learning was explored using research question five. The results of the study revealed that what pre-tertiary students' value in the learning of mathematics have statistically significant variation with respect to their grade levels. Similar finding was observed in a study in Eastern China on the attributes valued by Primary, JHS, and SHS students. In that study, Tang, Seah, Zhang and Zhang (2021) confirmed that students at different school grade levels have relatively different values in the learning of mathematics.

By comparing the pupils/students at these three grade levels (primary, JHS and SHS) in the Ghanaian context with respect to this study, statistically significant variations among pre-tertiary students on what they value in their mathematics learning were discovered in five (C3, C4, C5, C6 and C7) out of the seven value components (C1, C2, C3, C4, C5, C6 and C7). The spiral nature of the Ghanaian pre-tertiary mathematics curriculum which makes learners learn relatively the same concept from Primary school through JHS to SHS with increasing level of complexity of the topic (Bruner, 1960) may have introduced variations in what students' value in mathematics learning across these three grade levels.

Again, the phenomenon of "high school pressure" might have introduce inner consciousness and induced grade level differences among learners as progress from JHS to SHS. Pieces of advice and authentic teaching offered by teachers coupled with parents' effort in arranging extra tuition opportunity for their children at JHS to enhance their chances of getting admission in "good schools" might have brought about valuing differences in mathematics learning among learners from JHS to SHS.

Although the value attribute, *ICT – C5* (Using the calculator to calculate, Using the calculator to check the answer, Learning maths with the computer and Learning maths with the internet) was valued by all the students at the different grade levels, it was valued most by SHS students, followed by JHS students and primary school pupils in that order. This order of valuing in mathematics learning among students across grade levels is consistent with the findings of the work of Davis, Carr and Ampadu (2019) and this was highly expected. JHS students and primary school pupils have not been exposed to the use of calculator and other ICT tools in their lessons, albeit the mathematics curriculum of these cohorts advocate for their usage. However, at the SHS, students are encouraged to use calculators in their mathematics learning. They have access to government's free internet connectivity together with some appreciable level of technological infrastructure to support mathematics teaching and learning in schools which is almost non-existent in most public primary and JHSs.

The effect of sex (male and female) on valuing in students' mathematics learning among mathematics teachers was explored. The findings of the study showed an increase in the estimated marginal mean from females to males for the valuing of *Versatility* (using calculator to check the answer, using calculator to calculate, mystery of mathematics, verifying theorems/hypothesis, stories about recent development in mathematics, explaining where rules/formulae came from, learning mathematics with the internet and learning the proofs). Male teachers valued attributes such as mystery of mathematics, stories about recent development in mathematics, verifying theorems/hypothesis, explaining where rules/formulae came from, learning the proofs more than their female counterpart. These attributes generally relate to the sociological values of mystery and openness in values in western mathematics (Bishop, 1988). In the same way, female teachers valued attributes such as using calculator to check the answer, using calculator to calculate, learning mathematics with the internet less than their male colleagues. In general, these value items relate to the value attribute of ICT. However, the use of technological tools and other ways of learning mathematics is embraced by both male and female mathematics teachers in their students' mathematics learning.

The effect of sex (male and female) of students' on the attributes they value in their mathematics learning was also explored. There was significant differences between sex and what students' value in their mathematics learning for *Fluency – C1* and *Learning strategies – C7*. Thus, valuing of *C1* is lower in females than males and valuing of *C7* is lower in males compared to females. The former affirms the research studies that girls are taught to do mathematics in a more 'rote' manner using rules, algorithms and more conventional approaches and boys are taught to be more autonomous and use a more independent method and unconventional approaches (Kimball, 1989). However, the latter confirmed research by Kaldo and Oun (2020) that although both sex find learning strategies in mathematics learning to be important because they enable students to learn new information easier and better, female learners may have effective learning strategies than male learners. They further reported that female students make effort to organize the material in a way that will be simple for them to recollect. They review their notes and prioritize their points more than men do. Females showed more powerful organising skills and have better repeating strategies than their male counterpart.

The researcher explored the effect of teachers' academic qualification on the attributes they value in students' mathematics learning. There was no s*ignificant difference in what mathematics teachers' value across their academic qualifications* with respect to the value of *Achievement – C3*. This means that teachers who teach mathematics at different grade levels of the pre-tertiary level of education recognize that achievement of students in mathematics is paramount to their success now and in their future. The finding agrees with a study by Goe (2007) who reported that teachers' academic qualifications particularly their special area and content knowledge does not affect their value in students mathematics achievement.

However, for the values of *Understanding – C1* and *Versatility – C2*, significant differences among mathematics teachers with different academic qualifications in terms of what they value in students' mathematics were observed. This finding may not be so surprising because these in-service teacher cohorts offered different academic programmes that prepared them to espouse unique values in mathematics learning. The estimated marginal mean of the teachers' academic qualification on what they value in students' mathematics learning shows that teachers who hold Bachelor degree (Math) with teacher training value *Understanding* the least. This is another major surprise because teachers who have gone through teacher training in both mathematics content and methods and so one would expect that they will appreciate the importance of understanding to the academic success of students compared to teachers whose initial training was in the area of mathematics content only.

In furtherance to that, most of these teachers are found teaching mathematics at the SHS where deeper understanding is needed to overcome the ever challenging and demanding content of mathematics at that level. Again, teachers who hold Master degree (Math) with no teacher training value *Versatility – C2* most in mathematics learning with teachers who hold Teacher's Certificate 'A' valuing *Versatility– C2* the least. These teachers have had series of exposure to more content courses at both the undergraduate and postgraduate levels. One would expect that these teachers will appreciate the affordance of teaching mathematics with different and deeper perspectives far more than teachers who are Teacher's Certificate 'A' and Diploma in Basic Education holders.

Again, the effect of teachers' teaching experience on the attributes they value in students' mathematics learning was explored. Statistically significant difference among pre-tertiary teachers with varied teaching experience was observed for *Versatility – C2*. Teachers are lifelong learners. They learn from their colleagues and students. For instance, if a teacher has forty students in his/her class, he learns about forty ways of solving a mathematics problem he has given to them. As teachers teach for many years, they get exposed to different curriculum materials and resources, meet several experts in the subject area through workshops and conferences which add to their repertoire of knowledge in the field. As teachers learn from diverse sources and perspectives, they become versatile in addressing the challenges students face in mathematics. With this, the more years a teacher teaches, the more flexible he becomes in dealing with students questions, challenges and misconceptions in mathematics (Graham & Thomas, 2005).

CHAPTER FIVE

SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

Overview

This chapter outlines the summary of the findings, conclusions and provides recommendations as well as suggestions for further research.

Summary

The purpose of the study was to explore what pre-tertiary mathematics teachers value in their students' mathematics learning and what their students' value in their own mathematics. Also, the study sought to determine if there is alignment between mathematics teachers and their students on what they find important in mathematics learning.

The study was directed by six research questions. They included:

1a. What do primary, Junior High School (JHS) and Senior High School (SHS) teachers' value in their students' mathematics learning?

1b. What do primary, Junior High School (JHS) and Senior High School (SHS) students' value in their mathematics learning?

2.How similar or different are values of teachers and their students in mathematics learning?

3a. What is the effect of grade level teachers teach (primary, JHS and SHS)) on the attributes they value in students' mathematics learning?

3b. What is the effect of grade level of students (primary, JHS and SHS) on the attributes they value in their mathematics learning?

4a. What is the effect of sex on valuing in students' mathematics learning among mathematics teachers?

4b. What is the effect of sex of students' on the attributes they value in their mathematics learning?

5. *What is the effect of teachers' academic qualification on the attributes they value in students' mathematics learning?*
6. *What is the effect of teachers' teaching experience on the attributes they value in students' mathematics learning?*

The study adopted a multistage sampling procedure to select participants. A stratified random sampling technique was used to select Primary schools and Junior High Schools based on urban and rural locations of schools. The Primary schools and JHSs in the selected metropolis were put into four strata. Urban Primary schools represented one stratum, Urban JHSs constituted another stratum, Rural Primary schools formed a stratum and Rural JHSs formed the last stratum. A stratified random sampling technique was used to select schools from each stratum. Simple random sampling with proportional allocation of samples was used to select students from each stratum. 200 and 136 primary school pupils were randomly selected from 7 urban and 5 rural schools respectively of the metropolis to participate in the study. 21 and 15 of the teachers from the urban and rural schools respectively who teach mathematics in these schools were purposely selected to partake in the study. On the other hand, 180 and 145 JHS students were randomly selected from 9 urban and 6 rural schools respectively of the metropolis to participate in the study. 20 and 14 of the teachers from the urban and rural schools respectively who teach mathematics in these schools were purposely selected to partake in the study.

Also, SHSs were put into strata according to school type (co-educational, single sex female and single sex male schools). Co-educational schools were put in one stratum, single sex female schools constituted a stratum and single sex male schools formed another stratum. 271 students from co-educational, 158 students from single sex female and 173 students from single sex male schools were randomly selected. The corresponding number of their teachers (45 from co-educational schools, 26 from single sex female schools and 36 from single sex male schools) were purposively selected to take part in the study from the seven selected SHSs.

The data was collected using questionnaires. The questionnaires focused on what Primary, Junior High School (JHS) and Senior High School (SHS) teachers and students value in mathematics learning. 1263 students from the selected schools completed the "What I Find Important" (WIFI) in my mathematics learning questionnaire adopted for the study. Also, 177 teachers from the selected schools completed the modified version of the WIFI questionnaire. A Principal Component Analysis (PCA) with Kaiser normalization and varimax rotation was used to identify attributes valued by teachers and students in mathematics learning. Descriptive statistics mainly means and standard deviations were used to assess the extent to which teachers and students value the components (factors or attributes) in mathematics learning. MANOVA was used to examine the effects of grade level and sex of students on the attributes they value in their mathematics learning. It was also used to explore the effects of grade level teachers teach, teachers' sex, teachers' academic qualification and teaching experience on the attributes they value in students' mathematics learning.

Key Findings

The study showed that none of the value items on the WIFI questionnaire was rated as either unimportant or absolutely unimportant by the pre-tertiary mathematics teachers. However, 12.50% $(n = 8)$ of the items were rated as absolutely important, 82.81% $(n = 53)$ were valued as important whereas 4.69% $(n = 3)$ were rated as neither important nor unimportant by the teachers.

The study also revealed that the seven most rated items on the modified version of the WIFI questionnaire by the pre-tertiary mathematics teachers included: *connecting mathematics to real life, working step-by-step, using concrete materials to understand mathematics, teacher giving feedback, examples to help students understand, problem solving* and *using diagrams to understand mathematics*.

However, the study revealed seven least rated value items by the pre-tertiary mathematics teachers: *shortcuts to solving a problem, using the calculator to calculate, being lucky at getting the correct answer, using calculator to check the answer, memorizing facts, mystery of mathematics* and *stories about mathematics*.

The study showed that pre-tertiary teachers valued three attributes in students' mathematics learning. These were: *Understanding*, *Versatility* and *Achievement*.

With respect to what pre-tertiary students value in their mathematics learning, none of the value items on the WIFI questionnaire was rated by the pre-tertiary students as either unimportant or absolutely unimportant. Thus, for the 64 items on the WIFI questionnaire that were rated by the pre-tertiary students, 15.63% $(n = 10)$ of them were rated as absolutely important, 82.81% $(n = 53)$ were rated as important whilst 1.56% $(n = 1)$ was rated as neither important nor unimportant. This is a manifestation that the pre-tertiary students see 98.44% $(n = 63)$ of the 64 value items on the WIFI questionnaire as either important or absolutely important to their mathematics learning.

The results of the study further revealed that *Remembering the work we have done*, *Knowing the steps of the solution*, *Examples to help students understand, Knowing which formula to use*, *Explaining by the teacher, Getting the right answer and Working step-by-step* were the seven most valued items by the pre-tertiary students on the WIFI questionnaire.

On the contrary, *Using the calculator to calculate, Stories about mathematics*, *Being lucky at getting the correct answer, Stories about recent developments in mathematics, Mathematics debates, Stories about mathematicians* and *Appreciating the beauty of mathematics* were the seven lowest ranked value items by the pre-tertiary students.

The study revealed that pre-tertiary students valued seven attributes in their mathematics learning. They included: *Fluency*, *Understanding*, *Instructional materials/activities*, *Connections*, *ICT*, *Feedback* and *Learning strategies*.

The study further reported that out of the three value attributes held by pre-tertiary mathematics teachers in students' mathematics learning and the seven value attributes held by pre-tertiary students in their mathematics learning, the only co-valued characteristic/attribute by both pre-tertiary mathematics teachers and students in mathematics learning was the value of *Understanding*.

The study further revealed that teachers who teach Primary school pupils mathematics value *Understanding – C1* in their students' mathematics learning more than their colleagues who teach mathematics in JHS and SHS. SHS mathematics teachers value *Versatility – C2* in students' mathematics learning more than teachers who teach mathematics at both Primary school and JHS. Teachers in JHS value *Achievement – C3* higher than their counterparts in Primary and SHS.

The study also found that the value attribute, *ICT – C5* (Using the calculator to calculate, Using the calculator to check the answer, Learning maths with the computer and Learning maths with the internet), was valued most by SHS students followed by JHS students and Primary school pupils in that order.

The findings of the study showed an increase in the estimated marginal mean from female to male teachers in students' mathematics learning for the valuing of *Versatility* (using calculator to check the answer, using calculator to calculate, mystery of mathematics, verifying theorems/hypothesis, stories about recent development in mathematics, explaining where rules/formulae came from, learning mathematics with the internet and learning the proofs).

The study found that there were significant differences between sex on what students' value in their mathematics learning for *Fluency – C1* and *Learning strategies – C2*. Marginal mean score for *Fluency* was greater for males than females. On the other hand, marginal mean score for *Learning strategies* was less for males than females.

The study found that statistically significant differences exist among pre-tertiary teachers who teach mathematics with varied academic qualifications in terms of what they value in students' mathematics learning for *Understanding – C1* and *Versatility – C2*. Estimated marginal means of valuing across teachers' academic qualifications showed that teachers who hold Master degree (Math) with no teacher training value *Understanding – C1* and *Versatility – C2* the most in students' mathematics learning. Teachers who hold Bachelor degree (Math) with teacher training value the former the least and teachers who hold Teacher's Certificate 'A' value the latter the least in students mathematics learning.

The study found that teachers with high teaching experience (long years of teaching mathematics) value attribute *Versatility – C2* the most in students' mathematics learning.

Conclusions

Based on the summary of the findings of the study, it can be concluded that pre-tertiary mathematics teachers value student centred approach in students' mathematics learning. Thus, the highest rated value items by the mathematics teachers are geared towards instilling conceptual and procedural knowledge, relational understanding, critical thinking, problem solving and control in students. In general, the study revealed that pre-tertiary mathematics teachers valued three attributes in students' mathematics learning; *Understanding, Versatility* and *Achievement.*

On the part of the students, the study concluded that they value instrumental understanding, relational understanding, know-how and control in their mathematics learning. Based on the findings of the study, it is further concluded that pre-tertiary students value aspects of mathematics learning that have direct bearing on their classroom mathematics lessons. In general, the study revealed that pre-tertiary students valued attributes: *Fluency, Understanding, Instructional materials/activities, Connections, ICT, Feedback* and *Learning strategies* in their mathematics learning.

The study further reported that the value of *Understanding* was co-valued by both pre-tertiary mathematics teachers and their students. It is important to emphasize that both teachers and students have equal stake in it to ensure that it achieved in the mathematics classroom. *Understanding* is the foundational value attribute which links up all the other value attributes that are valued singly by teachers and students.

The findings of the study also showed that teachers who teach mathematics at different pre-tertiary educational level value different attributes in students' mathematics learning. The expectation of teachers relative to what they think is important for their students' mathematics learning differ by grade levels.

Again, in Ghana, the kind of academic programme a prospective mathematics teacher pursues at the teacher training institutions informs the kind of content they go through and the grade level such a graduate will teach. The kind of training they go through may have influenced their valuing in students' mathematics learning. The fact that these teachers come from different socio-cultural backgrounds may contribute to differences in what they value in students' mathematics learning.

From the findings of the study, it is further concluded that the valuing of *versatility* by teachers in students' mathematics learning differs for males and females in favour of males. The stereotype roles society has placed on males and females make them exhibit different versatility in their day-to-day dealings of which mathematics classroom is not an exception.

The study concluded based on its findings that sex differences among pre-tertiary students in mathematics learning is significant for both *Fluency* and *Learning strategies*. Students' ability to apply, demonstrate and transfer knowledge of mathematics (fluency) in solving problems in mathematics is generally common to both male and female students. However, social aspect may provide explanation for sex difference in mathematics learning. There is a general belief in the society that mathematics is a male domain.

The study concluded that mathematics teachers with varied academic qualifications value *Understanding – C1* and *Versatility – C2* differently in students' mathematics learning. The study shows that teachers who hold Bachelor degree (Math) with teacher training value *Understanding* the least. These teachers per their training recognise two types of understanding in mathematics. Since the type of understanding was not defined, the teachers who hold Bachelor degree (Math) with teacher training holders might see the understanding being portrayed here as instrumental whose limitations supersede that of its advantages and so will not be highly valued by them. Again, teachers who hold Master degree (Math) with no teacher training value *Versatility – C2* most in students' mathematics learning. This is because the advanced nature of their mathematics training may have influenced their high valuing of *Versatility* in their students' mathematics learning.

Again, the effect of teachers' teaching experience on the attributes they value in students' mathematics learning was observed for *Versatility – C2*. The number of years teachers teach mathematics have brought about variations in how they become flexible in their approach to solving problems in mathematics. Many years of teaching mathematics lead to mathematical exposure which is a requirement for success as a mathematics teacher.

Recommendations

In light of the study's key findings, the following recommendations are made for practice and policy.

1. Mathematics teacher training institutions such as Colleges of Education and Universities should reform their curriculum in mathematics education to include education on values. This change will ensure that mathematics teachers become conscious of their valuing stances and how it impacts on their classroom instructions.

2. Students should be encouraged to exhibit what they value in mathematics learning in the mathematics classroom during mathematics lessons so that teachers can identify those values that will not promote their academic success and help them to unlearn them.
3. Ghana Education Service (GES) should organise seminars and workshops to train teachers on values to enable them to be aware of their own values and that of their students. This will enable teachers to ensure value alignment between them and their students.
4. Also, workshops and conferences could be organised for in-service pre-tertiary mathematics teachers to enable them learn the values in mathematics learning that are important to the grade level they teach. This will enable the teachers to instill the right values into their respective students.
5. Again, Curriculum developers at the pre – tertiary level of education in Ghana should ensure that the values espoused by the pre–tertiary mathematics teachers and their students reflect those of the mathematics curriculum.
6. It is further recommended that mathematics teachers at the pre-tertiary level of education should instil in both their male and female students all the relevant values in mathematics learning needed to excel in mathematics.

7. The study recommends that female and male students should be encouraged to improve upon their valuing for *Fluency* and *Learning strategies* respectively in their mathematics learning.
8. Government of Ghana through the Ministry of Education should institute ways to keep the experienced teachers who have learnt and developed good values in mathematics learning to continue to teach mathematics at the pre-tertiary level. This will ensure that the schools have experienced teachers to inculcate the right values in mathematics learning into their students.

Suggestions for Further Research

1. Values and valuing in mathematics education is generally divided into three major areas. They include teachers' values in mathematics, students' values in mathematics learning and values in mathematics curriculum or textbooks. The study combined the first two broad areas and explored what teachers and students value in mathematics learning with the latter still remains a grey area in research in mathematics education in Ghana. For this reason, future studies could go into values portrayed by mathematics textbooks and curriculum used in teaching mathematics and their effects on students' values.
2. Ghana is a big nation with different regions, metropolis, municipalities and districts with diverse geographical and population characteristics. A study on what teachers and students value in mathematics learning at the pre-tertiary level in a metropolis in the Western region of Ghana provides

the basis for further research in this affective construct. In this regard, further study could be carried out to look into values of mathematics teachers and students in mathematics learning while extending the population to include teachers and students across the schools in the remaining regions of Ghana.

3. Future researchers could undertake a study to explore why teachers and students value different attributes across grade levels using qualitative methods.
4. Longitudinal study on values could be carried out to compare the values of low and high achievers as they progress from primary to SHS.
5. Future research could compare what teachers and students value in mathematics across rural and urban schools.

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APPENDICES

APPENDIX A

TEACHER QUESTIONNAIRE (TQ)

Dear Colleague,

The purpose of this study is to find out what teachers find important in students' learning of mathematics. The study is therefore solely for academic pursuit and does not in any way evaluate either teachers or institutions, or to call for any administrative changes. Your genuine response is very much needed for the success of this study. Please give your opinion about all the statements by ticking [√], circling or writing in the space provided.

You are assured that the information provided will be treated confidentially. Thank you for your maximum co-operation.

SECTION A

GENERAL INFORMATION

1. Sex: Male [ ] Female [ ]

2. Age ……………………………………….

3. How many years have you been teaching?

i. 1 – 5 years [ ]

ii. 6 – 10 years [ ]

iii. 11 – 15 years [ ]

iv. 16 – 20 years [ ]

v. Above 20 years [ ]

4. How long have you been teaching Mathematics?

i. 1 – 5 years [ ]

ii. 6 – 10 years [ ]

iii. 11 – 15 years [ ]

iv. 16 – 20 years [ ]

v. Above 20 years [ ]

5. Which educational level do you teach?

i. Primary School [ ]

ii. Junior High School (JHS) [ ]

iii. Senior High School (SHS) [ ]

6. My school is

i. co-educational (boys and girls) [ ]

ii. single sex female [ ]

iii. single sex male [ ]

7. Which category does your school belong to?

i. Category A [ ]

ii. Category B [ ]

iii. Category C [ ]

8. Have you ever attended In-Service Training for Mathematics teachers?

Yes [ ] No [ ]

9. What is your Professional Qualification?

i. Teacher's Certificate 'A' [ ]

ii. Diploma in Basic Education [ ]

ii. Bachelor degree in Basic Education [ ]

iii. Bachelor degree (Math) with teacher training [ ]

iv. Bachelor degree (Math) with no teacher training [ ]

iv. Master degree (Math) with teacher training [ ]

vi. Master degree (Math) with no teacher training [ ]

vii. Other (Please specify) ………………………………………………

10. For those who offered Basic Education Programme at Certificate A / Diploma/Degree in Basic Education level, was mathematics your major, minor or core subject?

Major [ ] Minor [ ] Core Subject [ ]

11. a) Did you opt for the teaching of mathematics in your school?

Yes [ ] No [ ]

b) If No, please state the reason (s) why you have taken to the teaching of mathematics.

…………………………………………………………………………………
…………………………………………………………………………………
…………………………………………………………………………………
…………………………………………………………………………………

12. What is your rank in Ghana Education Service (GES)?

i. Senior Superintendent [ ]

ii. Principal Superintendent [ ]

iii. Assistant Director [ ]

iv. Director [ ]

v. Other (Please specify)………………………………………

SECTION B

For each of the items below, tick a box to tell how **important** it is to you in your students' learning of mathematics. **Absolutely Unimportant - AU**, **Unimportant - U, Neither Important nor Unimportant** – **NIU**, **Important - I**, **Absolutely Important** – **AI**

ITEM	AU	U	NIU	I	AI
1. Investigations					
2. Problem-solving					
3. Small-group discussions					
4. Using the calculator to calculate					
5. Explaining by the teacher					
6. Working step-by-step					
7. Whole – class discussions					
8. Learning the proofs					
9. Mathematics debates					
10. Relating mathematics to other subjects in school					
11. Appreciating the beauty of maths					
12. Connecting maths to real life					
13. Practising how to use maths					

formulae					
14. Memorising facts (e.g Area of a rectangle = length × breadth)					
15. Looking for different ways to find the answer					
16. Looking for different possible answers					
17. Stories about mathematics					
18. Stories about recent developments in mathematics					
19. Students explaining their solutions to the class					
20. Mathematics puzzles					
21. Students posing maths problems					
22. Using calculator to check the answer					
23. Learning maths with the computer					
24. Learning maths with the internet					

25. Mathematics games					
26. Relationships between maths concepts					
27. Being lucky at getting the correct answer					
28. Knowing the times tables					
29. Students making up their own maths questions					
30. Alternative solutions					
31. Verifying theorems / hypotheses					
32. Using mathematical words (e.g angle)					
33. Writing the solutions step-by-step					
34. Outdoor mathematics activities					
35. Teacher asking students questions					
36. Practicing with lots of questions					
37. Doing a lot of mathematics work					

38. Given a formula to use					
39. Looking out for maths in real life					
40. Explaining where rules/formulae came from					
41. Teacher helping students individually					
42. Students working out maths by themselves					
43. Mathematics tests / examinations					
44. Teacher giving students feedback					
45. Feedback from my friends					
46. Teacher asking students questions					
47. Using diagrams to understand maths					
48. Using concrete materials to understand mathematics					
49. Examples to help students understand					
50. Getting the right answer					

51. Learning through mistakes					
52. Hands-on activities					
53. Teacher use of keywords (e.g 'share' to signal division; contrasting 'solve' and simplify)					
54. Understanding concepts / processes					
55. Shortcuts to solving a problem					
56. Knowing the steps of the solution					
57. Mathematics homework					
58. Knowing which formula to use					
59. Knowing the theoretical aspects of mathematics (e.g proof, definition of triangles)					
60. Mystery of mathematics (e.g $111\ 111\ 111 \times 111\ \ 111\ \ 111 = 12345678987654321$)					
61. Stories about mathematicians					
62. Completing mathematics					

work					
63. Understanding why my students solution is incorrect or correct					
64. Remembering the work we have done					

65. Comments (if any): ……………………………………………………………...

………………………………..

SECTION C

For each pair of phrases below, circle a number to indicate how **important** one phrase is to you in your students' mathematics learning than the other phrase. If you circle the middle number labelled '3', it means that both phrases are equally important to you. If you circle the number labelled '2', it means that the phrase at the left hand side is **important** to you. If you circle the number labelled '1', it means that the phrase at the left hand side is **more important** to you. However, if you circle the number labelled '4', it means that the phrase at the right hand side is **important** to you. If you circle the number labelled '5', it means that the phrase at the right hand side is **more important** to you.

66. *How* the answer to a problem is obtained			*What* the answer to a problem is	
1	2	3	4	5
67. Feeling *relaxed* or having *fun* when doing maths			*Hard work* is needed when doing maths	
1	2	3	4	5
68. Leaving it to *ability* when doing maths			Putting in *effort* when doing maths	
1	2	3	4	5
69. Applying maths *concepts* to solve a problem			Using a *rule / formula* to find the answer	
1	2	3	4	5
70. *Truths and facts* which were discovered			*Mathematical ideas* and *practices* we normally use in life	
1	2	3	4	5
71. Someone *teaching* and *explaining* maths to students			*Exploring* maths themselves or with peers / friends / parents	
1	2	3	4	5
72. *Remembering* maths ideas, concepts, rules or formulae			*Creating* maths ideas, concepts, rules or formulae	
1	2	3	4	5
73. *Telling* students what a triangle is			Letting students see *concrete examples* of triangles first, so that they understand the properties of triangles	
1	2	3	4	5
74. *Demonstrating* and *explaining* maths			Keeping mathematics *magical / mysterious*	

concepts to others (e.g. proofs)				
1	2	3	4	5
75. Using maths to *predict* / *explain* events, that is, to stay in control			Using maths for *development/ progress*	
1	2	3	4	5

76. Comments (if any): ………………………………………………………….

…………………………………………………………..

SECTION D

A student asks you, I want to do well in mathematics at school. What three most important things do l need to do well?'

77. Most important: ………………………………………………

78. Second most important: …………………………………

79. Third most important: …………………………………………………

80. I have chosen the three words above because: …………………………

APPENDIX B

STUDENT QUESTIONNAIRE (SQ)

Dear Student,

The purpose of this study is to find out what you find important in the learning of mathematics. The study is therefore solely for academic pursuit and does not in any way evaluate you, your teachers or your school or to call for any administrative changes in your school. Your genuine response is very much needed for the success of this study. Please give your opinion about all the statements by ticking [√], circling or writing in the space provided.

You are assured that the information provided will be treated confidentially.

If you have any question, please kindly raise up your hand and I will attend to you.

Please kindly respond to all the items.

Thank you for your maximum co-operation.

SECTION A

GENERAL INFORMATION

1. Sex: Male [ ] Female [ ]
2. Age ……………………….
3. What is your current level of education?
    - i. Upper Primary School [ ]
    - ii. Junior High School (JHS) [ ]
    - iii. Senior High School (JHS) [ ]

4. My school is
   i. co-educational (boys and girls) [ ]
   ii. single sex female [ ]
   iii. single sex male [ ]
5. Which category does your school belong to?
   i. Category A [ ]
   ii. Category B [ ]
   iii. Category C [ ]

SECTION B

For each of the items below, tick a box to tell how **important** it is to you in your students' learning of mathematics. **Absolutely Unimportant - AU**, **Unimportant - U, Neither Important nor Unimportant** – **NIU**, **Important - I**, **Absolutely Important** – **AI**

ITEM	AU	U	NIU	I	AI
1. Investigations					
2. Problem-solving					
3. Small-group discussions					
4. Using the calculator to calculate					
5. Explaining by the teacher					
6. Working step-by-step					
7. Whole – class discussions					

8. Learning the proofs					
9. Mathematics debates					
10. Relating mathematics to other subjects in school					
11. Appreciating the beauty of maths					
12. Connecting maths to real life					
13. Practising how to use maths formulae					
14. Memorising facts (eg Area of a rectangle = length × breadth)					
15. Looking for different ways to find the answer					
16. Looking for different possible answers					
17. Stories about mathematics					
18. Stories about recent developments in mathematics					
19. Students explaining their solutions to the class					
20. Mathematics puzzles					
21. Students posing maths problems					
22. Using calculator to check the answer					

23. Learning maths with the computer					
24. Learning maths with the internet					
25. Mathematics games					
26. Relationships between maths concepts					
27. Being lucky at getting the correct answer					
28. Knowing the times tables					
29. Students making up their own maths questions					
30. Alternative solutions					
31. Verifying theorems / hypotheses					
32. Using mathematical words (eg angle)					
33. Writing the solutions step-by-step					
34. Outdoor mathematics activities					
35. Teacher asking us questions					
36. Practicing with lots of questions					
37. Doing a lot of mathematics work					
38. Given a formula to use					
39. Looking out for maths in real life					

40. Explaining where rules/formulae came from					
41. Teacher helping me individually					
42. Working out the maths by myself					
43. Mathematics tests / examinations					
44. Teacher giving students feedback					
45. Feedback from my friends					
46. Me asking questions					
47. Using diagrams to understand maths					
48. Using concrete materials to understand mathematics					
49. Examples to help me understand					
50. Getting the right answer					
51. Learning through mistakes					
52. Hands-on activities					
53. Teacher use of keywords (eg 'share' to signal division; contrasting 'solve' and simplify)					
54. Understanding concepts / processes					
55. Shortcuts to solving a problem					
56. Knowing the steps of the solution					
57. Mathematics homework					
58. Knowing which formula to use					

59. Knowing the theoretical aspects of mathematics (eg proof, definition of triangles)					
60. Mystery of mathematics (eg 111 111 111 × 111 111 111 = 12345678987654321)					
61. Stories about mathematicians					
62. Completing mathematics work					
63. Understanding why my solution is incorrect or correct					
64. Remembering the work we have done					

65. Comments (if any): ……………………………………………………
…………………………………………………….................

SECTION C

For each pair of phrases below, circle a number to indicate how **important** one phrase is to you in your mathematics learning than the other phrase. If you circle the middle number labelled '3', it means that both phrases are equally important to you. If you circle the number labelled '2', it means that the phrase at the left hand side is **important** to you. If you circle the number labelled '1', it means that the phrase at the left hand side is **more important** to you. However, if you circle the number labelled '4', it means that the phrase at the right hand side is **important** to you. If you circle the number labelled '5', it means that the phrase at the right hand side is **more important** to you.

	66. *How* the answer to a problem is obtained		*What* the answer to a problem is	
1	2	3	4	5
	67. Feeling *relaxed* or having *fun* when doing maths		*Hardwork* is needed when doing maths	
1	2	3	4	5
	68. Leaving it to *ability* when doing maths		Putting in *effort* when doing maths	
1	2	3	4	5
	69. Applying maths *concepts* to solve a problem		Using a *rule / formula* to find the answer	
1	2	3	4	5
	70. *Truths and facts* which were discovered		*Mathematical ideas* and *practices* we normally use in life	
1	2	3	4	5
	71. Someone *teaching* and *explaining* maths to me		*Exploring* maths myself or with peers / friends / parents	
1	2	3	4	5
	72. *Remembering* maths ideas, concepts, rules or formulae		*Creating* maths ideas, concepts, rules or formulae	
1	2	3	4	5

73. *Telling* students what a triangle is			Letting students see *concrete examples* of triangles first, so that they understand the properties of triangles	
1	2	3	4	5
74. *Demonstrating* and *explaining* maths concepts to others (e.g. proofs)			Keeping mathematics *magical / mysterious*	
1	2	3	4	5
75. Using maths to *predict / explain* events, that is, to stay in control			Using maths for *development/ progress*	
1	2	3	4	5

76. Comments (if any): ………………………………………………………

………………………………………………………..

SECTION D

A student asks you, I want to do well in maths at school. What three most important things do l need to do well?'

77. Most important: ……………………………………………………

78. Second most important: ………………………………………

79. Third most important: ………………………………………..

80. I have chosen the three words above because: …………………………

…………………………

APPENDIX C: INTRODUCTORY LETTER

UNIVERSITY OF CAPE COAST
COLLEGE OF EDUCATION STUDIES
FACULTY OF SCIENCE AND TECHNOLOGY EDUCATION
DEPARTMENT OF MATHEMATICS AND I.C.T EDUCATION

Telephone: 0332096951
Telex: 2552, UCC, GH
Telegrams & Cables: University, Cape Coast
Email: dmicte@ucc.edu.gh

University Post Office
Cape Coast, Ghana

Your Ref:

Our Ref: DMICTE/P.3/V.3/032

Date: 8th September, 2021

TO WHOM IT MAY CONCERN:

Dear Sir/Madam,

RESEARCH VISIT

The bearer of this letter, **Mr. Bright Dapaah Addae** with registration number ET/DME/18/0007 a Ph.D. (Mathematics Education) Student of the Department of Mathematics and ICT Education, College of Education Studies, University of Cape Coast.

As part of the requirements for the award of a doctorate degree, he is required to undertake a research visit at your outfit with the purpose of collecting data on the topic "**VALUING IN MATHEMATICS TEACHING AND LEARNING AMONGST GHANAIAN TEACHERS AND STUDENTS: HOW DOES IT LOOK LIKE ACROSS GRADE LEVELS?**".

I would be grateful if you could give him the necessary assistance he may need.

Thank you for your usual support.

Yours faithfully,

Boateng

Dr (Mrs) Christina Boateng
HEAD

APPENDIX D: ETHICAL CLEARANCE

UNIVERSITY OF CAPE COAST

INSTITUTIONAL REVIEW BOARD SECRETARIAT

TEL: 0558093143 / 0508878309
E-MAIL: irb@ucc.edu.gh
OUR REF: UCC/IRB/A/2016/1140
YOUR REF:
OMB NO: 0990-0279
IORG #: IORG0009096

4TH NOVEMBER 2021

Mr. Bright Dapaah Addae
Department of Mathematics and ICT Education
University of Cape Coast

Dear Mr. Addae,

ETHICAL CLEARANCE – ID (UCCIRB/CES/2021/84)

The University of Cape Coast Institutional Review Board (UCCIRB) has granted Provisional Approval for the implementation of your research titled **Valuing in Mathematics Teaching and Learning amongst Ghanaian Teachers and Students: What Does It Look Like Across Grade Levels?** This approval is valid from 4th November 2021 to 3rd November, 2022. You may apply for a renewal subject to submission of all the required documents that will be prescribed by the UCCIRB.

Please note that any modification to the project must be submitted to the UCCIRB for review and approval before its implementation. You are required to submit periodic review of the protocol to the Board and a final full review to the UCCIRB on completion of the research. The UCCIRB may observe or cause to be observed procedures and records of the research during and after implementation.

You are also required to report all serious adverse events related to this study to the UCCIRB within seven days verbally and fourteen days in writing.

Always quote the protocol identification number in all future correspondence with us in relation to this protocol.

Yours faithfully,

Samuel Asiedu Owusu, PhD
UCCIRB Administrator

ADMINISTRATOR
INSTITUTIONAL REVIEW BOARD
UNIVERSITY OF CAPE COAST

APPENDIX E: TEACHERS

Total Variance Explained

Component	Initial Eigenvalues			Extraction Sums of Squared Loadings		
	Total	% of Variance	Cumulative %	Total	% of Variance	Cumulative %
1	14.180	22.156	22.156	14.180	22.156	22.156
2	4.613	7.208	29.364	4.613	7.208	29.364
3	3.052	4.769	34.134	3.052	4.769	34.134
4	2.579	4.030	38.164	2.579	4.030	38.164
5	2.183	3.411	41.575	2.183	3.411	41.575
6	2.066	3.228	44.803	2.066	3.228	44.803
7	2.028	3.169	47.972	2.028	3.169	47.972
8	1.838	2.872	50.844	1.838	2.872	50.844
9	1.762	2.753	53.597	1.762	2.753	53.597
10	1.585	2.476	56.073	1.585	2.476	56.073
11	1.466	2.290	58.364	1.466	2.290	58.364
12	1.450	2.266	60.630	1.450	2.266	60.630
13	1.398	2.184	62.813	1.398	2.184	62.813
14	1.311	2.049	64.862	1.311	2.049	64.862
15	1.161	1.814	66.677	1.161	1.814	66.677
16	1.131	1.768	68.444	1.131	1.768	68.444
17	1.081	1.690	70.134	1.081	1.690	70.134
18	1.010	1.578	71.713	1.010	1.578	71.713
19	.980	1.530	73.243			
20	.941	1.471	74.714			
21	.911	1.424	76.138			
22	.884	1.381	77.519			
23	.867	1.354	78.873			
24	.825	1.290	80.163			
25	.784	1.224	81.387			
26	.731	1.142	82.529			
27	.674	1.053	83.582			
28	.632	.987	84.569			
29	.624	.975	85.544			
30	.575	.898	86.442			
31	.556	.869	87.311			
32	.538	.841	88.152			
33	.518	.810	88.962			
34	.481	.751	89.713			
35	.444	.693	90.407			

36	.428	.669	91.075			
37	.405	.633	91.708			
38	.392	.612	92.320			
39	.381	.595	92.914			
40	.348	.544	93.459			
41	.323	.505	93.963			
42	.311	.486	94.450			
43	.308	.481	94.931			
44	.292	.457	95.387			
45	.268	.419	95.806			
46	.255	.399	96.205			
47	.251	.392	96.597			
48	.230	.359	96.956			
49	.215	.335	97.291			
50	.192	.300	97.591			
51	.184	.287	97.879			
52	.173	.271	98.150			
53	.164	.256	98.406			
54	.149	.233	98.639			
55	.142	.222	98.860			
56	.127	.199	99.059			
57	.106	.166	99.225			
58	.101	.158	99.383			
59	.092	.143	99.526			
60	.088	.138	99.664			
61	.068	.106	99.770			
62	.056	.087	99.857			
63	.049	.077	99.934			
64	.042	.066	100.000			

Extraction Method: Principal Component Analysis.

APPENDIX F: STUDENTS

Total Variance Explained

Component	Initial Eigenvalues			Extraction Sums of Squared Loadings		
	Total	% of Variance	Cumulative %	Total	% of Variance	Cumulative %
1	14.529	22.702	22.702	14.529	22.702	22.702
2	3.031	4.736	27.438	3.031	4.736	27.438
3	2.683	4.193	31.631	2.683	4.193	31.631
4	2.446	3.822	35.453	2.446	3.822	35.453
5	2.130	3.328	38.782	2.130	3.328	38.782
6	1.528	2.387	41.169	1.528	2.387	41.169
7	1.402	2.191	43.360	1.402	2.191	43.360
8	1.366	2.135	45.495	1.366	2.135	45.495
9	1.234	1.929	47.424	1.234	1.929	47.424
10	1.217	1.902	49.325	1.217	1.902	49.325
11	1.176	1.838	51.163	1.176	1.838	51.163
12	1.160	1.812	52.975	1.160	1.812	52.975
13	1.095	1.711	54.686	1.095	1.711	54.686
14	1.039	1.623	56.309	1.039	1.623	56.309
15	1.027	1.604	57.913	1.027	1.604	57.913
16	1.002	1.565	59.478	1.002	1.565	59.478
17	.937	1.463	60.941			
18	.928	1.451	62.392			
19	.910	1.422	63.814			
20	.877	1.371	65.185			
21	.860	1.344	66.529			
22	.830	1.297	67.827			
23	.808	1.262	69.089			
24	.787	1.230	70.319			
25	.769	1.202	71.521			
26	.755	1.180	72.701			
27	.732	1.144	73.845			
28	.708	1.106	74.951			
29	.687	1.074	76.025			
30	.663	1.036	77.060			
31	.647	1.011	78.071			
32	.631	.986	79.057			
33	.614	.959	80.016			
34	.606	.947	80.962			
35	.580	.907	81.869			

36	.576	.900	82.769			
37	.570	.890	83.659			
38	.546	.853	84.512			
39	.512	.800	85.312			
40	.505	.789	86.101			
41	.490	.766	86.867			
42	.479	.748	87.615			
43	.470	.735	88.350			
44	.468	.731	89.081			
45	.449	.701	89.782			
46	.445	.696	90.478			
47	.435	.680	91.158			
48	.411	.642	91.799			
49	.410	.641	92.441			
50	.407	.635	93.076			
51	.393	.614	93.690			
52	.378	.590	94.280			
53	.358	.560	94.840			
54	.354	.553	95.393			
55	.344	.538	95.931			
56	.327	.512	96.443			
57	.315	.492	96.935			
58	.308	.481	97.416			
59	.301	.471	97.886			
60	.293	.458	98.345			
61	.284	.443	98.788			
62	.268	.419	99.207			
63	.255	.399	99.606			
64	.252	.394	100.000			

Extraction Method: Principal Component Analysis.

APPENDIX G: PARALLEL ANALYSIS – TEACHERS

Monte Carlo PCA for Parallel Analysis
Version .

1/8/2024 4:00:08 PM
Number of variables: 64
Number of subjects: 177
Number of replications: 100

Eigenvalue #	Random Eigenvalue	Standard Dev
1	2.9314	.0780
2	2.7015	.0661
3	2.6957	.0457
4	2.5818	.0438
5	2.4373	.0424
6	1.9697	.0373
7	1.9059	.0338
8	1.8456	.0323
9	1.7872	.0302
10	1.7288	.0283
11	1.6782	.0287
12	1.6298	.0317
13	1.5841	.0301
14	1.5402	.0290
15	1.4937	.0250
16	1.4478	.0244
17	1.4056	.0231
18	1.3659	.0226
19	1.3237	.0223
20	1.2884	.0216
21	1.2523	.0204
22	1.2149	.0212
23	1.1821	.0214
24	1.1442	.0216
25	1.1116	.0191
26	1.0784	.0194
27	1.0443	.0204
28	1.0147	.0191
29	0.9856	.0184
30	0.9544	.0167
31	0.9280	.0179
32	0.8979	.0166
33	0.8681	.0167
34	0.8427	.0161
35	0.8138	.0158
36	0.7863	.0158
37	0.7609	.0155
38	0.7345	.0164
39	0.7113	.0170
40	0.6872	.0161
41	0.6588	.0151
42	0.6363	.0136
43	0.6127	.0138
44	0.5891	.0149
45	0.5674	.0170
46	0.5428	.0150
47	0.5205	.0149
48	0.5000	.0152
49	0.4797	.0143
50	0.4574	.0135
51	0.4388	.0131
52	0.4187	.0119
53	0.3996	.0115
54	0.3806	.0126
55	0.3596	.0132
56	0.3390	.0112
57	0.3202	.0123
58	0.3002	.0112
59	0.2815	.0126
60	0.2639	.0121
61	0.2447	.0124
62	0.2247	.0115
63	0.2032	.0145
64	0.1754	.0143

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Monte Carlo PCA for Parallel Analysis

APPENDIX H: PARALLEL ANALYSIS – STUDENTS

Monte Carlo PCA for Parallel Analysis
Version .

1/8/2024 4:03:48 PM
Number of variables: 64
Number of subjects: 1263
Number of replications: 100

Eigenvalue #	Random Eigenvalue	Standard Dev
1	1.5696	.0205
2	1.5324	.0161
3	1.5041	.0152
4	1.4792	.0132
5	1.4583	.0121
6	1.4390	.0107
7	1.4013	.0109
8	1.3916	.0116
9	1.3848	.0097
10	1.3673	.0092
11	1.3522	.0081
12	1.3364	.0091
13	1.3237	.0091
14	1.3098	.0086
15	1.2950	.0085
16	1.2818	.0078
17	1.2682	.0080
18	1.2557	.0083
19	1.2433	.0080
20	1.1296	.0082
21	1.1169	.0077
22	1.1052	.0076
23	1.0923	.0069
24	1.0801	.0068
25	1.0681	.0073
26	1.0564	.0068
27	1.0455	.0070
28	1.0339	.0070
29	1.0235	.0072
30	1.0114	.0073
31	1.0001	.0062
32	0.9901	.0062
33	0.9783	.0063
34	0.9663	.0064
35	0.9560	.0069
36	0.9449	.0063
37	0.9348	.0064
38	0.9240	.0065
39	0.9126	.0071
40	0.9030	.0068
41	0.8924	.0065
42	0.8822	.0063
43	0.8713	.0063
44	0.8615	.0069
45	0.8509	.0073
46	0.8403	.0078
47	0.8293	.0072
48	0.8179	.0063
49	0.8090	.0066
50	0.7991	.0066
51	0.7881	.0063
52	0.7770	.0067
53	0.7667	.0061
54	0.7553	.0081
55	0.7444	.0084
56	0.7323	.0077
57	0.7209	.0080
58	0.7087	.0086
59	0.6957	.0088
60	0.6824	.0079
61	0.6697	.0093
62	0.6552	.0096
63	0.6369	.0105
64	0.6166	.0118

1/8/2024 4:06:12 PM

Monte Carlo PCA for Parallel Analysis

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APPENDIX I: COMPONENT MATRIX (FACTOR LOADING) –TEACHERS

Component Matrix[a]

	Component																	
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Alternative solutions	.665			-.295		.139				-.153		.186					.146	
Understanding concepts / processes	.659	-.185			.129	-.272		-.370	.163		-.166			-.183				-.102
Knowing the theoretical aspects of mathematics (e.g proof, definition of triangles)	.639	.230	.306				.102				-.195	.247						
Knowing which formula to use	.637	-.331	.141		.158		-.172		.187	.130						.142		
Writing the solutions step-by-step	.637	-.209			.131	.141	.253	-.207	.184	-.234			-.123		-.154	-.139		.102
Mathematics puzzles	.623		-.229		-.108	.141			-.300		-.118	-.108					-.309	
Completing mathematics work	.589	-.265		-.164	-.158	-.148	-.175	.227			-.138	-.103		-.164				-.120
Knowing the steps of the solution	.580	-.360	.227				-.262								-.220	.166	.124	.119
Using mathematical words (e.g angle)	.577		-.210	.393			.194	-.209		-.190								
Teacher asking students questions	.576	-.233	.214		.155	.111		.236		.127					.269			

Stories about mathematicians	.570				-.190		-.388		.101		.214	-.256			-.181	-.134	-.137	
Students working out maths by themselves	.566	-.133	.117	-.207				.151	-.102	.143	-.202		.327		-.162		-.175	
Students posing maths problems	.559	.171	-.255			.224	-.231	-.112	.257		-.162					-.205	-.145	
Practising how to use maths formulae	.559	-.134	.109	.368	-.163											-.140		.165
Getting the right answer	.558	-.143	.189	.290	.162	-.234		-.167	.160		-.149	-.178				.122		-.217
Students making up their own maths questions	.553	.124	-.269	.146		.207		-.117	-.240	.208		.196		.218		-.156		-.110
Teacher use of keywords (e.g 'share' to signal division; contrasting 'solve' and simplify)	.550	-.183	-.116	.103		-.337		-.383				-.190						-.125
Students explaining their solutions to the class	.548	-.123	-.206	-.245	-.123	.351	-.180	.120		-.117	-.114		.197	-.143			-.154	
Looking out for maths in real life	.548	.125	-.171	-.144	-.233	.121		-.108	-.154	.108	-.149	.167		.125	.134	.166	.115	
Examples to help students understand	.537			.112	-.146	.183	.386	.260	.104	.125			-.166					-.197
Appreciating the beauty of maths	.533		-.197		-.341			.129				-.181	.300				.105	.183
Relationships between maths concepts	.532	.236	.117		.144	.113		-.217	-.207			-.173			-.345			
Mathematics tests / examinations	.530	-.378	.116					.238			.157		-.109	.216	.248		-.150	

Using concrete materials to understand mathematics	.528	-.237		-.383	.253				.273	.181				.251			.138	
Small-group discussions	.518	.109	.144		-.183	-.145			.264	-.163	.226	.138				-.197		
Understanding why my students solution is incorrect or correct	.509	-.147		-.240		-.180		.168	-.239	-.151			.223	-.106	.113			-.134
Practicing with lots of questions	.501	-.460	-.152	-.120	.148	.353						-.172		-.138				.117
Learning maths with the computer	.495	.431		-.258	.115		-.164	-.173	-.113		.238	-.159				.186		
Verifying theorems / hypotheses	.492	.432	-.170		.246			.179	-.174	-.126	-.153	.163	-.220	-.163				.128
Learning the proofs	.490	.401	.258	-.128	.201	.115	.206		-.176		-.142		.125					.223
Connecting maths to real life	.489		-.262	-.128			.139		.135	.128		-.437			.167		.294	.111
Hands-on activities	.480	-.233	-.163	-.115		-.255	-.139	-.231	.146	.289		.209				-.280		
Mathematics debates	.477	.201	-.243		.149	-.148	-.163	.227	.196	-.115		.406		-.139				-.115
Mathematics homework	.468	-.212		.237	-.155	-.125	.185	.104			.265	-.189	-.256		-.144	.126	.129	
Remembering the work we have done	.467	-.156	.226			-.150	-.142			.128		.264	-.129	-.161		.132	.268	.243
Explaining where rules/formulae came from	.454	.376	.292	.222				-.100	-.281		.124	-.131		-.164	-.207			
Using diagrams to understand maths	.428	-.142	.272		.224		.143	.158		.283		-.153	-.271	.379		-.168	.178	
Relating mathematics to other subjects in school	.426	.328			-.365		.259				-.221		.110	.246		-.305	.153	
Whole – class discussions	.411			.272	-.166	-.243	-.211		-.215	.197	-.209	.201	.114		.408			

Teacher asking students questions	.411	-.313	-.148	.165		.364	.258		-.195	-.130		-.149		-.207				-.241
Mathematics games	.394		-.377	.152		.204	.127	-.306	-.159	.182	.334					.260		-.107
Doing a lot of mathematics work	.394	-.338	-.357			.285	.134	-.245			.183	.180	-.127					.222
Using calculator to check the answer	.285	.657	.219			.228		-.140	.191	-.148		-.154	.120		.116			
Stories about recent developments in mathematics	.370	.540			-.206		-.364		.250	-.136	.231		-.183	.138			.112	
Using the calculator to calculate		.527	.197			.253		-.241	.433				.223		.152	.198	.131	
Learning maths with the internet	.421	.520	-.133		.315	-.279		-.234	-.183							.105		.218
Mystery of mathematics (e.g 111 111 111 × 111 111 111 = 12345678987654321)	.464	.482	.272		-.109		.135				-.193		-.133				-.125	-.162
Shortcuts to solving a problem	.201	.106	.567	.109	.102		-.137		-.317	-.177			-.153	.126	.346			
Outdoor mathematics activities	.257	.249	-.451	.226	.343			.346	-.226								.241	-.125
Given a formula to use	.348	-.350	.384	.120	.207		-.185		-.154	-.312	.134			.183			-.158	-.132
Investigations	.352	.240	-.380	.148	.341	-.223	.182	.245	.137			.177		.127	-.146		-.170	
Feedback from my friends	.215	.277	.375	-.232	-.169	.139		.150	-.179	.320	.348	.107		-.160		-.185	-.160	
Explaining by the teacher	.261		.374	.266	.366		.104	.208	.170	.216		-.201	.226	-.306		-.109		

Being lucky at getting the correct answer	.122		.269	.539	-.159	.337	-.271		.159					.157				
Memorising facts (e.g Area of a rectangle = length × breadth)				.503	.262		.163	.192		-.109	.167	.195	.493	.115		.102	.216	
Stories about mathematics	.135	.408	-.275	.426			-.409	.283	.153		.111				.116	.172		
Looking for different possible answers	.424	.213	.159		-.480	-.108	.124	.230		-.281					-.211		.216	
Learning through mistakes	.402		.105		-.252	-.512		-.105	-.174		.244		.199		.165	-.242		
Working step-by-step	.400			.148	-.102		.467		.216		.131		-.161			.189	-.126	.404
Looking for different ways to find the answer	.529	-.147	.188	-.271						-.562								
Teacher giving students feedback	.275		.106	-.336			.333			.215	.355	.239	.128	-.104		.157	-.229	-.310
Knowing the times tables	.368	-.343	.114	.250		.254	-.157	-.151				.177	.370	.214	-.202	.124		
Problem-solving	.365	.240	-.168	-.204	.243			.119			.210		.154	.410			-.150	
Teacher helping students individually	.212			.127	-.370	-.123	.185	.128		.201	-.334			.297	-.198	.374	-.299	

Extraction Method: Principal Component Analysis.

a. 18 components extracted.

APPENDIX J: COMPONENT MATRIX (FACTOR LOADING) – STUDENTS

Component Matrix[a]

	Component															
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Knowing which formula to use	.603															
Mathematics tests / examinations	.600			-.349												
Completing mathematics work	.599															
Students posing maths problems	.585															
Teacher asking students questions	.580															
Understanding concepts / processes	.580															
Teacher use of keywords (e.g ‘share’ to signal division; contrasting ‘solve’ and simplify)	.576															
Mathematics homework	.570															
Teacher asking students questions	.570			-.346												
Relationships between maths concepts	.567															
Looking out for maths in real life	.557															
Knowing the steps of the solution	.556															
Teacher giving students feedback	.547				-.304	-.304										
Practising how to use maths formulae	.544	.341														
Getting the right answer	.543															
Using mathematical words (e.g angle)	.541															
Stories about recent developments in mathematics	.539				.307											
Students explaining their solutions to the class	.538															

Using diagrams to understand maths	.536												-.307			
Explaining where rules/formulae came from	.535		-.315													
Knowing the theoretical aspects of mathematics (e.g proof, definition of triangles)	.534	.302	-.361													
Remembering the work we have done	.534															
Practicing with lots of questions	.526			-.305												
Writing the solutions step-by-step	.519															
Learning the proofs	.517		-.382													
Verifying theorems / hypotheses	.511		-.304			-.327										
Doing a lot of mathematics work	.507							.301		-.343						
Examples to help students understand	.506															
Alternative solutions	.500					-.368										
Mathematics puzzles	.497										.400					
Whole – class discussions	.496					.315										
Outdoor mathematics activities	.492			.337												
Stories about mathematicians	.483		-.399	.402												
Mystery of mathematics (e.g 111 111 111 × 111 111 111 = 12345678987654321)	.482		-.398													
Appreciating the beauty of maths	.471				.344											
Given a formula to use	.469	.390														
Understanding why my students solution is incorrect or correct	.466															
Connecting maths to real life	.465	-.317														
Looking for different ways to find the answer	.464															-.336
Knowing the times tables	.461															
Hands-on activities	.459															

Students making up their own maths questions	.454															
Problem-solving	.454								.303		.349					
Mathematics debates	.438				.333											
Looking for different possible answers	.431				.344											
Students working out maths by themselves	.424									.301						
Feedback from my friends	.421				-.321		.326									
Learning through mistakes	.413		.310									-.304				
Using concrete materials to understand mathematics	.407			.372												
Memorising facts (e.g Area of a rectangle = length × breadth)	.405	.352														
Explaining by the teacher	.395			-.358												
Investigations	.387													-.320		
Relating mathematics to other subjects in school	.377				.303											
Teacher helping students individually	.374		.315							.326					.311	
Working step-by-step	.328			-.304											.304	
Using the calculator to calculate		.647	.400													
Using calculator to check the answer		.630	.339													
Shortcuts to solving a problem		.516														
Learning maths with the internet			.579		.379											
Learning maths with the computer			.566													
Stories about mathematics	.410	-.322		.507												
Small-group discussions	.351					.400	.426									
Being lucky at getting the correct answer								.427				.380				
Mathematics games	.365										.390		.410		.301	

Extraction Method: Principal Component Analysis.
a. 16 components extracted.