UNIVERSITY OF CAPE COAST

THE EFFECT OF DIAGRAMS IN ADDRESSING LEARNERS' DIFFICULTIES IN SOLVING WORD PROBLEMS INVOLVING ALGEBRAIC EXPRESSIONS

SAMUEL KENNEY

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By

SAMUEL KENNEY

Thesis submitted to the Department of Basic Education of the Faculty of Educational Foundations, College of Education Studies, University of Cape Coast, in partial fulfilment of the requirements for award of Master of Philosophy Degree in Basic Education.

November 2023

DECLARATION

Candidate's Declaration

I hereby declare that this thesis is the result of my own original research and that no part of it has been presented for another degree in this university or elsewhere.

Candidate's Signature: ……………………….. Date: ……………………….

Name: ………………………………………………………………………..

Supervisors' Declaration

We hereby declare that the preparation and presentation of the thesis were supervised in accordance with the guidelines on supervision of thesis laid down by the University of Cape Coast.

Principal Supervisor's Signature: …………………… Date: ……..………….

Name: ……………………………………………………………………….

ABSTRACT

This study sought to assess the effectiveness of using diagrams in assisting learners in Basic Seven in solving word problems involving algebraic expressions. The multistage sampling technique was employed to select 171 participants from various schools, and a two-group pretest-posttest design with a delayed posttest was implemented. The participants were divided into low-achieving and high-achieving classes, each comprising control and experimental groups. Mathematics Achievement Test and Think-Aloud Protocols were used to obtain relevant data and were analysed using descriptive statistics, content analysis, and analysis of covariance. The study revealed that the experimental groups, which received instruction with the use of diagrams, displayed a significantly higher level of improvement compared to the control groups. Furthermore, the delayed posttest demonstrated the sustainability of the diagram usage's positive effects, further consolidating this instructional approach's long-term effectiveness. In conclusion, this study contributes to the existing literature by establishing the effectiveness of diagrams in assisting learners in the Basic Seven in solving word problems involving algebraic expressions. The findings underscore the importance of incorporating visual representations into mathematics instruction, particularly in solving word problems. The implications of this research for mathematics educators suggest that the incorporation of diagrams can serve as a valuable instructional strategy to foster learners' problem-solving skills and enhance their comprehension of algebraic word problems.

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DEDICATION

To my wife and children

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CHAPTER ONE

INTRODUCTION

Society's economic success highly depends on mathematical ability (Lipnevich, MacCann, Krumm, Burrus, & Roberts, 2011). Mathematical skills serve as an engine upon which the understanding of other disciplines in the sciences, engineering, social sciences, and arts ride (Patena & Dinglasan, 2013; Phonapichat, Wongwanich, & Sujiva, 2014). The subject, mathematics, is key in several curricula due to its numerous importance. The prime focus of every mathematics curriculum is to offer learners with the needed skills and knowledge that are essential in effecting significant changes in a technological world (Ngussa & Mbuti, 2017).

Almost all of the other disciplines needed to steer the economic success of every society depend on some specific aspects of mathematics and algebra is chiefly seen among these mathematical aspects (Niss,1994). Demme (2018) described Algebra as the pointlessness of mathematics yet it is the mathematical concept that moves us beyond the basic forms of mathematics and prepares us for other higher aspects of mathematics such as calculus and statistics. Algebra positions mathematics as a concept that is not just about numerical computations but a concept that requires understanding the fundamental logic behind it before proceeding toward solving the problem. One major concept that provokes people's logical thinking in algebra is a word problem, which is also seen as a major challenge for most learners across the globe (Verschaffel, Schukajlow, Star, & Van Dooren, 2020)

Background to the Study

Today, mathematics is a high-antecedence and a requirement in every field, necessitating high-level mathematics teaching approaches to support learners' long-term and practical learning (Smith & Brown, 2022). To be advanced and proficient in mathematics, people who know and love mathematics are required to help meet this target. Owing to this assertion, Coskun (2013) outlined that, basic school mathematics teachers have a lot of responsibilities in nurturing mathematics learners. This is because, studies indicate that, negative attitudes toward the study of mathematics emerge at an early age and continue to increase exponentially (Altintas, 2018).

A study by Brown, Brown, and Bibby (2008) indicates that learners perceive mathematics as a difficult discipline and lack the confidence to pursue mathematics at a higher level. Notwithstanding, McDonald (2018) also believes that almost everything said about mathematics being it from professionals and non-professionals is flat-out wrong. McDonald further explained that mathematics is defined wrongly, incorrectly explained, incorrectly taught and studied with the wrong approach.

It is important to recognise that the study of mathematics serves as the prime vehicle for nurturing learners' logical reasoning and advanced cognitive abilities. Beyond the fundamental numeracy skills, mathematics assumes a critical role by preparing learners to connect with real-world activities, equipping them with problem-solving abilities. Furthermore, mathematics plays a pivotal role in numerous scientific disciplines, including physics, engineering, and statistics.

Consequently, cultivating a positive attitude toward mathematics among learners is a primary goal in mathematics education worldwide. Considering the significance of the subject, what effective strategies can teachers and researchers implement to help learners develop a positive attitude toward mathematics, which will ultimately enhance their academic performance in the subject?

A review of the various aspects of mathematics shows that the study of algebra serves as the foundation for most of the other mathematical areas widely used in the aforementioned disciplines. Notably, regular arithmetic focuses on number computations, while algebra introduces the concept of replacing some numbers with letters for learners to identify the missing element (Pang & Kim, 2018). This concept of finding the missing piece to create a complete whole is a key component of problem-solving. Therefore, algebra serves as the basis for problem-solving in primary education and beyond. It stimulates learners' logical thinking and reasoning skills in finding solutions to problems. Additionally, algebra connects the real world to the classroom by presenting real-world problems in an educational setting. Many of these algebraic problems are presented as word problems, which are a crucial aspect of mathematics that enrich a learner's mental skills, improve logical analysis, and enhance creative thinking.

The term "word problem" refers to any mathematical exercise where key information about the problem is presented in text form rather than mathematical notation (Krawec, 2014). This requires learners to tackle problems and develop strategies for solving them, which is essential for preparing individuals for real-life situations. The primary objective of mathematics is to instill a problem-solving mindset in learners, which depends on their ability to solve word problems. Unfortunately, many learners find word problems challenging. For instance, Shin and Bryant (2013) report that over 68% of learners struggle with applying multiple steps in word problems, while Hecht, Close, and Santisi (2003) indicate that learners face difficulties in selecting and using the correct steps to solve these problems.

Strategic competency is regarded as a key element for learners to achieve success and proficiency in mathematics. It broadly refers to the ability to formulate, represent, and solve mathematical problems (National Research Council, 2001). This includes an understanding of various strategies and representations that can be applied to solve mathematical problems, as well as the ability to use these strategies effectively and switch between them according to the problem's requirements (National Research Council, 2001). In essence, representations or diagrams are fundamental to strategic competence.

The incorporation of diagrams in teaching or concept development holds significance in transitioning the problem-solving process from an abstract, symbolic form to a more tangible and practical one (Agrawal & Morin, 2016). Educational researchers assert that tapping into learners' existing knowledge is crucial for establishing a solid foundation for new learning. Diagramming proves to be one of the most effective tools and strategies in the classroom for achieving this purpose (Hembree, 1992; Uesaka, Manalo & Ichikawa, 2007). However, research findings indicate that a majority of learners do not spontaneously resort to using diagrams (Uesaka & Manalo, 2012). Nevertheless, substantial evidence suggests that learners exhibit increased spontaneity in diagram usage when provided with sufficient knowledge about diagrams (Ayabe & Manalo, 2018; Manalo & Uesaka, 2016).

The argument posits that, before the existence of algebra, geometry represented the most intricate facet of mathematics (Seidenberg, 1978). The author contends that despite the inherent difficulty in grasping geometric concepts, the utilisation of graphs, diagrams, and other visual representations has enhanced human understanding of the subject compared to plain text. It is noteworthy that the diagrammatic element of mathematics presents mathematical concepts in less abstract forms, facilitating easier comprehension for learners. Given this reasoning, one might question whether diagrams constitute the most effective method for teaching and learning word problems. A survey of research studies in the field of diagrams reveals a diverse array of definitions spanning various disciplines. For instance, Purchase (2014) defines a diagram as a composite collection of visual elements on a two-dimensional plane, serving as a representation of a concept or object in the observer's mind. This effectively positions the problem solver to solve the problem. This is because diagrams can record information about a problem early in the process and serve as a guide to the solution process.

After learners gain a comprehension of a topic, diagrams can function as a reasoning tool for tackling mathematical problems. In this regard, they can be employed for scrutinising problems and devising solutions, adjusting and elucidating actions, forecasting consequences, monitoring and assessing progress, and synthesising and conveying results in formats beneficial to others (Pape &

Tchoshanov, 2001). Considering all these aspects, the question arises: Is diagramming the most efficient tool for interpreting and resolving word problems in algebra? Could the introduction of diagrams impact learners' attitudes toward learning algebra? These inquiries, among others, motivated the study, which aims to investigate the use of diagrams in addressing issues related to algebraic expressions. The researcher's focus is on exploring the utilisation of diagrams, recognised as an effective and potent visualisation strategy in the learning of word problems in algebra (Agrawal & Morin, 2016; Uesaka, Manalo & Ichikawa, 2007).

Statement of the Problem

The overarching educational objective of a nation is to equip learners with problem-solving abilities. Consequently, learners undergo various skills and techniques aimed at cultivating these problem-solving skills. The enhancement of problem-solving skills is viewed as the ultimate aim of the New Ghanaian Mathematics Curriculum (Ministry of Education [MoE], 2019a, 2019b, 2020), aligning with the broader goal of preparing individuals to address challenges in their daily lives. Mathematics emerges as the primary discipline emphasising problem-solving in most basic schools, leading to the introduction of the concept of word problems.

As defined by Powell, Namkung, and Lin (2022), word problems represent a combination of numbers and words, requiring learners to apply reading, comprehension, and mathematical skills in solving them. The study of word problems is recognised for playing a significant role in the teaching and learning of mathematics. Notably, the Common Core State Standards (CCSS) for mathematical practice argues that word problems position learners to evolve into problem solvers capable of reasoning, applying, justifying, and effectively using appropriate mathematical vocabulary to demonstrate understanding in everyday situations (CCSSI, 2010). However, as argued by Matthews (2018), McDonald and Smith (2020), and Di Leo and Muis (2020), word problems remain a source of apprehension and increasing difficulty for many learners as they progress through different grades in mathematics.

According to Stern (2009), mathematics performance is very important at the basic level since it serves as a strong predictor of achievement at the higher levels which significantly influences their future socio-emotional well-being. Studies also indicate that mathematics scores obtained by various learners appear to be lower than the scores in other subjects (Pearce, Bruun, Skinner, & Lopez-Mohler, 2013). For example, Peltier and Vannest (2017) reported that 60% of basic four learners and 67% of basic eight learners failed to reach mathematical proficiency after analysing the National Assessment Data in the USA. The results later indicated that most of the learners scored very low marks, particularly in the area of problem-solving. Similarly, findings from the Ministry of Education [MoE] (2016) by Ghana National Education Assessment [NEA] on basic four and five learners indicate that 29% to 45% of Ghanaian learners fail to achieve minimum competency levels with less than 25% showing some level of emergence in competency level as established by the Ghana Education Service [GES] 2005's Mathematics Test Mark.

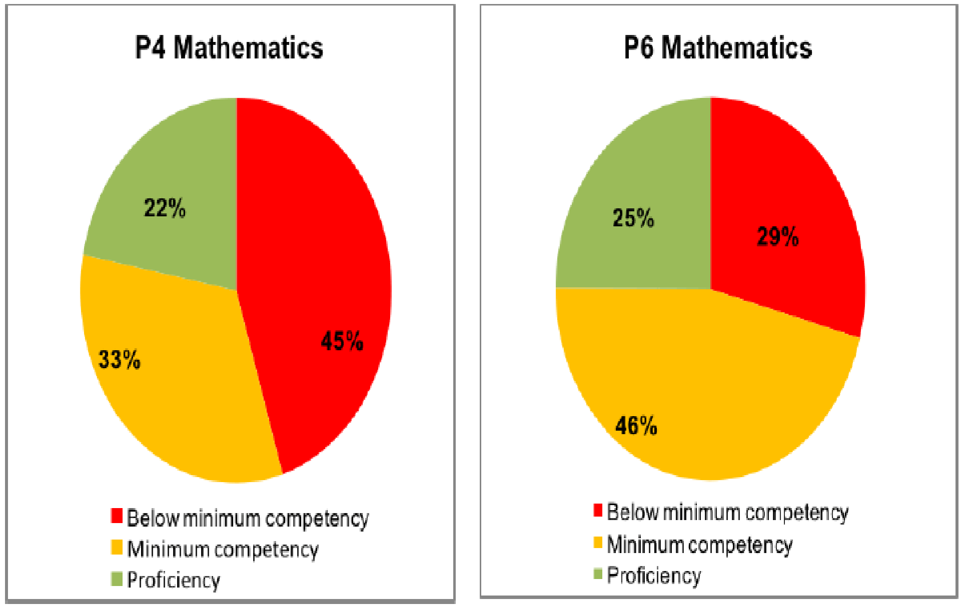


Figure 1: National Education Assessment (2016)

Some empirical studies have attributed some of the challenges learners face in the study of mathematics to teacher's actions or inactions (Ampadu, 2012; Kaiser & Konig, 2019; Ren & Smith, 2018), content, experience, and activities in mathematics (Clarkson, Ntow, Chidthachack & Crotty, 2015; Ntow, Clarkson, Chidthachack & Crotty, 2017), choice of the textbook (Pratama & Retnawati, 2018; van den Ham & Heinze, 2018) and environmental factors (Mazana, Suero Montero, & Olifage, 2019). Despite the various studies conducted in mathematics with several models and interventions implemented to curb this challenge, recent studies indicate that the difficulty in the subject area persists among learners. For example, the former minister of Education Dr, Mathew Opoku-Prempeh once expressed his upset with the low grades Ghanaian learners continue to score in mathematics in their final examination of the Basic Education Certificate Examination (BECE) and the West African Senior Secondary Certificate Examination (WASSCE) (Ghana Business News [GBN], 2017).

In addition, a review of the last ten decades of the mathematics Chief Examiners' reports by the West African Examinations Council [WAEC] (2015, 2017 & 2019) have consistently indicated that Basic Education Certificate Examination (BECE) candidates lack an in-depth understanding of mathematics questions presented in prose. In addition, Kintsch and Greeno (1985) argue that learners perform from 10 to 30 per cent worse on word problems than when the same problem is presented in mathematical form. Furthermore, Anamuah-Mensah and Mereku (2005) indicated that the first time Ghana participated in the Trends in International Mathematics and Science Study (TIMSS), Ghanaian learners found constructed response items more difficult than the direct forms of questions. This is to mean that, Ghanaian learners struggled most when mathematical questions are presented in the text compared to the arithmetic forms of questions. Could this be the main reason learners are having lower scores in mathematics compared to other subjects?

This challenge is not different from the case of the learners in the Assin Central Municipality. This is because previous BECE results have constantly shown low marks obtained by learners in the Assin Central Municipality. For example, during the Municipality's 2020 School Performance and Appraisal Meeting (SPAM), the Municipality's Director of Education based on the 2019 Mathematics Chief Examiner's report indicated that most learners lack basic computational skills. The Director later recommended that mathematics teachers pose mathematical problems that link the learner to their immediate environment.

In Ghana, the study of mathematics is an unavoidable subject` for all learners at the basic level and a basic demand for entering into any tertiary institution. This means learners would have to pass mathematics before they can move up the grade levels. Nonetheless, several researchers who have analysed the Basic Education Certificate Examination (BECE) which is a standardised test in Ghana taken by Junior High School (JHS) learners to assess their proficiency in various subjects and determine their eligibility for progression to senior high school have reported learners are underperforming in the subject area. For example, a study by Ansah (2017). indicated that about 36,849 candidates, which comprise 8% of the 2017 (BECE) candidates, were not placed into any of the country's Senior High School (SHS) owing to them obtaining grade 9 in either English or Mathematics or both. In addition, Nugba, Quansah, Ankomah, Tsey, and Ankoma-Sey (2021) also highlighted that education at the basic level is witnessing a downturn based on performance in the BECE. Their assumption is based on the records from 2006 to 2016 that indicated that 1,562,270, which is 43% of the total 3,669,138, failed to make the required grade for progression into senior high schools. The last decade of the Ghanaian mathematics chief examiners report by the West African Examination Council [WAEC] (2013, 2015, 2017, 2018, 2019, & 2020) has consistently indicated that Basic Education Certificate Examination (BECE) candidates lack an comprehensive understanding of word problems. For example, the 2013 Mathematics Chief Examiner highlighted that 'poor knowledge and understanding of the English Language suspected in the unpopularity of questions written in prose" (WAEC, 2013. P.1). The Examiner goes on to suggest that teachers should "intensify the teaching and learning of comprehension in English Language" (WAEC, 2013. P.1) as a remedy to help learners overcome this challenge.

An additional insight from the report by Anamuah-Mensah and Mereku (2005) indicates that the official mathematics curriculum and textbooks primarily focus on numbers and addition, prompting responses in the lowest cognitive domain (involving knowledge of facts and procedures) rather than encouraging higher-level reasoning skills. Furthermore, the approach of searching for keywords is often employed in solving word problems, which, as suggested by the researchers, might hinder the development of learners' abilities.

In light of this, there is a requirement for teaching methods and procedures that enable learners to acquire precisely the knowledge they need, avoiding extraneous information (Coskun, 2013). Additionally, there is a demand for instructional resources and materials that impart meaningful learning experiences, linking concepts to everyday life, and instilling motivation in learners to take responsibility for their learning (Demircioglu, Demircioglu, & Ayas, 2006). An approach that can be employed to achieve this level of engagement in mathematics is the use of a self-generated diagram, as opposed to relying solely on text comprehension or keywords.

Purpose of the Study

The purpose of the study was to determine the effect of diagrams in addressing learners' difficulties in solving word problems involving algebraic expressions.

Objectives:

- identify the errors learners make when solving word problems before being introduced to the use of diagrams
- identify the challenges learners encounter in solving word problems before being introduced to the use of diagrams
- examine the influence of diagrams on the performance of the experimental and control groups of the high-achieving class in solving word problems.
- examine the influence of diagrams on the performance of the experimental and control groups of the low-achieving class in solving word problems.
- Identify the errors learners make when solving word problems after being introduced to the use of diagrams.

Research Questions and Hypotheses

Research questions

The research was directed by the subsequent research questions:

1. What errors learners do make when solving word problems before being introduced to the use of diagrams?

2. What challenges do learners experience in solving word problems before being introduced to the use of diagrams?
3. What errors do learners make when solving word problems after being introduced to the use of diagrams

Hypotheses

1. H_0: The use of diagrams has no statistically significant effect on the performance in solving word problems between the experimental group and the control group of the high-achieving class.
2. H_0: The use of diagrams has no statistically significant effect on the performance in solving word problems between the experimental group and the control group of the low-achieving class.

Significance of the Study

The outcomes of this study are anticipated to enhance learners' problem-solving strategies, ultimately improving their performance in algebra and fostering positive attitudes towards the broader study of mathematics. Teachers stand to gain valuable insights from this research, acquiring more effective methods for teaching algebra involving word problems. This, in turn, can assist key stakeholders in education, including policymakers, in fortifying existing policies and integrating the outlined methods into curriculum development. Both the Ghana Education Service (GES) and other authors of mathematics textbooks can benefit by incorporating empirically proven findings into their educational materials for learners. Moreover, the study is expected to contribute to the existing literature and may offer pertinent information for future research endeavours.

Delimitation

The study of mathematics has several aspects and units but this study is delimited to the study of algebra and specifically solving word problems in algebra. It is worth noting that, solving word problems in algebra is considered a broad aspect of mathematics and therefore, this research work concentrated on solving word problems in linear equations. Furthermore, in terms of whom to study, this research work targeted Ghanaian learners at the basic level. To be precise, the study focused mainly on Ghanaian learners at the Junior High Schools located in the Assin Central Municipality.

Limitations

The study acknowledges certain limitations that should be considered when interpreting the results. For example, the study focused solely on the use of diagrams, and future research could explore the effectiveness of other visual aids or multimodal approaches in supporting learners' word problem-solving abilities. In addition, the sample size for the study may be limited, and future research could involve a larger and more diverse participant pool to ensure wider generalisability. Furthermore, diagrams cannot be used in all situations in word problem solving, particularly in areas such as higher-level algebra where abstract manipulation of variables is required. Finally, the study only considered learners in the public schools in the Municipality. Therefore, the findings may not be typical among learners in private schools and, thus, not generalisable to all learners in the country.

Definition of Terms

For this study, word problem, attitude, learner, teacher, and diagrams were defined as follows:

Word problem: This pertains to a mathematical problem that is completely expressed or articulated in words. Word problems are mathematical problems presented in a statement form or in a story form for learners to decipher the mathematics in the story and solve. In short, they are questions raised for inquiry.

Attitude: This refers to an established way of thinking or feeling about something. It refers to a consistent orientation to something which develops into a settled way of thinking or feeling about that thing and most often occurs in the mind.

Learner: An individual undergoing formal education in a school or recognised institution. A learner is someone who receives instruction or guidance from a teacher within an educational setting (Adom, 2021).

Teacher: An individual who facilitates and imparts knowledge to a learner within an educational environment (Adom, 2021). A teacher is someone who aids learners in acquiring knowledge within a specific content area.

Diagrams: This refers to a simplified drawing representing something. It is a schematic or symbolic representation of information depending on one's visual techniques or skills.

Organisation of the Study

This research is organised into five chapters. Chapter One provides an introduction to the study, including the background, problem statement, purpose, significance, scope, delimitations, and limitations. Chapter Two explores the generative theory of diagram construction as the main theoretical framework, reviews additional relevant concepts, examines previous research on word problems, and investigates self-directed diagrams as a heuristic strategy. Chapter Three details the research methods, design, population, sample, sampling techniques, data collection procedures and instruments, as well as data processing and analysis. Chapter Four presents and discusses the study's results. Finally, Chapter Five offers a summary of the study, along with conclusions and recommendations.

CHAPTER TWO

LITERATURE REVIEW

Overview

This study aims to explore the use of diagrams in addressing learners' difficulties in solving word problems in algebraic expressions in some selected basic schools in the Assin Central Municipality. This chapter presents a review of related literature. The literature comprises the theoretical backdrop of the study, conceptual issues and some empirical reviews. The chapter is organised under the following sub-headings:

1. Theoretical Framework:
   a. Generative Theory of Drawing Construction [GTDC] (van Meter & Garner, 2005)
2. Conceptual Review:
   a. Algebraic expressions
   b. Word problem in algebra
   c. Visual representations in the study of algebra
   d. Self-directed diagrams as a schema and heuristic strategy
3. Empirical Review:
   a. Challenges learners experience in solving word problems.
   b. Errors learners make in solving word problems.
   c. Effects of the use of diagrams in solving word problems.

Theoretical Framework

It is hard to envision teaching, learning, or performing mathematical calculations without the use of visual aids. The spatial and temporal aspects of mathematics, similar to those in the fields of science, technology, engineering, and mathematics (STEM), cover a vast range and involve complexities that challenge human understanding. Visual representations serve as effective tools for making the invisible visible and simplifying the complex (Quillin & Thomas, 2015). Based on this premise, this section of the study introduces Van Meter and Garner's (2005) Generative Theory of Drawing Construction (GTDC) as the theoretical framework for the research.

The GTDC is considered an extension of Mayer's Generative Theory of Textbook Design (GTTD), which focuses on learning from clarified text (Mayer, Steinhoff, Bower, & Mars, 1995). While Mayer et al.'s theory is centred on textbook design, both theories share a method of generating drawings involving selection, organisation, and integration, which are also supported by Paivio's dual-coding theory (1986, 1991). In each of these theories, referential connections between concepts within verbal representations are used to activate pre-existing nonverbal representations, known as "imagens" (Paivio, 1991), when the learner engages in the drawing task.

The Generative Theory of Drawing Construction (GTDC)

The Generative Theory of Drawing Construction (GTDC), developed by Van Meter and Garner (2005), highlights the importance of using drawings created by learners as a teaching method. They argue that drawings made by learners act as learning tools, where students read written materials and then create visual representations that capture the key elements and relationships from each section of the text. This method is seen as a thorough, activity-based approach designed to actively engage learners and improve their understanding of the content being taught (Mason, Lowe, & Tornatora, 2013). Therefore, in this study, the use of learner-generated drawings is employed as a strategy to improve students' comprehension of textual information.

Considerable evidence suggests that learners exhibit improved retention and the ability to apply what they've learned when they are instructed to produce visual illustrations while reading a passage, as opposed to relying solely on textual comprehension (Fiorella & Mayer, 2015; Fiorella & Zhang, 2018; Leutner & Schmeck, 2014). Additionally, other scholars contend that when learners are actively engaged in the process of creating drawings to depict the content, they become part of the generative process of the text (Hellenbrand, Mayer, Opfermann, Schmeck, & Leutner, 2019). This is attributed to the fact that the act of drawing involves the selection of crucial and relevant elements, their organisation into descriptive and illustrative representations, and their subsequent integration with prior knowledge to construct a coherent mental model.

In the GTDC framework, diagrams or drawings created by learners are described as intentionally crafted visual representations designed to accurately depict objects mentioned in a text (Hellenbrand et al., 2019). The GTDC posits that when learners are required to produce external visual representations of key elements from instructional material, they transition from being passive recipients of information to actively engaging in cognitive processes such as selection, organisation, and integration. Van Meter and Firetto (2013) expanded on GTDC by introducing the Cognitive Model of Drawing Construction (CMDC), which upholds the core principles of GTDC. These principles, based on Mayer et al.'s Generative Theory of Textbook Design (GTTD) (1995) and Paivio's dual-coding theory (1986, 1991), focus on selecting and organising descriptive features and integrating descriptive and illustrative representations, all of which are crucial for self-regulated learning processes (van Meter & Firetto, 2013).

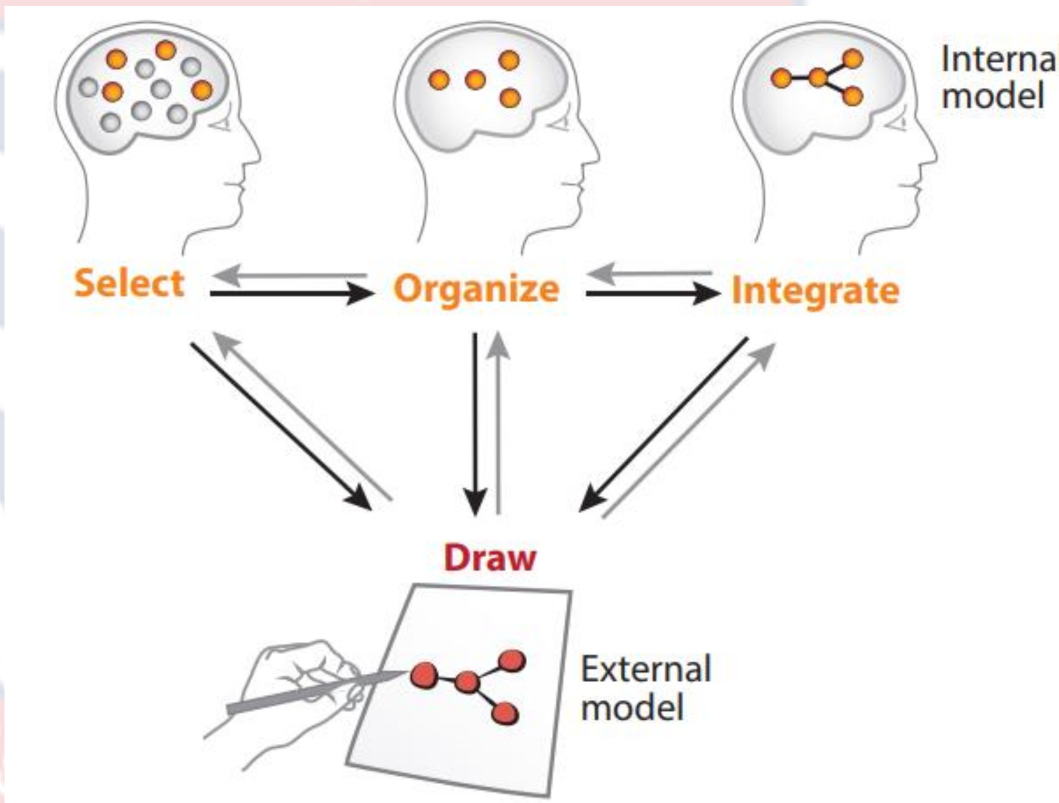


Figure 2: The GMDC processes (Quillin & Thomas, 2015)

Hellenbrand et al. (2019) avow that the generative drawings made by learners are not a linear sequence. This assertion is buttressed by van Meter and Firetto (2013) who maintain that for the learners to produce a drawing, they will have to go through numerous iterations of the GMDC processes, switching between numerous internal and external representations. From the illustration in Figure 2, the GMDC proposes that learner-generated drawings begin with the selection of key elements. The second phase of the drawing process is organisation whilst integration, which is the construction of the mental model, is the third phase.

The reader, through a process of selection, identifies key elements within both verbal and visual representations. This involves independently organising internal mental images to create a coherent representation of the text and illustrations, using previously established components. During this process, connections between internal concepts are either activated from prior knowledge or formed as new links. This organisation is supported by a third step called integration, which involves creating referential connections. As a result, the learner forms a mental model of the text that includes both verbal and nonverbal information. This mental model is thought to improve the learner's ability to understand concepts and solve problems (Mayer & Gallini, 1990; Mayer & Sims, 1994).

Empirical research on learner-generated drawing has yielded somewhat inconclusive results (Alesandrini, 1984; van Meter & Garner, 2005). In other words, some studies have demonstrated favourable outcomes associated with learner-generated drawings, indicating improved text comprehension and general problem-solving abilities (Hall, Bailey, & Tillman, 1997; van Meter, 2001; van Meter, Aleksic, Schwartz, & Garner, 2006), while other studies have not observed such effects (Leutner, Leopold, & Sumfleth, 2009). The inconsistent implementation of the drawing strategy's approach is a factor in these contradictory empirical findings. In several studies, the drawing technique was utilised in its simplest version; learners were simply told to create an image based on the key concepts of the text (Alesandrini, 1984; Leutner et al., 2009).

In other research, instructors supported learners' drawing activities. For example, Wammes, Meade, and Fernandes (2016) provided learners with cut-out figures as models. Similarly, van Meter (2001) and van Meter et al. (2006) studies involved instructors giving learners illustrations to refer to after they had completed their own drawings. Studies that included instructional support consistently demonstrated the benefits of the sketching method. The most convincing evidence that the advantages of learner-generated drawing techniques may depend on instructional support comes from the studies by van Meter et al. (2006). However, the research by Leutner et al. (2009) suggests that learners did not benefit from drawing instruction that lacked such support, likely because managing drawing activities independently imposes higher cognitive demands on learners.

Conceptual Framework

The conceptual framework is regarded as the foundation of a research problem (Chapagain, 2019). It provides clarity and focus, allowing us to understand and organise the research questions better. Owing to this assertion Figure 3 served as the conceptual framework for the study.

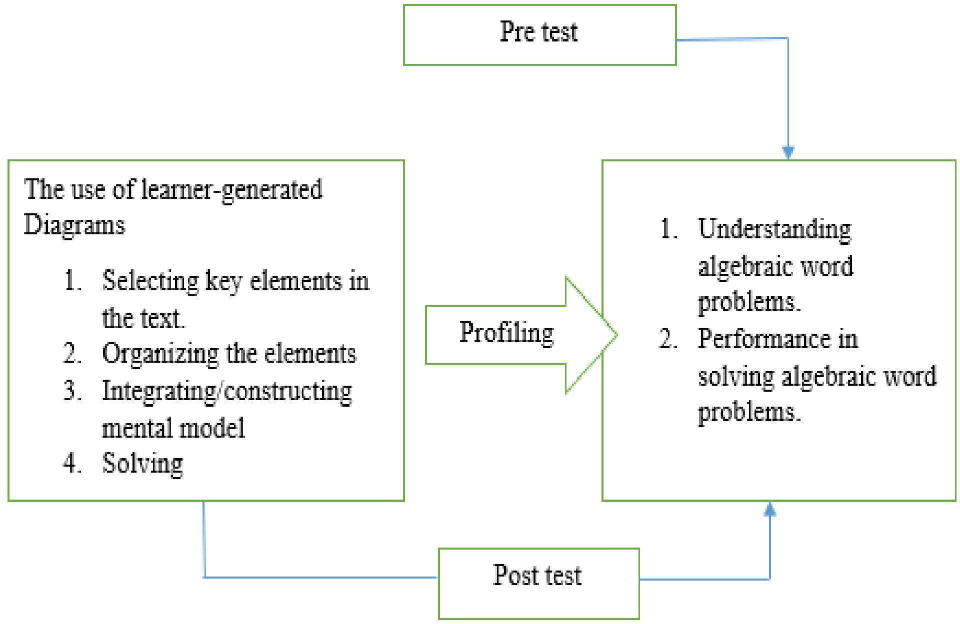


Figure 3: Conceptual framework. (Researcher's construct)

In Figure 3, the researcher assisted the learners in how to use the GTDC to construct their diagrams before solving a given word problem. The activities included how to identify and select key elements in a given text, how to organise and form mental models or images and then construct the mental models on paper. The learners then solve the word problem with the mental model generated as their guide. These activities are geared toward assisting learners to understand word problems and also improve their performance.

Conceptual Review

This section addresses definitional matters related to certain concepts or variables used in this research.

Algebraic expression

According to Kieran (2020), algebra has conventionally been taught to students aged 12-18 years, particularly in secondary schools. Kieran notes that the focus has typically been on working with polynomial and rational expressions, converting word problems into algebraic expressions, and applying axioms to solve equations involving variables and unknowns. However, there have been significant changes in the approach to teaching and learning algebra. For instance, Arcavi, Drijvers, and Stacey (2017) describe the goals of algebra as encompassing generalising concepts, expressing relationships, solving problems, exploring properties, proving theorems, and performing calculations.

Various authors have put forth diverse attributes of algebra and algebraic thinking, as exemplified by the works of Lins (2002) and Mason (2017). Numerous articles, books, and research papers have explored this subject to varying degrees. The definitions of algebra and algebraic thinking carry substantial implications for the advancement of classroom methodologies and educational materials, as emphasised by Lins and Gimenez (2000). Schoenfeld (1995) labels the study of algebra as the new literacy requirement because algebra serves the same purpose as reading and writing did throughout the Industrial Revolution. This assertion is supported by Moses (1993) who labels algebra as the new civil right. Both authors perceive algebra as a primary concept needed to understand science, statistics, commerce, or modern technology.

Conventionally, algebra has been viewed as a course focused on symbolic manipulation, designed to equip learners with the tools needed for future calculus studies (Butler, Jackiw, Laborde, Lagrange, & Yerushalmy, 2009). This narrow perception of algebra and mathematics, in general, has been a significant concern, prompting the development of the National Council of Teachers of Mathematics

(NCTM) Standards for Curriculum and Evaluation (Cai & Howson, 2012). In their effort to define algebra more comprehensively, the NCTM team aimed to address several key questions: What is algebra, and why is it important? How should it be taught? And how should it be assessed? The standards outline the algebraic processes learners should master by the end of their core curriculum. According to Kaput, Carraher, and Blanton (2017), defining algebra is challenging because curriculum leaders, educators, and mathematicians have varying perspectives on its meaning.

Nonetheless, several authors and researchers have offered their own definitions of algebra: Sheffield (2017) describes it as a course with a mythical body language; Morneau-Guérin (2022) sees it as a branch of mathematics; Schifter and Russell (2022) view it as generalised arithmetic; Sethy (2021) defines it as a symbolic language; Anwar and Rahmawati (2017) describe it as a language using verbal, tabular, graphical, and symbolic forms; Lee, Bull, Pe, and Ho (2011) see it as the study of relationships, patterns, and functions; Sun-Lin and Chiou (2019) refer to it as a series of problem-solving strategies; Pedemonte (2008) characterises it as a modelling process; Kaput (2017) considers it a way of reasoning; and Lo (2001) describes it as a formal structure. These various definitions converge around the idea that algebra involves the symbolic representation of situations and the manipulation of these symbols in ways that make sense both in relation to the symbols themselves and the situations they represent.

Russell (2018) defines algebra as the branch of mathematics that replaces numbers with letters. This means algebra involves working with symbols and performing arithmetic operations on these symbols. The symbols, known as variables, do not have fixed values and typically represent unknown quantities that need to be determined. Algebra is concerned with finding these unknowns or inserting real-life variables into equations and solving them. Examples of algebra include real and complex integers, matrices, and vectors. For instance, a scale can be represented by an algebraic equation where operations performed on one side must be mirrored on the other, with numbers acting as constants.

The study of patterns is central to mathematics. Defining mathematics as the study of numerical, geometric, logical, and structural patterns highlights the importance of learning algebra. Ginsburg, Leinwand, Anstrom, and Pollock (2005) argue that algebra is a powerful tool for expressing and structuring critical aspects of mathematics through logic and abstraction. Mathematics involves identifying patterns, hypothesising, testing, discussing, expressing, and generalising these patterns. Mathematicians analyze key elements of patterns, develop conceptual and relational understandings, create a language to describe them, and differentiate between various patterns.

Exploring relationships between quantities (variables) within patterns leads to an understanding of essential mathematical relationships and functions. Algebra can thus be defined as the use of symbols to represent phenomena, describe patterns, formulate equations involving geometric, physical, economic, and other relationships, and manipulate variables to uncover and describe additional patterns and variations. The development of symbolic reasoning, pattern recognition, and representational skills are key objectives in algebra instruction (Jack & Thompson, 2017).

Teaching algebra is also a crucial skill that mathematics educators need to master. Gibson (2021) argues that a deep understanding of the elements involved in teaching algebra greatly benefits learners in numerous ways. Newton, Star, and Lynch (2010) suggest that the study of algebra is not just about learning formulas and equations but also about exploring various explanations and strategies for solving problems. They emphasize that a key aspect of teaching algebra is to encourage students to understand that there are multiple methods for solving algebraic problems, thereby allowing them to explore different approaches.

Additionally, Star, Foegen, Larson, McCallum, Porath, Zbiek, and Lyskawa (2015) propose three recommendations for teaching algebra. First, they recommend using solved problems to engage students in analysing and identifying effective strategies for solving algebraic problems. This involves allowing students to discuss the structure and solutions of these problems to understand the connections between strategies and identify potential errors. The second recommendation is to help students apply the identified structures and language to solve algebraic problems. The third recommendation is to teach students to deliberately select from various algebraic strategies for problem-solving, encouraging them to explain their choice of strategy and its effectiveness. Overall, students are advised to evaluate and compare the results of different strategies.

Word problem

In fostering mathematics proficiency among learners in various schools, the NCTM (2000) outlined six guiding principles, with two of them directly related to mathematics teaching and learning. According to the NCTM, effective mathematics instruction requires teachers to understand students' existing knowledge and then both support and challenge them in learning new concepts. The NCTM also highlights that learning mathematics involves building and actively constructing new knowledge based on what learners already know and have experienced. Understanding is a central theme in both principles.

To fully grasp a concept indicates that one possesses some level of knowledge about it. Dossey, McCrone, Giordano, and Weir (2002) classify mathematical knowledge into three interconnected areas: concepts, procedures, and problem-solving. They argue that understanding concepts involves comprehending the fundamental essence of what something represents. Procedural knowledge entails being familiar with selecting and applying the correct procedures. The third category, problem-solving, requires learners to identify scenarios, discern their underlying structure, navigate the related relationships, manipulate these relationships, and effectively communicate the results. These elements are crucial components of word problems.

A key focus of this study is on word problems, which are considered an important part of mathematics education and curriculum (Moschkovich, 2002). Word problems are recognised for their educational value in mathematics teaching and learning (Dayal & Chandra, 2016). Many researchers and educators share a common understanding of word problems. For instance, Lester (1983) defines a mathematical word problem as a task that engages learners in problem-solving. Cawley and Miller (1986) describe word problems as incorporating linguistic and structural aspects of mathematics that challenge learners. Both perspectives view word problems as complex mathematical concepts that present various difficulties.

Baroody (1987) describes word problems as mathematical tasks that involve analysing unknowns or tasks with excessive incorrect data, requiring multiple approaches to find a solution. Mayer (1982) adds that word problems contain both relevant and irrelevant information, which can act as distractors. These definitions clarify why Wyndhamn and Saljo (1997) argue that word problems require learners to apply mathematical knowledge to real-life situations rather than relying solely on rote learning. Parkins and Hayes (2006) similarly describe word problems as reflections of real-life scenarios involving mathematical concepts. Word problems are essentially mathematical challenges presented in ordinary language rather than mathematical symbols or notations, and one major difficulty is translating them into mathematical equations before solving them.

Features of a word problem

Recognising that mathematical word problems are usually presented in written or textual formats, Martiniello (2008) and Xin, Wiles, and Lin (2008) argue that these problems have a structure that includes variables, quantities, and grammatical components. According to De Corte, Greer, and Verschaffel (1996), the structure of a word problem encompasses known and unknown quantities, as well as the mathematical operations and relationships described in the text. This structure can range from simple to complex linguistic features. A key aspect of this structure is the use of variables to denote unknown quantities, which may appear as letters or words within sentences and can vary in length and number (Lepik, 1990; Martiniello, 2008).

Additionally, word problems can be categorised into known, desired, and auxiliary quantities (Lepik, 1990). Known quantities are those with given values, such as prices, distances, or ages, and are used for computation. Desired or unknown quantities, identified within the problem statement, are the ones for which solutions need to be found.

Grammar is another essential structural element in word problems, as it influences how the problem is understood (Xin, Wiles, & Lin, 2008). Ambrose (2010) points out that grammar can direct attention to important elements, including mathematical terms, quantities, and context. Thus, grammar acts as a guide for learners to better understand the problem. Errors in grammar can, therefore, cause confusion or misinterpretation of the problem statement.

The Importance of Word Problems in Mathematics Learning.

Mathematical word problems come in various categories and levels of difficulty, challenging learners to engage in deep thinking and apply their mathematical knowledge to find solutions. Since the 1980s, the importance of word problems in mathematics has been discussed by numerous authors, as noted by Xin (2008). Ambrose (2010) observes that historically, word problem exercises mainly involved applying addition and subtraction algorithms.

Wyndhamn and Saljo (1997) highlight that mathematics word problems have significant implications for both teachers and learners, such as providing learners with insights into real-world applications. Similarly, Verschaffel, Greer, and De Corte (2000) argue that solving word problems encourages learners to use mathematical tools effectively, making connections between mathematics and real-life scenarios and fostering deeper thinking. Thus, word problems are crucial for linking mathematical concepts to real-world applications in mathematics education.

Xin (2008) argues that exposing learners to a larger number of mathematical word problems significantly enhances their critical thinking and reasoning skills in mathematics. He further suggests that these opportunities are vital for developing a broader conceptual understanding of mathematics. This view aligns with Ambrose (2010), who notes that solving word problems encourages learners to think mathematically and develop robust problem-solving strategies, rather than relying on memorised procedures. Jitendra, DiPipi, and Grasso (2001) support this, stating that these skills are essential for modern employment. Consequently, improving the teaching and learning of mathematical word problems in Ghanaian schools is crucial for advancing higher education and future career prospects.

Types of Word Problems

Word problems have been categorised into three main types by several scholars and researchers including Christou and Phillipou (1998), Fuchs, Seethaler, Fuchs, Powell, Hamlet, and Fletcher (2008), Garcia, Jimenez, and Hess (2006), and Kintsch and Greeno (1985). These three types are change, compare and combine word problems. It is noteworthy that, all these word problems are reflected in the new Ghanaian mathematics curriculum (MoE, 2020). The following are the in-depth explanations of the three types of word problems.

Change word problems.

Ambrose (2010) points out that problems involving change are among the most common types of mathematical word problems. According to Fuchs et al. (2008), Garcia et al. (2006), and Riley, Greeno, and Heller (1983), these problems typically follow a specific pattern: they start with an initial quantity and then describe an action that either increases or decreases that quantity. Griffin and Jitendra (2009) categorise change word problems into three key pieces of information: the initial amount, the change, and the final amount. These problems usually involve adding or subtracting from the initial quantity to determine a new total. For example, a problem might state: "Akwasi has five pencils. His friends Kojo and Kwame each took one pencil from Akwasi. How many pencils does Akwasi have now?"

In change word problems, the main operations are addition and subtraction, which either increase or decrease the original amount (Carpenter & Moser, 2020; Fuchs et al., 2021). The final quantity depends on whether the action results in an increase or decrease from the initial amount. As Kalyuga (2006) notes, these problems focus on situations where a variable undergoes a sustained change over time.

Combine word problems.

The second category of word problems in mathematics is known as combined word problems. According to Powell and Fuchs (2018), these problems can also be referred to as total or part-part-whole problems. Combined problems typically require learners to calculate the total amount or to determine one of the parts within a given problem. Fuchs et al. (2008) also note that combined word problems involve situations where multiple quantities are integrated into a single problem. For example, if *there are 32 oranges and 16 apples in a box, the problem would ask: "How many fruits are there in total in the box?* Garcia, Jimenez, and Hess (2006) suggest that combined word problems illustrate the relationship between a whole set and two distinct subsets.

From the example, the two disjoint subsets are propositions 32 oranges and 16 apples. The unknown quantity which is the third proposition is the total set to be calculated. Ambrose (2010) posits that this type of word problem usually requires addition and subtraction depending on the relationship between the unknown set and the two disjoint sets. In this case, the phrase '*how many fruits are there altogether*'? warrants that the operation to be employed to obtain the third set is 'addition' (+). Conversely, questions with a proposition like "remaining" or "left" require the subtraction operation to obtain the third set.

Compare problems.

Powell and Fuchs (2018) refer to this category of word problem as a 'difference problem.' This type of problem involves comparing two sets to determine the difference between them, with the difference being the third quantity or set (Schumacher & Fuchs, 2012). A key aspect of compare word problems is that any of the three quantities can be the unknown that learners need to find. According to Munez, Orrantia, and Rosales (2013), Schumacher and Fuchs (2012), and

Verschaffel, De Corte, & Pauwels (1992), compare word problems are often more cognitively challenging and difficult for learners compared to change and combine problems. Schumacher and Fuchs (2012) explain that the complexity of compare word problems arises from their depiction of static relationships and the integration of relational expressions or complex schemas (Shum & Chan, 2020).

In constructing compare word problems, Schumacher and Fuchs (2012) identify three subtypes based on the quantity to be calculated by the learner: the referent set, the compared set, and the difference set. For example, in the problem *Peter has 17 marbles and Paul has 9 marbles. How many fewer marbles does Paul have than Peter?* Peter's marbles represent the referent set, Paul's marbles represent the compared set, and the difference is described by the question "How many fewer marbles does Paul have than Peter?" Although all three subtypes describe a comparative relationship, Carpenter and Moser (2020) note that problems with unknown referent sets are the most challenging for learners, followed by problems with unknown compared sets, with problems involving unknown difference sets being generally easier.

Research indicates that change, combine, and compare word problems involving addition and subtraction are typically designed for primary school students (Carpenter & Moser, 1983). However, Ambrose (2010) suggests that these problems can also be suitable for secondary school students depending on their construction. Schumacher and Fuchs (2012) also note that the difficulty level depends greatly on the relational terminology used, with more complex relational statements being more challenging for learners. In the context of the Ghanaian

Mathematics curriculum, these types of word problems are included for Junior High School (JHS) students and are introduced in basic seven (MoE, 2020).

Essential Forms of Knowledge for Solving Word Problems

Solving word problems as earlier hinted positions learners to think creatively and develop new mathematical concepts and skills. This form of training rides on some form of knowledge and skills expected of the learner to acquire before solving word problems. This knowledge is needed for the conceptual understanding of word problems (Ambrose 2010). Literature suggests that there are two basic forms of knowledge to be acquired by learners and they are vocabulary knowledge and syntactic knowledge (Ambrose 2010).

Vocabulary knowledge. Vocabulary knowledge involves two main components: the range of words a learner knows and the depth of their understanding of these words (Dong, Tang, Chow, Wang, & Dong, 2020; Read, 1993). Each academic field has its own specialized vocabulary, ranging from everyday language to technical jargon (Halliday, 1978, as cited by Abbaszadeh, 2022). Mastery of the vocabulary in a particular discipline reflects one's proficiency in that area, making it crucial for mathematics students to develop thorough vocabulary knowledge, as it significantly impacts their mathematical abilities.

Nation (2001) distinguishes vocabulary knowledge into two categories: receptive and productive. Receptive vocabulary refers to understanding and comprehension through listening or reading, while productive vocabulary involves using language through speaking and writing. These aspects include recognizing text forms, meanings, and the application of specific words (Nation, 2001; Qian, 2002; Zhang & Annual, 2008). Mastery in mathematics requires proficiency in both receptive and productive vocabulary, as these are critical for understanding, interpreting, and solving mathematical word problems, which involve specific grammatical functions and constraints (Nation, 2001).

Researchers also differentiate vocabulary knowledge into breadth and depth concerning the words used in word problem tasks (Nassaji, 2004; Nation, 2001; Qian, 1999, 2002; Read, 1989, 2000; Wallace, 2007; Wesche & Paribahkt, 1996). The "breadth" of vocabulary knowledge refers to the number of words a learner knows, while Nation (2001) describes it as the number of words a learner recognizes at a certain proficiency level. In contrast, the "depth" of vocabulary knowledge relates to how well a learner understands various aspects of a word, including its pronunciation, spelling, register, and its relationships with other words (Qian, 1999; Nassaji, 2004). Deep vocabulary knowledge goes beyond merely knowing a word's definition, as emphasized by Ambrose (2010), and includes understanding how the word functions within different contexts.

Syntactic knowledge: Syntactic knowledge is another essential form of understanding required for solving mathematical word problems (Ambrose, 2010). Effectively addressing these problems requires a deep grasp of the text, which involves understanding the role of each word and how it relates to others in the statement. This aspect of understanding is referred to as syntactic knowledge. Syntactic knowledge encompasses the ability to combine words to create meaningful sentences, phrases, or utterances (Alexander, 2015). Shiotsu and Weir (2007) describe it as recognizing whether a sentence or its components, like clauses or phrases, are well-formed or not. Essentially, syntactic knowledge involves knowing how words and sentences are structured in a language (Writer, 2020).

To solve mathematical word problems, one must be able to understand the relationships between words in a sentence, a skill known as syntactic awareness (Sedita, 2020). Sedita notes that syntactic awareness develops from early oral language experiences and further grows with exposure to written text as students advance through grades. Ambrose (2010) emphasizes that syntactic knowledge is crucial for understanding mathematics word problems, highlighting the importance for learners to develop strong syntactic skills. Mastery of syntactic knowledge is vital, as it leads to better comprehension, a key step in solving mathematical problems (Qian, 1999).

Visual Representations in the Study of Algebra

Visual representations have long been integral to mathematical practice, with their use dating back to ancient Mesopotamia and classical Greece. Zimmermann and Cunningham (1991) highlight that visual representations, including diagrams, have been an inherent part of mathematics from its earliest stages. Geometry, for instance, has historically relied heavily on drawings, and for a time, so did other areas of mathematics (Rival, 1987). These representations can be direct or symbolic, taking forms such as photos, diagrams, images, memes, and graphics to illustrate people, objects, locations, or situations. Their importance for both mathematics educators and students is well-documented.

Debrenti (2015) argues that visual representations help learners gain a comprehensive understanding of mathematical problems and foster unique mathematical thinking. Research also shows that students are six times more likely to solve math problems correctly when using accurate visual aids compared to when they do not use them (Boonen, van Wesel, Jolles, & van der Schoot, 2014; Krawec, 2014). Conversely, learners using incorrect visual representations are less successful in solving math problems (Boonen, van Wesel, Jolles, & van der Schoot, 2014). This underscores the importance of effectively using visual representations to tackle mathematical problems, as they aid in grasping abstract mathematical concepts.

While visual representations offer significant benefits, it's also important to recognize potential challenges in their classroom use. Teachers should not assume that students interpret these representations in the same way that teachers do (Hall, 1998). The value of a representation to a teacher may differ greatly from its perceived value by a student (Cobb, Yackel, & Wood, 1992). Therefore, when incorporating representations into lessons, teachers need to help students learn how to interpret them effectively by providing "effective transitional experiences" (Boulton-Lewis, 1998) to facilitate their use and understanding.

Self-directed Diagrams as a Schema and Heuristic Strategy

Piaget (1971) maintains that any piece of knowledge is connected to an action. This is to mean that, to understand a concept, action needs to be performed and that action must be repeated to aid understanding and retention. This assertion resonates with that of a heuristic strategy which is a problem-solving technique that uses practical methods or hands-on activities to solve a problem (Martí & Reinelt, 2022). Every practical activity has in its backdrop a framework, a plan or a theory that guides the activity. The practical nature of the heuristic strategy also exhibits itself in the schema theory. This is because a schema is simply defined as an outline of a plan (Michael, Hornby, Wehmeier & Ashby, 2005) or a structured framework (Villanyi, Martinek, & Szikora, 2010) or a diagrammatic presentation (Merriam-Webster, 1998) that makes an idea easier to understand.

The use of diagrams or visual representations in learning mathematics has already been established as an effective strategy or a schema for learning or solving mathematical problems. They are often regarded as adjunct aids to texts in schools and research studies (McLure, Won & Treagust, 2022). Despite the current mathematics curriculum for basic schools laying emphasis on constructivist approaches to learning, diagrams are often offered to learners with the premise that the existence of the diagrams alone should improve learning. Conversely, McLure, Won, and Treagust (2022) maintain that the majority of the diagrams presented to learners which are to serve as a framework or a plan for solving mathematical problems in schools are merely illustrative rather than playing explanatory roles. Illustrative diagrams just portray specific characteristics of the text whilst explanatory diagrams on the contrary strive to explain some aspect of the subject matter.

Briefly stated, research indicates that using diagrams alongside text can help with comprehension and recall (Guo, Zhang, Wright, & McTigue, 2020), but for conceptual understanding at a higher level, diagrams should be used to explain all the elements presented in the text (Gobert & Clement, 1999). Conceptual understanding in this regard requires learners to build mental models of the concept being presented to them. Although studies indicate learners may produce simple and single representations of a phenomenon (Davidowitz, Chittleborough, & Murray, 2010) which may be a limitation to learners generating their diagrams nonetheless McLure, Won, and Treagust (2020) maintain that diagrams generated by learners show their understanding and explanations to a text. In this regard, in planning what and how to solve a word problem, learners will have to draw a diagram that will represent the problem statement and also serve as a guide in solving the problem.

In this study, however, rather than presenting learners with predesigned diagrams, learners were guided to generate their diagrams to represent a given word problem before solving it. Self-generated diagrams therefore simply involve learners building a meaningful and appropriate mental representation of a given text. This meaningful representation however serves as a heuristic strategy or a schema/framework for the learner in deciphering the text and also guiding the learner to solve the problem.

Empirical Review

This chapter presents an empirical review of past studies conducted with the view of attempting to find answers to similar questions posed by the researcher. This section however will specifically highlight studies on:

1. Errors learners make in solving word problems.
2. Challenges learners experience in solving word problems.
3. Effects of the use of diagrams in solving word problems

Errors Learners Make in Solving Word Problems.

The Ministry of Education (MOE) (2019) stressed the importance of learners being able to articulate mathematical concepts, develop mathematical skills, and apply these skills to real-life situations. Mastery of mathematical concepts and problem-solving skills can lead to fewer mistakes in problem-solving. Research has shown that learners often make various types of mistakes when solving mathematical problems. If these mistakes are not addressed, they can turn into errors, indicating gaps in understanding. Therefore, researchers are interested in examining the different errors learners make with word problems.

One such study by Adu, Assuah, and Asiedu-Addo (2015) aimed to identify and analyze the nature of errors learners make and to propose solutions for improving classroom teaching. They used a descriptive survey design and purposive sampling to involve 130 first-year senior high school students. They administered 10 diagnostic linear equation test items and applied the Newman Error Analysis framework to categorize the errors. The study found that only about 2% of the learners provided correct answers, indicating widespread difficulties with word problems. Specifically, 75% of participants had comprehension errors, 86% made transformational errors, 84% had process skills errors, and less than 30% reached the final encoding stage. The researchers concluded that the main issue was learners' inability to understand and interpret the problem statements before applying their computational skills. They recommended that teachers participate in in-service training to improve students' problem-solving efficiency.

Similarly, Abdullah, Abidin, and Ali (2015) conducted a study focused on analyzing errors in solving Higher-Order Thinking Skills (HOTS) tasks related to fractions. They used a descriptive study with a quantitative approach, involving 96 Basic Seven students. Four HOTS test items were used, and the Newman Error Analysis framework was applied to analyze the results. The study revealed that 27.58% of errors were encoding errors, followed closely by process skills errors at 27.33%. Transformation and comprehension errors were found at 24.17% and 20.92%, respectively. The researchers noted that encoding errors were prevalent because many learners did not progress to this stage. They concluded that these errors stemmed from learners' inability to connect test information with the appropriate strategies for solving tasks.

Both studies significantly contribute to the existing knowledge. The study by Adu, Assuah, and Asiedu-Addo (2015) clearly defined the problem and purpose and detailed the sampling procedure and instrument validation. In contrast, the study by Abdullah, Abidin, and Ali (2015) did not provide information on how test items were validated and only mentioned the sample size. Knowledge of the sampling procedure is crucial as it supports the generalizability of the findings. Additionally, Abdullah, Abidin, and Ali (2015) explored learner performance across three achievement levels (high, medium, and low), whereas Adu, Assuah, and Asiedu-Addo did not consider achievement levels. Research suggests that examining achievement levels is beneficial for decision-making (Armstrong, Laird & Mulgrew, 2008).

Challenges Learners Experience in Solving Word Problems

Recognising the difficulties learners face with a particular concept is crucial for achieving teaching and learning objectives. Understanding each learner's unique challenges enables teachers to guide them more effectively towards a better understanding. In this regard, the study on challenges learners experience in solving word problems has been conducted using two approaches by the researchers. One group of researchers examined and reported on the challenges learners encounter when solving word problems in general whereas the other group identified a particular challenge and investigated it in detail.

Among the researchers who focused on identifying and describing the challenges learners encounter when solving word problems, Talikan (2021) adopted the descriptive survey and examined the challenges learners encounter in solving word problems. He adopted the simple random sampling technique to engage 40 grade 11 learners for the study. The study indicated that learners have diverse challenges which influence their attitude towards mathematics. The study found that learners struggle to convert word problems into mathematical notation. The researcher suggested that this difficulty often results in learners resorting to guessing the answers to many of the word problems presented to them. The study again showed that, though the teachers have mastery of the concept, chalkboards and worksheets are the only instructional materials used in teaching word problems at the expense of models, PowerPoint presentations and diagrams.

The findings from Talikan (2021) are distinctive as the author explored the challenges learners face regarding their attitudes towards studying word problems and mathematics in general, as well as their interactions in the classroom. However, while the researcher clearly articulated the study's purpose, the statement of the problem was not specified. In scientific research, a majority of scholars believe the problem statement is what triggers the entire research process (Weick, 1989). In addition, generalising from a descriptive survey with a sample of 40 Fry, Barrett, Seiling, and Whitney (2014) label it as contentious. Finally, though the authors adopted an instrument for data collection, the study failed to determine its reliability. In all, the study can be said to have contributed to knowledge, despite some flaws associated with its conduct. The study may not be generalised enough since much evidence was not provided to support the findings of the study.

Among the other studies, Intsiful and Davis (2019) used a stratified random sampling technique to involve 187 JHS 2 learners in examining their performance on mathematics word problems in relation to their linguistic skills. They employed a Mathematics Achievement Test (MAT) and an interview guide for data collection, and utilised a sequential mixed methods design. The study's findings revealed a decline in learners' performance as the difficulty of word problems increased from primary to JHS level. The authors identified linguistic challenges as the primary obstacle for learners in solving word problems. They recommended that mathematics teachers focus more on teaching word problems and suggested that service providers be included in professional development programmes for teaching word problems.

In the study, the authors outlined the purpose and clearly articulated the problem statement. They also provided the sample size, which helps support the generalisation of the study's findings. However, Tipton, Hallberg, Hedges, and Chan (2017) suggest that generalising the results from a study conducted in a single geographical area with a sample size of 187 might not accurately reflect the broader situation. Furthermore, the authors' primary attention was identifying challenges regarding the linguistic features of the participants ignoring their process skills, and their transformation skills among other challenges. The findings however have added to existing knowledge, the gaps this study seeks to fill.

Effects of the Use of Diagrams in Solving Word Problems

The use of diagrams and other visual aids is recognised as a highly effective strategy for helping learners tackle mathematical word problems. Poch, van Garderen, and Scheuermann (2015) conducted research in this area, providing a framework designed to assist learners with learning disabilities in understanding and using diagrams to solve problems. They employed purposive sampling to work with 10 learners, using diagnostic assessments and interviews to identify their challenges and propose effective support methods. The study's findings revealed that diagrams greatly enhanced the participants' mathematical skills. The researchers concluded that while the framework was developed for learners with learning disabilities, it could also benefit other learners.

The framework outlined by Poch, van Garderen, and Scheuermann (2015) can be said to have added knowledge to existing literature. Nonetheless, in a quick refutation, the authors adequately introduced the topic and clearly stated the problem. The authors however failed to fuse the current study with existing studies to buttress the problem stated. In addition, it was noted that the research instrument was adopted, but the authors failed to determine its reliability, though the instrument was originally tested after its development. It is essential to redo the reliability test since the instrument was developed in a different environment. Also, the parameters adopted in selecting learners with learning disabilities were not stated which may affect the generalisability of the findings.

Other researchers have explored the application of diagrams, specifically the bar model, as a strategy for solving word problems. Notably, Madani, Tengah, and Prahamana (2018) carried out action research to help 50 Grade 9 learners with problems involving profit, loss, and discount. The data collected were analysed using a paired sample t-test. The test results revealed a significant difference in the mean score at $p < 0.05$ between the pretest (m=4.76) and posttest (m=6.10) after using the bar model. The researchers later recommended that the study provide a fertile ground for understanding the effectiveness of the bar model.

Related studies on the use of bar models in mathematics have also been conducted by Osman, Yang, Abu, Ismail, Jambari, and Kumar (2018). Their research aimed to assess learners' mathematical performance in problem-solving after using the bar model and to explore their experiences with the technique. The researchers employed a convenience sampling method to involve 32 Grade 9 participants. Data were collected using semi-structured interview guides and test items, and analysed through thematic analysis and paired-sample t-tests. Comparing the marks obtained in both tests, the pretest showed 29 learners obtaining low marks (0-4) with only 3 learners obtaining medium marks (5-7), and no learners obtaining a high mark (8-10). However, the posttest showed 7 learners obtaining low marks, with 25 learners obtaining medium marks and 10 learners recording high marks. The researchers ultimately concluded that the bar model is an effective method for enhancing learners' mathematical problem-solving skills.

In a different study, Morin, Watson, Hester, and Raver (2017) applied the same model to evaluate students with mathematical challenges. They employed a multiple-baseline design replicated across various groups to assess the impact of the bar model on helping learners solve word problems. The study focused on six third-grade students who had scored below the 16th percentile (one standard deviation beneath the mean) on word problem tests from the Standards of Learning (SOL) assessments. Eight mathematics word problem questions representing the eight levels of instructional concept outlined in the study were used to assist the participants in overcoming the identified challenge. The results later outlined that; all six learners showed maximum improvement in their problem-solving abilities. The researchers later recommended the bar model as an effective modelling tool and strategy for increasing elementary learners' problem-solving abilities.

The findings of Madani, Tengah, and Prahamana (2018) are notable for their use of an action research design to evaluate the effectiveness of the bar model in solving word problems. However, the authors mentioned the study's purpose but did not provide a clear statement of the problem. It is important to emphasise that problem statements are crucial components of scientific research. In addition, generalising from an action research with a sample of 50 Fry, Barrett, Seiling, and Whitney (2014) label it as contentious. Furthermore, it was observed that though the authors indicated the sample size they adopted for the study, they, however, failed to indicate the type of sampling procedure they used. Knowledge of the sampling procedure is essential since it provides some kind of evidence for the generalisation of the findings of the study (Polit & Beck, 2010). Finally, though the authors adopted an instrument for data collection, the study failed to determine its reliability. In all, the study can be said to have contributed to knowledge, despite some flaws associated with its conduct. The study may not be generalised enough since much evidence was not provided to support the findings of the study.

Similarly, Osman et al. (2018) used convenience sampling to determine their sample size but did not specify the population from which the sample was drawn. Taherdoost (2016) argues that clarifying the study population is essential for establishing the eligibility of participants. Additionally, while the study mentioned using tests and semi-structured instruments for data collection, it did not address their reliability and validity. Moreover, the study found that the bar model is an effective technique for solving change or non-routine problems. This means that solving problems which require higher-order thinking like routine or compare word problems may not be suitable for this technique.

Chapter Summary

The chapter reviewed key concepts relevant to this study, including algebraic expressions, word problems, visual representations in algebra, and self-directed diagrams as heuristic strategies. It also covered literature on the bar model, the challenges learners face, and the errors they make in solving word problems. The literature suggests that while the bar model is an effective strategy, it is primarily suited for routine problems that assess lower-order thinking skills.

Additionally, the importance of reading comprehension in solving word problems was highlighted, though it was noted that understanding the problem statement alone does not guarantee correct solutions. Some studies also pointed out that relying on 'hunting' for keywords to understand problem statements can lead to overgeneralisation.

The current study explores the use of diagrams, specifically learner-generated diagrams, as a strategy to address the challenges learners face when solving word problems. The study will use the GTDC (General Theory of Diagram Construction) as its theoretical framework. The aim is to help learners create their own diagrams from a given text to provide additional information that facilitates better understanding and problem-solving with reduced difficulty. The following chapter will outline the research methods employed in the study.

CHAPTER THREE

RESEARCH METHODS

Overview

The study aimed to assess the effectiveness of using diagrams to address learners' challenges in solving algebraic expression word problems in Basic Schools within the Assin Central Municipality of the Central Region. This chapter outlines the research methods employed to conduct the study, including the practical considerations involved in designing the research to address the identified problems. It details the action plan used for the study (Jonker & Pennink, 2010), including the research paradigm or philosophy, research design, approach, study area, population, sample, and sampling procedures. Additionally, it covers the data collection instruments, pilot study, data collection procedures, data validity, and ethical considerations.

Research Paradigm

Thomas Kuhn's 1962 book *The Structure of Scientific Revolutions* provides evidence supporting Mouton's (1996) assertion that the concept of a research paradigm is rooted in Kuhn’s work. When Kuhn released the second edition of his book in 1970, it significantly heightened awareness of the importance of paradigms in scientific inquiry. Research indicates that various scholars and authors, such as Babbie (2010), Collis and Hussey (2009), Creswell (2007), De Vos, Strydom, Fouche, and Delport (2011), Mouton (1996), and Neuman (2011), have historically employed the term to underpin their ideas, greatly influencing their philosophical approaches, methodologies, and study frameworks (Collis & Hussey, 2009).

A research paradigm, or worldview, encompasses accepted theories, empirically established models, techniques, frameworks, traditions, research works, and methodologies. It serves as a foundation for observation and understanding (Babbie, 2010; Babbie & Rubin, 2010; Creswell, 2013). A researcher's worldview, known as their research paradigm, shapes how they approach their study (Mackenzie & Knipe, 2006). De Vos, Strydom, Schulze, and Patel (2011) describe a research paradigm as a comprehensive system of interconnected procedural and cognitive processes related to the nature of inquiry, including epistemology, ontology, and methodology. Essentially, the research paradigm is the philosophical framework that supports a study and provides a basis for the theories and practices employed.

It is crucial to acknowledge that a researcher's perspectives, mental models, and beliefs significantly affect their ability to present valid arguments and reliable findings. Gliner, Morgan, and Leech (2016) argue that the research paradigm outlines the approach to conducting research, while Cohen, Manion, and Morrison (2017) and Vveinhardt (2018) affirm that it guides the study's philosophical orientation. The research paradigm links the research questions with the study's objectives. Both the research objectives and questions are derived from the study's background, which is itself grounded in the research paradigm. The research paradigm and philosophy collaboratively shape the study based on theories and varying beliefs about reality (Vveinhardt, 2018).

Research studies are governed by various paradigms and philosophies, including positivism, interpretivism, realism, constructivism, and pragmatism, among others. This study, in particular, is approached from a pragmatic paradigm perspective. The pragmatic paradigm embraces the idea that reality is uncertain and that knowledge is gained through experience. Researchers using this paradigm assign meaning to activities or situations based on observations and identified actions at specific points in time.

The Pragmatism Paradigm

Research indicates that the pragmatic paradigm first emerged in the United States in the late 19th century (Maxcy, 2003). According to Maxcy, Charles Sanders Peirce (1839-1914) was instrumental in founding this paradigm. Subsequently, William James (1842-1910), a colleague of Peirce, made significant contributions to its early development by defending and advancing it. Over the past century, other philosophers such as John Dewey and George Herbert Mead have further developed and refined the concept (Maxcy, 2003; Morgan, 2013; Ormerod, 2006). It is also believed that the rise of the pragmatism paradigm was prompted by a consensus among researchers to reject traditional notions about reality, knowledge, and inquiry (Kaushik & Walsh, 2019). Pragmatist scholars openly contested the idea that research can only access reality through a single scientific method (Maxcy, 2003). Instead, pragmatists argue that knowledge is both socially and scientifically constructed and that there are multiple practical approaches to acquiring knowledge (Cresswell & Clark, 2017).

Researchers within the pragmatism paradigm hold that an objective reality exists independently of human perception or experiences (Goles & Hirschheim, 2000; Morgan, 2014). For pragmatists, reality is considered valid if it facilitates meaningful interactions with various aspects of our experiences (Baker & Schaltegger, 2015). The pragmatist views the interaction between individuals and their environmental experiences as central to their practical approach. In essence, pragmatists employ various methods and strategies to evaluate the effectiveness of an experience before recognising it as a reality.

Subsequently, the practical nature of the pragmatism paradigm is identified to accept a flexible approach to finding answers to an identified research problem (Revez & Borges, 2018). It involves the researcher enjoying the freedom to choose the type of method to adopt and employ in conducting the study (Mertens, 2010). All the same, the practical nature of the paradigm subjects itself to processes in conducting a research study. Given that, Morgan (2014) outlined a five-step model used by pragmatists in conducting research. Notably, these steps are also involved in the approach adopted in conducting this study. For example, for learners to translate text into mathematical notation, they will need to identify the requirements of the question and plan before solving the problem.

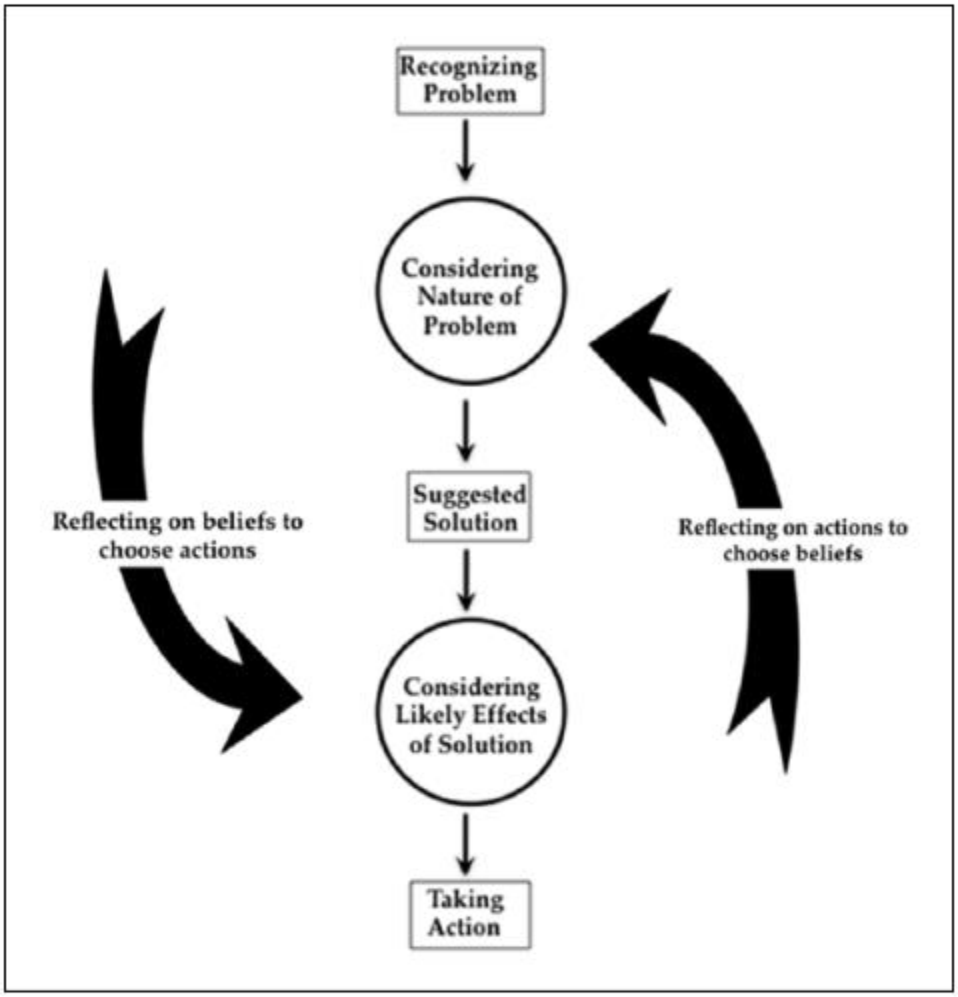


Figure 4: Dewey's five-step model of inquiry (Dewey as cited in Morgan, 2014)

From Figure 4, the first step taken to conduct any research according to a pragmatist is to first identify the problem. Dewey defines the problem in this context as finding yourself in an unfamiliar circumstance with no experience and appropriate action (Morgan, 2014). This however is seen in the first research question, thus seeking to identify the challenges learners have in solving word problems. The second step involves reflecting on existing beliefs and nature to understand why the issue identified is a problem. The ability to suggest a possible action to help solve the problem becomes the third step whilst reflecting on the effects of the suggested action forms the fourth step. The fifth step, however, is referred to as action, which means that the offered solution must be implemented to fix the problem. The second to the last step is the reflection of what the learners go through in generating diagrams to solve the problem which is the backbone of this study.

An examination of the steps depicted in Figure 4 reveals elements of the scientific approach to problem-solving. This characteristic is a key reason for choosing this paradigm for the study. Pearce (2020) defines the pragmatism paradigm as a framework for elucidating the meaning of a theory by identifying its practical implications. Additionally, this study is grounded in the pragmatic paradigm as it integrates diverse ideas, methodologies, approaches, and principles to address its research questions. The paradigm's focus on practicality also ensures that attention is directed only towards what is effective or feasible.

Moreover, Pool and Laubscher (2016) argue that pragmatic research often utilises mixed methods approaches, enabling researchers to delve deeper into data sets for a better understanding of their significance and to use one method to validate findings from another. This paradigm also emphasises the practical application of research, with data analysis and interpretation considering multiple perspectives or worldviews. Overall, this concept significantly impacts the researcher's subjective and objective perspectives when assessing data findings. Each of these factors strongly supports the adoption of the pragmatic research paradigm. The ideology of pragmatism according to Onwuegbuzie and Leech (2005), also fits nicely with the concurrent research approach.

Research Approach

A concurrent mixed-methods design, also referred to as a concurrent triangulation design, was utilised as the research approach for this study. This approach involves collecting both quantitative and qualitative data simultaneously, with equal emphasis, to address a research question or problem (Creswell & Creswell, 2017; Creswell & Clark, 2017). Researchers gather and analyse these data separately but integrate them during the interpretation phase to offer a more comprehensive understanding of the issue at hand (Teddlie & Tashakkori, 2003). The aim is to gain a well-rounded view of the research problem by examining it from various perspectives using diverse data collection methods and strategies.

The researcher chose this approach as it was suitable for investigating both the quantitative impact (effectiveness) of learner-generated drawings and the qualitative aspects, such as understanding the reasons behind their effectiveness or shortcomings. Educational research often involves complex variables, and relying solely on quantitative data might not fully capture the issue being studied (Teddlie & Tashakkori, 2003). Including qualitative data helps uncover the nuances and context-specific factors affecting learning outcomes, as highlighted in other studies.

Johnson and Onwuegbuzie (2004) found that mixed methods are beneficial for gaining a comprehensive understanding of educational research questions. Their findings support the use of concurrent mixed methods in educational research. Similarly, Rezaei and Dilmaghani (2016) applied a concurrent mixed-methods approach in mathematics education to study the impact of concept mapping on problem-solving skills. Their study demonstrated that this approach effectively measured quantitative performance effects while also providing qualitative insights into how and why these effects occurred. This aligns with the current study's objective of evaluating the effectiveness of learner-generated diagrams in solving algebraic word problems.

Regarding validity and reliability, Teddlie and Tashakkori (2003) argue that data triangulation enhances construct validity. This approach allows researchers to examine a research question from multiple perspectives, which reduces biases and improves external validity (Morse, 2009). Onwuegbuzie and Leech (2005) also suggest that convergence of quantitative and qualitative findings strengthens the study's credibility and validity. Such convergence provides robust evidence that the research question is well-founded.

This approach was selected because it is supported by empirical evidence in educational research and addresses the complexities and ethical considerations of education studies. It allows for the integration of quantitative and qualitative data, offering a rich understanding of the intervention's impact and the factors influencing it.

Research Design

Cresswell (2013) defines research design as the structured approach used by an investigator to conduct an inquiry. Kumur (2011) views it as a systematic plan to accurately and precisely address research questions. Many researchers agree that a research design should include various approaches, offering researchers different options to best fit their specific study area. Zikmund, Babin, Carr, and Griffin (2010) suggest that a research design should outline the procedural techniques and methods, including sampling techniques. It should also consider how data will be collected and analysed.

For this study, a two-group pretest-posttest design with a delayed posttest was employed to assess the impact of using diagrams on solving algebraic word problems. Reichardt (2019) notes that this quasi-experimental research design is commonly used, involving pretesting subjects, applying an independent variable or treatment, and then conducting a posttest. In this study, four classes were selected based on their test performance and divided into two high-achieving and two low-achieving groups, each with experimental and control subgroups. The participants were pretested before introducing the use of diagrams to the experimental groups to measure the intervention's effects.

Bell (2010) and Dimitrov and Rumrill (2003) argue that while pretest-posttest designs generally offer high external validity, they face challenges with internal validity due to potential extraneous variables such as maturation, regression towards the mean, historical factors, attrition, testing effects, and instrumentation. However, Bell highlights that the two-group pretest-posttest design with a delayed posttest can mitigate these threats. Rohrer (2015) suggests that administering a delayed posttest within three weeks of the intervention helps reduce most internal validity issues. Additionally, comparing the posttest results of control and experimental groups in both low-achieving and high-achieving classes aids in addressing internal validity concerns.

Area of Study

This study was conducted in the Assin Central Municipality, located in the Central Region of Ghana. The municipality is one of the 22 districts within this region. It was formerly part of the larger Assin District until the southern portion became the Assin South District on February 18, 2004. The northern section, however, was granted municipality status in February 2008. Subsequently, it was divided into Assin North District and Assin Central Municipality in March 2018, with Assin Foso serving as the capital of the municipality.

Population

The population for this study comprises 4778 JHS learners in the six circuits within the Assin Central Municipality. Out of the total population, 2345 were males and 2433 were females. The data was retrieved from the Assin Central Educational Municipality Directorate's statistics department. The accessible population for the study, however, comprises 418 JHS 1 learners in the selected circuit (Foso station B). The distribution of the JHS 1 learners within the selected circuit is presented in Table 1.

Table 1: Distribution of JHS 1 learners in Assin Foso station B circuit

School	Male	Female	Total
School A	20	22	42
School B	21	16	37
School C	18	21	39
School D	23	27	50
School E	29	22	51

School F	19	26	45
School G	23	27	50
School H	29	31	60
School I	24	20	44
Total	206	212	418

Source: GES, Assin Central Municipality Directorate (2022)

The study targeted learners who were in JHS 1 because of the topic under study. The study of word problems is presented as a sub-topic under linear equations and inequalities, which is to be studied in JHS 1 according to the new Ghanaian curriculum (MoE, 2020). This informed the selection of the JHS 1 learners to help identify and address the challenges under study.

Sampling and Sampling Procedure

A survey of the Assin Central Municipality indicates that all the schools are within the same geographical and ethnic background and are assessed using the same test items at the end of each term. Again, learners in the basic seven classes in the Municipality have an average age of 13 years which indicates some level of homogeneity across all the schools within each circuit. Given this, the multi-stage sampling technique which involves taking samples in stages was adopted and used. Firstly, the cluster sampling technique was used to randomly select the Foso station 'B's circuit to form the accessible population to represent the district.

Secondly, a purposive sampling approach was employed to specifically select two main classes based on their performance in a pretest, to categorise them into high and low-achieving classes. In this case, two schools with the highest average scores on the pretest were selected and categorised as the high-achieving classes and two schools with the lowest average scores on the pretest were also selected and categorised as the low-achieving classes. The rationale behind selecting low and high-achieving classes lies in the desire to examine the effect of using diagrams on both ends of the achievement spectrums.

By including learners from both high and low-achieving classes, it becomes possible to explore the effect of diagrams on learners with diverse levels of proficiency in problem-solving. This approach provides a comprehensive understanding of how diagrams can benefit learners across the performance spectrums and allows for the identification of any differential effects based on initial proficiency.

In addition, the simple random sampling technique was used to randomly place the two high-achieving classes (schools) into experimental and control groups and the same exercise was repeated for the low-achieving classes (schools). Finally, the census sampling technique, however, was used to engage all learners found under the classes identified for the study. In all, 171 learners formed the sample size engaged in the study with 80 of them representing the high-achieving classes and 91 learners representing the low-achieving classes.

Data Collection Instruments

To capture quality evidence that seeks to answer the questions posed in the study, two major data collection instruments were obtained and used for this purpose. These instruments were Mathematics Achievement Test (MAT) and Think Aloud Protocols (TAP). The MAT (pretest and posttest) were adopted from past BECE questions and GES-accredited textbooks. Ten test items were selected in all with seven of them taken from the past BECE questions. For example, with the pretest items, questions 1, 3, 4, and 5 were from the BECE past questions whereas question 2 was from their textbook. In addition, with the posttest items, questions 2, 3, and 5 were from the BECE past questions and questions 1 and 4 were from the GES-accredited textbooks.

These questions were subjected to expert review and were confirmed to be test items with the same level of difficulty. The test items covered areas under sharing, fractions, equations, measurements, ratios, and patterns and relations. These areas were selected because they run through all the strands and sub-strands of the new JHS mathematics curriculum. Ghauri, Gronhaug, and Strange (2020) argue that an instrument selected for data collection should cover the actual area of investigation. Given this, for an instrument to measure what it is intended to measure, the instrument must be valid.

The Think-Aloud Protocol (TAP) however is a method of data collection employed to gain insights into an individual's cognitive processes while they perform a task or solve a problem. In this case, the learners were asked to express their thoughts out loud, describing their understanding of the task, the steps they plan to take, and any challenges they encounter along the way. This technique is believed to assist the researcher in observing and analysing the participant's thought processes and helps the researcher understand how individuals approach and solve problems (Ericsson, 2017). In addition, the TAP questions used to engage the learners were based on the Newman Error Analysis prompts adopted from White (2010).

Pilot Testing

Before the pilot test, although questions from WAEC were considered valid and reliable the data collection instruments were still presented to my supervisors and other experts in the area of mathematics and test items construction for review. This was to help determine the content, criterion and construct validity of the instruments. Adefioye (2015) argues that the three validity types highly validate test items by assuring that the instruments measure accurately all the aspects of the content and the construct of the study and also match the level of learners' understudy. The instruments were later pilot-tested using a sample of 64 learners in Abura Dunkwa Methodist basic 'A' and 'B' schools. The expert reviews, along with the results and feedback from the pilot test, were utilised to improve the instrument for the final data collection.

Results of pilot testing

This section of the study outlines the results of the pilot test. The researcher randomly selected and used 64 learners for the pilot testing. The test items consisted of ten test items that cover areas like sharing, fractions, equations, measurements, ratios, and patterns and relations. Each school was made to answer five of the questions at a sitting. School A was made to answer the pretest items whereas School B answered the posttest items. These test items were given to the learners to solve individually. Learners were given a maximum of 45 minutes to solve the questions. It is important to note that, the test items were adopted from the BECE past questions which are deemed to be valid since subject area experts are employed in the construction of these test items. Also, all questions and examples found in the Ghanaian-accredited textbooks are vetted for their validity and reliability by the GES and National Council for Curriculum Assessment (NaCCA). Therefore, obtaining test items from these accredited sources indicates that the test items are within the scope of the study.

In addition, to measure how reliable the test items were, the learners were made to write the test twice within three weeks specifically 15 days after the first test. Therefore, the test-retest reliability was employed in determining how consistent and stable the scores were over time. The reliability coefficient after conducting the first test was .08 the second test however indicated a test-retest reliability coefficient of 0.9 for both classes. This according to Noble, Scheinost, and Constable (2021) shows that the test items are reliable.

In addition, feedback from the pilot test gave the impression that the 45-minute time allocation was not enough for a majority of the learners. In that regard, an hour was decided to be the time allocation during the main study. Extending the duration allowed learners ample time to read, reflect more deeply, and solve the questions, as using diagrams was somewhat time-consuming.

Data Collection Procedures

The data collection procedures encompassed several steps including gaining access to schools, fostering positive relationships with administrators and learners, the data collection process and tools, the pilot study, administering the measuring instrument, ensuring data validity, ethical considerations, addressing limitations, and drawing conclusions. The researcher secured letters of introduction from the Basic Education Department at the University of Cape Coast, the Ghana Education Service in the Assin Central Municipality, and received ethical approval from the Institutional Review Board at the University of Cape Coast.

Intervention Process

The intervention was conducted over three weeks, before administering the posttest. Sessions were held three times a week, each lasting a maximum of two hours. The intervention was structured into several key steps:

Introduction of Word Problem Tasks: Learners were introduced to specific word problem tasks. This step aimed to familiarise learners with the types of problems they would encounter and set the stage for deeper engagement with the content.

Modelling the Process: The researcher demonstrated how to identify key elements in a given text. This included guiding learners through the process of reading and understanding the text, highlighting important information, and disregarding irrelevant details. The modelling phase was crucial for showing learners how to approach word problems systematically.

Guided Practice: Learners practised making connections between the text, their personal experiences, and prior knowledge. This practice helped them to form a mental model or picture of the problem, facilitating a deeper understanding of the problem's context and requirements.

Independent Practice: Learners independently drew their mental models and wrote the mathematical representation of the text before solving the problem.

This step reinforced their ability to translate textual information into mathematical form and solve problems independently.

Reflection: This is where learners base on their generated diagrams to verify if it has helped them to organise their thoughts and enhance their understanding. Reflecting on their work through visual aids enabled learners to see the problem's structure and their approach more clearly, promoting better problem-solving strategies.

Feedback: The researcher provided corrective feedback, addressing any misconceptions learners had. This feedback was essential for helping learners understand their mistakes and learn from them, ensuring they could apply the correct methods in future problems.

Extension Activities: Additional activities were provided to extend learners' understanding and application of the concepts. These activities challenged learners to apply what they had learned in new and varied contexts, solidifying their skills and knowledge.

Experimental and Control Groups

Experimental Group: The researcher personally took the experimental group through the detailed intervention process outlined. This group underwent the structured steps which included the introduction of word problem tasks, modelling the process, guided practice, independent practice, reflection using diagrams, feedback to address misconceptions, and extension activities for deeper understanding. The direct involvement of the researcher ensured consistency and adherence to the intervention protocol.

Control Group: The control group was taught by their regular teachers using the keyword method as outlined in the mathematics curriculum. This method focused on 'hunting' for keywords in the problems to help translate the text and also guide the solution process. This instruction also spanned three weeks with sessions held three times a week for a maximum of two hours each.

Steps Involved in Using the Keyword Method

The keyword method is a structured approach to solving word problems in mathematics. It entails recognising key words or phrases in the problem that indicate the operations required to determine the solution. Here are the steps involved in using the keyword method:

Read the problem carefully: Begin by reading the entire problem carefully to get a general understanding of the scenario. At this stage, learners are advised to avoid focusing on numbers or calculations initially but to just understand the context and what is being asked.

Identify and highlight keywords: Learners are guided on how to 'hunt' for keywords or phrases that indicate specific mathematical operations.

Common keywords include:

Addition: sum, total, altogether, increase, more than

Subtraction: difference, less, minus, decrease, fewer

Multiplication: product, times, of, multiplied by

Division: quotient, per, divided by, out of, ratio

Determine the operation(s): Based on the identified keywords, learners decide which mathematical operation(s) will be used to solve the problem.

Sometimes multiple operations may be needed, so learners are informed to ensure all keywords are considered.

Extract the relevant information: Learners again are guided on how to identify and write down the numbers and other relevant information from the problem. Through this, learners set up the mathematical expressions or equations.

Solve the problem: Learners then perform the calculations as per the setup equation or expression. Learners re-read the problem and ensure that their solution answers the question asked. Opportunity is also provided to help them verify their calculations and ensure no steps were missed or errors made. They finally write down the final answer, ensuring it is in the correct form and includes any necessary units.

Pretest Administration

The pretest was administered prior to the intervention to assess the learners' initial level of knowledge and skills. This initial assessment allowed for the evaluation of the participants' proficiency levels before any instructional activities took place. By administering the pretest, the researcher was able to identify the specific areas where learners needed improvement and tailor the intervention accordingly. The pretest results also facilitated a comparison with subsequent posttest scores, thereby helping to measure the effectiveness of the use of diagrams.

Delayed Posttest Administration

The delayed posttest was administered two weeks after the initial posttest. This allowed for the assessment of the long-term retention of the concepts and skills taught during the intervention. The delayed posttest provided additional insights into the effectiveness of the intervention by evaluating how well learners retained and applied the skills over time.

Field Assistants

Eight field assistants, consisting of five mathematics education students and three basic education students majoring in mathematics, were recruited to support the intervention. The recruitment process involved selecting candidates based on strong academic performance, effective learner engagement skills, and relevant educational experience. After an interview process, the selected field assistants underwent a comprehensive training programme. This included an orientation session on the research objectives and intervention specifics, workshops on identifying learners' errors, communication strategies, and data collection protocols, as well as role-playing exercises to practice interacting with learners and providing feedback.

During the intervention, the field assistants engaged learners in productive conversations to identify their errors, challenges, and strengths while solving word problems. They provided immediate, constructive feedback, supported learners throughout the intervention, and assisted the researcher in administering the intervention steps. An essential task involved gathering and documenting precise and thorough data on learners' performance and progress to ensure the study's findings were reliable and valid. Recruiting and training qualified field assistants were crucial for effectively supporting the intervention and collecting high-quality data. By organising the interventions and assessments this way, the study sought to gain a thorough understanding of how various teaching methods assist learners in overcoming difficulties with solving word problems.

Data Processing and Analysis

This section outlines the data collection instruments used for each research question or hypothesis, along with the corresponding analysis tools for interpreting the results. For the first research question, which aimed to identify the challenges learners face in solving word problems before the introduction of diagrams, data were collected using the MAT and TAP. The data were then analysed with descriptive statistics and content analysis, respectively. Content analysis is a method used to systematically examine and interpret written or verbal communication (Neuendorf, 2017). Content analysis just like descriptive statistics involves identifying and categorising patterns, themes, symbols or other elements within the content to draw meaningful insights or conclusions. Both statistical tools were adopted because they help in identifying the patterns associated with the learners' challenges and provide a more objective interpretation of the data collected.

Additionally, research questions two and three were examined using descriptive statistics and a diagnostic method called the Newman Error Analysis (NEA) Framework. This framework divides the process of solving word problems into five stages: reading, comprehension, transformation, processing, and encoding. During the reading stage, the learner is expected to read and understand the question's sentence. Errors at this stage may include learners' inability to recognise the words or symbols in the sentence. The comprehension stage involves gaining a thorough understanding of what the question is asking the learner to do. Errors at this stage may occur if learners struggle to identify the requirements of the question or if they are unfamiliar with the terms and symbols used in the text.

The transformation stage, also known as modelling, involves learners using various methods, strategies, or the appropriate formula to solve the problem. Errors at this stage occur when learners fail to interpret or convert the text into a mathematical form. The processing however is the stage where learners apply the rules, procedures or appropriate algorithms in solving the problem. Computational errors and misinterpretation of mathematical rules are some of the basic errors at this stage. The final stage which is encoding is where the learner is expected to provide the final answer correctly with the right unit(s). The inability to provide the correct answer or provide the answer with wrong units are considered errors at the final stage. This framework also provided the foundation for the think-aloud protocol probing questions, which included five interview prompts adapted from White (2010). Each prompt aimed to identify particular errors at each stage of solving a word problem. In obtaining relevant data to analyse learners' errors, the MAT and the think-aloud protocols were used as data collection tools.

To analyse hypotheses one and two across both proficiency levels, the test scores of the experimental and control groups were examined using Analysis of Covariance (ANCOVA). This method was employed to determine if the intervention (using diagrams) influenced participants' performance in solving word problems. ANCOVA is an inferential statistical tool used to assess the effect of an independent variable on a dependent variable while accounting for one or more extraneous variables (Gaddis, 1998). It is also considered an ideal statistical method for pretest-posttest designs (Wu & Lai, 2015). A summary of the data collection and analysis process is provided in Table 2.

Table 2: A summary of how data was collected and analysed.

Research Question/Hypotheses	Data Collection Instrument	Data Analysis
1. What are the errors learners make in solving word problems before being introduced to the use of diagrams?	Mathematics Achievement Test Think-aloud protocol	Frequencies Newman Error Analysis
2. What are the challenges learners experience in solving word problems before being introduced to the use of diagrams?	Mathematics Achievement Test Think-aloud protocol	Content Analysis Frequencies
3. What are the errors learners make when solving word problems after being	Mathematics Achievement Test Think-aloud protocol	Frequencies Content Analysis

	introduced to the use of diagrams		
4.	The use of diagrams has no statistically significant effect on the performance in solving word problems between the experimental group and the control group of the high-achieving class.	Mathematics Achievement Test	ANCOVA Paired-sampled t-test
5.	The use of diagrams has no statistically significant effect on the performance in solving word problems between the experimental group and the control group of the low-achieving class.	Mathematics Achievement Test	ANCOVA Paired-sampled t-test

Ethical Considerations

Researchers who want to collect data that is not in the public domain have always had to cope with ethical issues like accessibility, privacy, confidentiality, and informed consent. These ethical difficulties in research must be acknowledged to respect and preserve the rights of participants. Obtaining the participants' consent for their participation is necessary for a researcher during data collection. In doing so, consent forms were given out for learners to fill out before administering the instruments. Again, participation in this study will be voluntary.

Furthermore, pseudonyms were employed to protect the confidentiality and anonymity of respondents. All necessary permissions were obtained from the Department of Basic Education and the Institutional Review Board (IRB) to secure an introductory letter for introducing the researcher to the district and schools participating in the study. In addition, documents retrieved from the participants were filed and locked in a cabinet. The researcher also ensured strict adherence to the World Health Organisation's COVID-19 protocols.

Chapter Summary

The aim of this chapter was to detail the methodology and design of the study. It covered the research paradigm, approach, design, study area, population, sample, and sampling techniques, as well as the data collection tools, the validation of instruments and data, and the ethical considerations relevant to the research. This chapter established the framework to ensure that the study's goals and objectives were met. The following chapter will present an accurate reflection of the study's findings, outcomes, and interpretations.

CHAPTER FOUR

RESULTS AND DISCUSSION

The study aimed to investigate the difficulties learners face in solving word problems and evaluate the effectiveness of self-generated diagrams in addressing these challenges. It employed a two-group pretest-posttest design with a delayed posttest. Data was collected using a Mathematics Achievement Test (MAT) and the Think-Aloud Protocol (TAP). The data collection tools were reviewed by experts in test item construction and pretested to ensure their validity. This chapter outlines the results and discussions, beginning with the demographic characteristics of the respondents and followed by the findings related to the research questions.

Demographics of Respondents

This section provides an overview of the respondents' demographic distribution, focusing on the gender and ages of the two primary classes. Table 3 presents details of the demographic features of the high-achieving class.

Table 3: *Demographic features of the high-achieving class*

Variable	Age (Years)			Total
Gender	Below 13	13-15	Above 15	
Male	2 (2.5%)	34 (42.5%)	6 (7.5%)	42 (52.5%)
Female	1 (1.25%)	35 (43.75%)	2 (2.5%)	38 (47.5%)
Total	3 (3.75%)	69 (86.25%)	8 (10%)	80 (100%)

Source: Field survey (2022)

Table 4 also displays the demographic characteristics of the low-achieving class. It is important to note that learners were assigned to either the high-achieving or low-achieving class based on their performance in the pretest.

Table 4: Demographic features of the low-achieving class

Variable	Age (Years)			Total
Gender	Below 13	13-15	Above 15	
Male	0 (0%)	40 (43.95%)	7 (0.07%)	47 (51.64%)
Female	1 (0.01%)	36 (39.56%)	7 (0.07%)	44 (48.35%)
Total	1 (0.01%)	76 (83.51%)	14 (0.15%)	91 (100%)

Source: Field survey (2022)

From both demographic features of the two main classes, it was identified that 145 (84.79%) of the learners were found to be in the age group which conforms with the National Pre-Tertiary Education Curriculum Framework's age limits (MoE, 2018). In the next section, I present the results of research question one.

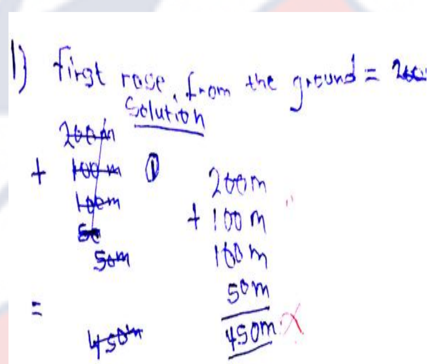
Research Question 1

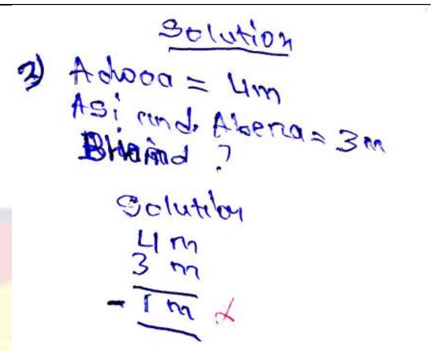
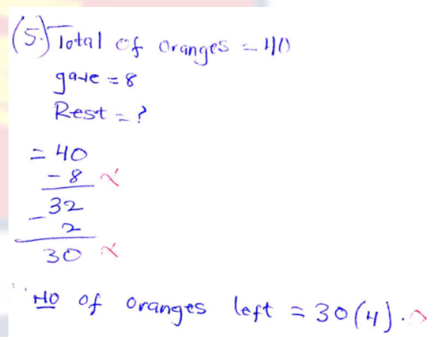
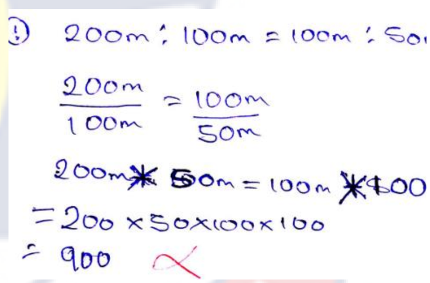
What errors do learners make when solving word problems before being introduced to the use of diagrams?

The first research question sought to identify the common errors learners make when solving word problems. Descriptive statistics and the Newman Error Analysis (NEA) diagnostic procedure which breaks down the steps involved in solving word problems into five stages: reading, comprehension, transformation, processing, and encoding were employed to analyse the data gathered. This helped identify specific areas where errors occur during problem-solving.

Some of the errors identified during the marking included computational errors and conceptual errors, which included the sign to adopt and use at a particular time. For example, most of the learners were just adding all the numbers found in the text. Others were also seen using the wrong mathematical computational signs and others also had their computations wrongly done. About 17 of the learners from both classes did not answer any of the questions at all and when they were confronted, they were identified as having reading difficulty. A summary of the parameters for identifying learners' errors is presented in Table 5.

Table 5: Parameters for Newman's Error Analysis Framework

S/N	Error Code	Descriptors	Excerpt from learners' work
1	Reading error	The learner was not able to recognise the words or symbols in the problem	
2	Comprehension error	The learner doesn't know what the task requires of him/her or the learner does not understand or is not familiar with some of the terms and symbols in the text.	

3	Transformation error	The learner was not able to interpret or transform the text into a mathematical statement.	
4	Process skill error	The learner does not know how to perform the calculation, or the learner makes mistakes in his/her computations	
5	Encoding error	The learner fails to express the answer in the correct unit or the learner cannot present the final answer based on their calculations.	

The MAT consisted of ten test items of which five were used for the pretest whereas the remaining five were also used during the posttest. In each test item, the five levels of the NEA are employed to determine the number of errors a learner makes. For example, if a learner is unable to read, understand, translate, calculate and present a correct answer for item one the learner is adjudged as committing all the errors in that question. Notwithstanding, some of the learners may commit just two or three of the errors depending on their abilities on a particular question. For example, most of the high-achievement group could read but had challenges with their transformation skills causing them to commit a lot of errors. Details of the NEA findings after administering the pretest in both groups are presented in Table 6.

Table 6: *Descriptive Analysis of Errors Made by Learners from Both Groups*

S/N	Type of Error	High-achieving	low-achieving
1	Reading	18 (22.5%)	61 (67.03%)
2	Comprehension	32 (40%)	71 (78.02%)
3	Transformation	67 (83.75%)	89 (97.8%)
4	Process skills	30 (37.5%)	69 (75.82%)
5	Encoding	16 (20%)	10 (10.98%)

Source: Field Survey (2022)

From Table 6, it was realised that transformation errors were the most common errors that were made by the learners from both groups. The low-achieving class alone had 89 (97.8%) learners falling prey to this error and 67 (83.75%) learners from the high-achieving class also committed this error. It was observed that both groups had about 91.22% of their members making transformational errors.

For example, for question four which is; "*on one side of a scale are three honey pots and a 100g weight. On the other side, there are 200g and 500g weights.*

If the scale is balanced, what is the mass of a pot of honey"? For this question, the learners were expected to assume the mass of a pot of honey is say, **x** grams. Therefore, the total mass on the left side of the scale is $3x + 100$ grams. In this case, the total mass on the right side of the scale is $200 + 500 = 700$ grams. Since the scale is balanced, the equation could have been written as:

$$3x + 100 = 700$$

Solving for x, we get:

$$x = \frac{(700 - 100)}{3}$$

x = 200 grams, meaning the mass of a pot of honey is 200 grams.

However, the following conversation between the researcher and a learner shows evidence of how some of the learners solved the questions:

Researcher: So how were you able to arrive at this answer?

B10: Sir, the question wanted us to find the mass of one honey pot so I added all the masses of the other items and I got 800g.

Researcher: So, if 800g is the mass of one honey pot then what is the mass of all three honey pots?

B10: Smiled (She did not provide any answer)

Researcher: Do you think the scale would balance if one honey pot is 800g?

B10: Yes sir.

Evidence of the learner's computation is shown in Figure 5.

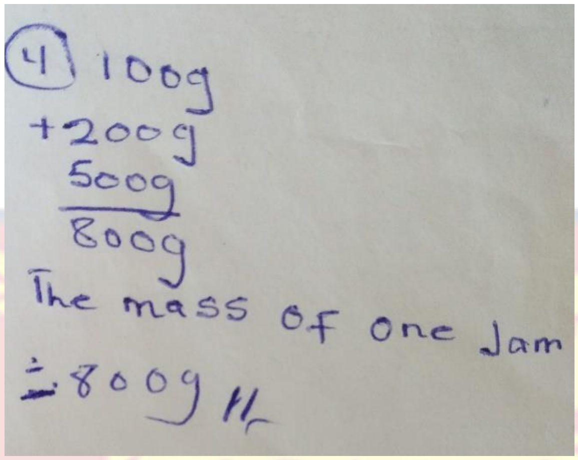


Figure 5: Evidence of transformational errors

Similarly, close to a third of the participants were seen just manipulating the figures in the text. Evidence of their manipulations is shown in Figure 6.

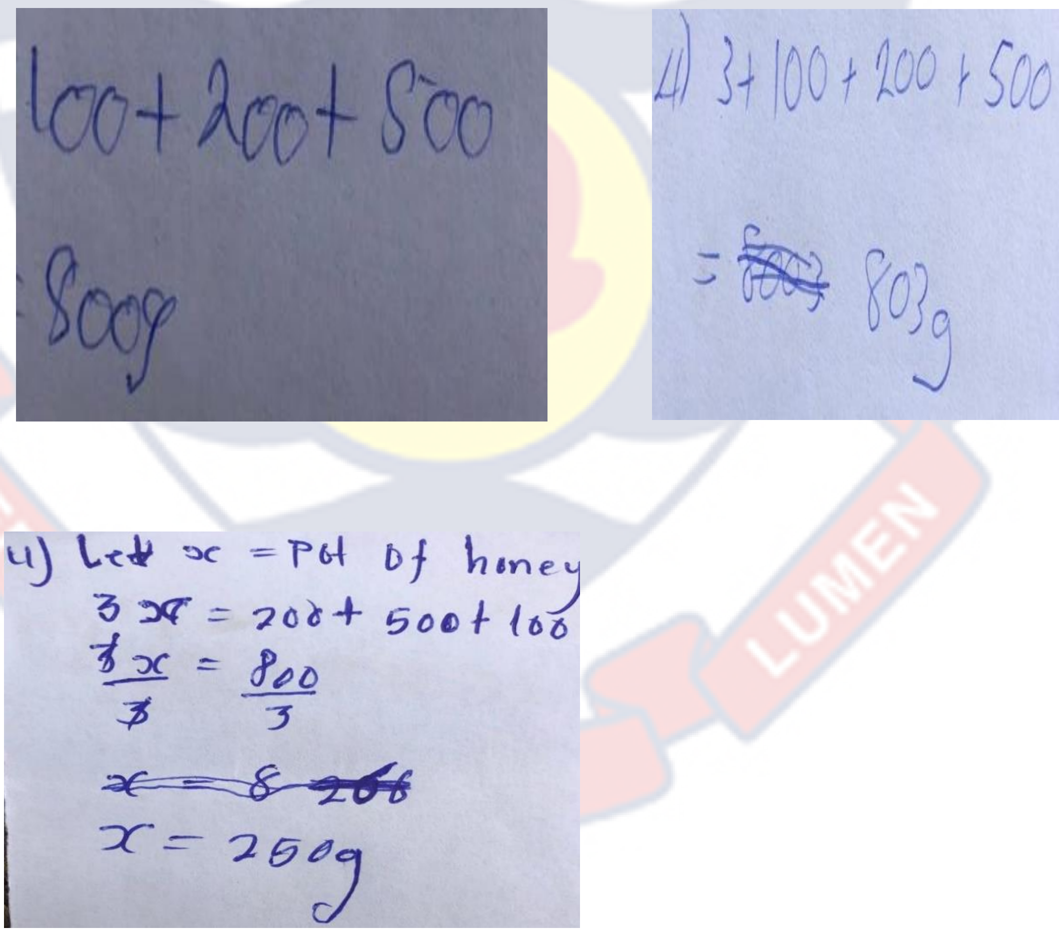


Figure 6: Evidence of learners' wrongful manipulations

From Figures 5 and 6, they showed a noteworthy trend that emerged which indicated transformational errors were notably attributable to a lack of heuristic strategies for converting textual information into mathematical notation. This deficiency led learners to primarily engage in numerical manipulation, neglecting the deeper mathematical structures and relationships embedded within the word problems. Rather than employing heuristic methods to identify relevant variables, equations, or problem-solving approaches, learners resorted to simplistic manipulation of numerical figures presented in the text. Consequently, this limited approach often resulted in inaccuracies in problem-solving outcomes and hindered the development of robust mathematical reasoning skills. Addressing this issue underscores the vital importance of fostering heuristic skills in mathematics education, as it empowers learners to approach word problems with greater comprehension and analytical depth, thereby mitigating the risk of transformational errors and cultivating a more robust problem-solving proficiency grounded in conceptual understanding and strategic reasoning.

For the same example, the following conversation went on between the researcher and the learners:

Researcher: so, tell me how you translated the question into this expression.

A7: Sir, it is 100 plus the pots of honey equal to the 200g and the 500g weights.

Researcher: Okay, so show me how you wrote that. (The learner showed her expression to the researcher.) So tell me why you multiplied the 100 by an x?

A7: (She looked at the researcher for a while and replied with a question) "Sir, or should I add it"?

Researcher: (both laughed).

Evidence of how the learner translated and solved the question is presented in Figure 7.

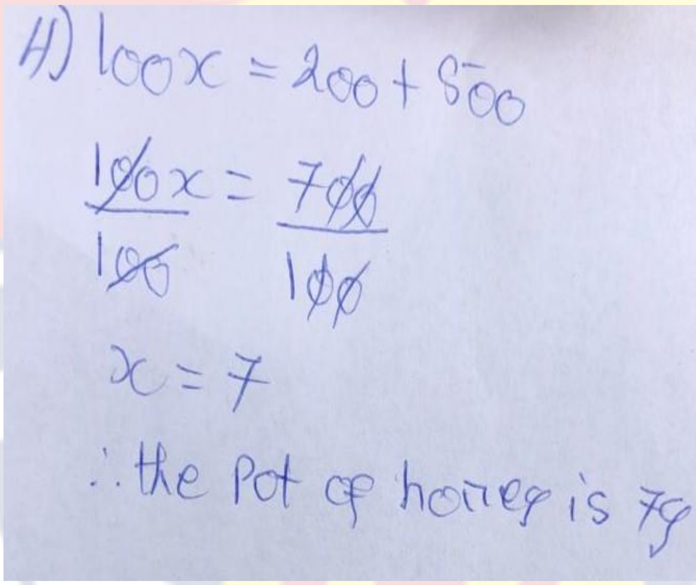


Figure 7: Evidence of how the learner translated and solved the question

Comprehension errors came next to transformation errors with 60.23% of the learners making these errors. However, 71(78.02%) and 32(40%) of the low-achieving class and the high-achieving class respectively exhibited these errors. For example, question five reads: *"One morning, Janet bought forty oranges at a supermarket. She gave* $\frac{1}{8}$ *of them to her neighbour. After lunch, she ate two oranges. How many oranges had she left?"* From this question, learners were expected to find one-eighth of 40 which will represent what Janet gave to her neighbour ($\frac{1}{8} \times 40 = 5$). That will imply that the total number of oranges left after

Janet gave out five and eating two will be 40 - 7 which is equal to 33 oranges (40-5-2 = 33).

However, about 23 of the learners expressed one-eighth as a whole number thereby representing one-eighth as eight whereas about 37 of the learners were seen manipulating the figures in the text. This suggests that this group of learners did not understand the mathematical notations and terminologies used in the text. Evidence of learners' computations is presented in Figure 8.

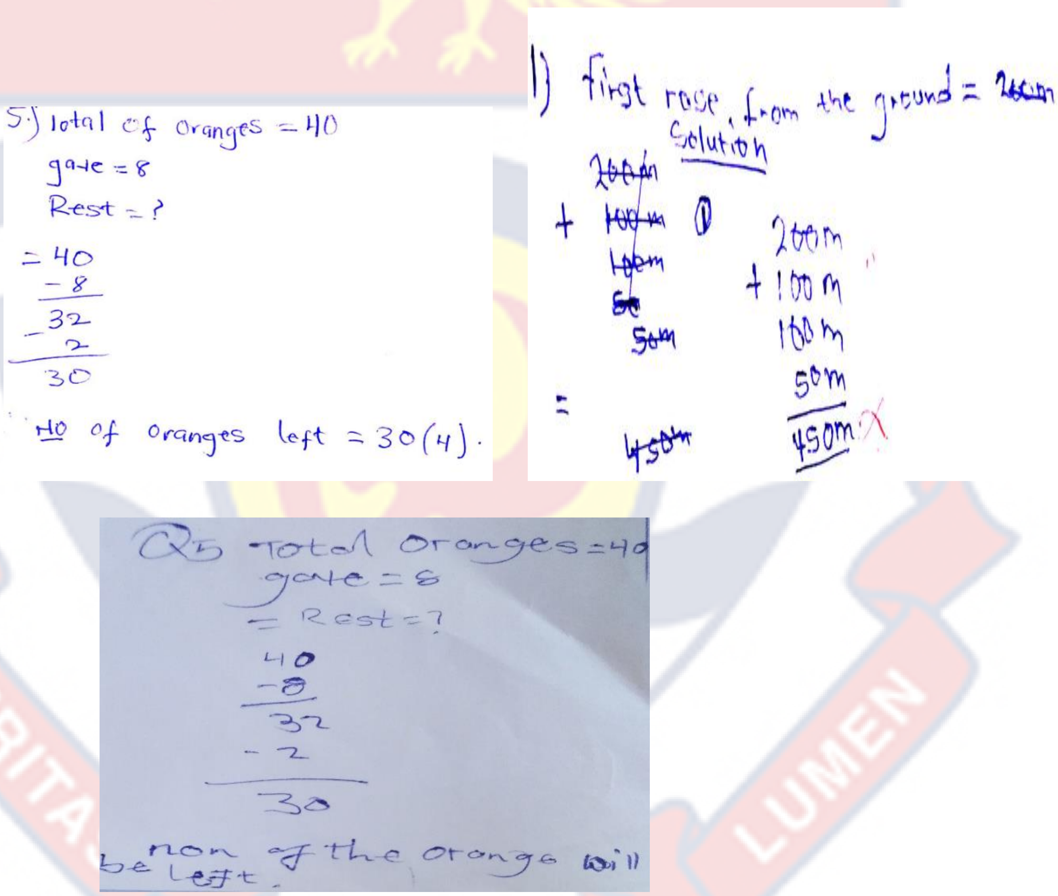


Figure 8: Evidence of comprehension error

The data also revealed that the majority of the learners had problems with computations. The third in terms of ranking of the errors learners committed is the process skill errors with 69 (75.82%) of the learners from the low-achieving class and 30 (37.50%) of the learners in the high-achieving class making this type of error. In all, 57.89% of the learners from both classes made these errors. This could have emerged from the lack of an effective heuristic strategy which could have helped learners to track their computations.

For example, from the same question five which reads: "*One morning, Janet bought forty oranges at a supermarket. He gave* $\frac{1}{8}$ *of them to his neighbour. After lunch, he ate two oranges. How many oranges had he left?"* Evidence showed that some of the learners exhibited a series of process skills errors. Figure 9 shows a learner solved the question:

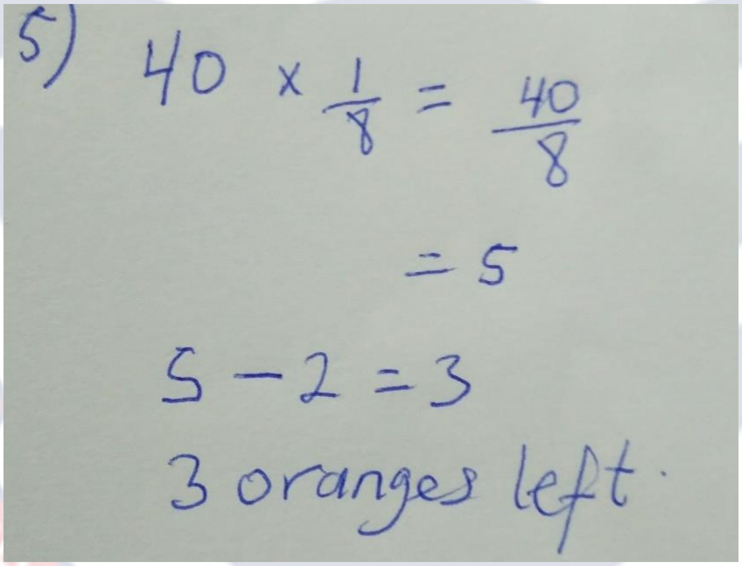


Figure 9: Evidence of learner's process skill error

In an attempt to outline the ideas behind the learners' solution processes, the learners were engaged in a conversation and the following vignette outlines some of the occurrences between the researcher and some of the learners:

Researcher: Good work, you were able to find one-eighth of the total number of oranges, but how come you obtained 3?

D19: I got 5 from dividing 40 by 8 so I subtracted the two she ate after launch from the 5 to obtain 3.

Researcher: oh! I see

It is quite obvious the learner made a mistake in his computations. Instead of subtracting the 5 which is the result of finding the one-eight of the 40 from the 40 before subtracting the 2 out of the remaining figure, the learner rather subtracted the 2 from the 5. This and other examples were seen among other several learners in the study.

In the same example, another learner got a negative 75 as the answer. So, I went on to ask how he got that answer. The conversation between the learner and the researcher is below:

Researcher: Can you please explain how you obtained -75?

B13: Sir, I first tried to clear the fractions so I found the L.C.M. of eight and one and I got eight.

Researcher: okay continue.

B13: (laughed) And I worked it out to obtain $\frac{-15}{8}$, after that I multiplied 40 to the $\frac{-15}{8}$ and got -75.

Researcher: -75 oranges? How possible? Have you heard of a negative orange before?

B13: (both laughed) No sir.

Evidence of how the learner solved the question is shown in Figure 10.

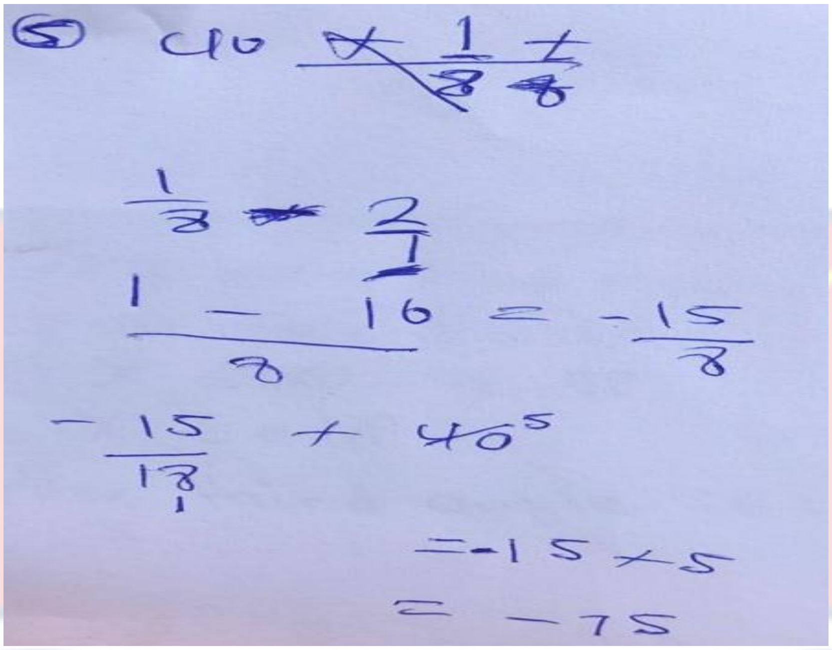


Figure 10: Evidence of learner's process skills error

In a similar fashion, from question 2: *"Bodwoase D/A JHS 1 learners were given an amount of GH¢42.00 to share. If each learner received* $\frac{2}{7}$ *of the amount, how many learners received a share?"*. The majority of the learners had their computations wrong. Some could not compute for each share in cedis nor compute for the total number of learners who received a share. Some of the learners were again expressing the fractions as whole numbers, and others also presented the value they obtained after finding the two-seventh of the GH¢42.00 as the final answer. Meanwhile, the learners were expected to find two-seventh of GH¢42.00 and divide GH¢42.00 by the value they obtained to arrive at the number of learners who received a share. Evidence of learners' computations is presented in Figure 11.

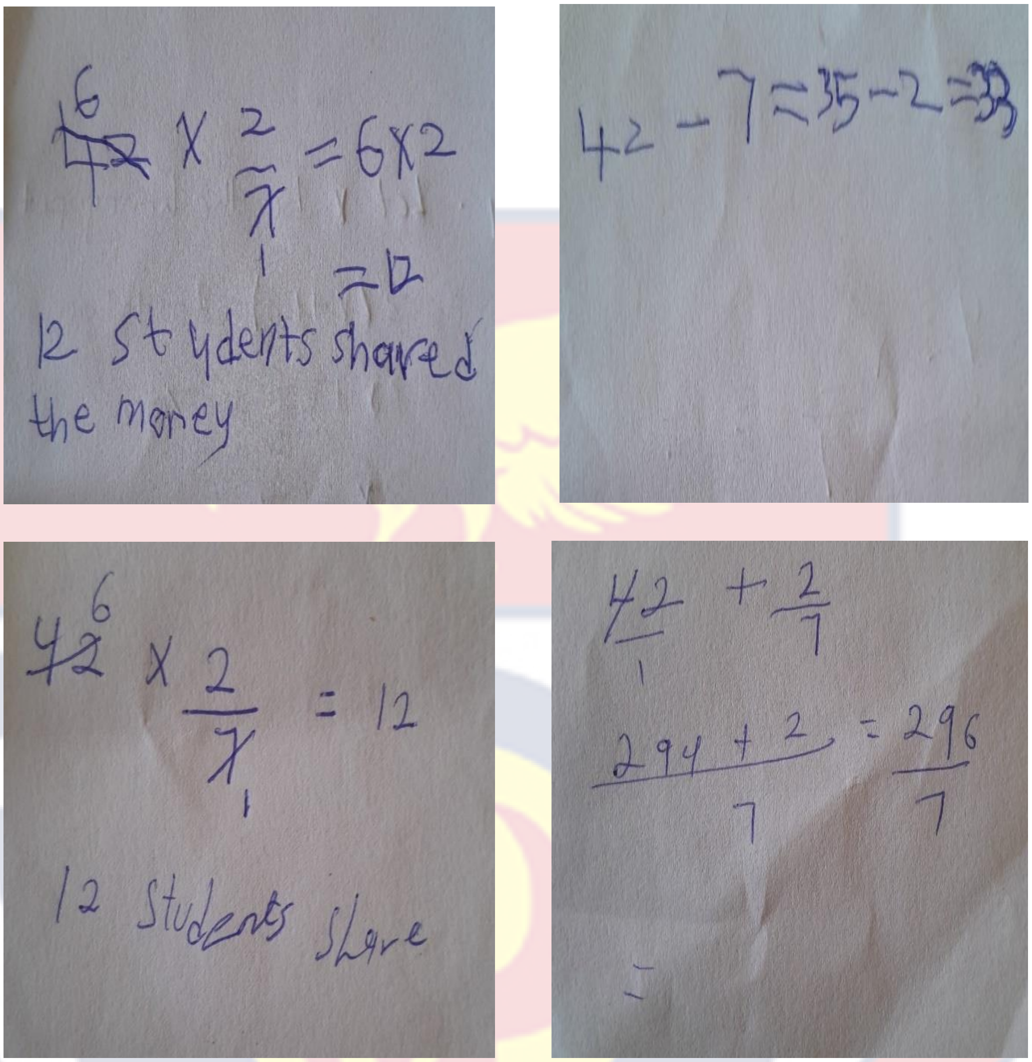


Figure 11: Evidence of learner's process skills error

Reading errors were also identified as the fourth most common error the learners committed. This error was identified when the learners were made to read the text. The data indicated that 18(22.5%) of the high-achieving class and 61(67.03) of the low-achieving class could not read the text properly. In the course of obtaining evidence for this error, it was realised that the learners could not pronounce most of the words in the text though most of the items were taken from their textbooks. For encoding errors, 15.20% of the learners from both classes were seen committing these errors. It was realised that about 27 learners of the high-achieving class and 15 of the low-achieving class were able to present their final answer although some of the learners had had some issues with their computations. Out of this number, 16 (20%) of the learners from the low-achieving class and 10 (10.98%) from the high-achieving class committed this error.

Research Question 2

What are the challenges learners experienced when solving word problems before being introduced to the use of self-generated *diagrams?*

The primary focus of this research question was to unwrap the various challenges learners have concerning solving word problems. The pretest questions were made up of five MAT questions. The main purpose of the MAT was to evaluate the learners' mathematical proficiency and comprehension of algebraic word problems. The test was marked out of 20 with each question carrying four marks. A learner's ability to translate, use the correct process, do the correct computations and show the correct answer with its appropriate unit fetches a learner all four marks for a question. This was the marking strategy used throughout the study. Learners were also asked to explain their choice of method and setbacks using the TAP technique as they solved each of the questions in the test. The various challenges identified were then analysed using descriptive statistics and content analysis.

Table 7 revealed that the learners' major challenge in solving word problems is interpreting or translating the text into mathematical form. A total of 156 (91.22%) learners were identified as having this challenge. It was closely followed by 124 (72.51%) learners having the challenge of obtaining a definite formula to solve the word problem. Notwithstanding, 68 (85%) and 88 (96.70%) of the high-achieving and low-achieving classes respectively indicated that translating the text into mathematical form is their most difficult challenge in solving word problems. With regards to those who asserted that obtaining a formula is another difficult task, 51 (63.75%) of them were in the high-achieving class and 73 (80.21%) of them were in the low-achieving class.

Some of the learners also highlighted that the terminologies used in the construction of the problem statement also posed a challenge to them. With this challenge, the data indicated a total of 135 (78.94%) from both classes, with 56 (70%) being in the high-achieving class and 79 (86.81%) in the low-achieving class. However, according to the learners, reading and comprehension were the least challenging tasks in solving word problems. However, the low-achieving class recorded a total number of 69 (75.82%) of them having reading and comprehension difficulties with the high-achieving class having 23 (28.75%). This indicates that although the data outlines the frequency of reading and comprehension for both groups was small, learners in the low-achieving class have reading and comprehension as a major challenge.

It is worth noting that, each theme was analysed holistically taking into consideration the total number of learners within the classes. During the probe, some of the learners were of the view that they were unable to read and interpret the written text given to them and believed they would have been able to do better if the texts had been read and interpreted to them. Others also opined that word problem tasks are presented in diverse forms and are highly unpredictable. Some also said some of the keywords or terms may be misleading. The outcome of that survey is presented in Table 7.

Table 7: Learners' Challenges in Solving Word Problems

Challenges	Groups		Total
	High-achieving (80)	Low-achieving (91)	
Unfamiliarity with the terminologies	56 (70%)	79 (86.81%)	135 (78.94%)
Obtaining a definite formula for solving the problems presented	51 (63.75%)	73 (80.21%)	124 (72.51%)
Reading and comprehension	23 (28.75%)	69 (75.82%)	92 (53.80%)
Interpretation or translation of the text	68 (85%)	88 (96.70%)	156 (91.22%)

Source: Field survey (2022).

In addition to the output in Table 7, the TAP technique was also employed to identify the learners' challenges by the researcher. The TAP is a data collection procedure in which participants or learners verbalise their thoughts and problem-solving strategies while working on a task. It seeks to reveal the cognitive processes and decision-making involved in problem-solving, providing insights into how individuals approach and tackle such problems. The following vignette occurred between the researcher and some of the learners.

Researcher: Do you understand word problems?

A8: No sir

Researcher: Do you do well on word problems?

A8: (shakes his head and replies) No

Researcher: Why

A8: They are very tricky and difficult

Researcher: What makes it tricky and difficult?

A8: Sir, they don't have one formula.

Researcher: Can you explain that further?

A8: Sir, it seems every task has its own formula; getting them is very tricky and difficult.

Another learner was also engaged in the same conversation, and this is what transpired between the researcher and the learner.

Researcher: Do you enjoy solving word problems?

B17: No, please

Researcher: why?

B17: Sir, sometimes I do not understand some of the terms they use in the questions.

Researcher: Do you find word problems challenging?

C12: Yes (nods heavily and smiles)

Researcher: Why?

C12: 'Sir when you read you don't know what the question is saying'

Researcher: can you please explain further?

C12: 'Sir, reading and understanding what the question wants you to do is my major problem'.

From the two conversations, the learners' most challenging task has to do with translating the text into mathematical notations. Some blame it on their reading and comprehension abilities. Reading and comprehension could be the main cause because another challenge that was raised by the learners was Unfamiliarity terminologies. This, the researcher perceives may be rooted in reading and comprehension difficulties. Again, obtaining a definite formula and translating the text into mathematical notations could also be seen as a lack of effective strategies. Therefore, from the vignettes and the findings in Table 7, the learners' challenges in solving word problems could be placed under two main umbrellas which are reading and comprehension and lack of proper heuristics to help translate text into mathematical notations.

Hypothesis 1

H_0: The use of diagrams has no statistically significant effect on the performance in solving word problems between the experimental group and the control group of the high-achieving class.

This hypothesis aimed to determine the effectiveness of the use of diagrams in solving word problems in the high-achieving class. It is important to note that the control group of the high-achieving class were taught using the conventional method that is the 'hunt for keywords' method by their regular mathematics teachers whereas the experimental group was guided on how to develop diagrams to solve word problems by the researcher. Because of that, the test scores of the experimental and control groups of the high-achieving class were analysed using the Analysis of Covariance (ANCOVA) to testify to whether the intervention (the use of diagrams) had an effect on the performance of the participants in solving word problems.

Before running the ANCOVA tests, the various assumptions such as normality tests, homogeneity tests and homogeneity of regression tests were performed first on the test scores of both groups to determine whether the sample data taken met the assumptions for running ANCOVA. In addition, these tests were performed to help generalise the findings or analysis of a study (Siswono, Hartono, & Kohar, 2018). The outputs of the normality test for the experimental group of the high-achieving class are presented in Table 8.

Table 8*: Normality Test of the Experimental Group of the High-Achieving Class*

Shapiro-Wilk			
Group	Statistic	Df	Sig.
Experimental	.958	43	.173

a. Lilliefors Significance Correction

As shown in Table 8, the Shapiro-Wilk Test was adopted to test the normality of the data for the experimental group of the high-achieving class. With an alpha level of .05 and a sample size of 43, the output displayed a p-value of .173. This indicates that the Shapiro-Wilk Test did not show evidence of non-normality of the data retrieved for the experimental group.

In addition, the normality test was performed for the control group of the high-achieving class and the output is presented in Table 9.

Table 9: *Normality Test of theControl Group of the High-achieving Class*

Shapiro-Wilk			
Group	Statistic	Df	Sig.
Control	.919	37	.380

a. Lilliefors Significance Correction

From the output in Table 9, the outcome for a sample size of 37 with an alpha level of .05 also indicated a p-value of .380. This also indicates that the Shapiro-Wilk Test did not show evidence of non-normality of the data for the control group. Based on these outcomes, a parametric test was adopted and the mean was used to summarise the data.

Levene's test for equality of variances was also adopted to test for the assumption of equality of variance among the test scores of the high-achieving class. Table 10 shows the output of Levene's test for the high-achieving class.

Table 10: *Levine Test Results for the High-Achieving Class*

Levene Statistic	df1	df2	Sig.
.224	1	78	.637

Source: Field survey (2022)

A Levene's test was performed on a sample of 80 learners belonging to the high-achieving class to ascertain their level of homogeneity. From Table 10, the Levene statistic was .22 with a significant value of .63 at an alpha level of .05. The Levene's test reported a significant. value greater than .05 which indicated that the data does not violate the equality of variance assumption.

Furthermore, the homogeneity of regression test which is also one of the major assumptions of ANCOVA was also performed on both groups of the high-achieving class to check for the interaction between the covariate and the experimental manipulation. The result of the test is presented in Table 11.

Table 11*: Homogeneity of Regression Test of the High-Achieving Class.*

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	2604.001[a]	3	868.000	138.913	.000
Intercept	1934.470	1	1934.470	309.589	.000
Group	860.581	1	860.581	137.726	.000
Pretest	695.601	1	695.601	111.323	.000
Group * pretest	.013	1	.013	.002	.964
Error	474.886	76	6.249		
Total	11019.000	80			
Corrected Total	3078.888	79			

a. R Squared = .846 (Adjusted R Squared = .840)

In addition, from Table 11, the output of homogeneity of regression test also indicated a significant value of .96 which is greater than .05. This also indicates that the dataset has not violated the assumption of homogeneity of regression. This supports the other assumption for conducting ANCOVA analysis to identify the effect of the use of diagrams in solving word problems among the two groups of the high-achieving class.

In this regard, a one-way between-groups analysis of covariance (ANCOVA) was used to show whether the use of diagrams has an effect on the control and experimental groups of the high-achieving class. The results of the data analysis are presented in Table 12.

Table 12: *ANCOVA Test of the High-Achieving class*

Source	Type III Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared
Corrected Model	2603.989[a]	2	1301.994	211.105	.001	.846
Intercept	1737.525	1	1937.525	314.150	.001	.803
Pretest	669.575	1	699.575	113.429	.001	.596
Group	1517.952	1	1617.952	262.334	.001	.675
Error	474.899	77	6.168			
Total	11019.00	80				
Corrected Total	3078.888	79				

Source: Field survey (2022)

A one-way between-groups analysis of covariance (ANCOVA) was performed to evaluate the effectiveness of using diagrams to solve word problems between the experimental and control groups within the high-achieving class. The dependent variable was the posttest scores of participants in the high-achieving class after the intervention. Pre-intervention scores were used as covariates in the analysis. Preliminary checks confirmed that the assumptions of normality, homogeneity of variances, and homogeneity of regression were not violated. After adjusting for pre-intervention scores, a statistically significant difference was found between the experimental and control groups regarding the use of diagrams in solving word problems [F(1,77)= 262.3, p=.00, partial eta squared=.675]. This indicates significant differences in learning outcomes between the experimental and control groups in the high-achieving class.

In addition to the ANCOVA results, the estimated marginal means of the experimental and control groups were compared to further support the ANCOVA findings. Estimated marginal means are statistical outputs that allow for comparison of group means while accounting for other influencing variables, providing a clearer picture of the relationship between independent and dependent variables. Table 13 displays the estimated marginal means for the two groups within the high-achieving class.

Table 13: *Estimated Marginal Means of the High-Achieving Class*

			95% Confidence Interval	
Group	Mean	Std. Error	Lower Bound	Upper Bound
Experimental	11.163[a]	.380	13.407	14.920
Control	6.080[a]	.410	4.264	5.897

a. Covariates appearing in the model are evaluated at the following values: pretest = 3.2750.

According to the results shown in Table 13, the estimated marginal mean for the experimental group was 11.16, whereas the control group had a mean of 6.08. This indicates that learners who used diagrams to solve word problems outperformed those who did not use diagrams. The substantial mean difference supports the conclusion that this result is not likely due to chance. Thus, it can be concluded that the use of diagrams significantly enhances performance in solving word problems among the high-achieving class, leading us to reject the null hypothesis.

Along with the statistical report, Figure 12 shows examples of tasks with which learners employed the use of diagrams in solving. For example, for question one of the posttest which reads; *Daniel was given thirty biscuits on his birthday. He gave* $\frac{3}{5}$ *to his friends who attended his birthday party. He ate 3 of the remaining biscuits the next day. Calculate the number of biscuits he had left,* this is how a learner solved the problem using a diagram.

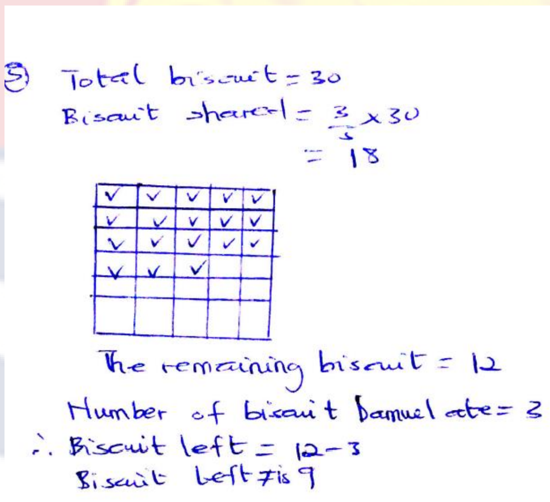


Figure 12: Evidence of the high-achieving class using diagrams in solving word problems

In this case, the learner used the diagram as a guide and a means of tracking his solution. This is one of the strengths of the use of diagrams. In addition, the diagram he adopted helped the learner to distinguish between the whole and its fractional parts, fostering a deep understanding of fractions and their place within the context of the problem.

In another example, from question three of the posttest; *The following items were found on a balanced scale. On one side is a 400g and a 600g of metal blocks. The other side had a 100g of beads and three different pieces of stones. Calculate* *the mass of one stone on the balance,* Figure 13 shows how a learner solved the question.

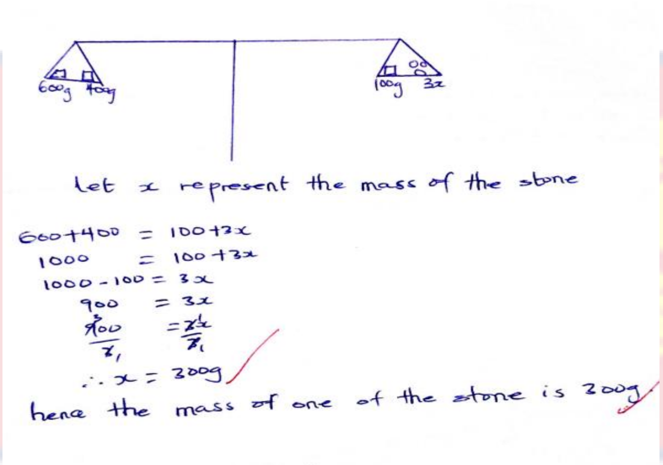


Figure 13: Evidence of the high-achieving class using diagrams in solving word problems

The learner used a beam balance to show how balanced the scale was with the various items specified in the question. He further used that as a guide to translate the text into mathematical notation and solve. This learner exhibited strengths in the use of diagrams in solving word problems which includes the ability to understand the problem statement, translate, and track solutions to promote learners' efficiency in solving word problems.

Hypothesis 2

H_0: The use of diagrams has no statistically significant effect on the performance in solving word problems between the experimental group and the control group of the low-achieving class.

The objective of this hypothesis was to assess the effectiveness of using diagrams in solving word problems for the low-achieving class. To achieve this, Analysis of Covariance (ANCOVA) was employed to analyse the test scores of both the experimental and control groups within this class. It is important to note that the high-achieving class control group was instructed using the traditional 'hunt for keywords' method by their usual teachers, while the experimental group received guidance from the researcher on how to use diagrams to tackle word problems. This approach was intended to evaluate the impact of the intervention on the performance of the low-achieving class in solving word problems. Normality, homogeneity, and homogeneity of regression tests were conducted on their test scores to ensure the data met the assumptions required for ANCOVA. The results of these tests for the low-achieving class are detailed in Table 14.

Table 14: *Normality Test of the Experimental Group of the Low-Achieving Class*

Shapiro-Wilk			
Group	Statistic	Df	Sig.
Experimental	.977	41	.550

a. Lilliefors Significance Correction

From Table 14, a Shapiro-Wilk test of normality was conducted to identify whether the data obtained from the experimental group of the low-achieving class is normally distributed. Reading from Table 14, a sample size of 41 showed a test statistic value of .97 with a significant value of .55. Since the significant value is greater than .05, it can be concluded that the Shapiro-Wilk test did not show evidence of the dataset violating the assumption of normality.

Table 15: *Normality Test of the Control Group of the Low-Achieving Class*

Shapiro-Wilk			
Group	Statistic	Df	Sig.
Control	.913	50	.480

Furthermore, the Shapiro-Wilk test was performed on the control group of the low-achieving class to determine the normality of their test scores. The result as shown in Table 15 indicated that, with a sample size of 50, the test statistic was .91 and a significant value of .48. This also reported a significant value greater than .05 which also indicates that the Shapiro –Wilk test conducted for the control group of the low-achieving class did not show evidence of non-normality in the dataset. Given that, in summarising the data, a parametric test with the mean was adopted.

In addition, Levene's test for equal variance was used to check the assumptions of equal variances across groups for the dataset of the low-achieving class. This was also to help determine whether the data obtained are homogeneous and hence do not violate the assumption of equal variance. Table 16 shows the results of the test of homogeneity across both groups of the low-achieving class.

Table 16*: Levine Test Results for the Low-Achieving Class*

Levene Statistic	df1	df2	Sig.
.249	1	89	.619

Source: Field Survey (2022)

In addition to the normality test, Table 16 shows the results of the test conducted to check for equal variance across the groups in the low-achieving class. The Levene's test yielded a statistic of .24 with a significance value of .61 for a sample size of 91 participants. Since this significance value of .61 exceeds the threshold of .05, it indicates that the assumption of equal variance across both groups is not violated.

Alongside the homogeneity of variance test, a homogeneity of regression test was also conducted to assess the interaction between the covariate and the experimental manipulation for both groups in the low-achieving class. The results of this test are presented in Table 17.

Table 17: *Homogeneity Test of Regression of the Low-Achieving Class*

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	1749.527[a]	3	583.176	72.266	.000
Intercept	1555.038	1	1555.038	192.697	.000
Group	680.260	1	680.260	84.296	.000
pretest	146.034	1	146.034	18.096	.000
Group * pretest	.829	1	.829	.103	.749
Error	702.078	87	8.070		
Total	6827.000	91			
Corrected Total	2451.604	90			

Source: Field Survey (2022)

The homogeneity of regression test was performed to augment the other assumptions before running the ANCOVA test. The interaction effect between the groups and the covariate indicated a significant value of .74. This also showed that the assumption of homogeneity of regression was not violated. The various outputs all indicated that the major assumptions for running ANCOVA were met.

Given that, a one-way between-groups analysis of covariance was conducted to help identify the effect of the use of diagrams in solving word problems on both the control and experimental groups of the low-achieving class. The result of the analysis is presented in Table 18.

Table 18: *ANCOVA Test of the Experimental and Control Groups of the Low-Achieving Class*

Source	Type III Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared
Corrected Model	1748.698[a]	2	874.349	109.464	.000	.713
Intercept	1556.384	1	1556.384	194.851	.000	.689
pretest	151.211	1	151.211	18.931	.000	.177
Group	1263.664	1	1263.664	158.204	.000	.563
Error	702.906	88	7.988			
Total	6827.000	91				
Corrected Total	2451.604	90				

Source: Field Survey (2022)

A one-way ANCOVA was conducted to evaluate the effectiveness of using diagrams in solving word problems between the experimental and control groups of the low-achieving class. After performing normality tests, Levene's test, and homogeneity of regression checks, all assumptions were confirmed to be met. According to Table 18, there was a statistically significant difference in the posttest scores [F(1,88)= 158.2, p=.00, partial eta squared=.563] between the experimental and control groups of the low-achieving class. This result indicates a significant effect of the diagram intervention on solving word problems. To further explore this relationship, the adjusted means (marginal means) of both groups were compared, with the results presented in Table 19.

Table 19: *Estimated Marginal Means of the Low-Achieving Class*

			95% Confidence Interval	
Group	Mean	Std. Error	Lower Bound	Upper Bound
Experimental	9.188[a]	.450	10.295	12.082
Control	4.446[a]	.406	2.639	4.252

Source: Field Survey (2022)

The marginal means for the experimental and control groups of the low-achieving class were 9.19 and 4.45, respectively. This significant mean difference indicates that the use of diagrams had a notable impact on the performance of the experimental group compared to the control group. Given that this difference is unlikely to have occurred by chance, it can be concluded that diagrams significantly improved the performance of the low-achieving class in solving word problems. Consequently, we reject the null hypothesis, affirming the significant effect of using diagrams.

Along with the statistical report, Figure 14 shows evidence of tasks with which learners employed the use of diagrams in solving. For example, from question four of the posttest; *Essel flies a Kite as high as 300m from the ground, the kite moves 150m to the east, then dropped 30m. It then travelled 30m to the east and finally dropped straight to the ground. How far was the kite from Essel's position?* The way the learner solved the question is presented in Figure 14.

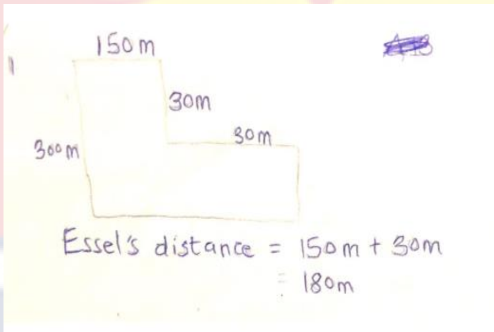


Figure 14: Evidence of the low-achieving class using diagrams in solving word problems

From the learner's diagrammatic illustration, the learner comes to identify the key numbers to compute and those that are detractors. Before being introduced to the use of diagrams a majority of the learners were seen adding all the figures in the text. Nonetheless, the diagrammatic representation guided the learner to only compute how far the kite was away from Essel. This again is another evidence of the strength of the use of diagrams in solving word problems which helps learners to identify the key element in the text and also guides the solution process.

To demonstrate that the differences in mean scores between the experimental groups from the two classes were not due to chance, a delayed posttest was conducted to support the findings. This approach aligns with the research methodology used in the study. The delayed posttest aimed to determine if the observed differences would continue over time. Consequently, participants in the experimental groups were tested again three weeks after the initial posttest. The results for the delayed posttest of the low-achieving class are shown in Table 21, while Table 20 displays the paired sample statistics for this delayed posttest.

Table 20: *Delayed Posttest Statistics of the Experimental Group of the LAC*

		Mean	N	Std. Deviation	Std. Error Mean
Pair 1	Experimental Posttest	9.188	41	2.97510	.46463
	Delayed Posttest	9.198	41	3.06753	.47907

Source: Field Survey (2022)

A paired-sample t-test was performed to assess the differences between the mean scores of the post-test and the delayed posttest for the experimental group of the low-achieving class. The results indicated no statistically significant difference [df (40) = -1.09, sig. = .27] between the posttest scores (M = 9.188, SD = 2.97) and the delayed posttest scores (M = 9.198, SD = 3.067) for this group.

Table 21: *Paired Samples Test for the Delayed Posttest of the Experimental Group of the Low-Achieving Class*

	Paired Differences							
				95% Confidence Interval of the Difference				
	Mean	Std. Deviation	Std. Error Mean	Lower	Upper	t	df	Sig. (2-tailed)
‘ Pair 1 Experimental Posttest – Delayed Posttest	.14634	.85326	.13326	-.41566	.12298	-1.098	40	.279

Source: Field Survey (2022)

The paired sample t-test showed a p-value of .28 which is greater than the alpha value of .05. This also indicated that there is no statistically significant difference between their posttest scores and the delayed posttest scores. All these outputs indicate that the intervention had a significant effect on the learners' ability to solve word problems and hence the new skill identified persists among the learners.

Likewise, the same exercise was repeated for the high-achieving class and the result for the delayed posttest is presented in Table 23. Again, the test statistics of the delayed posttest for the high-achieving class are presented in Table 22.

Table 22: *Delayed Posttest Statistics of the Experimental Group of the HAC*

		Mean	N	Std. Deviation	Std. Error Mean
Pair 1	Experimental Posttest	11.163	43	3.49386	.53281
	Delayed Posttest	11.258	43	3.73722	.56992

Source: Field Survey (2022)

Table 23: Paired Samples Test for the Delayed Posttest of the Experimental Group of the High-Achieving Class

		Paired Differences							
					95% Confidence Interval of the Difference				
		Mean	Std. Deviation	Std. Error Mean	Lower	Upper	T	df	Sig. (2-tailed)
Pair 1	Experimental Posttest – Delayed Posttest	-.02326	.91257	.13917	-.30410	.25759	-.167	42	.868

Source: Field Survey (2022)

A paired-sample t-test was conducted to compare the mean scores of the posttest and the delayed posttest for the experimental group of the high-achieving class. The results revealed no statistically significant difference [df (42) = -0.167, sig. = .86] between the posttest scores (M = 11.16, SD = 3.49) and the delayed posttest scores (M = 11.25, SD = 3.73). These findings suggest that the use of diagrams significantly impacted the learners' ability to solve word problems and that this new skill was maintained over time.

Research Question 3

What are the errors learners make after being introduced to the use of diagrams in solving word problems?

This research question sought to identify the strengths and weaknesses of the intervention beyond the statistical effect. This includes the errors the learners made when using diagrams and also identifying the areas in the intervention that helped the learners overcome the challenges they were initially facing. Such an analysis would enable prospective teachers to know the kinds of challenges they may face and plan appropriate strategies to address them. Provided that, the posttest of the experimental groups of both the high-achieving and low-achieving classes were analysed to achieve this objective. The errors that were identified before the intervention as per the Newman Error Analysis included: reading errors, comprehension errors, transformational errors, process skills errors and encoding errors. This research question was analysed using content analysis. Content analysis was adopted because it helps to determine concepts within a given data and quantify them by analysing the meanings and relationships that exist in the concepts identified.

First and foremost, the challenges and/or errors learners made after being introduced to the use of diagrams were analysed and presented before the strengths of the use of diagrams were presented. It is important to note that, this analysis was carried out on only the experimental groups since they were the groups that received the treatment. Details of the content analysis findings of the errors the learners made after administering the intervention in both groups are presented in Table 24.

Table 24: Content Analysis of Errors Made by Learners after the Intervention

Category	Frequency	Percentage (%)

Misinterpretation of the problem (Unmatched diagrams)	29	34.52
Failure to identify or understand the relationships between variables	21	25
Incorrect labelling of the diagram	12	19.04
Failure to use the diagram to solve the problem	22	26.19
Total	84	

Source: Field Survey (2022)

The results of the content analysis show that the most common error that the learners made after being introduced to the use of diagrams to solve word problems is misinterpreting the problem (f = 29, 34.52%), followed by failure to use the diagram to solve the problem (f = 22, 26.19%). These results suggest that learners may still have difficulties in understanding and translating the problem text into a diagrammatic representation. This the researcher suggests may be a result of the difficulty the learners have in reading and understanding. The least common error identified was incorrect labelling of the diagram to solve the problem (f=16, 19.04%), which indicates that learners may have some skills in applying the diagram to find the solution. These results have implications for future research on how to support and scaffold learners' comprehension and representation skills when using diagrams in mathematics education.

The researcher in an attempt to identify the learners' challenges with the use of diagrams engaged some of the learners in a conversation. Some of these conversations are in this vignette:

Researcher: Hello, so tell me how were you able to generate this diagram?

A15: The question is about triangles so I drew my triangle and represented the angles in it.

Researcher: why did you indicate two of the angles as 98^0 each?

A15: (She pointed at the question and said) 'Two of the angles are 98^{0}'.

Researcher: So how did you obtain the third angle?

A15: 'Sir, is 180-98-98 which is equal to 16'.

Researcher: 16 what?

A15: 16 degrees

Researcher: Are you sure? Is it positive 16 or negative 16

A15: Sir, it is positive 16. Our teacher said we do not have negative degrees so all angles are positive.

Researcher: Oh I see. Thank you.

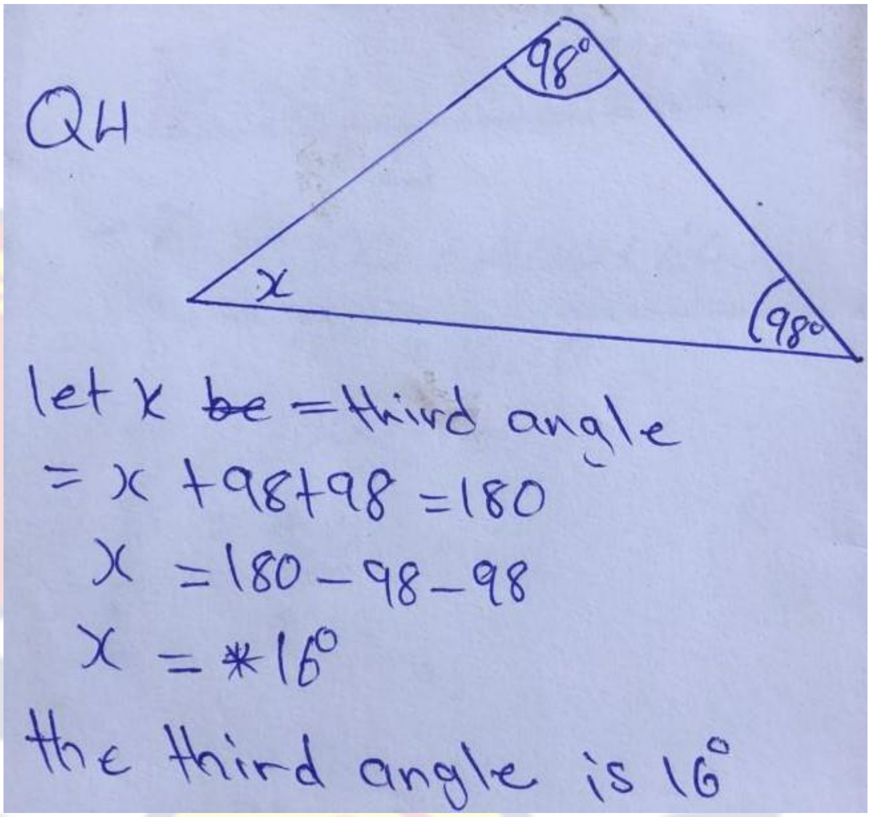


Figure 15: Evidence of learner's challenge when using diagrams

From the vignette, the learner exhibited almost all the challenges most learners experienced when using diagrams. From question 3 of the posttest which reads: "*Two angles of a triangle add up to 98^0. What is the size of the third angle?*", the learner was expected to represent two angles as 98^0 but represented each of the two different angles as measuring 98^0. This means she failed to interpret the text properly as well as incorrectly labelling the diagram. This error also affected her final answer. Some of these common mistakes were observed in most of the learners' works.

Apart from those who had good drawing skills but couldn't represent the text with appropriate diagrams, some of the learners were having difficulty in drawing. For some, their diagrams did not match the text items. Although it is argued that the use of diagrams as a step of the process can be a good starting point for solving word problems, there exist some challenges with its usage. For example, for question 2 of the posttest items; '*A bottle contains 500ml of water. How many bottles are needed to fill a tank that can hold 10 litres of water?'* Figure 16 shows how some of the learners made their diagrams to solve the problem:

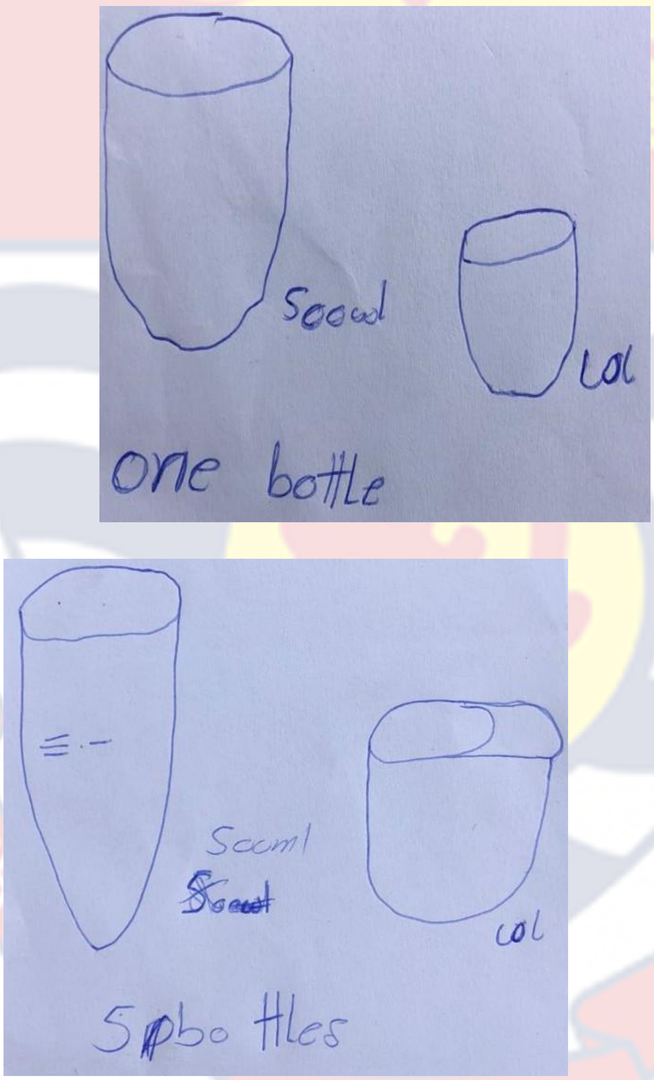


Figure 16: Evidence of learner's challenge when using diagrams

From Figure 16 most of the learners conceptualise the 500-millilitre bottle as bigger than the 10-litre bottle. However, the question demanded the learners to find how many times a half-a-litre bottle will fill a 10-litre bottle. Similarly, from question 5 of the posttest; '*A boy was asked to fetch water and fill a 1 litres gallon. He was to use a small gallon which was* $\frac{1}{8}$ *litres capacity of the big gallon. How many times will he use the gallon to fetch the water?'* most of the learners had their diagrams unmatched to the demands of the question. The learners were expected to find how many times one-eighth will fill a whole. An example of a learner's drawings is illustrated in Figure 17.

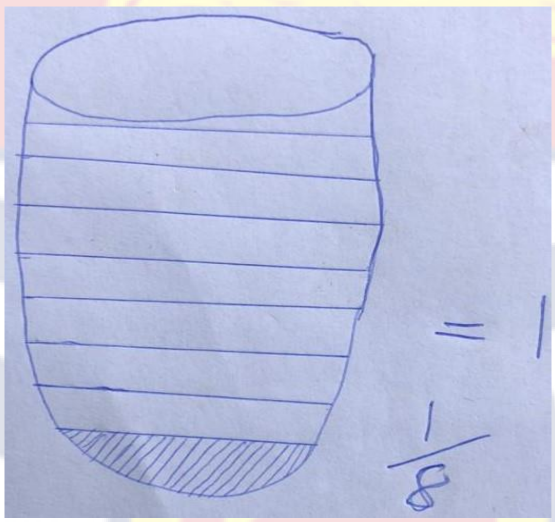


Figure 17: Evidence of learner's challenge when using diagrams

In addition, this research question also outlined the areas the use of diagrams helped the learners to overcome the challenges and errors they were making before the intervention. Again, content analysis was employed to analyse the posttest of the experimental groups of both the low and high-achieving classes. The result is presented in Table 25.

Table 25: Content Analysis of the Strengths of the Use of Diagrams

Category	Frequency (84)	Percentage
Ability to interpret the text	52	61.90

Ability to identify or understand the relationships between variables	61	72.62
Ability to use the diagram to solve the question	49	58.33
Ability to verify and present the final answer with appropriate units	57	67.86

Source: Field survey (2022)

The results of the content analysis show that the use of diagrams in solving word problems has several strengths. The most common strength was an improved understanding of the problem (f = 61, 72.62%) followed by the ability to verify and present the final answer with appropriate units (f = 57, 67.86%). The least common strength was the improved ability to use the diagram to solve the problem (f = 49, 58.33%), which indicates that learners may still need guidance and practice on how to apply the diagram to find the solution. These results have implications for future research on how to design and implement effective instructional strategies for teaching and learning with diagrams in mathematics education.

Some of the learners were asked to mention the areas the use of diagrams helped them in solving the problems presented to them. Some of their answers included: diagrams provide clues that words cannot provide, diagrams guide the solution process, diagrams help to verify if answers are correct or not and again diagrams provide additional information which helps to understand the problem statement. For example, from the question; '*A boy was asked to fetch water and fill a 1 litres gallon. He was to use a small gallon which was* $\frac{1}{8}$ *litres capacity of the big* *gallon. How many times will he use the gallon to fetch the water?'* the learner drew the one-eighth litre gallon and also drew a bigger gallon and divided it into eight parts showing how many times to fill the whole. Evidence is shown in Figure 18.

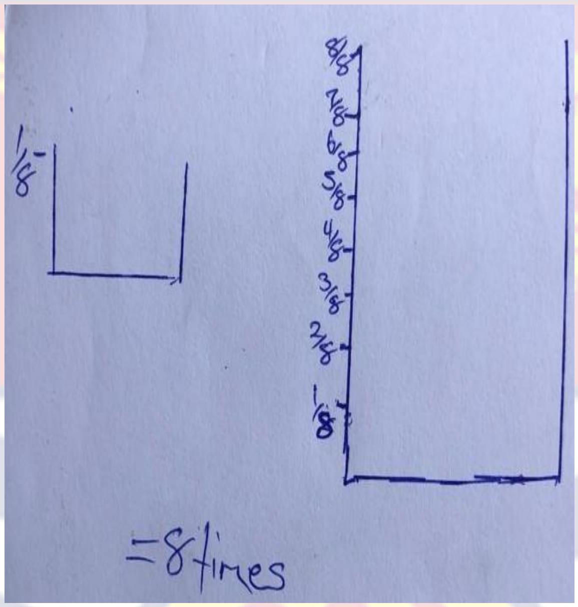


Figure 18: Evidence of the strengths of the use of diagrams

The positive impacts of the use of diagrams were exhibited in other questions. Some of the learners' diagrammatic representations are in Figure 19.

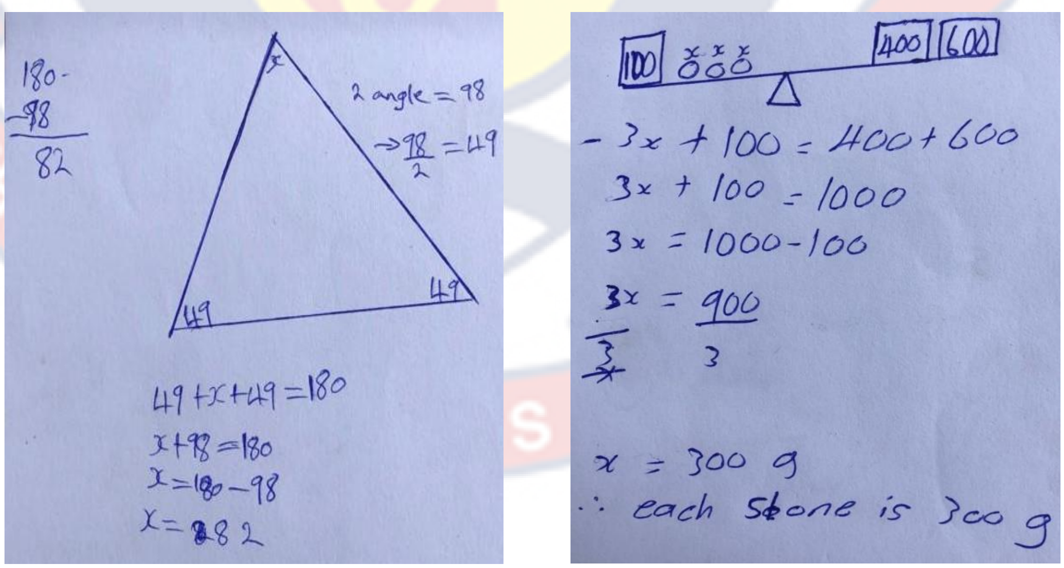


Figure 19: Evidence of the strengths of the use of diagrams

Discussion

This section discusses the results of the study. The discussion was ordered under the following relevant issues:

Errors learners made in solving word problems before being introduced to the use of diagrams in solving word problems.

Challenges learners experience in solving word problems before being introduced to the use of diagrams in solving word problems.

Effects of the use of diagrams in solving word problems.

Error learners made after being introduced to the use of diagrams in solving word problems

Errors Learners Make in Solving Word Problems.

One objective of this study was to identify the errors learners make when solving word problems. According to the NEA framework, the study revealed various errors committed by learners, with the low-achieving class making the most errors. Since teaching and learning are interconnected (Borromeo Ferri, 2018; Atkinson, 2002), understanding learners' thought processes and their grasp of specific concepts is crucial. The study found that both groups of learners made all five types of errors, with the low-achieving class making the most frequent errors. Errors in reading and comprehension occurred when learners struggled to understand what was required by the questions. Transformation errors happened when learners had difficulty converting text into mathematical expressions, while process skills and encoding errors were observed during computation and final answer presentation, respectively.

The most common errors identified were transformation errors, followed by comprehension errors, process skills errors, encoding errors, and reading errors. This aligns with findings from Abdullah, Abidin, and Ali (2015), Adu, Assuah, and Asiedu-Addo (2015), and Suseelan, Chew, and Chin (2022). Jiang, Li, Xu, and Lei (2020) also suggested that many transformational errors result from improper use of learned heuristics. However, these results contrast with Raduan's (2010) study, which found comprehension errors to be the most frequent. This discrepancy may be due to differences in word problem types, test items, and sample populations between studies.

Unlike Raduan's (2010) study, which included learners with various abilities working on routine word problems in real-world contexts, this study focused on low- and high-achieving learners tackling high-cognitive-demand, compare-type (non-routine) problems. These problems often contain extraneous information and do not explicitly state the mathematical relationships. Low-achieving learners, who often struggle with mathematical representation (Montague, 2014), may find it particularly challenging to translate word problems into mathematical sentences without proper heuristics (Xin, Jitendra, & Deatline-Buchman, 2005).

Reading errors were the least common among all errors, with 46% of the low-achieving class encountering this issue. This finding is consistent with Abdullah et al. (2015) and Suseelan, Chew, and Chin (2022), which indicated that while learners could read the text, they often did not understand what was required, hindering their progress. The study's test items, adapted from GES-approved textbooks and BECE past questions, were validated to ensure they matched learners' levels. Unfamiliar terms were simplified based on pretest feedback, resulting in fewer reading errors compared to other types. Limited vocabulary, dyslexia, and similar factors may have contributed to the reading errors and overall poor performance.

In summary, learners made more errors in transformation, process skills, and encoding than in reading and comprehension. Chan and Kwan (2021) found that learners' content knowledge and reading skills impacted their ability to solve word problems. Fuchs et al. (2018) suggested that transformation, process skills, and encoding errors are linked to content knowledge, while reading and comprehension errors relate to language factors (Lin, 2021; Singh, Rahman, & Hoon, 2010). The study found that content-knowledge-related errors were more prevalent than language-related errors, consistent with research by Clements and Ellerton (1996) and Suseelan, Chew, and Chin (2022). Poor mathematical content knowledge likely contributed to the high rate of transformation, process skills, and encoding errors (Lin, 2021; Singh et al., 2010).

Challenges Learners' Experience in Solving Word Problems

The study revealed that learners come across diverse challenges when confronted with word problem tasks. It is worth mentioning that, more than 50% of the learners from both groups have challenges in each of the major challenges identified in the study. This revelation may have several implications for teachers' methods of teaching, the materials adopted in teaching, the textbook employed in teaching and learning, and the choice of words or terminologies used in the word problem test item construction among other classroom-related activities. This is because the challenges identified have their roots in these elements involved in teaching and learning.

Obtaining and understanding the challenges learners have in a particular concept Ozdemir (2018) and Zhou (2019) maintain that they are a good step in solving learners' challenges. Interpretation or translating the text to mathematical form according to the learners was the most challenging aspect of dealing with word problems. This finding is in harmony with a couple of studies (Cruz, & Lapinid, 2014; Daroczy, Wolska, Meurers, & Nuerk, 2015; Ibrahim, & Yaw 2019; Talikan, 2021). For example, Cruz and Lapinid (2014) opine that translating word problems into mathematical form is the most difficult task for learners, especially at the elementary level. This, therefore, makes it seem like a hindrance to learning mathematics in general.

A review of this challenge reveals that learners often misinterpret the problem statement or may lack an in-depth understanding of the problem posed as suggested by Yeo (2009). However, other factors like incorrect operations, the use of appropriate symbols to represent certain terminologies, carelessness, interchanging values and unfamiliar words were identified as other factors contributing to this challenge. Being able to interpret the terms, and representing them with correct and appropriate mathematical notations and symbols is translating the text into mathematical form.

Unfamiliarity with terminologies presented in the text, however, was identified as the second most challenging issue when dealing with word problems.

A review of the old Ghanaian syllabus introduces learners to the use of keywords in translating a text into mathematical form whereas the new curriculum uses a combination of keywords and predesigned diagrams to assist learners in translating text into mathematical forms. Surprisingly, most of the Ghanaian-approved textbooks continue to introduce the use of keywords to assist learners in solving word problems. Words like 'altogether' are taught as addition, 'less than' are also explained as subtraction, whereas words like 'of' and 'share' are explained as multiplication and division respectively. It is important to note that, not all keywords work in all instances and that the keywords just provide pathways but are not a sure way to solve the problem.

This unfamiliarity with keywords has become a challenge because learners may have internalised or generalised keywords as the specific operation in solving word problems. Hence learners find word problems challenging when new terms are presented to them. Again, some of the keywords may be deceptive. For example, expressions like 'how many more' may seem as using the addition sign rather than the subtraction sign. The danger with this technique is that it teaches learners to bypass the context of word problems (Karp, Bush, & Dougherty, 2019; Mandal & Naskar, 2021). Furthermore, the concept of teaching word problems in schools is not just about computing correct responses but rather guiding learners to be grounded in mathematical thinking. Notwithstanding, word problem tasks must also be presented in a language that is within the learners' developmental level.

The learners also noted that word problems are particularly challenging because there is no definite formula to follow, making it difficult to derive one for a given task. According to the findings, this is the third most challenging aspect for the learners. The researcher attributes this difficulty to the way mathematics is presented in textbooks and taught in Ghanaian schools. Students are often encouraged to memorise formulas and perform computations rather than develop strategies for solving problems (Haghighi Siahgorabi, 2021). This teaching approach tends to keep learners at an operational or procedural level of mathematical understanding. As a result, they perceive mathematics as being primarily about formulas and calculations, similar to the level of understanding during the Renaissance (Wigderson, 2019).

The results also indicated that reading and comprehension pose a major challenge in solving word problems. This particular challenge has been reported by several Ghanaian authors as the main reason learners underperform in word problem tasks (Adu, Assuah, & Asiedu-Addo, 2015; Davis, 2010; Intsiful & Davis, 2019; Mereku & Cofie, 2008). For example, in Adu, Assuah, and Asiedu-Addo (2015), it was reported that 75% of 130 senior high school first-year learners were able to read the test items but had challenges with comprehension. These findings confirm Intsiful and Davis's (2019) findings that junior high school learners also have challenges with comprehension.

The current study, however, indicated that 28.75% of the 80 learners from the high-achieving class couldn't read or understand the test whereas 75.82% of the 91 learners from the low-achieving class could not do the same. Reading and comprehension have been identified to pose a challenge to learners because at that level two basic processes are involved. These processes are word recognition and acquisition of the meaning of the word in the context of its usage in the text item. Again, at that level, learners are expected to identify the syntactic and semantic relationships between words and sentences and then based on previous knowledge integrate them to form meaning out of the text. This has oftentimes been the challenge learners with reading and comprehension face.

Teachers are therefore urged to identify learners' challenges before mounting a strategy to teach the concept. Additionally, mathematics teachers are urged to give learners the chance to participate in excellent discussions about the interpretation and meaning of texts in multiple academic fields as one crucial strategy to enhance their reading comprehension. For example, judging from the results, teachers in the low-achieving class would have to assist learners with strategies that will enable them to read and understand before being introduced to word problems since 80% of the learners identified reading and comprehension as a major challenge in solving word problems.

The findings of this study, concerning the various challenges identified, are in harmony with a couple of studies (Ibrahim, & Yaw, 2019; Siniguian, 2017; Talikan, (2021). For example, the study by Ibrahim and Yaw (2019) showed that learners in Tamale have challenges in solving word problems. Although the study used learners from secondary school the researchers highlighted fear, unclear or obscure formulas, Unfamiliarity with terminologies, and time-wasting as learners' major challenges in solving word problems. These revelations were made known after the participants were interviewed after the pretest. In principle, fear and time-wasting as stated by the researchers as challenges may seem to be a perception and not rather a challenge therefore may be subjected to review. Notwithstanding, learners being unfamiliar with some of the terminologies used in the text construction and not obtaining a definite formula for solving a word problem may be seen as a major challenge in solving word problems. The researcher do not dispute the previous research findings in any way; instead, the researcher analysed their findings within the framework of their investigation. In some ways, this study tackles the inefficiencies of the challenges learners face in solving word problems.

Effect of the Use of Diagrams in Solving Word Problems

Having identified that transformation errors were the most encountered challenge or error by learners, it was therefore imperative to identify a remedy to help learners translate text items into mathematical forms hence this sub-purpose. Several researchers have come out with various heuristics such as Polya's problem-solving model (Asoma, Ali, Adzifome, & Eric, 2022; Nashiru, Alhassan, & Sadiq, 2018), the use of the cubes method (Tibbitt, 2016), the use of keywords method (Powell & Fuchs, 2018; Powell, Namkung, & Lin, 2022), and the use of the bar model (Anane, Awudetsey, Sedegah, Mishiwo, & Awuitor, 2016; Baysal, 2019) among others to help solve this challenge. On the other hand, current literature suggests that transformational errors remain the most occurring error for most learners across the globe (Jiang, Li, Xu, & Lei, 2020; Suseelan, Chew, & Chin, 2022).

Findings from this current study indicated the use of diagrams has a significant effect on the performance of learners in solving word problems. This was realised after comparing the pretest and posttest findings of learners from both the high-achieving and the low-achieving classes. This simply means the use of diagrams has a great ability in assisting learners in translating text items into mathematical form hence the increase in performance of the learners. These findings conform to the findings of Poch, van Garderen, and Scheuermann (2015) and van Garderen, Scheuermann, and Jackson, (2013). However, their studies were solely on the use of diagrams and other visual representations like pictures in assisting learners with learning disabilities in solving word problems. Notwithstanding, learning disabilities are exhibited differently over time, both in severity and with varying settings and environments (Ahmad, 2015). Again, studies also indicated a majority of learners face this challenge and therefore concentrating only on learners with learning disabilities may put other learners at a disadvantage since their challenges may not be identified and dealt with.

Errors Learners Made After Being Introduced to the Use of Diagrams in Solving Word Problems

While diagrams can be a useful tool for solving word problems, their effectiveness has limitations. Research shows that learners with learning disabilities often use diagrams less effectively compared to their peers (Van Garderen, Scheuermann, & Poch, 2014). This is because the diagrams they create tend to be of lower quality. Fiorella, Stull, Kuhlmann, and Mayer (2020) identified several weaknesses in learner-generated diagrams, including issues such as being time-consuming, inaccurate, or incomplete if the learner lacks the skills to create them, and difficulties in interpretation. Despite these limitations, Kim and Kim (2018) argue that the benefits of learner-generated diagrams outweigh their drawbacks.

The challenges and errors observed after learners were introduced to diagram generation may be attributed to several factors. These include difficulties in reading, unfamiliarity with mathematical terms, poor drawing skills, and inadequate mathematical proficiency. Many learners struggled to solve problems even when presented in mathematical notation. The researcher suggests that implementing various strategies could help learners tackle everyday problems more effectively.

This study aimed to address gaps in the literature regarding the use of diagrams for solving word problems by using self-directed diagrams as an intervention. Although other scholars have noted some challenges associated with diagrams, MacCormack and Matheson (2022) argue that diagrams remain one of the most effective strategies for solving word problems. According to the findings, diagrams serve as a valuable tool for exploring alternative problem-solving approaches and for monitoring and evaluating solutions.

CHAPTER FIVE

SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

The aim of this study was to assess how effective diagrams are in solving word problems at selected junior high schools in the Assin Central Municipality. This chapter provides a summary of the study's findings, draws conclusions, and offers recommendations based on the results.

Summary

The study investigated how effectively diagrams aid in solving word problems among selected junior high school students in the Assin Central Municipality. It was guided by four objectives, which were formulated into three research questions and two hypotheses. The study employed a concurrent mixed-methods approach and utilised a two-group pretest-posttest design with a delayed posttest. The target population included all junior high school students in the Assin Central Municipality, with a focus on JHS 1 students due to the study's subject matter. Using purposive sampling, two groups were selected based on their pretest performance and categorised into low-achieving and high-achieving classes. Within each class, participants were randomly assigned to control and experimental groups. The final sample comprised 171 students.

A Think-Aloud Protocol (TAP) and Mathematics Achievements Test (MAT) items from accredited textbooks and WAEC past questions were adapted and used as data collection instruments. The instruments were subjected to expert review and were later pilot-tested to determine their validity and reliability. The data collected were analysed using descriptive statistics; frequencies, percentages, themes and content analysis. The ANCOVA and paired-sample t-test were also used for all the inferential analyses.

Findings

The following findings came to light from the study:

1. Translating text items into mathematical form was identified as the most difficult challenge learners encounter when presented with word problem tasks.
2. The use of self-directed diagrams has a positive significant effect on learners' ability to solve word problem tasks.
3. Learners were able to use the diagrams constructed to serve as a guide and an evaluation tool in tracking the results.
4. The use of diagrams was more effective for text items which involved measurements, distances, heights, and sharing wholes.
5. There was a significant difference in the mean scores of the low-achieving class and the high-achieving class when their pretest results were compared to their posttest results.
6. The delayed posttest conducted indicated that the skill obtained in using diagrams to solve word problems aids in retention.

Conclusion

From the findings, it can be concluded that translating text items into mathematical notations is the most difficult task learners encounter when solving word problems. This challenge could be a result of the difficulty in reading and understanding what the task requires of the learners. In addition, getting the necessary heuristics to help learners decipher and translate the text items was the major cause of this challenge hence the majority of the learners failed to obtain good grades when solving word problems.

It is then concluded that learners coming out with their own diagrams in an attempt to solve word problems not only helps them to translate the text but also assists learners in tracking their computations and verifying whether their answers are correct or not. Again, assisting learners to identify key elements in a text and also using diagrams provide learners with more information to understand the requirements of the questions presented to them and solve the questions with less difficulty. Concisely, the use of diagrams can be said to be an effective strategy in assisting learners to overcome the challenges that come as a result of solving word problems.

Recommendations

Based on the findings of this study, the following recommendations are made:

1. Heads of various basic schools as part of their refresher programmes or workshops for teachers should guide mathematics teachers on how to assist learners in using self-generated diagrams to describe a given text item.
2. The mathematics curriculum and textbooks should offer various strategies to help learners develop diagrams for solving word problems.

3. Mathematics teachers should be encouraged to involve learners in a series of word problem tasks to improve their skills in solving word problems.
4. Mathematics teachers are urged to use real-life problems when teaching word problems to help learners sync well with the task.
5. Learners are also entreated to solve more challenging mathematical word problems and approach experts in the field for assistance when necessary.

Suggestions for Further Research

1. A future study is suggested to identify other heuristic strategies that can help learners easily translate word problem tasks into mathematical notations.
2. It is also suggested that this study be carried out in various Ghanaian districts and regions because different teaching approaches and word problem assignments may provide different outcomes.
3. Further studies in this area are suggested to identify whether this strategy applies to learners in the lower classes.

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APPENDICES

APPENDIX A: PRETEST

UNIVERSITY OF CAPE COAST

FACULTY OF EDUCATIONAL FOUNDATIONS

DEPARTMENT OF BASIC EDUCATION

CAPE COAST

MATHEMATICS ACHIEVEMENT TEST ON USING DIAGRAMS IN ADDRESSING LEARNERS' DIFFICULTIES IN SOLVING WORD PROBLEMS IN ALGEBRAIC EXPRESSIONS.

Dear respondent, this test aims to explore the use of diagrams in addressing learners' difficulties in solving word problems in algebraic expressions. This research is conducted for purely academic purposes. All information provided will be treated with strict confidentiality and will only be used for the intended purpose of this study. Kindly provide sincere and objective answers to the questions. Thank you for participating.

SECTION A: DEMOGRAPHIC DATA OF RESPONDENTS

A. School……………………………………………………………..………… ………………

B. Gender: Male [ ] Female [ ]

C. Age: ___________________________

1. A balloon first rose 200m from the ground, then moved 100m to the east, then dropped 100m. It then travelled 50m to the east and finally dropped straight to the ground. How far was the balloon from its original starting point?
2. Bodwoase D/A JHS 1 learners were given an amount of GH₵42.00 to share. If each learner received $\frac{2}{7}$ of the amount, how many learners received a share?
3. Two angles of a triangle add up to 98^0. What is the size of the third angle?
4. On one side of a scale are three honey pots and a 100g weight. On the other side, there are a 200g and a 500g weights. The scale is balanced. What is the mass of a pot of honey?
5. One morning, Janet bought forty oranges at a supermarket. She gave $\frac{1}{8}$ of them to his neighbour. After lunch, she ate two oranges. How many oranges had he left?

Appendix B: POSTTEST

UNIVERSITY OF CAPE COAST

FACULTY OF EDUCATIONAL FOUNDATIONS

DEPARTMENT OF BASIC EDUCATION

CAPE COAST

MATHEMATICS ACHIEVEMENT TEST ON USING DIAGRAMS IN ADDRESSING LEARNERS' DIFFICULTIES IN SOLVING WORD PROBLEMS IN ALGEBRAIC EXPRESSIONS.

Dear respondent, this test aims to explore the use of diagrams in addressing learners' difficulties in solving word problems in algebraic expressions. This research is conducted for purely academic purposes. All information provided will be treated with strict confidentiality and will only be used for the intended purpose of this study. Kindly provide sincere and objective answers to the questions. Thank you for participating.

SECTION A: DEMOGRAPHIC DATA OF RESPONDENTS

A. School…………………………………………………………...…………………………

B. Gender: Male [ ] Female [ ]

C. Age: ________________________

1. Daniel was given thirty biscuits on his birthday. He gave $\frac{3}{5}$ to his friends who attended his birthday party. He ate 3 of the remaining biscuits the next day. Calculate the number of biscuits he had left.
2. A bottle contains 500ml of water. How many bottles are needed to fill a tank that can hold 10 litres of water?
3. The following items were found on a balanced scale. On one side is a 400g and a 600g of metal blocks. The other side had a 100g of beads and three different pieces of stones. Calculate the mass of one stone on the balance.
4. Essel flies a Kite as high as 300m from the ground, the kite moved 150m to the east, then dropped 30m. It then travelled 30m to the east, and finally dropped straight to the ground. How far was the kite from Essel's position?
5. A boy was asked to fetch water and fill a 1 litres gallon. He was to use a small gallon which was $\frac{1}{8}$ litres capacity of the big gallon. How many times will he use the gallon to fetch the water?

Appendix C: DELAYED POSTTEST

UNIVERSITY OF CAPE COAST

FACULTY OF EDUCATIONAL FOUNDATIONS

DEPARTMENT OF BASIC EDUCATION

CAPE COAST

MATHEMATICS ACHIEVEMENT TEST ON USING DIAGRAMS IN ADDRESSING LEARNERS' DIFFICULTIES IN SOLVING WORD PROBLEMS IN ALGEBRAIC EXPRESSIONS.

Dear respondent, this test aims to explore the use of diagrams in addressing learners' difficulties in solving word problems in algebraic expressions. This research is conducted for purely academic purposes. All information provided will be treated with strict confidentiality and will only be used for the intended purpose of this study. Kindly provide sincere and objective answers to the questions. Thank you for participating.

SECTION A: DEMOGRAPHIC DATA OF RESPONDENTS

A. School……………………………………………………………...…………
………………

B. Gender: Male [ ] Female [ ]

C. Age: ______________________

1. A bottle contains 400ml of water. How many bottles are needed to fill a tank that can hold 20 litres of water?
2. The following items were found on a balanced scale. On one side is a 200*g* and a 400*g* of metal blocks. The other side had a 50g of beads and two different pieces of stones. Calculate the mass of one stone on the balance.
3. A balloon first rose 200m from the ground, then moved 100m to the east, then dropped 100m. It then travelled 50m to the east and finally dropped straight to the ground. How far was the balloon from its original starting point?
4. Kwaata D/A JHS 1 learners were given an amount of GH₵52.00 to share. If each learner received $\frac{3}{7}$ of the amount, how many learners received a share?
5. A boy was asked to fetch water and fill a 1 litres gallon. He was to use a small gallon which was $\frac{1}{8}$ litres capacity of the big gallon. How many times will he use the gallon to fetch the water?

Appendix D: ETHICAL CLEARANCE

UNIVERSITY OF CAPE COAST
COLLEGE OF EDUCATION STUDIES
ETHICAL REVIEW BOARD

UNIVERSITY POST OFFICE
CAPE COAST, GHANA

Our Ref: CES-ERB/ucc.edu/v6/22-62

Date: 27th July, 2022

Your Ref: ....

Chairman, CES-ERB
Prof. J. A. Omotosho
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Vice-Chairman, CES-ERB
Prof. K. Edjah
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Secretary, CES-ERB
Prof. Linda Dzama Forde
lforde@ucc.edu.gh
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Dear Sir/Madam,

ETHICAL REQUIREMENTS CLEARANCE FOR RESEARCH STUDY

The bearer, Samuel Kenney, Reg. No. EF/BEP/20/0004 is M.Phil. / Ph.D. student in the Department of Basic Education in the College of Education Studies, University of Cape Coast, Cape Coast, Ghana. He / ~~She~~ wishes to undertake a research study on the topic:

Using diagrams in addressing learners' difficulties in solving word problems in algebraic expressions.

The Ethical Review Board (ERB) of the College of Education Studies (CES) has assessed his/her proposal and confirm that the proposal satisfies the College's ethical requirements for the conduct of the study.

In view of the above, the researcher has been cleared and given approval to commence his/~~her~~ study. The ERB would be grateful if you would give him/her the necessary assistance to facilitate the conduct of the said research.

Thank you.
Yours faithfully,

[signature]

Prof. Linda Dzama Forde
(Secretary, CES-ERB)

APPENDIX E: FIELDWORK ACTIVITIES

S/N	WEEK		RESEARCH ACTIVITIES
1	Before the study	-	Mapping
2	1st week	-	Pilot testing
3	2nd week	-	Administering Pretest
4	3rd week	-	Training of field assistants
5	4th week	-	Pretest
6	5th week	-	Intervention
7	6th week	-	Intervention
8.	7th week	-	Intervention
8	8th week	-	Posttest
9	11th week	-	Posttest
10	14th week	-	Delayed posttest