UNIVERSITY OF CAPE COAST

STATISTICAL MODELING OF THE GHANAIAN

WEATHER USING MARKOV CHAINS:

A Case Study of Accra, Kumasi and Tamale, 2003-2007

ALEXANDER OKYERE- BOAKYE

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USING MARKOV CHAINS:

A Case Study of Accra, Kumasi and Tamale, 2003-2007

BY

ALEXANDER OKYERE- BOAKYE

A Dissertation submitted to the Department of Mathematics & Statistics of the School of Physical Sciences, University of Cape Coast in partial fulfillment of the requirements for the award of Master of Science degree in Statistics.

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DECLARATION

Candidate's Declaration

I hereby declare that this dissertation is the result of my own original work and that no part of it has been presented for another degree in this University or elsewhere.

Candidates Signature:………………………………… Date:………………

Candidate Name:……………………………………………..

Supervisor's Declaration

I hereby declare that the preparation and presentation of the dissertation were supervised in accordance with the guidelines on supervision laid down by the University of Cape Coast.

Supervisor's Signature:…………………………........ Date:…………………

Supervisor's Name:………………………………………………………..

ABSTRACT

In this dissertation, Markov chain was used to model the rainfall pattern and sunshine duration for three regional capitals in Ghana (Tamale, Kumasi and Accra). Data on mean monthly averages and daily figures of measured parameters (sunshine duration and rainfall) for five years were used. To achieve these goals, a statistical basis was presented before adopting the three main approaches for modeling. Transition Matrix shows the short-run behavior of Markov chain; the steady-state probability, indicating the long-run behavior of Markov chain; and finally, the mean first passage times. The chain for the transition matrix for each region was found to be ergodic. The steady-state probability gave the possibility of weather change in a particular day on the condition that it was rainy or sunny for the last two days or the previous day. The mean first passage times of the weather gave the average number of daily rainfall before sunshine and an average number of daily sunshine before rainfall.

The study showed that among the three towns, Kumasi has the highest probability of rainfall followed by Tamale and then Accra. Secondly, Tamale has the highest probability of duration of sunshine followed by Accra and then Kumasi. Kumasi has the highest average number of occurrence of daily rainfall compared to Tamale and Accra. Finally, Tamale has the highest average number of occurrence of daily sunshine compared to Accra and Kumasi.

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I must express my sincere gratitude to Mr. Mahmud Ibrahim, a lecturer of University of Cape Coast, Department of Mathematics and Statistics. Firstly, for accepting to be my supervisor for this research, the invaluable technical advice and material support. Indeed, without his willing and timely support the entire project could not have been possible within the time frame. Secondly, my sincere thanks also go to the entire staff of the Ghana Metrological Agency Head Office, Legon for their immense assistance during the data collection and finally, to Projector Plus Ventures, Aburi.

DEDICATION

To my family

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CHAPTER ONE

INTRODUCTION

Background

Weather change has been of great concern to policy makers. Changes of weather conditions have resulted in a lot of media reports in recent times. People of all walks of life have added their voice to the problem by trying to find the causes and solutions. Governments of developing countries like Ghana have relied solely on Ghana Meteorological Agency for their weather forecast. Because Ghana Meteorological Agency is not well resourced, it has become difficult for the country to predict the weather conditions in the long-run.

Weather change affects natural resources and economic activities. The bio-productive systems can be affected by changes in temperature, rainfall and sea-level rise. Effects will vary from one region to another. Studies elsewhere have established expected responses of bio-productive systems to changes in carbon dioxide, temperature and rainfall. Temperature influences plant growth and development, and the higher the temperature, the faster the plants grow and mature. Increased temperature will enhance the growth and productivity of plants. However, high temperature leads to high evaporation and evapotranspiration. This increases water demand for plant growth. Under such circumstances, dry land farming will be limited and irrigation schemes will suffer from the problems of salinisation and water loss through evaporation.

High temperatures and high atmospheric humidity will favour development of animal and crop pests and diseases. It is therefore expected that increase in temperatures would lead to increase in animal and plant pests and decreases. This is likely to lead to a reduction in agricultural production. It is generally expected that an increase in precipitation in the humid regions would lead to an increase in agricultural and forest production. However, more rainfall, high temperatures and higher concentration of carbon dioxide could aggravate the production of weeds in cultivated areas, and the resultant competition could reduce crop yields. High rainfall, especially in the humid regions, would increase nutrient leaching, problems of soil erosion, flooding and soil losses. Decreased rainfall will lead to a reduction in soil moisture thereby affecting dry land farming potential, fuel wood availability and forage material for livestock and wildlife. Weather change, high population growth rates and a lack of significant investment have made Africa most vulnerable to the impacts of projected changes as widespread poverty limits adaptation capabilities.

Lakes and major dams have reached low levels, thus threatening industrial activity. Power rationing in Ghana in 2007 was due to the low level of the Akosombo Dam. This affected many industries, leading to high cost of production, lost of jobs and low profit. It also increased the government expenditure on energy, increase in inflation rate and security problems. Climate change will increase the frequency of such low storage events.

The academic interest in the topic originates primarily from the country climate changes that have remained high on the government agenda.

Objectives of Study

There have been several statistical and mathematical approaches in modeling weather conditions in Ghana. This shows the level of importance government and researchers have placed on the climate in the region.

The objectives of the research are as follows:

(i) To use Markov chain to model the rainfall and sunshine pattern in Accra, Kumasi and Tamale.

(ii) To determine transient (or short-run) behavior of a Markov chain of rainfall and sunshine in the three regions.

(iii) To use the steady-state probabilities to describe the long-run behavior of rainfall and sunshine patterns in the three regions.

(iv) To use the mean first passage time to find how long it takes to move from one state to another. That is, how long it takes rainfall to occur before sunshine and vice versa.

The identified pattern could then be used to predict future rainfall and sunshine in each of the three regions.

Literature Review

Research on various applications of Markov chains on rainfall, sunshine, marketing, student admission, work-force planning, accounts receivable situation, and others have been carried out by many researchers.

According to Winston (1993), a random variable which changes over time may be applied in knowing how the price of a share of stock or a firm's market price evolves over time. He explained how types of Markov chains have been applied in areas such as education, marketing, health services, finance, accounting and production.

Bessent and Bessent (1980) applied Markov chain to model the path of a student admission through the State College. Each student was observed at the beginning of each Fall semester. For example, if a student is a junior at the beginning of the Fall semester, there is percentage chance that he will be a senior at the beginning of the next Fall semester, a percentage chance that he will still be a junior and a percentage chance that he will quit. This was modeled with the use of Markov chain.

Pegels and Jelmert (1970), talks on how fresh blood obtained by a hospital will spoil if it is not transfused within the same day. Markov chain was used to determine two policies that were possible for determining the order in which blood is transfused. Thompson and McNeal (1967) were also able to model sales planning and control using absorbing Markov chains.

In a work-force planning, for example, a law firm of Mason and Burger employs three types of lawyers: junior lawyers, senior lawyers and partners. What is the probability that a newly hired junior lawyer will leave the firm before becoming a partner? How long does a newly hired junior lawyer stay with the firm? The answers to these were obtained with the use of Markov chain. Hence, the firm was able to predict the number of employees of each type who will be available from the steady-state probabilities.

In Deming and Glassers (1968) article, a markovian analysis of the life of newspaper subscriptions was conducted. This enabled them to predict those who would cancel the subscription, those who would subscribe for one year and new entries.

The accounts receivable situation of a firm (Cyert, Davidson and Thompson 1963) is often modeled as an absorbing Markov chain. Suppose a firm assumes that an amount is uncollectable if the account is more than three months over due. Then at the beginning of each month, each account may be collapsed into one of the following areas:

State 1: New account.

State 2: Payment on account is one month over due.

State 3: Payment on account is two month’s over due.

State 4: Payment on account is three month’s over due.

State 5: Account has been paid.

State 6: Account is written off as bad debt.

The Markov chain was applied to describe how the status of an account changes from one month to the next month.

From the literature review, it can be seen that various researchers have made use of Markov chains to model different areas of interest. These have informed the use of Markov chain to model rainfall and its corresponding duration of sunshine in Ghana (a case study of Accra, Kumasi and Tamale).

Data Collection

The relevant data for the research are rainfall and sunshine figures (refer to appendix). Below is the mean monthly rainfall of Kumasi from January 2003 to December 2007.

Kumasi Monthly Rainfall Total (mm)

Year	Jan	Feb	March	April	May	June	July	Aug	Sept	Oct	Nov	Dec
2003	32.9	74.5	73.1	129.5	188.8	254.6	95.3	26.8	99.5	180.1	163.2	30.9
2004	25.8	70.8	164.3	101	72.3	41.1	229.4	115	243.5	232.4	43.5	76.5
2005	12.5	48.9	84.2	146.4	272.1	121.3	18.3	36.7	174.1	236.9	49.8	29.8
2006	111.1	98.4	112.8	66.9	187.3	145.4	66.7	65.2	111.4	158.4	32.5	3.7
2007	0.2	16.4	56.2	310.9	164.2	176.0	192.9	117.7	534.5	153.9	51.7	19.8

The data which were gathered from the Ghana Meteorological Agency are secondary type. They included daily and mean monthly rainfall and sunshine duration from 2003 to 2007. This data covered only three regions: Greater Accra was selected to represent the southern belt. This also represents coastal scrub, grassland, and strand and mangrove zone. Kumasi was selected to represent the middle belt of the country. It also represents moist-semi deciduous forest and rain forest. Tamale was selected to represent the Northern belt. It also represents the Guinea and Sudan savanna.

Outline of Dissertation

The dissertation is outlined as follows:

Chapter one gave the background of the study, the objectives of study, literature review, data collection and outline of dissertation.

Chapter Two deals with the review of basic theory and methods of Markov chain. The aims are: the transient (short run) behavior of Markov chain, the steady-state probabilities to describe the long-run behavior or a Markov chain. The mean first passage time was used to find how long it takes to move from one state to another. Other statistical methods that can be used to model the weather for the three regions were also discussed.

Chapter Three is concerned with preliminary analysis of daily and monthly rainfall and sunshine in the five year period, from 2003 to 2007. The probability of occurrence of rainfall and duration of sunshine is put into a transition matrix which describes the short run behavior of the Markov chain.

In Chapter Four, both the management science software and MATLAB software were used to determine the steady-state probabilities and mean first passage times respectively of daily and monthly rainfall pattern and sunshine duration in the three towns.

Chapter Five looks at the summary of the report, the relevant discussion and necessary conclusions. All the important points concerning the objectives of the report are summarized in this chapter. That apart, the chapter tackles the probability of rainfall and sunshine in each of the three regions. Conclusion gives the justification of the use of Markov chain in modeling and analyzing the sunshine and rainfall patterns in Ghana.

CHAPTER TWO

REVIEW OF BASIC THEORY AND METHODS

Introduction

This chapter covers the basic theory and methods that would be used in modelling the rainfall and sunshine in the three regions. Other statistical methods that can also be used to model the data would also be discussed. These are Monte Carlo, Likelihood Estimation, Bayesian methods and Time Series- ARIMA.

Stochastic Process

Stochastic means random and probability models that describe a quantity that evolves randomly in time. A stochastic process is a sequence of random variables $\{X_u, u \in I\}$ which we will sometimes denote by *X*. u is called the index and most commonly denotes time. Thus we say, X_u is the state system in time u. I is the index set. This is the set of times we wish to define for the particular process under consideration. The index set *I* will be either a discrete or a continuous set. If it is discrete (e.g. $I = \{0, 1, 2 \ldots\ldots\ldots\ldots\}$) then we say that *X* is a discrete time stochastic process. If it is continuous (e.g. $I = [0, \infty)$) then we say *X* is a continuous time stochastic process. Whether the index set *I* is discrete or continuous is important in determining how we mathematically study the process.

State Space

The other fundamental component (besides the index set *I*) in defining the structure of a stochastic process is the state space. This is the set of all values that the process can take on. The concept of the sample space of a random variable is the sequence making up the process, which is denoted by S.

Expectation of a Random

Let *X* be a discrete random variable defined on a sample space S with probability mass function *f*(*x*). The expected value of *X*, also called the mean of *X* and denoted by E [*X*], is given by

$$E[X] = \sum_{x \in s} x f(x) = \sum_{x \in s} x \, P(X = x),$$

if the sum is absolutely convergent. Note that the sample space of a quantitative random variable is always a subset of R, the real line. Expectation is a very important quantity when evaluating a stochastic system. In financial markets, expected return is often to determine a 'fair price' for financial derivatives and other equalities (based on the notion of a fair game, for which a fair price to enter the game is where the expected net return is zero). When devising strategies for investment, expected profit is often used as a guide for developing optional strategies (e.g. a financial portfolio).

Conditional Expectation

Given two events A and B with P (B) > 0, the conditional probability of A given B is written, P (A /B) = $\dfrac{P(A \cap B)}{P(B)}$

Given two (discrete) random variables X and Y the conditional expectation of X given $Y = y$, where $P(Y = y) > 0$ is defined to be

$$E(X / Y = y) = \sum_{x} xP(X = x / Y = y)$$

Note that this is just the mean of the conditional distribution of X given $Y = y$.

Conditioning on an event has the interpretation in that knowing the event $\{Y = y\}$ occurred gives us information about the likelihood of the outcomes of X.

Markov Chains

A discrete-time stochastic process X is said to be a Markov chain if it has the Markov Property.

Markov Property (Version 1)

For any s, $i_0 .. i_{n-1} \in S$ and any n ≥ 1

$$\sum_{J \in S} P_{ij} = \sum_{J \in S} P(X_1 = j / X_0 = i) = 1$$

Each row of P has entries that sum to one. In other words, we say the distribution of X_n given the entire past of process only depends on the immediate past.

Note that we are not saying that, for example X_{10} and X_1 are independent. They are not. However, given X_9, for example, X_{10} is conditionally independent of X_1.

The Markov property is that the distribution of where one goes to next depends only on where one is now, not on where one has been. This property is a reasonable assumption for many, though certainly not all, real world processes.

Note that, as with the notion of independence, in applied modeling the Markov property is not something we usually try to prove mathematically. It usually comes in the model as an assumption.

Markov Property (Version 2)

For any s, $i_{0,} i_1 .. i_{n-1} \in S$ and any n> 1 and m > 0

$$P(X_{n+m} = s / X_0 = i_{0,} ..., X_{n-1} = i_{n-1})$$
$$= P(X_{n+m} = s / X_{n-1} = i_{n-1})$$

In words, this says that the distribution of the process at any time point in the future given the moist recent past is independent of the earlier past.

Classification of States

Some basic definitions are outlined below:

(i) A state *j* is reachable from state *i* if there is a path leading from state *i* to state *j*.

(ii) Two states *i* and *j* are said to communicate if state *j* is reachable from state *i*, and state *i* is reachable from state *j*.

(iii) A set of states S in a Markov chain is a closed set if no state outside of is reachable from any state in S.

(iv) A state *i* is an absorbing state if $P_{ij} = 1$.

(v) A state i is a transient state if there exists a state j that is reachable from state i, but the state i is not reachable from state j.

(vi) If a state is not transient; it is called a recurrent state.

(vii) A state j is periodic within period K> 1 if K is the smallest number such that all paths leading from state i back to state i have a length that is a multiple of K. If a recurrent state is not periodic, it is referred to as aperiodic.

(viii) If all states in a chain are recurrent, aperiodic, and communicate with each other the chain is a said to be ergodic.

An example on how to describe the states in a markov chain is shown below. The Winston's approach would be used to find the recurrent, transient and absorbing states for the following transition matrix.

$$\begin{array}{c} \\ 1\\2\\3\\4\\5\\6\\7\\8\\9\\10 \end{array} \begin{array}{c} \begin{array}{cccccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0.3 & 0.3 & 0.1 & 0.3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.6 & 0 & 0 & 0 & 0 & 0.4 & 0 & 0 \\ 0 & 0 & 0 & 0.5 & 0 & 0.5 & 0 & 0 & 0 & 0 \\ 0 & 0.7 & 0 & 0 & 0 & 0.3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.9 & 0 & 0 & 0 & 0 & 0.1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0.8 & 0 & 0 & 0 & 0 & 0 & 0.2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Below is the state transition diagram for the transition probability matrix.

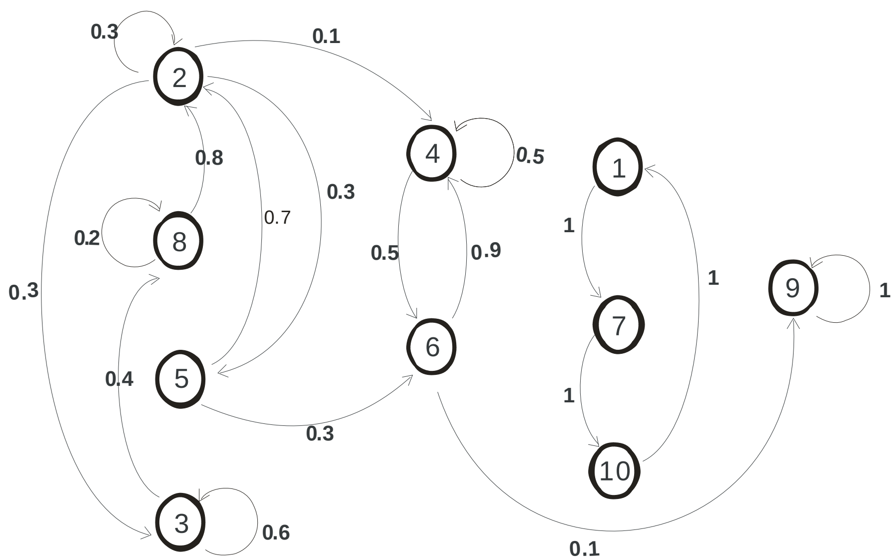


Figure 1: Graphical Representation of Transition Matrix.

From Figure 1, the graphical representation of the transition matrix, of the given the states 1 and 10, a path from 1 to 10 is a sequence of transitions that begins in state and ends in state 10, such that each transition in the sequence has a probability of occurring.

From the transition probability matrix as represented in Figure 1, state 9 is reachable from state 3 (via the path 3-8-2-5-6-9), thus states 2,3,5,6 and 8 are transient states. Also it is possible to go along the path 2-3-8-2-5 but state 5 is not reachable from state 6. Similarly, there is no way to return to state 2 from state 4. States 4 and 6 are said to communicate because, state 4 is reachable from 6, and state 6 is reachable from 4.

Also, from the Markov Chain with transition matrix in Figure 1, the states 1, 7 and 10 are closed states because, from observation once we enter a closed set we can never leave the closed set. The states 1, 7 and 10 also form a periodic chain because, each state has period 3. For example if we begin in state 1, the only way to return to state 1 is to follow the path 1-7-10-1 for some number of times (say, m). Hence, any return to state 1 will take 3m transitions, so state 1 has period 3. Wherever we are, we are sure to return three periods later. Hence states 1, 7 and 10 is periodic markov chain with k=3.

State 9 is an absorbing state because P_{ii}=1. Whenever enter an absorbing state, we never leave the state. State 9 can also be said to be a closed state containing only one state. State 9 is not transient hence; it is called a recurrent state. Finally, it can be said that the chain is not ergodic.

An Ergodic Chain

The transition matrix below represents an example of an ergodic chain.

$$R = \begin{bmatrix} \frac{1}{2} & \frac{2}{3} & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & \frac{1}{4} & \frac{3}{4} \end{bmatrix}$$

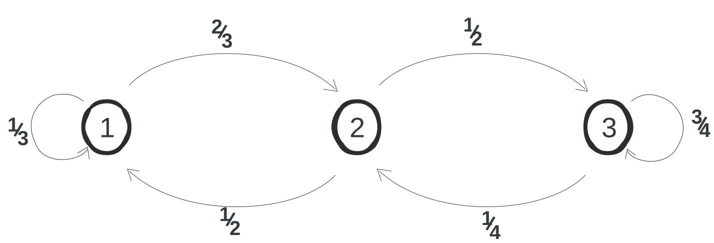


Figure 2: Graphical Representation of Transition Matrix R of an ergodic chain

Firstly, the three states 1, 2 and 3 are said to communicate because, state 1 is reachable from 2 and 3. State 2 is reachable from 1 and 3 while state 3 is reachable from 1 and 3. Secondly, none of the states is a transient state because state 1 is reachable from state 2, and state 2 is reachable from state 1. Hence, the states are a recurrent state. Thirdly, because the states are not periodic, they are referred to as aperiodic. In conclusion since all the states in the chain are recurrent, aperiodic and communicate with each other the chain is said to be ergodic.

A Nonergodic Chain

The transition matrix below represents an example of a nonergodic chain.

$$T = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{2}{3} & \frac{1}{3} \\ 0 & 0 & \frac{1}{4} & \frac{3}{4} \end{bmatrix}$$

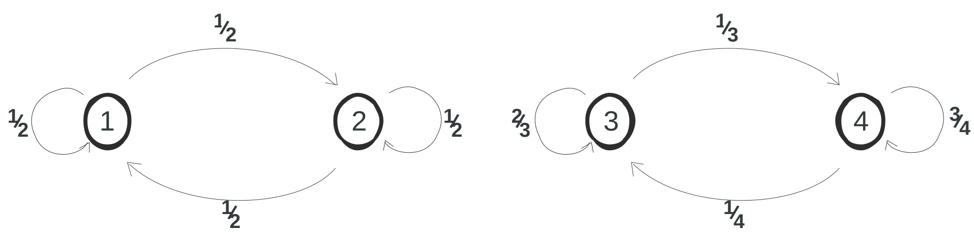


Figure 3: Graphical Representation of Transition Matrix T of a nonergodic chain

Firstly, the two states 1 and 2 are said to communicate because, state 1 is reachable from 2 while state 2 is reachable from 1. States 1 and 2 are not reachable from 3 and 4 and vice versa. Secondly, the states 1, 2, 3 and 4 are not a transient state because state 1 is not reachable from state 3, and state 2 is reachable from state 4. Hence, the states are not recurrent state. In conclusion since all the states in the chain are not recurrent and communicate with each other the chain is said to be nonergodic.

The One-Step Transition Matrix

We think of putting the 1-step transition probabilities p_{ij} into a square matrix called the 1– step transition matrix, also called the transition probability matrix of the Markov chain. We will usually denote this matrix by P. The $(i,j)^{th}$ entry of P is p_{ij}.

Furthermore, since

$$\sum_{J\in S} P_{ij} = \sum_{J\in S} P(X_1 = j / X_0 = i) = 1$$

each row of P has entries that sum to one. In other words, each row of P is probability distribution over s (indeed, the i^{th} row of P is the conditional distribution of X_n =*j* given that $X_{n-1} = i$). For this reason we say that P is a stochastic matrix.

It turns out that the transition matrix P gives an almost complete mathematical specification of the Markov Chain. In general, we would say that a stochastic process is specified mathematically, once we specify the state space and the joint distribution of any subset of random variables in the sequence making up the stochastic process. These are called the Finite-dimensional distribution of the stochastic process. So for a Markov chain, there is quite a lot of information we can determine from the transition matrix P.

One thing that is relatively easy to see is that the 1-step transition probabilities determine the n-step transition probabilities, for any value of n. This fact is contained in what are known as the Chapman-Kolmogorov Equations.

The Chapman-Kolmogorov

Let $P_{ij}(n)$ denote the n-step transition probabilities

$$P_{ij}(n) = P(X_n = j / X_0 = i)$$

Let P^n denote that n-step transition probability matrix whose (i, j)th entry is $P_{ij}(n)$

Then $P^{m+n} = P^m P^n$ which implies that

$$P_{ij}(m+n) = \sum_{k \in s} P_{ik}(m) P_{kj}(n)$$

That is, the probability of going from *i* to *j* in *m*+*n* steps is the sum over all *k* of the probability of going from *i* to *k* in m steps, then form *k* to *j* in n steps. The Chapman–Kolmogorov equations explain the fact that the 1-step transition probabilities determine the n-step transition probabilities.

According to Karush Jack (1961) of the University of California, Berkeley who wrote an article on ‘The Chapman-Kolmogorov equation’ gave a partial answer to the question of whether every Markov random function comes from a system of transition probabilities satisfies the Chapman-Kolmogorov equation. He said a given Markov random function determines the transition probabilities up to sets of probability zero and for any choice of the transition probabilities the Chapman-Kolmogorov equation holds up to sets of probability zero. The problem then is one of selecting appropriate versions of the transition probabilities so that the Chapman- Kolmogorov equation holds everywhere. It can shown that such selections exist whenever the time parameter set is countable whenever the joint distribution of any two of the random variables is absolutely continuous with respect to the product of the marginal distributions. Although the latter condition is always satisfied when the state space is countable, or more generally, when each random variable assumes a countable number of values with probability one, this case, being especially sample, is treated separately. The results are based on exploiting the device of using the marginal distribution when in doubt about what the conditional probability distribution should be.

Steady-State Probabilities Theorem

Let P be the transition matrix for an s-state ergodic chain. Then there exists a vector $\pi = [\pi_1 \pi_2 .. \pi_s]$ such that

$$\lim_{n\to\infty} P^n = \begin{bmatrix} \pi_1 & \pi_2 & . & . & . & \pi_s \\ \pi_1 & \pi_2 & . & . & . & \pi_s \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ \pi_1 & \pi_2 & . & . & . & \pi_s \end{bmatrix}$$

Recall that the ij^{th} element of P^n is $P_{ij}(n)$.

The above theorem tells us that, for any initial state $\lim_{n\to\infty} P_{ij}(n) = \pi_j$

Observe that for large n, P^n approaches a matrix with identical rows. This means that after a long time, the Markov chain settles down, and (independent of the initial state *i*) there is a probability π_j that we are in state j. That vector

$\pi = [\pi_1 \pi_2 .. \pi_s]$ is often called the steady-state distribution, or equilibrium distribution for the Markov chain.

From the theorem above, it can be observed that for large n and all i,

$$P_{ij}(n+1) \cong P_{ij}(n) \cong \pi_j \qquad (1)$$

Since $P_{ij}(n+1) =$ (row i of P^n) (column j of P)

We may write

$$P_{ij}(n+1) = \sum_{k=1}^{k=s} P_{ik(n)} P_{kj} \qquad (2)$$

If n is large, substituting (1) into (2) yields

$$\pi_j = \sum_{k=1}^{k=s} \pi_k P_{kj} \qquad (3)$$

In matrix form, the equation (3) may be written as

$$\pi = \pi P \qquad (4)$$

Unfortunately, the system of equations specified in (3) has an infinite number of solutions, because the rank of the P matrix always turns out to be less than s-1.

To obtain unique values of the steady–state probabilities, it should that noted that for any n and any i,

$$P_{i1}(n) + P_{i2}(n) + ... + P_{is}(n) = 1 \qquad (5)$$

Letting n approach infinity in (5), we obtain

$$\pi_1 + \pi_2 + ... + \pi_s = 1 \qquad (6)$$

Thus, after replacing any of the equations in (4) by (6), we may use (4) to solve for the steady-state probabilities.

Mean First Passage Times

For an ergodic chain, let M_{ij} = expected number of transitions before we first reach state j , given that we are currently in state i , M_{ij} is called the mean first passage time from state *i* to state *j* . Assume that we currently in state *i*. Then with probability P_{ij} , it will take one transition to go from state *i* to state *j*. For $k \neq j$, we next go with probability P_{ik} to state *k*. In this case, it will take an average of $1 + M_{kj}$ transitions to go from *i* to *j*. This implies that

$$M_{ij} = P_{ij}(1) + \sum_{k \neq j} P_{ik}(1 + M_{kj})$$

$$\text{Since } P_{ij} + \sum_{k \neq j} P_{ik} = 1$$

We may rewrite the last equation as

$$M_{ij} = 1 + \sum_{k \neq j} P_{ik} M_{kj} \qquad (7)$$

By solving equation (7), we may find all the mean first passage times. It can be shown that $M_{ii} = \dfrac{1}{\pi_i}$

This can simplify the use of (7).

The equations below show the mean first passage times for the four-state matrix for Tamale, Kumasi and Accra. Because the four-state matrix will involve

16 equations with 16 unknown variables, the MATLAB software will be used to determine the mean first passage times.

Mean First Passage Times for Two-State Ergodic Chain

The analysis using the mean first passage times for a two-state chain will enable us to determine the average number of rainfall or number of sunshine duration tomorrow with respect to the average number of rainfall or number of sunshine duration today.

The equations below show how the mean passage time is calculated for a two-state chain.

Since $M_{ij} = 1 + \sum_{k \neq j} P_{ij} M_{kj}$

For i=1; j=1; k=2

$$M_{11} = 1 + P_{12} M_{21}$$

For i= 1; j= 2; k=1

$$M_{12} = 1 + P_{11} M_{12}$$

For i=2; j=2; k=1

$$M_{22} = 1 + P_{11} M_{12}$$

For i= 2; j= 1; j = 2

$$M_{21} = 1 + P_{22} M_{21}$$

Also $M_{11} = \dfrac{1}{\pi_{11}}$ $\qquad M_{22} = \dfrac{1}{\pi_{22}}$

Mean First Passage Times for Four-State Ergodic Chain

The equations below show how the mean first passage time is calculated.

$$M_{ij} = 1 + \sum_{k \neq j} P_{ij} M_{kj}$$

For, *i*=1; *j*=1; *k*=2, 3, and 4:

$$M_{11} = 1 + P_{12} M_{21} + P_{13} M_{31} + P_{14} M_{41}$$

For, *i*=1; *j*=2; *k*=1, 3, and 4:

$$M_{12} = 1 + P_{11} M_{12} + P_{13} M_{32} + P_{14} M_{42}$$

For, *i*=1; *j*=3; *k*=1, 2, and 4:

$$M_{13} = 1 + P_{11} M_{13} + P_{12} M_{23} + P_{14} M_{43}$$

For, *i*=1; *j*=4; *k*=1, 2, and 3:

$$M_{14} = 1 + P_{11} M_{14} + P_{12} M_{24} + P_{13} M_{34}$$

For *i*=2; *j*=1; *k*=2, 3, and 4:

$$M_{21} = 1 + P_{22} M_{21} + P_{23} M_{31} + P_{24} M_{41}$$

For, *i*=2; *j*=2; *k*=1, 3, and 4:

$$M_{22} = 1 + P_{21} M_{12} + P_{23} M_{32} + P_{34} M_{42}$$

For, *i*=2; *j*=3; *k*=1, 2, and 4:

$$M_{23} = 1 + P_{21} M_{13} + P_{22} M_{23} + P_{24} M_{43}$$

For, *i*=2; *j*=4; *k*=1, 2, and 3:

$$M_{24} = 1 + P_{21} M_{14} + P_{22} M_{24} + P_{23} M_{34}$$

For, *i*=3; *j*=1; *k*=2, 3, and 4:

$$M_{31} = 1 + P_{32} M_{21} + P_{33} M_{31} + P_{34} M_{41}$$

For, *i*=3; *j*=2; *k*=1, 3, and 4:

$$M_{32} = 1 + P_{31}M_{12} + P_{33}M_{32} + P_{34}M_{42}$$

For, i=3; j=3; k=1, 2, and 4:

$$M_{33} = 1 + P_{31}M_{13} + P_{32}M_{23} + P_{34}M_{42}$$

For, i=3; j=4; k=1, 2, and 3:

$$M_{34} = 1 + P_{31}M_{14} + P_{32}M_{24} + P_{33}M_{34}$$

For, i=4; j=1; k=2, 3, and 4:

$$M_{41} = 1 + P_{42}M_{21} + P_{43}M_{31} + P_{44}M_{41}$$

For, i=4; j=2; k=1, 3, and 4:

$$M_{42} = 1 + P_{41}M_{12} + P_{43}M_{32} + P_{44}M_{42}$$

For, i=4; j=3; k=1, 2, and 4:

$$M_{42} = 1 + P_{41}M_{13} + P_{42}M_{23} + P_{44}M_{43}$$

For, i=2; j=3; k=1, 2 and 4:

$$M_{44} = 1 + P_{41}M_{14} + P_{42}M_{24} + P_{43}M_{34}$$

Also $M_{11} = \dfrac{1}{\pi_1}$ $\quad M_{22} = \dfrac{1}{\pi_2}$ Also $M_{33} = \dfrac{1}{\pi_3}$ $\quad M_{44} = \dfrac{1}{\pi_4}$

To determine the Mean First Passage Time for the weather of each of the three regional capitals (Tamale, Kumasi and Accra) the MATLAB software were used to compute for each month by solving as system of equations.

Other Statistical Methods that can be used to Model the data

Apart from Markov chains, there are other statistical methods that can be used in this research. Some are Maximum likelihood, Monte Carlo Method, ARIMA-Time series and Bayesian Estimation.

The method of Maximum Likelihood

In two papers published by R.A.Fisher, the prominent statistician proposed a general method of estimation called the method of maximum likelihood. He also demonstrated the advantages of this method by showing that it yields sufficient estimators whenever they exist and that maximum likelihood estimators are asymptotically minimum variance estimators.

To help understand the principle on which the method of maximum likelihood is based, suppose that four letters arrive in somebody's morning mail, but unfortunately one of them is misplaced before the recipient has a chance to open it. If, among the remaining three letters, two contain credit card billings and the other one does not, what might be a good estimate of k, the total number of credit-card billings among the four letters received? Clearly, k must be two or three, and if we assume that each letter had the same chance of being misplaced, we find that the probability of the observed data (two of the remaining letters contain credit-card billings) is

$$\frac{\binom{2}{2}\binom{2}{1}}{\binom{4}{3}} = \frac{1}{2}$$

for k = 2 and

$$\frac{\binom{3}{2}\binom{1}{1}}{\binom{4}{3}} = \frac{3}{4}$$

for k = 3. Therefore, if we choose as our estimate of k the value which maximizes the probability of getting the observed data, we obtain k = 3.We call this estimate a maximum likelihood estimate, and the method by which it is obtained, the method of maximum likelihood.

Definition: If $x_1, x_2, \dots, x_n$ are the values of a random sample from a population with the parameter θ, the likelihood function of the sample is given by $L(\theta) = f(x_1, x_2, \dots, x_n; \theta)$ for values of θ within a given domain. Here $f(x_1, x_2, \dots, x_n; \theta)$ in the value of the joint probability density of the random variables $X_1, X_2, \dots, X_n$ at $X_1 = x_1, X_2 = x_2, \dots, X_n = x_n$

Thus, the method of maximum likelihood consists of maximizing the likelihood function with respect to θ, and we refer the value of θ which maximizes the likelihood function as the maximum likelihood estimate of θ.

Case 1: Given x 'successes' in n trails, and we want to find the maximum likelihood estimate of the parameter θ of the corresponding binomial distribution. Firstly, we find the value of θ which maximizes $L(\theta) = \binom{n}{x}\theta^x(1-\theta)^{n-x}$ it will be convenient to make use of the fact that the value of θ which maximizes $L(\theta)$ will also maximize In L(θ) =In$\binom{n}{x} + x$.Inθ + (n-x).In(1-θ)

Thus, we get $\frac{d[In\ L(\theta)]}{d\theta} \equiv \frac{x}{\theta} - \frac{n-x}{1-\theta}$.

Secondly, equating this derivative to 0 and solving for θ, we find that the likelihood function has a maximum at $\theta = \frac{x}{n}$. This is the maximum likelihood estimate of the binominal parameter θ, and we refer to $\hat{\theta} = \frac{x}{n}$ as the corresponding maximum likelihood estimator.

Case 2: If $x_1, x_2, .., x_n$ are the values of a random sample from an experimental population, and we are to find the maximum likelihood estimator of its parameter θ.

Firstly, since the likelihood function is given by

$$L(\theta) = f(x_1, x_2, \dots, x_n; \theta)$$

$$= \prod_{i=1}^{n} f(x_i ; \theta)$$

$$= \left(\frac{1}{\theta}\right)^n . e^{\frac{1}{\theta}(\sum_{i=1}^{n} x_i)}$$

differentiation of In $L(\theta)$ with respect to θ yields

$$\frac{d[In\ L(\theta)]}{d\theta} = \frac{-n}{\theta} + \frac{1}{\theta^2} \sum_{i=1}^{n} x_i$$

Secondly, equating this derivative to zero and solving for θ, we get the maximum likelihood estimate $\hat{\theta} = \frac{1}{n} . \sum_{i=1}^{n} x_i = \bar{x}$

Hence, the maximum likelihood estimator is $\hat{\theta} = \bar{x}$

Case 3: If $X_1, X_2, \dots, X_n$ constitute a random sample from a uniform population with $\alpha = 0$, we can be shown that largest sample value (that is, the nth order statistic, Y_n) is a biased estimator of the parameter β. Also, we can modify this estimator β to make it unbiased.

Firstly, substituting into the formula for

$$g_n(y_n) = n.f(y_n)[\int_{-\infty}^{y_n} f(x)dx]^{n-1} \text{ for } -\infty < y_n < \infty \text{ we find that}$$

the sampling distribution of Y_n is given by

$$g_n(y_n) = n.\frac{1}{\beta}(\int_0^{y_n} \frac{1}{\beta}dx)^{n-1}$$

$$= \frac{n}{\beta^n} y_n^{n-1} \qquad \text{for } 0 < y_n < \beta \text{ and } \quad g_n(y_n) = 0 \text{ elsewhere, and}$$

hence that $E(Y_n) = \frac{n}{\beta^n}\int_0^{\beta} y_n^n \, dy_n$

$$= \frac{n}{n+1}\beta$$

Thus, $E(Y_n) \neq \beta$ and the n^{th} order statistic is a biased estimator of the parameter β. However, since $E\left(\frac{n+1}{n}.Y_n\right) = \frac{n+1}{n}.\frac{n}{n+1}.\beta$

$$= \beta$$

It follows that $\frac{n+1}{n}$ times the largest sample value is an unbiased estimator of the parameter β.

Finally, from the results above we can say that maximum likelihood estimators need not be unbiased.

Monte Carlo method

A Monte Carlo method is a technique that involves using random numbers and probability to solve problems. The term Monte Carlo method was coined by S. Ulam and Nicholas Metropolis in reference to games of chance, a popular attraction in Monte Carlo, Monaco (Hoffman, 1998; Metropolis and Ulam, 1949).

Monte Carlo simulation is a method for iteratively evaluating a deterministic model using sets of random numbers as inputs. This method is often used when the model is complex, nonlinear, or involves more than just a couple uncertain parameters.

A Simulation can typically involve over 10000 evaluations of the model, a task which in the past was only practical using super computers.

By using random inputs, you are essentially model into a stochastic model.

The Monte Carlo method is just one of many methods for analyzing uncertainty propagation, where the goal is to determine how random variation, lack of knowledge, or error affects the sensitivity, performance, or reliability of the system that is being modelled. Monte Carlo simulation is categorized as a sampling method because the inputs are randomly generated from probability distributions to simulate the process of sampling from an actual population. So we try to choose a distribution for the inputs that most closely matches data we already have, or best represents our current state of knowledge. The data generated from the simulation can be represented as probability distributions (or histograms) or converted to error bars, reliability predictions, tolerance zones, and confidence intervals.

STEPS: The Steps in Monte Carlo simulation corresponding to the uncertainty propagation are fairly simple, and can be easily implemented in Excel for simple models. All we need to do is to follow the five simple steps listed below:

Step 1: Create a parametric model, $y = f(x_1, x_2, .., x_q)$,

Step 2: Generate a set of random inputs, $X_{i1}, X_{i2}, ..., X_{iq.}$

Step 3: Evaluate the model and store the results as Y_i.

Step 4: Repeat steps 2 and 3 for $i = 1$ to n

Step 5: Analyze the results using histograms, summary statistics, confidence intervals, etc.

Monte Carlo Sampling: Random variations are represented using probability distributions. The procedure for generating random variables from given probability distributions is known as random variable generation or Monte Carlo Sampling. The principle of sampling is based on the frequency interpretation of probability and requires a steady stream of random numbers. We generate random numbers for this procedure using congruential methods. The most commonly used of the methods is the linear congruential method. Random numbers generated from a linear congruential generator use the following relation.

$$x_{i+1} = (ax_{i+1} + c) \text{ modulo m (i=0,1,2,…)}$$

This gives us the remainder from the division of $(ax_{i+1} + c)$ by m. The random numbers are delivered using the relation $R_i = \frac{x_i}{m} (i = 1,2,3,...)$.

For discrete distributions Monte Carlo sampling is achieved by allowing ranges of the random numbers according to the probabilities in the distribution.

For continuous distributions, we generate random variables using one several algorithms, using the inverse transformation method and the acceptance-rejection method. The inverse transformation method requires a cumulative distribution function (c.d.f.) in closed form and consists of the following steps:

Step 1: Given a probability density function *f(x),* develop the cumulative distribution function as $F(x) = \int_{-\infty}^{x} f(t)dt$

Step 2: Generate a random number r.

Step 3: Set *F(x)* = *r* and solve for *x*. The variable *x* is then a random variate from the distribution whose probability density function (p.d.f.) is given by *f(x)*.

The acceptance-rejection method requires the p.d.f. to be defined over a finite interval. Thus, given a probability density function *f(x)* over the interval $a \leq x \leq b$, we execute the acceptance-rejection algorithm as follows:

Step 1: Select a constant M such that M is the largest value of *f(x)* over the interval [a, b].

Step 2: Generate two random numbers, r_1 and r_2.

Step 3: Compute $x^* = a + (b - a)r_1$.

Step 4: Evaluate the function f(x) at the point x^*. Let this be $f(x^*)$.

Step 5: If $r_2 < \frac{f(x^*)}{M}$, deliver x^* as a random variate. Otherwise, reject x^* and go back to step 2.

Between these two methods, it is possible to generate random variates from almost all of the commonly used distributions. The one exception is the normal distribution. For the normal distribution, we generate random variates directly by transforming the random numbers r_1 and r_2 into standardized variates,

$$Z_1 = (-2\, In\, r_1)^{1/2} \sin 2\pi r_2 \quad and \quad Z_2 = (-2\, In\, r_1)^{1/2} \cos 2\pi r_2$$

ARIMA-Time Series

The concept of time series is a time dependent sequence $Y_1, Y_2 \ldots Y_n$ or $\{Y_t\}, t \varepsilon N$ where 1, 2…..N denote time steps.

If a time series can be expressed as a known function then, it is said to be a deterministic time series, that is $Y_t = F(t)$

If a time series can be expressed as $Y_t = X(t)$, where X is a random variable then $\{Y_t\}$ is a stochastic time series.

There are several possible objectives analyzing a time series. These objectives may be classified as description, explanation prediction and control.

Traditional methods of time series analysis are mainly concerned with decomposing the variation in a series into the various components of trend, periodic and stochastic.

If $Y_t = Y_{t+T} + e_t \quad \forall t \in N$ then the time series has a periodic component of period T.

If $Y_t = y + \beta_t + e_t$ then there exists a linear trend with the slope being β.

If $Y_t = y + e_t$ the e_t is the stochastic component of the time series

A time series is said to be strictly stationary if the joint distribution of $X_{t1} \ldots X_{tn}$ is the same as the joint distribution of $X_{t1+T} \ldots X_{tn+T}$ for all $t_{1+T} \ldots t_{n+T}$

In other words, shifting the time origin by an amount T has no effects on the joint distributions, which must therefore depend only on the intervals between $t_1 \ldots t_n$.

If there is trend in the mean then differencing the time series data will remove the trend and seasonality will be achieved. For non- seasonal data, first order differencing is usually sufficient to attain seasonality, so that the new series.

$Y_1 Y_2 \ldots\ldots Y_{n-1}$ is formed from the original series

$$\{y_t = X_{t+1} - X_t = \nabla X_{t+1}\}$$

First- order difference is widely used. Occasionally second- order differencing is required using the order, ∇^2 where

$$\nabla^2 X_{t+2} = \nabla X_{t+2} - \nabla X_{t+1} = X_{t+2} - 2X_{t+1} + X_t$$

If there is a trend in variance, the series is made stationary by transforming the data as follows.

$$Y_t = U_t \qquad \text{where } U_t = \log X_t$$

Autoregression

An Autoregression is a regression where the right- hand side variables are merely the values of the dependents variable in provision periods; they are time-logged observations. A complete random or white noise series has no discernible pattern or structure. By measuring and examining auto correlations for time tags of more than one period, evidence is provided on how values of a given series are related.

If a time series is stationary, we can easily model it via an equation with fixed coefficients estimated from horizontal observations. A non stationary is one where the structural relationships of the model change over time. If a nonstationary series is differenced one or more times and the differenced series become stationary, the series is said to be homogeneous.

The autocorrelation function is extremely useful for describing the generating process used to develop a forecasting model. First-order autocorrelation is the autocorrelation of a time series in period t with time series observation in period t-1. Similarly, autocorrelation of higher orders refers to correlation with periods t-2, t-3 etc.

Important questions to ask about a time series are as follows

1. Are the data random? Randomness
2. Are the data non stationary? If non stationary, the level of differencing at which the series become stationary.
3. Are the data seasonal? If seasonal, the length of seasonality

Knowledge of these properties will permit us to produce better forecast.

Autocorrelation Function

Autocorrelation function measures the degree of correlation between neighboring observations in a time series.

The autocorrelation function at lag k is defined as

$$\rho_k = \frac{E[(Y_t - \mu_Y)(Y_{t+k} - \mu_Y)]}{[E(Y_t - \mu_Y)^2 (Y_{t+k} - \mu_Y)^2]}$$

$$\rho_k = \frac{\operatorname{cov}(Y_{t,} Y_{t+k})}{\sigma_{Y+K}}$$

The autocorrelation coefficient is estimated from sample observations using the

formula $$r_K = \frac{\sum_{t=2}^{n} (Y_t - Y_t)(Y_{t+k} - Y_{t+k})}{\sum_{t=1}^{n} (Y_t - Y_t)^2}$$

Partial Autocorrelation Function

Partial Autocorrelation function measures the degree of association between Y_t and Y_{t+k} when the effects of other time lags on Y are held constant. We study partial autocorrelation when we are unaware of the appropriate order of the autoregressive process to fit the time series.

The partial auto correlation function PACF denoted by $\{\Phi_{kk} : k = 1,2,...\}$ the set of partial auto correlation at various lags are defined by

$$\Phi_{kk} = \frac{|P_k^*|}{|P_k|}$$

where P_k is the k x k auto correlation matrix and P_k^* is P_k with the last column replaced by $[\rho_1 \rho_2 ... \rho_k]^T$ and

$$\rho_k = \begin{bmatrix} 1 & \rho_1 & \rho_2 & \rho_{k-1} \\ \rho_1 & 1 & \rho_1 & \rho_{k-2} \\ . & . & . & . \\ . & . & . & . \\ . & . & . & . \\ \rho_{k-1} & \rho_{k-2} & \rho_{k-3} & 1 \end{bmatrix}$$

And estimates of $\phi_{11} = \phi_1 = \rho_1$ and $\phi_{22} = \dfrac{\begin{vmatrix} 1 & \rho_1 \\ \rho_1 & \rho_2 \end{vmatrix}}{\begin{vmatrix} 1 & \rho_1 \\ \rho_1 & 1 \end{vmatrix}} = \dfrac{\rho_2 - \rho_1^{\,2}}{1 - \rho_1^{\,2}}$ and estimates of ϕ_{kk} can be obtained by replacing the ρ_1 by r_1.

Moving Average Models

A moving-average (MA) model provides predictions based on a linear combination of past forecast errors and thus is similar to experimental smoothing models. The order of the MA model refers to the number of parameters of the model. If the autocorrelation pattern in the time series residual requires a more complete model, a mixed model can be estimated. An auto regressive moving-average (ARMA) model of order (p, q) has auto correlation that diminish as the distance between residuals increase. Also, the patterns in the time series that can be described by ARMA processed are more general than those of either AR (p) or MA (q) models.

Identification of AR (p), MA (q) and mixed ARMA (p,q) time series requires the computation and plotting of the sample auto correlation function (ACF) (coefficients) of the time series data. Of the sample ACFs and PAFCFs for a time series appear to be generated by a particular time series model, then it is likely model generated the time series data.

Examining plots of theoretical models will enable for actual time series to be identified better. Through careful examination of the ACF plots we can discern the true underling ARIMA process that gives rise to the time series. For example, the influence of the constant term can be found when the ACF and PACF are drawn.

Differencing of time series for the purposes of identification Forecasters need only to identify the lowest level of differencing for which a stationary model is apparent seasonal time series often exhibit non stationary. In these situations, a forecaster must transform the time series to a stationary one by a mathematical transformation. One very useful transformation is natural logarithms.

AR (p) model cannot isolate certain data pattern when P is small. However, an alternative model the moving average model may isolate the pattern when AR (p) models fail.

Word (1954) showed that any discrete time series can be expressed as an AR model, an MA model, or a combination called an ARMA model.MA models provide predictions of Y_t based on linear combinations of past forecast errors. In contrast, AR model express Y_t as a linear function of actual past values of Y_t.

The general MA model is $Y_t = \mu + e_t - \theta_i e_{t-1} - \theta_e e_{t-2} - \cdots - \theta_q e_{t-q}$

$$If\ \mu = 0, we\ have$$

$$Y_t = e_t - \theta_1 e_{t-1} - \theta_2 e_{t-2} - \cdots - \theta_q e_{t-q}$$

Autoregressive Integrated Moving Average Model (ARIMA)

ARIMA models can be purely seasonal in that only seasonal parameters are present examples include SAR, SMA and seasonal ARIMA models where the model includes parameters for the seasonal terms. Identification of purely seasonal models is similar to the identification process.

For purely seasonal models only coefficients at the seasonal lags are examined. If a non-stationary time series which has variation in the mean is differenced to remove the variation the resulting time series is called an integrated time series. It is called an integrated model because the stationary model which is fitted to the differenced data has to be summed or integrated to provide a model for the non- stationary data.

Notational, all AR (p) and MA (q) models can be represented as ARIMA models: for example an AR (1) can be represented as ARIMA (1,0,0); that is no differencing and no MA part.

The general model is ARIMA (p,d,q), where p is the order of the AR part, d the degree of differencing and q the order of the MA part

Writing, $W_t = \nabla^d Y_t = (1-B)^d Y_t$

the general ARIMA process is of the form

$$W_t = \sum_{i=1}^{p} \alpha_t W_{t-1} + \sum_{j=1}^{q} \theta_t e_{t-1} + \mu + e_t$$

An example of ARIMA (p,d,q,) is the ARIMA (1,1,1) which has one autoregressive parameter, one level of differencing and one MA parameter is given by

$$W_t = \alpha_1 W_{t-1} + \theta_1 e_{t-1} + \mu + e_t$$
$$(1-B)Y_t = \alpha_1 (1-B) Y_{t-1} + \theta_1 e_{t-1} + \mu + e_t$$

which can be simplified further as

$$Y_t - Y_{t-1} = \alpha_1 Y_{t-1} - \alpha_1 Y_{t-2} + \theta_1 e_{t-1} + \mu + e_t$$
$$Y_t - Y_{t-1} = \alpha_1 (Y_{t-1} - Y_{t-2}) + \theta_1 e_{t-1} + \mu + e_t$$

Autoregressive Moving Average

A more general model is a mixture of the AR (p) and MA (q) models and is called an Autoregressive moving average model (ARMA) model.

$$Y_t = \mu + \emptyset_1 Y_{t-1} + \emptyset_2 Y_{t-2} + \cdots + \emptyset_p Y_{t-p} + e_t - \theta_1 e_{t-1} - \theta_2 e_{t-2} - \cdots - \theta_q e_{t-q}$$

$If\ \mu = 0,\ then\ we\ have$

$$Y_t = \emptyset_1 Y_{t-1} + \emptyset_2 Y_{t-2} + \cdots + \emptyset_p Y_{t-p} + e_t - \theta_1 e_{t-1} - \theta_2 e_{t-2} - \cdots - \theta_q e_{t-q}$$

Like the AR (p) model, the ARMA (p, q) model has autocorrelation that diminish as that distance between residuals increases. However, the patterns in the time series that can be described by ARMA (p, q) processes are more general than those of either AR (p) or MA (q) models.

$Y_t = \mu + \emptyset_1 Y_{t-1} + e_t - \theta_1 e_{t-1}$ ARIMA (1, 0, 1)

$Y_t = \mu + \emptyset_1 Y_{t-1} + \emptyset_2 Y_{t-2} + e_t - \theta_1 e_{t-1}$ ARIMA (2, 0, 1)

It can be shown that the mixed ARIMA (1, 1) model is stationary if $-1 < \emptyset_1 < 1$ and is invertible if $-1 < \emptyset_1 < 1$.

The constant model is $\frac{\mu}{(1-\emptyset_1)}$

The order of the model is Qs

For a mixed seasonal autoregressive moving average model, both SAR and SMA parameters are used in the same way that non seasonal AR and MA parameters are used. A mixed SAR and SMA model is written as follows

$$Y_t = \emptyset_s Y_{t-s} + \cdots + \emptyset_{ps} Y_{t-ps} - \theta_s e_{t-s} - \cdots - \theta_{es} e_{t-Qs} + e_t$$

The order of the seasonal ARMA model is expressed in terms of both Ps and Qs. The identification of seasonal parameters is accomplished by the plotting and careful examination of the autocorrelation and partial autocorrelation of the stationary time series. First the autocorrelation patterns associated with purely seasonal analogous to those for nonseasonal models. The only difference is that none zero autocorrelations that forms the patterns occur at lags that are multiples of the number of periods per season.

For purely SAR models, the autocorrelation die down and partial autocorrelation cut off after one seasonal lag of an SAR (2) model. Similarly the partial autocorrelation die down for SMA models. Also the autocorrelation cut-off after one lag of an SMA (1) model and after two lag for an SMA (2) model. Finally for a mixed model with one SAR and one SMA parameter, both the theoretical correlation function and the partial autocorrelation function die down.

In Table 1, the stationary and Invertibility conditions of specific time series models and the behaviour of their theoretical ACF and PACF functions are summarized.

Table 1: Stationary and Invertibility conditions of Specific Time Series

Model	Stationary condition	Invertibility conditions	Auto correlation coefficients	Partial autocorrelation coefficients
ARIMA(1,d,0)	$-1 < \phi_1 < 1$	none	dies down	cuts off after one lag
ARIMA(2,d,0)	$\phi_1 + \phi_2 < 1$ $\phi_1 - \phi_2 < 1$ $-1 < \theta_2 < 1$	none	dies down	cuts off after two lags
ARIMA(0,d,1)	none	$\theta_1 < 1$	cuts off after one lag	dies down
ARIMA(0,d,2)	none	$\theta_1 + \theta_2 < 1$ $\theta_2 - \theta_1 < 1$ $\theta_2 < 1$	cuts off after two lags	dies down
ARIMA(1,d,1)	$-1 < \theta_1 < 1$	$-1 < \theta_1 < 1$	dies down	dies down

It can be seen in the table 1 that the theoretical ACFs of both AR (1) and R (2) processes can die down in a damped exponential manner. Hence it is difficult to distinguish between AR (1) and AR (2) models without examining the sample PACF(s).

To whether distinguish between AR (1) and AR (2) models, it is important to determine sample PACF(s) after one of two lags.

In a similar way, we cannot look at only the sample PACF(s) of MA processes to distinguish between an MA(1) and an MA (2) process. Only by examining the behaviour of the sample ACF(s) to see when they cut off can we distinguish between them.

In general, to identify the particular AR (p) MA (q) or ARIMA (p,q) process which generated the time series, we must examine the behaviour of both the sample ACFs and the sample PACFs. We must try to determine whether the function is decaying or cuts off. Finally the number of lags of coefficients examined must be large enough to identify whether a particular pattern in the coefficients dominates the movements. In general, at least ten lags should be examined.

The seasonal part of an ARIMA model has the same structure as the non-seasonal part; it may have an AR factor, an MA factor, and or an order of differencing. In the seasonal part of the model, all of these factors operate across multiples of lags (the number of periods in a season).

A seasonal ARIMA model classified as an ARIMA (p,d,q) x (P,D,Q) model,

where P = number of seasonal autoregressive (SAR) terms.

D = number of seasonal differences

Q = number of seasonal moving average (SMA) terms.

In identifying a seasonal model, the first step is to determine whether or not a seasonal difference is needed, in addition to or perhaps instead of a non-seasonal difference. One should look at time series plots and ACF and PACF for all possible combination of zero and one non- seasonal difference and zero or one seasonal difference.

Box-Jenkins Method and ARIMA Modeling

The Box-Jenkins Method of Modelling Time Series is a statistically sophisticated way of analyzing and building a forecasting model which best represents a time series. This technique has a number of advantages over other methods of time series analysis.

Firstly it is logical and statistically accurate. Secondly, the method extracts a great deal of information from the historical time series data. Finally, the method result in an increase in forecast accuracy while keeping the number of parameters to a minimum in comparison with similar processes. At least 50 observations are usually required for Box- Jenkins estimation. Even more observation is recommended for a seasonal model.

Stage 1: Identification

The first stage is the identification of the appropriate ARIMA models through the study of the Autocorrelation and Partial Autocorrelation functions for raw data. When the autocorrelations is very large at first and do not trail off towards zero quickly. Secondly, when they appear to be forming a sine wave pattern, but because the damping process is so slow, we can conclude that the process is nonstationary. Upon differencing to achieve stationary ARIMA process forecasting, new diagrams of autocorrelation coefficients and partial autocorrelation coefficient are drawn. When these autocorrelations rapidly trail off towards zero and the partial autocorrelations out off after lag 1 then, both these patterns would indicate an ARIMA (p,r,q) model. However, this is only a tentative choice.

The purpose of the identification phase is to choose a specific ARIMA model from the general class of ARIMA (p, q) models donated as

$$Y_t = \emptyset_1 Y_{t-1} + \emptyset_2 Y_{t-2} + \cdots + \emptyset_p Y_{t-1} + e_t - \theta_1 e_{t-1} - \theta_2 e_{t-2} \qquad (8)$$

The selection of the appropriate p and q values requires examining the autocorrelation and partial autocorrelation coefficient calculated for the data

If in equation 8, we let q=0 and p=0, 1, 2, 3…p

Consecutive, equation 8 then becomes the equations stated below

$$Y_t = e_t$$

$$Y_t = \emptyset Y_{t-1} + e_t$$

$$Y_t = \emptyset_1 Y_{t-1} + \emptyset_2 Y_{t-2} + e_t$$

$$Y_t = \emptyset_1 Y_{t-1} + \emptyset_2 Y_{t-2} + \emptyset_3 Y_{t-3} + e_t$$

When the time order of equation (8) is p=0, the parameter $\emptyset_1$ will have a value that is not statistically different from zero. Thus the result would be an AR (0) process and $\emptyset_1 = 0$. Alternatively, if the true order is p=1, $\emptyset_2$ will not be statistically different from zero. Finally, in general, the p^{th} parameter of an AR (p) process will only be statistically different from zero when the auto regressive (AR) process is at least of order p or higher. Identically the order of an AR process can be done by examining its partial autocorrelation coefficient. The order is simply the same as the number of partial autocorrelation statistically different from zero. The partial autocorrelation up through p time lags will be statistically significant, while the remaining coefficients will be approximately equal to zero. This resulting value of p will be the order of the AR process.

Stage 2: Estimation

Once the preliminary model is chosen, the estimation stage begins. The estimates must be of the following model:$(1-B)(1-\emptyset_1 B)Y_t = e_t$ where the $(1-B)Y_t$ are the differences of the original values expressed in terms of deviations? The purpose of estimates is to find the parameter estimates that minimize the mean square error (MSE). The process is iterative, and the final value of the parameter estimates may be significantly different from the initialized values of the estimation procedure. However, the estimates will usually converge on an optimal value for the parameters with a small number of iterations. ARIMA models can be can be fitted by least squares. An iterative non linear least squares procedure is applied to parameter estimates of an ARMA (p, q) model.

The method minimizes the sum of squares of errors,$\sum e_t^2$, given the form of the model and the data. This is the least squares method for fitting a modal to data. Since the procedure in general is non linear because of the moving average terms, the least squares process is non linear.

Stage 3: Diagnostic Testing

Before the model is used for forecasting, it should be checked for adequacy. This diagnostic checking is done by examining the error terms e_t to be sure that they are random. If the error terms are statistically different from zero, the modal is not considered adequate. If several auto-correlations are large we should return to the initial stage to select an alternative model, and then continue the analysis.

To check for adequacy the autocorrelations of the residuals are diagnostically examined by calculating an X^2 statistic

The test statistic is the Q statistic

$$Q = n(n+2)\sum_i^k \left(\frac{r_i^2}{n-k}\right)$$

Which is approximately distributed as a X^2 with k-p-q degrees of freedom.

In this equation, n is the length of the time series

k= the first k autocorrelation being checked

P= the order of the AR process

q= order of the MA process

r = the estimated autocorrelation coefficient of the i^{th} residual term

When the first k=24 autocorrelations are used for the test, the null and alternative hypothesis are as follows respectively:

H_0: the model is adequate

H_1: the model is not adequate

The number of degrees of freedom (d.f.) = k-p-q.

When the calculated value is less than the tabulated value we accept the null hypothesis at the 0.05 level and conclude that the model is adequate

If the calculated value of Q is greater than X^2 for k-p-q degrees of freedom, then the model should be considered inadequate. The forecaster should then return to selecting an alternative model and continue the Box-Jenkins analysis until a satisfactory model is found.

Both diagnostic procedures aid the analysis to arrive at a final forecasting model batter neither procedure can be considered the final word. For example, if some large deviation from the forecast adequately can be explained as unusual and unrepeatable circumstances, these deviations can be ignored.

Finally if two or more models are judged to be about equal although no model is an exact fit, the principle of parsimony should be prevail.

Stage 4: Forecasting

We forecast for five steps ahead using the arrived ARIMA model with any observation number as the starting value. It is also where the analysis uses the model chosen to forecast the process ends.

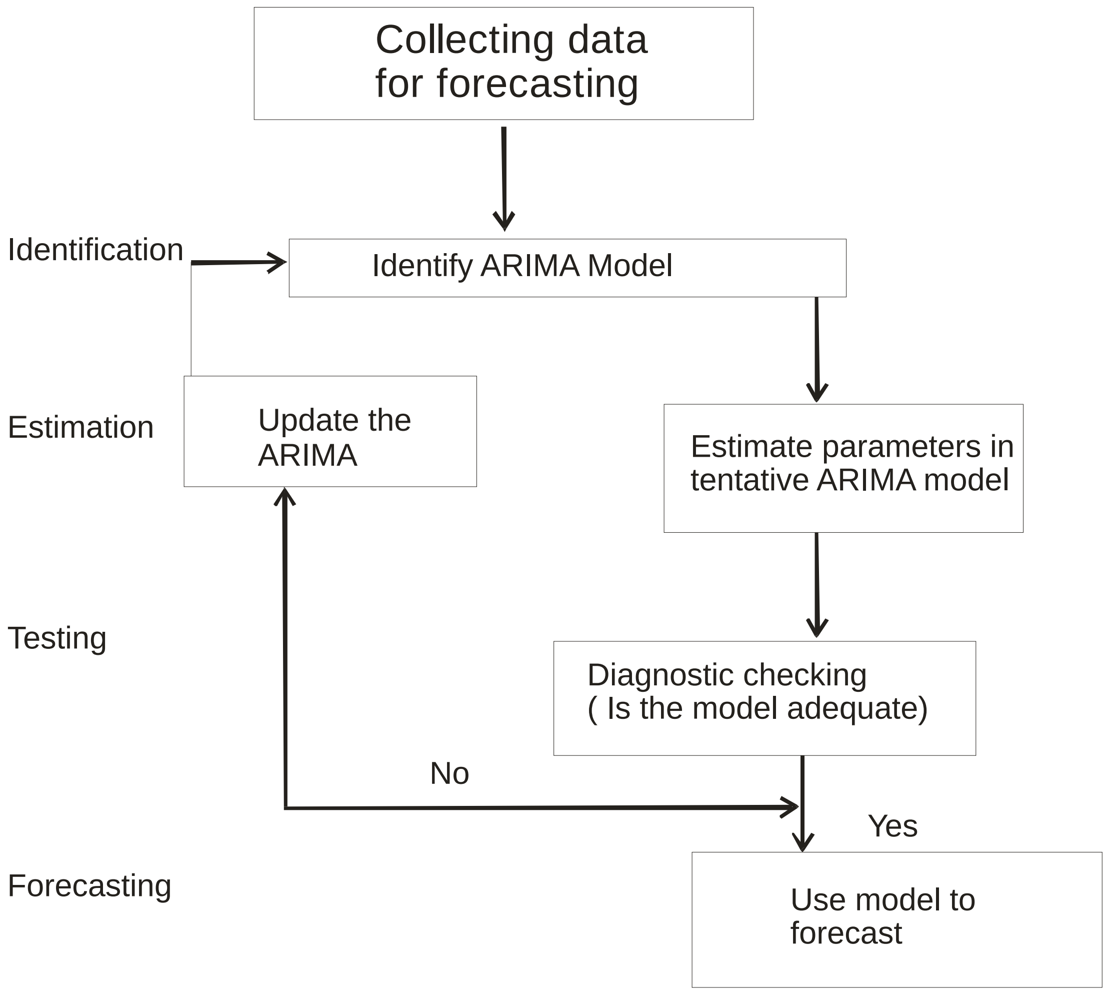


Figure 4: Schematic representation of Box- Jenkins Process

Tools for Arima Model Identification

Two tools for model identification were identified Akaike's information criteria and Schwarz's Bayesian information criterion.

Akaike's Information

The AIC which was proposed by Akaike uses sum of square error (SSE) method. In the implementation of the approach, a range of potential ARIMA models is estimated by sum of square error (SSE), and for each, the AIC is calculated given by

$In(SSE) + \frac{2K}{n}$ where n is the total number of observation

$k = p + q + P + Q + d + SD$

k = number of estimates + order of regular difference + product of seasonality and seasonal difference

Given two or more competing models the one with the smaller AIC value will be selected.

Schwarz's Bayesian Information Criterion(BIC)

Schwarz's BIC like AIC uses the sum of square error (SSE). It is given by

$In(SSE) + \frac{K\ In(n)}{n}$

where n is the total number of observation

$k = p + q + P + Q + d + SD$

k = number of estimates + order of regular difference + product of seasonality and seasonal difference

Bayesian Estimation

In Bayesian estimation the parameters are looked upon as random variables having prior distribution, usually reflecting the strength of ones belief about the possible values they can assume. The main problem of Bayesian estimation is that of combining prior feelings about a parameter with direct feelings about a parameter with direct sample evidence. This can be accomplished by determining $\varphi(\theta/x)$, the conditional density of θ given $X = x$. In contrast to the prior distribution of θ, this conditional distribution (which also reflects the direct sample evidence) is called the posterior distribution of θ.

In general, if h(θ) is the value of the prior distribution of θ at $small\ \theta$ and we want to combine the information which it conveys with direct sample evidence about θ, for instance, the value of a statistic $W = u(X_{1,}X_{2,} \dots, X_n)$ we determine the posterior distribution of $\theta$by means of the formula

$$\varphi(\theta/w) = \frac{f(\theta, w)}{g(w)} = \frac{h(\theta).f(w/\theta)}{g(w)}$$

Here $f(w/\ \theta)$ is the value of the sampling distribution of w given $\theta = small\ \theta$ at w, $f(\theta, w)$ is the value of the joint distribution of θ and W at $small\ \theta$ and w, and g(w) is the value of the marginal distribution of W at w. Note that the above formula for $\varphi(\theta/w)$ is, in fact, an extension of Bayes' theorem, which states that If $B_1, B_2, \dots, B_k$ constitute a partition of the sample space S and $P(B_i) \neq 0$ for $i=1,2,\dots k$ then for any event A in S such that $P(A) \neq 0$

$$P(B_r\ /A) = \frac{P(B_r).P(A/B_r)}{\sum_{i=1}^{k} P(B_i).P(A/B_i)}$$

for r =1,2,…,k to the continuous case. Hence, the term 'Bayesian estimation'. Once the posterior distribution of a parameter has been obtained, it can be used to make probability statements about the parameter.

CHAPTER THREE

PRELIMINARY ANALYSIS

Introduction

In this chapter, the various probabilities for each regional town will be transformed into a transition matrix. This will be used to analyze the short-run behavior of the Markov chain.

Two-State Transition Matrix Representation of Tamale weather

Analyzing Tamale weather for the five years, from 2003 to 2007, where the weather on the day depends on the previous day’s weather, for the months of January to December.

The table 1 describes the two-state transition matrix of Tamale weather.

From the table 1 below, state 1(RR) represents the probability of rainy today given that it rained yesterday. RS represents the probability of sunny today given that it rained yesterday. SR represents the probability of rainy today given that it was sandy yesterday. State (2) SS represents the probability of sunny today given that it was sunny yesterday.

Table 2: Two-State Transition Matrix of Tamale weather

	STATES PROBABILITY			
MONTH	RR	RS	SR	SS
January	0. 0060	0. 9940	0. 9136	0. 0864
February	0. 0050	0.9550	0. 9067	0.0933
March	0. 0320	0.9680	0.9190	0. 0809
April	0.0840	0.9160	0.9154	0.0846
May	0.1150	0.8850	0.9110	0.0890
June	0.1140	0.8660	0.9207	0.0793
July	0.1870	0.8130	0.9340	0.0660
August	0.1900	0.8100	0.9420	0.0580
September	0.1800	0.8200	0.9333	0.0667
October	0.0680	0.9320	0.9026	0.0974
November	0.0110	0.9890	0.8998	0.1002
December	0.0070	0.9930	0.9019	0.0981

Analysis of Figure 5 shows that the two-state 1 and 2 are reachable. They communicate with each other, form a closed set and there is no absorbing state. Also, the states are not periodic; hence, they are recurrent and aperiodic. Finally, since all the states in the chain are recurrent with each other, the graphical representation of transition matrix for each month is said to be ergodic.

Graphical representation of transition matrix for January and December of Tamale weather respectively

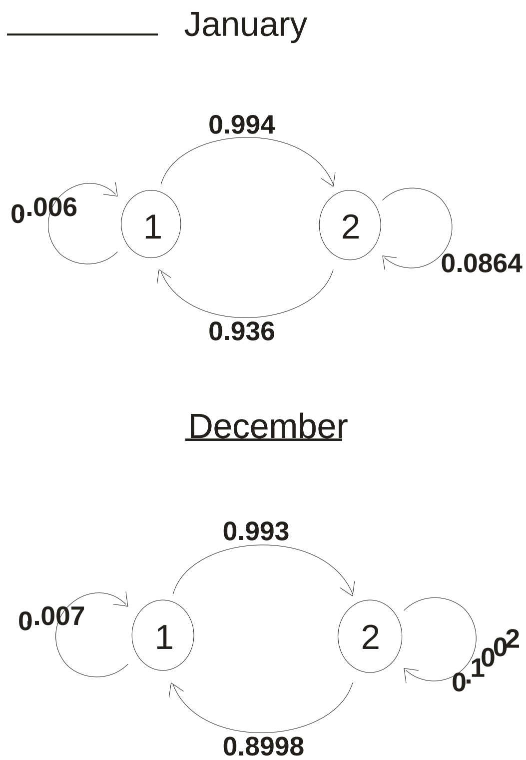


Figure 5: Markov chain for January and December of Tamale weather

Two-State Transition Matrix Representation of Kumasi weather

Analyzing Kumasi weather for the five years from 2003 to 2007, where the weather on the day depends on the previous day's weather.

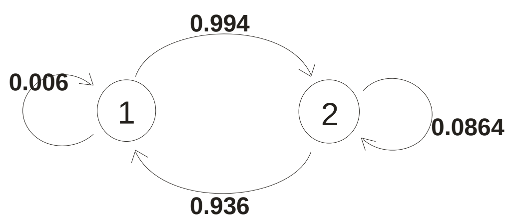
Table 3: The Two-State Transition Matrix of Kumasi weather

	STATES PROBABILITY			
MONTH	RR	RS	SR	SS
January	0. 0263	0. 9737	0. 9155	0. 0845
February	0. 0445	0.9555	0. 9011	0.0989
March	0. 0706	0.9294	0.9039	0. 0961
April	0.1086	0.8914	0.8977	0.1023
May	0.1273	0.8727	0.8983	0.1017
June	0.1062	0.8938	0.9284	0.0716
July	0.0867	0.9133	0.9481	0.0579
August	0.0520	0.9480	0.9579	0.0421
September	0.1673	0.8327	0.9438	0.0562
October	0.1384	0.8616	0.9039	0.0961
November	0.0490	0.9510	0.8925	0.1075
December	0.0231	0.9737	0.9088	0.0912

Preliminary analysis of the Kumasi weather using the graphical presentation of the transition matrix shows that all the states are recurrent, aperiodic and communicate with each other. Hence, the states are said to be ergodic.

Graphical representation of transition of transition matrix for January and December of Kumasi weather are as shown.

January



December

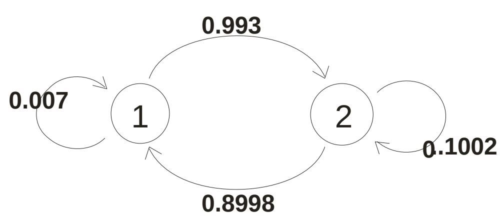


Figure 6: Markov chain for January and December of Kumasi weather

Two-State Transition Matrix Representation of Accra weather

Two-State analysis of Accra weather for five years has been represented in the transition matrix in the table 4

Table 4: The Two-State Transition Matrix of Accra Weather

	STATES PROBABILITY			
MONTH	RR	RS	SR	SS
January	0. 0069	0. 9931	0. 9275	0. 0725
February	0. 0154	0.9486	0. 9579	0.0421
March	0. 0654	0.9346	0.9160	0.0840
April	0.1052	0.8948	0.9075	0.0925
May	0.1822	0.8178	0.9088	0.0912
June	0.2316	0.7684	0.9337	0.0663
July	0.0751	0.9249	0.9420	0.0688
August	0.0421	0.9579	0.9402	0.0598
September	0.0814	0.9126	0.9230	0.0770
October	0.0520	0.9480	0.9579	0.0421
November	0.0545	0.9455	0.8973	0.1027
December	0.0215	0.9785	0.9045	0.0925

Analyzing Table 4 above, the two-state chain is ergodic. This is because the two states 1 and 2 are reachable. They communicate with each other. Secondly, they form a closed set and there is no absorbing state. Finally, the states are recurrent state and aperiodic, which implies that they are transition and not periodic. Hence, the states are said to be ergodic.

Graphical Representation of transition matrix for the Greater Accra

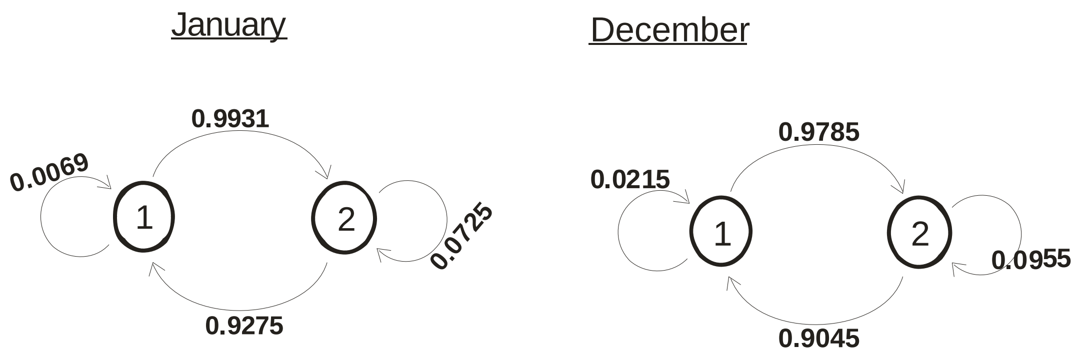


Figure 7: Markov chain for January and December of Accra weather.

Four-State Representation of Transition Matrix of Tamale weather

Analyzing Tamale weather for the five years from 2003 to 2007, where the weather on the day depends on the last two days weather.

If the last two days have been rainy, then 58% of the time, tomorrow will be rainy. If yesterday was rainy and today is sunny, then 52% of the time, tomorrow will be sunny. If yesterday was sunny and today is rainy, then 53% of the time, tomorrow will be sunny. If yesterday was sunny and today is rainy, then 53% of the time, tomorrow will be rainy. Finally if yesterday was sunny and today is sunny, then 75% of the time, tomorrow will be sunny.

The transition matrix of the weather is shown in the next page:

Let RS denote that yesterday was rainy and today is sunny. RR represents state 1, RS represents state 2, SR represents state 3 and SS represents state 4.

$$
\begin{array}{c c}
 & \begin{array}{cccc} RR & RS & SR & SS \end{array} \\
\begin{array}{c} RR \\ RS \\ SR \\ SS \end{array} & \begin{bmatrix} 0.58 & 0.42 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.48 & 0.52 \\ 0.53 & 0.47 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.25 & 0.75 \end{bmatrix}
\end{array}
$$

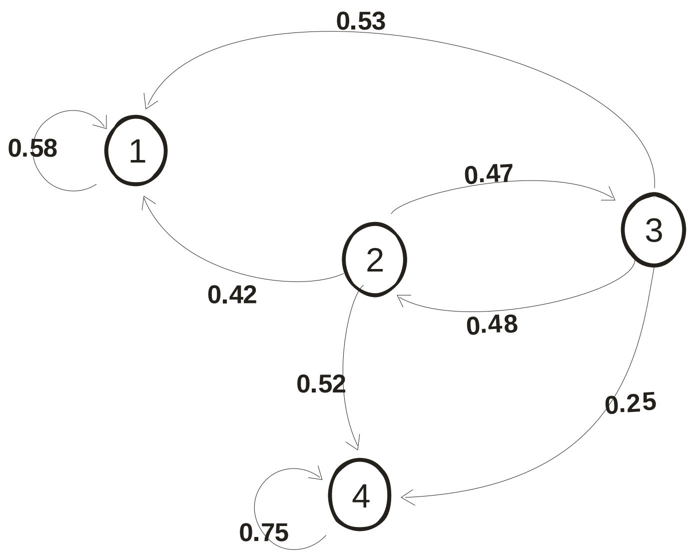


Figure 8: Markov chain of Tamale weather.

Relationships between states 1, 2, 3 and 4 are outlined. From the transition probability of Tamale Weather, states 4 and 3 are reachable from state 1 (through the paths 1 – 2 – 3 and, 1 – 2 – 4 respectively). States 2 and 3 communicate (from 2 to 3 and from 3 to 2). States 1, 2.3 and 4 form a closed set. There are no absorbing states; the states are not transient and periodic. Therefore the states are not recurrent and aperiodic. Since all the states are recurrent, aperiodic and communicate with each other, the chain is said to be ergodic.

Four-State Representation of Transition Matrix of Kumasi weather

A study of Kumasi weather for five years from 2003 to 2007, where the weather for the day depends on the last two previous day's weather. If the last two days have been rainy, then 67% of the time, tomorrow will be rainy. If yesterday was rainy and today is sunny, then 60% of the time, tomorrow will be sunny. If yesterday was sunny and today is rainy, then 65% of the time, tomorrow will be rainy. Finally, if yesterday was sunny and today is sunny, then 55% of the time, tomorrow will be sunny.

The transition matrix of the weather is shown below:

$$\begin{array}{c} \\ RR \\ RS \\ SR \\ SS \end{array} \begin{array}{c} \begin{array}{cccc} RR & RS & SR & SS \end{array} \\ \begin{bmatrix} 0.67 & 0.33 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.40 & 0.60 \\ 0.65 & 0.35 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.45 & 0.55 \end{bmatrix} \end{array}$$

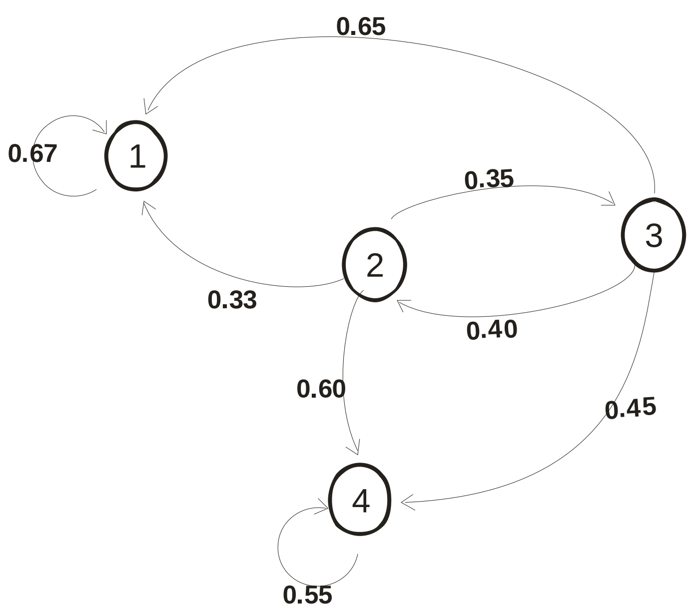


Figure 9: Markov chain of Kumasi weather

Analyzing the transition matrix, there exits a relationship between states 1, 2, 3 and 4. From the transition probability, states 4 and 3 are reachable from state 1(through the path 1 – 2 – 3, 1 – 2 – 4). States 2 and 3 communicate (from 2 to 3 and from 3 to 2). From the Markov Chain with transition matrix, states 1, 2, 3 and 4 are closed sets. There are no absorbing states; the states are not transient and not periodic. Therefore the states are recurrent and aperiodic. Since all the states are recurrent, aperiodic and communicate with each other, the chain is said to be ergodic.

Four-State Representation of Transition Matrix of Accra weather

The Accra weather for five years from 2003 to 2007, the weather for the day depends on the last two days weather. If the last two days have been rainy, then 41% of the time tomorrow will be rainy. If yesterday was rainy and today is sunny, then 57% tomorrow will be sunny. If yesterday was sunny and today is rainy, then 42% tomorrow will be rainy. Finally if yesterday was sunny and today is sunny, then 66% tomorrow will be sunny. Below is the transition matrix of ACCRA

$$\begin{array}{c c} & \begin{array}{cccc} RR & RS & SR & SS \end{array} \\ \begin{array}{c} RR \\ RS \\ SR \\ SS \end{array} & \begin{bmatrix} 0.67 & 0.33 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.40 & 0.60 \\ 0.65 & 0.35 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.45 & 0.55 \end{bmatrix} \end{array}$$

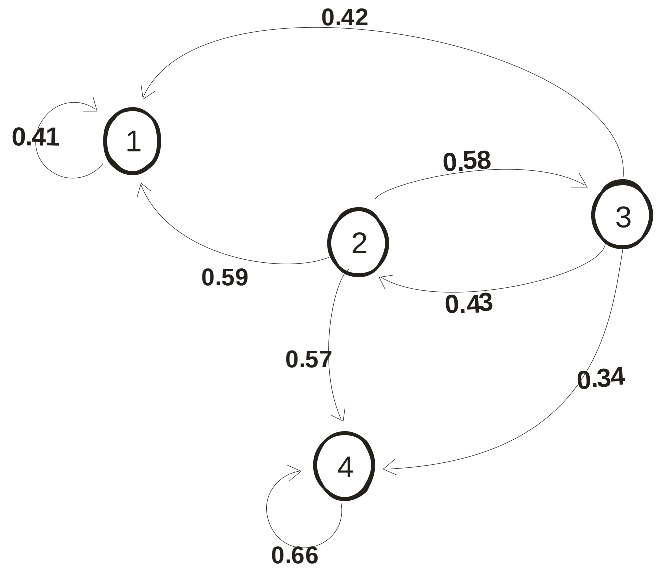


Figure 10: Markov chain of Accra weather

From the transition probability matrix of Accra weather, states 4 and 3 are reachable from state 1(through the path 1 – 2 – 3, 1 – 2 – 4). States 2 and 3 communicate (from 2 to 3 and from 3 to 2). From the Markov Chain with transition matrix, states 1, 2.3 and 4 are closed sets. There is no absorbing state. The states are not transient and not periodic. Therefore states are recurrent and aperiodic. Since all the states are recurrent, aperiodic and communicate with each other the chain are said to be ergodic.

Summary: The Markov chain for the two-state transition matrix (where the weather of the day depends on the previous day) for Tamale, Kumasi and Accra weather indicated an ergodic chain. Also, the Markov chain for the four-state transition matrix (where the weather of the day depends on the last two days weather) indicated an ergodic chain for Tamale, Kumasi and Accra weather.

CHAPTER FOUR

FURTHER ANALYSIS

Introduction

The understanding of steady state probabilities and the long–run behavior of Markov chains is vital. Hence, further analysis was conducted with the use of software (management science application software) to determine the steady state probabilities for the two-state and four-state for the Markov chain, and finally the use of MATLAB software to determine the mean first passage times. This chapter shall provide the steady-state probabilities and mean passage times for each of the three regional towns' rainfall pattern and sunshine duration.

Two-State steady State Probabilities of Tamale weather

The table 5 describes the two-state steady states probability of Tamale weather. From the table 5 below, state 1(RR) represents the long run probability of rainy today given that it rained yesterday. RS represents the long run probability of sunny today given that it rained yesterday. SR represents the long run probability of rainy today given that it was sandy yesterday. State (2) SS represents the long run probability of sunny today given that it was sunny yesterday.

Table 5: Two-State Steady State Probabilities of Tamale Weather

	STEADY STATES PROBABILITY			
MONTH	RR	RS	SR	SS
January	0. 479	0. 521	0. 479	0. 521
February	0. 477	0.523	0. 477	0.523
March	0. 487	0.515	0.487	0. 515
April	0.500	0.500	0.500	0.500
May	0.507	0.493	0.507	0.493
June	0.510	0.490	0.510	0.490
July	0.535	0. 465	0.535	0.465
August	0.538	0.462	0.538	0.462
September	0.532	0.468	0.532	0.468
October	0.492	0.508	0.492	0.508
November	0.476	0.524	0.476	0.524
December	0.476	0.524	0.476	0.524

Analyzing the Probabilities, it can be said that in the long run, the probability of it being rainy in any given day in the month of January is 0.479. The month of August has the highest probability, 0.538, of daily rainfall. Also, the months of November and December have the highest probability, 0.524, of sunshine. To the farmer in Tamale the month of August would be the appropriate month for planting and November or December the appropriate month for harvesting, storing of crops or cultivating of land.

Mean First Passage Times of Two-State of Tamale weather

Below shows how the mean first passage times of Tamale weather for the month of January were computed. From Table 1 in chapter 3, the transition matrix of Tamale weather is obtained as

$$P = \begin{pmatrix} 0.0060 & 0.9940 \\ 0.9136 & 0.0864 \end{pmatrix}$$

From Table 5, we obtain the steady state probability

$$\pi_1 = 0.479 \qquad \pi_2 = 0.521$$

where $\pi_1$and $\pi_2$are the steady state for the month of January

$$m_{11} = \frac{1}{\pi_1} = \frac{1}{0.479} = 2.0877$$

$$m_{22} = \frac{1}{\pi_2} = \frac{1}{0.521} = 1.9194$$

$$m_{12} = 1 + 0.006m_{12} \qquad , \qquad 0.994m_{12} = 1$$

$$\therefore m_{12} = \frac{1}{0.994} = 1.006$$

$$m_{21} = 1 + 0.0864m_{21} \qquad , \qquad 0.9136m_{21} = 1$$

$$\therefore m_{21} = \frac{1}{0.9136} = 1.0946$$

$$M = \begin{pmatrix} 2.0877 & 1.006 \\ 1.0946 & 1.9194 \end{pmatrix}$$

The rest of the months were generated by the same application as shown above.

The table 6 describes the Mean First Passage Time of two-state of Tamale weather. From the table 6 below, state 1(M_{11}) represents the average number of days of rainfall before any rainfall in a particular month. State 2 (M_{22}) represents the average number of days sunshine before any sunshine in a particular month.

Table 6: Mean First Passage Times of Two-State of Tamale weather

	Mean First Passage Time			
MONTH	M_{11}	M_{12}	M_{21}	M_{22}
January	2.0877	1.006	1.0946	1.9194
February	2.0964	1.005	1.1029	1.9120
March	2.0533	1.0331	1.1029	1.9493
April	2.0000	1.0917	1.0924	2.0000
May	1.9724	1.1299	1.0977	2.0284
June	1.9608	1.1287	1.0861	2.0408
July	1.8692	1.2300	1.0707	2.1505
August	1.8587	1.0616	1.2346	2.1645
September	1.8797	1.0870	1.0715	2.1368
October	2.0325	1.0730	1.1097	1.9685
November	2.1008	1.0111	1.1114	1.9084
December	2.1008	1.0070	1.0880	1.9084

From table 6, M_{21} for January is 1.0946. This implies that there will be an average of 1.0946 days sunshine duration before any rainfall in the month of January, whilst an average of 1.007 days rainfall before sunshine duration in the month of

December. From the findings, Tamale will experience rainfall in the month of January if the region have experience bright sunshine for a day in the month.

Two-State steady State Probabilities of Kumasi weather

Table 7: Steady State Probabilities of the two-state chain of Kumasi weather

	STEADY STATE PROBABILITY			
MONTH	RR	RS	SR	SS
January	0. 485	0. 515	0. 485	0. 515
February	0. 485	0.515	0. 485	0.515
March	0. 493	0.501	0.493	0. 507
April	0.502	0.498	0.502	0.498
May	0.507	0.493	0.507	0.493
June	0.509	0.491	0.509	0.491
July	0.509	0. 497	0.509	0.491
August	0.503	0.497	0.503	0.497
September	0.503	0.497	0.503	0.497
October	0.512	0.488	0.512	0.488
November	0.484	0.516	0.484	0.516
December	0.482	0.518	0.482	0.520

From the table 7, it can be said that the month of October has the highest probability of rain in a given day, whilst the month of December has the highest probability of sunshine in a particular day. From the findings the month of October will be appropriate for any individual or organization that needs rainfall in its activities, whilst the month of December for harvesting, drying and storing of crops in the region.

Mean First Passage Time of Two-State of Kumasi weather

Below shows the mean passage times of Kumasi weather for the month of January. From table 2 in chapter 3, the transition matrix of Kumasi weather is obtained as

$$P = \begin{pmatrix} 0.0263 & 0.9737 \\ 0.9155 & 0.0845 \end{pmatrix}$$

From table 7, we obtain the steady state probabilities

$$\pi_1 = 0.485 \qquad \pi_2 = 0.515$$

where π_1 and π_2 are the steady state for the month of January

$$m_{11} = \frac{1}{\pi_1} = \frac{1}{0.485} = 2.0619$$

$$m_{22} = \frac{1}{\pi_2} = \frac{1}{0.515} = 1.9417$$

$$m_{12} = 1 + 0.0263 m_{12} \qquad , \qquad 0.9737 m_{12} = 1$$

$$\therefore m_{12} = \frac{1}{0.9737} = 1.0270$$

$$m_{21} = 1 + 0.0845 m_{21} \qquad , \qquad 0.9155 m_{21} = 1$$

$$\therefore m_{21} = \frac{1}{0.9155} = 1.0923$$

$$M = \begin{pmatrix} 2.0619 & 1.0270 \\ 1.0923 & 1.9417 \end{pmatrix}$$

The rest of the months were generated by the same application as shown above.

Table 8: Mean First Passage Time of Two-State of Kumasi weather

	Mean First Passage Time			
MONTH	M_{11}	M_{12}	M_{21}	M_{22}
January	2.0619	1.0270	1.0923	1.9417
February	2.0619	1.0466	1.1098	1.9417
March	2.0284	1.0996	1.1063	1.9724
April	1.9920	1.1218	1.1140	2.0080
May	1.9724	1.1406	1.1132	2.0284
June	1.9646	1.1188	1.0771	2.0367
July	1.9460	1.0949	1.0547	2.0367
August	1.9880	1.0549	1.0440	2.0367
September	1.8832	1.2009	1.0595	2.1322
October	1.9531	1.1606	1.1063	2.0492
November	2.0661	1.0515	1.1204	1.9380
December	2.0747	1.0236	1.1004	1.9305

From the table 8, M_{11} for January is 2.0619. This implies that, there will be an average of 2.0619 days rainfall before another rainfall in the month of January, while average of 1.9305 days sunshine duration before sunshine in the month of December. From the findings it will take an average of 2 days of bright sunshine in the month of December before bright sunshine in the next day in the region.

Two-State Steady State Probabilities of Accra weather

Table 9: Steady State Probabilities of the two-state chain of Accra weather

	STEADY STATES PROBABILITY			
MONTH	RR	RS	SR	SS
January	0. 483	0. 517	0. 483	0. 517
February	0. 480	0.520	0. 480	0.520
March	0. 495	0.505	0.495	0. 505
April	0.504	0.496	0.504	0.496
May	0.526	0.474	0.526	0.474
June	0.549	0.451	0.549	0.451
July	0.502	0.498	0.502	0.498
August	0.495	0.503	0.495	0.497
September	0.503	0.497	0.503	0.497
October	0.504	0.496	0.504	0.496
November	0.487	0.513	0.487	0.513
December	0.480	0.520	0.480	0.520

The Steady State probabilities present a unique opportunity to streaming the long run behaviour of the weather of Accra. The month of April and October, have the highest probability, 0.504, of rainfall in a given day whilst the months of February and December have the highest probability, 0.520, of sunny conditions on a given day. From the findings the month of April and October will be appropriate month for the fisherman in the region in other to record a bumper harvest.

Mean First Passage Times of Two-State of Accra weather

The mean passage time of Accra weather for the month of January is shown below. From table 3 in chapter, the transition matrix of Accra weather is obtained as

$$P = \begin{pmatrix} 0.0069 & 0.9931 \\ 0.9275 & 0.0725 \end{pmatrix}$$

From table 9, we obtain the steady state probabilities

$$\pi_1 = 0.483 \qquad \pi_2 = 0.517$$

where $\pi_1$and $\pi_2$are the steady state for the month of January

$$m_{11} = \frac{1}{\pi_1} = \frac{1}{0.483} = 2.0704$$

$$m_{22} = \frac{1}{\pi_2} = \frac{1}{0.517} = 1.9342$$

$$m_{12} = 1 + 0.0069m_{12} \quad , \quad 0.9931m_{12} = 1$$

$$\therefore m_{12} = \frac{1}{0.9931} = 1.0069$$

$$m_{21} = 1 + 0.0725m_{21} \quad , \quad 0.9275m_{21} = 1$$

$$\therefore m_{21} = \frac{1}{0.9275} = 1.0782$$

$$M = \begin{pmatrix} 2.0704 & 1.0069 \\ 1.0782 & 1.9342 \end{pmatrix}$$

The rest of the months were generated by the same application as shown above.

Table 10: Mean First Passage Time of Accra weather

	Mean First Passage Time			
MONTH	M_{11}	M_{12}	M_{21}	M_{22}
January	2.0704	1.0069	1.0782	1.9342
February	2.0833	1.0156	1.1001	1.9231
March	2.0202	1.0670	1.0917	1.9802
April	1.9841	1.1176	1.1019	2.0161
May	1.9011	1.2228	1.1004	2.1097
June	1.8215	1.3014	1.0710	2.2173
July	1.9920	1.0812	1.0759	2.0080
August	2.0202	1.0440	1.0636	1.9802
September	1.9881	1.0982	1.0934	2.0121
October	1.9841	1.1269	1.1088	2.0161
November	2.0534	1.0516	1.1145	1.9493
December	2.0833	1.0220	1.1056	1.9231

From the table 10, M_{12} for January is 1.0069. This implies that after a long time, there will be an average of 1.0069 days of rainfall before sunshine, while an average of 1.0813 days sunshine duration before rainfall for the month of January. From the findings Accra will experience an average of 1 day of rainfall before bright sunshine in the month of January.

Four-State Steady State Probabilities of Tamale weather

We shall find the steady-state probabilities of Tamale weather given the transition matrix. The four-state representation of transition of Tamale weather was obtained in chapter three as

$$\begin{array}{c} \\ RR \\ RS \\ SR \\ SS \end{array} \begin{array}{c} \begin{array}{cccc} RR & RS & SR & SS \end{array} \\ \begin{pmatrix} 0.58 & 0.42 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.48 & 0.52 \\ 0.53 & 0.47 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.25 & 0.75 \end{pmatrix} \end{array}$$

From equation 4 (in chapter two) we obtain

$$\begin{bmatrix} \pi_1 & \pi_2 & \pi_3 & \pi_4 \end{bmatrix} = \begin{bmatrix} \pi_1 & \pi_2 & \pi_3 & \pi_4 \end{bmatrix} \begin{pmatrix} 0.58 & 0.42 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.48 & 0.52 \\ 0.53 & 0.47 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.25 & 0.75 \end{pmatrix}$$

$$\pi_1 = 0.58\pi_1 + 0.53\pi_3$$

$$\pi_2 = 0.42\pi_1 + 0.47\pi_3$$

$$\pi_3 = 0.48\pi_2 + 0.25\pi_4$$

$$\pi_4 = 0.52\pi_2 + 0.75\pi_4$$

$$\pi_1 + \pi_2 + \pi_3 + \pi_4 = 1$$

Using the management science software, we obtain

$\pi_1 = 0.236, \quad \pi_2 = 0.187, \quad \pi_3 = 0.187 \quad and \quad \pi_4 = 0.390.$

The value of $\pi_1 = 0.236$ means that, if it rained today, the probability of raining tomorrow is 0.236, regardless of whether it rained or was sunny yesterday.

Similarly, $\pi_2 = 0.187$ means that, if it rained today, the probability of a sunny tomorrow is 0.187, regardless of whether it rained or was sunny yesterday.

Also, $\pi_3 = 0.187$ means that, if it is sunny today, the probability of a rainy tomorrow is 0.187, regardless of whether it rained or was sunny yesterday. Finally, $\pi_4 = 0.390$ means that, if it is sunny today, the probability of a sunny tomorrow is 0.389, regardless of whether it rained or was sunny yesterday.

Mean First Passage Times of Four-State of Tamale weather

Using the transition matrix and steady state probabilities of Tamale weather, the mean first passage time can be calculated as follows.

$$M_{11} = \frac{1}{0.236} = 4.34$$

$$M_{22} = \frac{1}{0.187} = 5.35$$

$$M_{33} = \frac{1}{0.187} = 5.35$$

$$M_{44} = \frac{1}{0.390} = 2.56$$

$$
\begin{bmatrix}
0.42 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0.42 & 0 & 0 & -0.42 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0.42 & 0 & 0 & -0.42 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & -0.58 & 0 & 0 & -0.42 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & -0.42 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -0.58 & 0 & 0 & 0 \\
0 & 0 & 0 & -0.47 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
-0.53 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & -0.53 & 0 & 0 & -0.47 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -0.25 & 0 & 0 & 0.25 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.25 & 0 & 0 & 0.25 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.25
\end{bmatrix}
\begin{bmatrix}
M_{12} \\ M_{13} \\ M_{14} \\ M_{21} \\ M_{23} \\ M_{24} \\ M_{31} \\ M_{32} \\ M_{34} \\ M_{41} \\ M_{42} \\ M_{43}
\end{bmatrix}
=
\begin{bmatrix}
1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1
\end{bmatrix}
$$

Solving the matrix above as system of equations above using MATLAB software, we have

$$M = \begin{bmatrix} 4.3400 & 2.3810 & 5.0610 & 7.8855 \\ 6.9434 & 5.3500 & 2.6800 & 5.5045 \\ 4.2634 & 2.2619 & 5.3400 & 7.7664 \\ 8.2634 & 6.2619 & 4.0000 & 2.5600 \end{bmatrix}$$

Analysis of the result shows that after a long time, given that the last two days have been rainy, it will take an average of 4.34 days before we again have two rainy days. Similarly, given that the two days have been sunny, it will take an average of 2.56 days before we again have two sunny days.

Four-State Steady State Probabilities of Kumasi weather

The four-state representation of transition matrix of Kumasi weather was obtained in chapter three as

$$\begin{array}{c} \\ RR \\ RS \\ SR \\ SS \end{array} \begin{array}{c} \begin{array}{cccc} RR & RS & SR & SS \end{array} \\ \begin{pmatrix} 0.67 & 0.33 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.40 & 0.60 \\ 0.65 & 0.35 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.45 & 0.55 \end{pmatrix} \end{array}$$

From equation 4 (in chapter two) we obtain

$$\begin{bmatrix} \pi_1 & \pi_2 & \pi_3 & \pi_4 \end{bmatrix} = \begin{bmatrix} \pi_1 & \pi_2 & \pi_3 & \pi_4 \end{bmatrix} \begin{pmatrix} 0.67 & 0.33 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.40 & 0.60 \\ 0.65 & 0.35 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.45 & 0.55 \end{pmatrix}$$

$$\pi_1 = 0.67\pi_1 + 0.65\pi_3$$

$$\pi_2 = 0.33\pi_1 + 0.35\pi_3$$

Thus, $$\pi_3 = 0.40\pi_2 + 0.45\pi_4$$

$$\pi_4 = 0.60\pi_2 + 0.55\pi_4$$

$$\pi_1 + \pi_2 + \pi_3 + \pi_4 = 1$$

Thus, $\pi_1 = 0.371, \quad \pi_2 = 0.189, \quad \pi_3 = 0.189 \quad and \quad \pi_4 = 0.251$

$\pi_1 = 0.371$ means that, if it rained today, the probability of it raining tomorrow is 0.371, regardless of whether it rained or was sunny yesterday. Similarly, $\pi_2 = 0.189$ means that, if it rained today, the probability of it being sunny tomorrow is 0.189, regardless of whether it rained or was sunny yesterday. Also, $\pi_3 = 0.189$ means that, if it sunny today, the probability of it rainy tomorrow is 0.189, regardless of whether it rained or was sunny yesterday. Finally, $\pi_4 = 0.251$ means that, if it shined today, the probability of it sunny tomorrow is 0.251, regardless of whether it rained or was sunny yesterday.

Mean First Passage Times of Four-State of Kumasi weather

Using the transition matrix and steady state of Kumasi weather, the mean first passage times can be calculated as follows.

$$M_{11} = \frac{1}{0.371} = 2.695 \qquad M_{22} = \frac{1}{0.189} = 5.291$$

$$M_{33} = \frac{1}{0.189} = 5.291 \qquad M_{44} = \frac{1}{0.251} = 3.984$$

$$\begin{bmatrix} 0.33 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.33 & 0 & 0 & -0.33 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.33 & 0 & 0 & -0.33 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & -0.4 & 0 & 0 & -0.6 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & -0.6 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -0.4 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.35 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ -0.65 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -0.65 & 0 & 0 & -0.35 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -0.45 & 0 & 0 & 0.45 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.45 & 0 & 0 & 0.45 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.55 \end{bmatrix} \begin{bmatrix} M_{12} \\ M_{13} \\ M_{14} \\ M_{21} \\ M_{23} \\ M_{24} \\ M_{31} \\ M_{32} \\ M_{34} \\ M_{41} \\ M_{42} \\ M_{43} \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

By solving the above as system of equations using MATLAB software, we have

$$M = \begin{bmatrix} 2.695 & 3.0303 & 5.1212 & 6.6768 \\ 5.1282 & 5.291 & 2.0909 & 3.6465 \\ 2.7949 & 2.9697 & 5.291 & 6.6162 \\ 5.0171 & 5.1919 & 1.8182 & 3.984 \end{bmatrix}$$

Analysis of Kumasi weather shows that after a long time, given that the last two days have been rainy, it will take an average of 2.695 days before we again have two rainy days. Similarly, given that the last two days have been sunny, it will take an average of 3.984 days before we again have two sunny days.

Four-State Steady State Probabilities of Accra weather

The four-state representation of transition matrix of Accra weather was obtained in chapter three as

$$\begin{array}{c} \\ RR \\ RS \\ SR \\ SS \end{array} \begin{array}{c} \begin{array}{cccc} RR & RS & SR & SS \end{array} \\ \begin{pmatrix} 0.41 & 0.59 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.43 & 0.57 \\ 0.42 & 0.58 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.34 & 0.66 \end{pmatrix} \end{array}$$

From equation 4 (in chapter two) we obtain

$$\begin{bmatrix} \pi_1 & \pi_2 & \pi_3 & \pi_4 \end{bmatrix} = \begin{bmatrix} \pi_1 & \pi_2 & \pi_3 & \pi_4 \end{bmatrix} \begin{pmatrix} 0.41 & 0.59 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.43 & 0.57 \\ 0.42 & 0.58 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.34 & 0.66 \end{pmatrix}$$

Thus,

$$\pi_1 = 0.41\pi_1 + 0.42\pi_3$$

$$\pi_2 = 0.59\pi_1 + 0.58\pi_3$$

$$\pi_3 = 0.43\pi_2 + 0.34\pi_4$$

$$\pi_4 = 0.57\pi_2 + 0.66\pi_4$$

$$\pi_1 + \pi_2 + \pi_3 + \pi_4 = 1$$

Thus $\pi_1 = 0.162, \quad \pi_2 = 0.228, \quad \pi_3 = 0.228 \;\; and \quad \pi_4 = 0.382$

$\pi_1 = 0.162$ means that, if it rained today, the probability of it raining tomorrow is 0.162, regardless of whether it rained or was sunny yesterday. Similarly, $\pi_2 = 0.228$ means that, if it rained today, the probability of it sunny tomorrow is 0.228, regardless of whether it rained or was sunny yesterday. Also, $\pi_3 = 0.228$ means that, if it sunny today, the probability of it rained tomorrow is 0.228, regardless of whether it rained or was sunny yesterday. Finally, $\pi_4 = 0.382$ means that, if it shined today, the probability of it sunny tomorrow is 0.382, regardless of whether it rained or was sunny yesterday.

Mean First Passage Times of Four-State of Accra weather

Using the transition matrix and steady state of Accra weather, the mean first passage time can be calculated as follows.

$$M_{11} = \frac{1}{0.162} = 6.172 \qquad M_{22} = \frac{1}{0.228} = 4.386$$

$$M_{33} = \frac{1}{0.228} = 4.386 \qquad M_{44} = \frac{1}{0.382} = 2.618$$

$$\begin{bmatrix} 0.59 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.59 & 0 & 0 & -0.59 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.59 & 0 & 0 & -0.59 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & -0.43 & 0 & 0 & -0.57 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & -0.59 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -0.43 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.58 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ -0.42 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -0.42 & 0 & 0 & -0.58 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -0.34 & 0 & 0 & 0.34 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.34 & 0 & 0 & 0.34 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.34 \end{bmatrix} \begin{bmatrix} M_{12} \\ M_{13} \\ M_{14} \\ M_{21} \\ M_{23} \\ M_{24} \\ M_{31} \\ M_{32} \\ M_{34} \\ M_{41} \\ M_{42} \\ M_{43} \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

By solving the above matrix as system of equations using MATLAB software, we have

$$M = \begin{bmatrix} 6.1720 & 1.6949 & 4.4302 & 4.7407 \\ 8.7535 & 4.3860 & 2.7353 & 3.0458 \\ 6.0770 & 1.7119 & 4.3860 & 4.7577 \\ 9.0182 & 4.6530 & 2.9412 & 2.6180 \end{bmatrix}$$

The Accra weather shows that after a long time, given that the last two days have been rainy, it will take an average of 6.172 days before we again have two rainy days. Similarly, given that the last two days have been sunny, it will take an average of 2.618 days before we again have two sunny days.

Summary

Analyzing the Two-State Steady State Probabilities of Tamale weather, it can be said that in the long run, the probability of it being rainy in any given day in the month of January is 0.479. The month of August has the highest probability, 0.538, of daily rainfall. Also, the months of November and December have the highest probability, 0.524, of sunshine. The Two-State Mean First Passage Time for Tamale weather indicated M_{21} for January as 1.0946. This implies that there will be an average of 1.0946 days sunshine duration before any rainfall in the month of January, whilst an average of 1.007 days rainfall before sunshine duration in the month of December.

Analyzing the Two-State Steady State Probability of Kumasi weather, it can be said that the month of October has the highest probability of rain in a given day, whilst the month of December has the highest probability of sunshine in a particular day. The Two-State Mean First Passage Time of Kumasi weather indicated M_{11} for January as 2.0619. This implies that, there will be an average of

2.0619 days rainfall before another rainfall in the month of January, while average of 1.9305 days sunshine duration before sunshine in the month of December.

The Two-State Steady State probabilities of Accra weather present a unique opportunity to streaming the long run behaviour of the weather of Accra. The month of April and October, have the highest probability, 0.504, of rainfall in a given day whilst the months of February and December have the highest probability, 0.520, of sunny conditions on a given day. The Two-State Mean First Passage Time of Accra weather indicated M_{12} for January as 1.0069. This implies that after a long time, there will be an average of 1.0069 days of rainfall before sunshine, while an average of 1.0813 days sunshine duration before rainfall for the month of January.

Finding of Four-State Steady Probability of Tamale weather shows that if it rained today, the probability of raining tomorrow is 0.236, regardless of whether it rained or was sunny yesterday. If it rained in Tamale today, the probability of a sunny tomorrow is 0.187, regardless of whether it rained or was sunny yesterday. If it is sunny Tamale today, the probability of a rainy tomorrow is 0.187, regardless of whether it rained or was sunny yesterday. If it is sunny Tamale today, the probability of a sunny tomorrow is 0.389, regardless of whether it rained or was sunny yesterday. Further analysis of the Tamale weather shows that after a long time, given that the last two days have been rainy, it will take an average of 4.34 days before we again have two rainy days. Similarly, given that the last two days have been sunny, it will take an average of 2.56 days before we again have two sunny days.

Finding of Four-State Steady Probability of Kumasi weather shows that, if it rained today, the probability of it raining tomorrow is 0.371, regardless of whether it rained or was sunny yesterday. If it rained Kumasi today, the probability of it being sunny tomorrow is 0.189, regardless of whether it rained or was sunny yesterday. Also, if it sunny in Kumasi today, the probability of it rainy tomorrow is 0.189, regardless of whether it rained or was sunny yesterday. Finally, if it shined today in Kumasi, the probability of it sunny tomorrow is 0.251, regardless of whether it rained or was sunny yesterday. Analysis of Kumasi weather shows that after a long time, given that the last two days have been rainy, it will take an average of 2.695 days before we again have two rainy days. Similarly, given that the last two days have been sunny, it will take an average of 3.984 days before we again have two sunny days.

Finding of Four-State Steady Probability of Accra weather shows that if it rained today, the probability of it raining tomorrow is 0.162, regardless of whether it rained or was sunny yesterday. If it rained today in Accra, the probability of it sunny tomorrow is 0.228, regardless of whether it rained or was sunny yesterday. Also, if it sunny today in Accra, the probability of it rained tomorrow is 0.228, regardless of whether it rained or was sunny yesterday. Finally, if it shined today in Accra, the probability of it sunny tomorrow is 0.382, regardless of whether it rained or was sunny yesterday. The Accra weather shows that after a long time, given that the last two days have been rainy, it will take an average of 6.172 days before we again have two rainy days. Similarly, given that the last two days have been sunny, it will take an average of 2.618 days before we again have two sunny days.

CHAPTER FIVE

SUMMARY, DISCUSSION AND CONCLUSIONS

Summary

In order to achieve the objectives, a theoretical basis was presented before adopting the three main approaches of Markov chain models which are: Transition Matrix, Steady- State Probabilities and Mean First Passage Times. The transition matrices allow us to predict the probability for the short-run for weather changes in the three towns. Also, out of the transition matrix the classifications of states are made for each Markov chain. Graphical representations of the transition matrix were drawn for each weather. The Steady-State Probabilities predict the long-run behavior of Markov chain. This was based on the condition that each of transition matrices for each month weather is an ergodic chain. Analysis was done by the application of Management science software and MATLAB. The Mean First Passage Times are the extension of the transition matrix and the Steady State Probabilities. The Mean First Passage Times enables the researcher to model the expected number of transitions before we first reach state j, given that we are currently in state *i*. This is found by the application of MATLAB software. Thus, the forecaster is able to predict the number of days of rainfall before sunshine in a particular month and vice versa. Finally, applications of all these three methods enable us to forecast the probability of rainfall in each day of a month or the entire month and the corresponding sunshine duration in the entire month. Thus the forecast for short-run and long-run behaviour of rainfall pattern and sunshine duration for each of the three towns can be found in chapters three and four respectively. The average number of days of rainfall before the occurrence of sunshine and vice versa for each day or month of the three towns can be found in Chapter four.

The study has shown that the states in each of the transition matrix for the weather of the three towns are recurrent, a periodic and communicate with each other. Thus the chains are ergodic.

The two-state steady state probabilities of Tamale weather as discussed in table 5 shows that, the probability of it being rainy in any given day in the month of January is 0.479. The month of August has the highest probability, 0.538 of daily rainfall. The months of November and December have the highest probability, 0.524 of daily sunshine. The two-state mean first passage times of Tamale weather as shown in Table 6 indicated an average of 1.0946 days sunshine duration before any rainfall in the month of January, whilst an average of 1.007 days rainfall before sunshine duration in the month of December.

The two-state steady state probabilities of Kumasi weather as discussed in Table 7 indicated the month of September has the highest probability of rainfall a given day, whilst the month of December has the highest probability of sunshine in a particular day. The two-state mean first passage time for Kumasi as shown in Table 8, shows that there will be an average of 2.0619 days rainfall before another rainfall in the month of January, whilst average of 1.9305 days sunshine duration before sunshine in the month of December.

The two-state steady state probabilities of Accra weather as shown in Table 9 indicated the month of April and October as having the highest probability, 0.504 of rainfall in a given day whilst, the month of February and December as having the highest probabilities, 0.520 of sunny conditions on a particular day. The two-state mean first passage time for Kumasi as discussed from Table 10 indicated in the month of January, there will be an average of 1.0069 days rainfall before sunshine whilst, an average of 1.0813 days sunshine before rainfall.

The four-state steady state probabilities for Tamale weather shows that states 1, 2, 3 and 4 are 0.236, 0.187, 0.187 and 0.390 respectively. As discussed Table 8, state 1 indicated that if it rained today, the probability of it raining tomorrow is 0.236, regardless of whether it rained or was sunny yesterday. The mean first passage times of the four-state of Tamale weather indicated that it will take an average of 4.34 days before we again have two rainy days. Similarly, given that the two days have been sunny, it will take an average of 2.56 days before we again have two sunny days.

The four-state steady state probabilities of Kumasi weather shows that states 1, 2, 3 and 4 are 0.371, 0.189, 0.189 and 0.251 respectively. State 2 indicated that if it rained today, the probability of it being sunny tomorrow is 0.189 regardless of whether it rained or was sunny yesterday. The mean first passage times of the four-state of Kumasi weather as indicated in chapter 4 shows that after a long time, given that the last two days have been rainy, it will take an average of 2.695 days before we again have two rainy days. Similarly, given that the last two days have been sunny, it will take an average of 3.984 days before we again have two sunny days.

The four-state steady state probabilities for Accra weather shows that states 1, 2, 3 and 4 are 0.162, 0.228, 0.228 and 0.382 respectively. State 3 indicated that if it sunny today, the probability of it raining tomorrow is 0.228 regardless of whether it rainy or was sunny yesterday. The mean first passage time of four-state of Accra weather shows that after a long time, given that the last two days have been rainy, it will take an average of 6.172 days before we again have two rainy days. Similarly, given that the last two days have been sunny, it will take an average of 2.618 days before we again have two sunny days.

Further discussion of two-state and four-state steady state probabilities for each month of the three towns and its corresponding analysis of mean first passage times for the two-state and four-state has been provided in Chapter Four. Finally, from the two-state and the four-state will enable any forecaster to predict daily and monthly weather change for the three towns.

Discussion

Knowledge of the Transient (or short run behaviour) of Markov Chain, the Steady-State probabilities (the long- run behaviour) and the Mean First Passage Time (how long it takes to move from one state to another) of rainfall pattern and hours of bright sunshine in each of the three regions (Accra, Kumasi and Tamale) farmers, fishermen, industries, transport, communication, educational institutions and government will be able to include these two elements of weather any policy formulation.

Apart from the Markov Chain other advanced statistical modeling technique like the Auto Regressive Integrated Moving Average (ARIMA)

Times Series Analysis, Monte Carlo Method, Bayesian Estimation and Method of Maximum Likelihood can be used as discussed in chapter two.

Recommendations for further research should be on the following areas: Correlation between temperatures and solar irradiation in Ghana. Correlation between sunshine, rainfalls and bio-productive systems in Ghana. A Markov Analysis of any elements of weather and related diseases in Ghana. A Markovian Analysis of the life of Newspaper. A Markovian Analysis of purchase of a particular soft drink or alcoholic beverage with relation to any elements of weather.

Conclusions

From all the analysis of this research, it could be concluded that Kumasi has the highest rainfall pattern follow by Tamale and Accra. Also Tamale has the highest duration of sunshine followed by Accra and then Kumasi. Secondly, Kumasi has the highest number of occurrence of daily rainfall compared to Tamale and Accra. Tamale has the highest average number of daily bright sunshine compared to Kumasi and Accra. This finding confirms what other researchers have made on the weather of the country. It would be recommended if further research is made to find whether factors such as deforestation, mining, construction or farming have any role to the findings.

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APPENDICES

Appendix A

Kumasi Monthly Rainfall Total (mm)

Year	Jan	Feb	March	April	May	June	July	Aug	Sept	Oct	Nov	Dec
2003	32.9	74.5	73.1	129.5	188.8	254.6	95.3	26.8	99.5	180.1	163.2	30.9
2004	25.8	70.8	164.3	101	72.3	41.1	229.4	115	243.5	232.4	43.5	76.5
2005	12.5	48.9	84.2	146.4	272.1	121.3	18.3	36.7	174.1	236.9	49.8	29.8
2006	111.1	98.4	112.8	66.9	187.3	145.4	66.7	65.2	111.4	158.4	32.5	3.7
2007	0.2	16.4	56.2	310.9	164.2	176.0	192.9	117.7	534.5	153.9	51.7	19.8

Kumasi Mean Daily Duration of Bright Sunshine (hours)

Year	Jan	Feb	March	April	May	June	July	Aug	Sept	Oct	Nov	Dec
2003	6.3	7.3	7.4	7.1	7.4	4.3	4.2	3.3	3.8	6.7	7.2	6.5
2004	6.0	6.3	6.1	6.4	6.2	4.4	2.5	2.5	4.3	6.1	6.8	5.5
2005	6.2	6.2	6.7	6.7	7.0	3.7	2.6	1.9	3.8	6.9	7.1	6.0
2006	6.3	6.5	5.1	6.7	6.6	5.9	3.8	3.2	3.1	5.7	7.2	5.9
2007	2.7	5.9	6.0	6.4	5.9	5.0	3.8	2.8	3.3	5.9	6.7	5.8

Tamale Monthly Rainfall Total (mm)

Year	Jan	Feb	March	April	May	June	July	Aug	Sept	Oct	Nov	Dec
2003	17.5	10.4	40.6	123.5	151.0	206.7	150.0	289.0	215.3	69.8	15.5	0.0
2004	14.5	0.8	34.1	67.2	133.2	147.0	209.3	264.8	149.1	33.9	48.1	0.0
2005	0.2	14.2	69.9	138.5	81.2	109.1	340.2	88.1	190.7	69.5	tr	40.9
2006	0.0	3.4	4.3	58.5	138.0	87.9	179.4	211.2	164.7	135.6	0.0	0.0
2007	0.0	0.0	29.1	82.8	137.4	85.0	160.3	204.6	278.9	68.5	0.0	0.0

Tamale Mean Daily Duration of Bright Sunshine (hours)

Year	Jan	Feb	March	April	May	June	July	Aug	Sept	Oct	Nov	Dec
2003	8.4	9.1	7.7	7.7	8.7	6.4	6.1	4.5	6.2	8.8	8.6	9.0
2004	8.4	7.0	5.7	6.9	7.6	6.7	5.3	5.1	5.7	9.1	8.8	8.2
2005	6.3	6.8	7.3	7.8	8.4	6.6	4.9	5.0	6.6	8.1	8.5	9.0
2006	8.5	9.0	7.5	8.2	7.2	7.5	6.5	5.7	5.3	8.3	8.9	8.4
2007	6.1	8.8	7.1	6.3	6.9	7.4	6.0	5.0	5.3	8.2	8.9	8.2

Accra Mean Daily Duration of Bright Sunshine (hours)

Year	Jan	Feb	March	April	May	June	July	Aug	Sept	Oct	Nov	Dec
2003	7.1	8.0	7.2	7.4	8.4	5.0	7.3	5.5	5.6	8.6	8.5	7.4
2004	6.0	6.3	6.1	7.2	7.1	5.1	4.9	4.3	7.3	7.1	8.1	8.0
2005	4.9	7.0	6.8	7.1	7.2	4.3	5.1	4.6	6.2	8.2	8.4	8.1
2006	7.4	8.0	7.3	7.5	7.1	7.1	4.9	4.7	5.7	8.0	8.4	7.6
2007	3.7	7.2	6.3	7.9	6.8	5.1	5.4	4.9	6.1	7.7	7.8	7.2

Accra Monthly Rainfall Total (mm)

Year	Jan	Feb	March	April	May	June	July	Aug	Sept	Oct	Nov	Dec
2003	2.6	9.4	25.1	215.0	71.2	302.0	36.8	25.9	39.8	102.6	41.1	15.5
2004	14.7	17.7	5.3	26.4	121.5	116.5	38.6	18.4	100.2	87.5	22.6	4.8
2005	Tr	7.5	127.0	32.3	109.0	167.2	46.5	27.7	28.0	91.0	85.5	56.2
2006	8.7	6.0	25.8	37.0	236.6	118.1	42.1	16.1	71.1	75.6	11.2	0.1
2007	0.0	17.4	62.5	84.4	145.7	166.0	117.9	70.1	89.1	66.1	44.4	4.0

Appendix B

Table of probability of rainfall for each month in each region for a five year period (2003-2007)

MONTH	TAMALE	KUMASI	ACCRA
JANUARY	0.0060	0.0263	0.0069
FEBRUARY	0.0050	0.0445	0.0154
MARCH	0.0320	0.0706	0.0654
APRIL	0.0840	0.1086	0.1052
MAY	0.1150	0.1273	0.1822
JUNE	0.1140	0.1062	0.3216
JULY	0.1870	0.0867	0.0751
AUGUST	0.1900	0.0520	0.0421
SEPTEMBER	0.1800	0.1673	0.0874
OCTOBER	0.1680	0.1384	0.1126
NOVEMBER	0.0110	0.0490	0.0545
DECEMBER	0.0070	0.0231	0.0215
TOTAL	1	1	1

Table of probability of long hours of bright sunshine for each region (2003 to 2007)

MONTH	TAMALE	KUMASI	ACCRA
JANUARY	0.0864	0.0845	0.0725
FEBRUARY	0.0933	0.0989	0.0910
MARCH	0.0809	0.0961	0.0840
APRIL	0.0846	0.1023	0.0925
MAY	0.0890	0.1017	0.0912
JUNE	0.0793	0.0716	0.0663
JULY	0.0660	0.0519	0.0688
AUGUST	0.0580	0.0421	0.0598
SEPTEMBER	0.0667	0.0562	0.0770
OCTOBER	0.0974	0.0961	0.0987
NOVEMBER	0.1002	0.1075	0.1027
DECEMBER	0.0981	0.0912	0.0955
TOTAL	1	1	1

Appendix C

The two-state steady state probabilities of Tamale weather

Transition matrix for January

$$P = \begin{array}{c} \\ R \\ S \end{array} \begin{array}{c} \begin{array}{cc} R & S \end{array} \\ \begin{pmatrix} 0.0060 & 0.9940 \\ 0.9136 & 0.0864 \end{pmatrix} \end{array}$$

$$\begin{bmatrix} \pi_1 & \pi_2 \end{bmatrix} = \begin{bmatrix} \pi_1 & \pi_2 \end{bmatrix} \begin{pmatrix} 0.0060 & 0.9940 \\ 0.9136 & 0.0864 \end{pmatrix}$$

Steady state equation

$\pi_1 = 0.0060\,\pi_1 + 0.9136\,\pi_2$

$\pi_2 = 0.\,9940\,\pi_1 + 0.0864\,\pi_2$

$\pi_1 + \pi_2 = 1$

Steady state probabilities are shown below

State	Probability
1	0.479
2	0.521

$$P = \begin{array}{c} \\ R \\ S \end{array} \begin{array}{c} \begin{array}{cc} R & S \end{array} \\ \begin{pmatrix} 0.479 & 0.521 \\ 0.479 & 0.521 \end{pmatrix} \end{array}$$

There is a probability of 0.479 that it will be raining tomorrow given that it is rainy today. There is a probability of 0.521 that if will sunny tomorrow given that it is sunny today.

The two-state steady state probabilities of Kumasi weather

Transition matrix for January

$$P = \begin{array}{c c} & \begin{array}{cc} R & S \end{array} \\ \begin{array}{c} R \\ S \end{array} & \begin{pmatrix} 0.0263 & 0.9737 \\ 0.9155 & 0.0845 \end{pmatrix} \end{array}$$

$$\begin{bmatrix} \pi_1 & \pi_2 \end{bmatrix} = \begin{bmatrix} \pi_1 & \pi_2 \end{bmatrix} \begin{pmatrix} 0.0263 & 0.9737 \\ 0.9155 & 0.0845 \end{pmatrix}$$

Steady state equation

$\pi_1 = 0.0263\ \pi_1 + 0.9155\ \pi_2$

$\pi_2 = 0.9737\ \pi_1 + 0.0845\ \pi_2$

$\pi_1 + \pi_2 = 1$

Steady state probability is shown below

State	Probability
1	0.485
2	0.515

$$P = \begin{array}{c c} & \begin{array}{cc} R & S \end{array} \\ \begin{array}{c} R \\ S \end{array} & \begin{pmatrix} 0.485 & 0.515 \\ 0.485 & 0.515 \end{pmatrix} \end{array}$$

There is a probability of 0.485 that it will be raining tomorrow given that it is rainy today. There is a probability of 0.515 that if will sunny tomorrow given that it is sunny today.

The two-state steady state probabilities for Accra weather for January

Steady state equations

$$\begin{array}{c c} & \begin{array}{cc} R & S \end{array} \\ \begin{array}{c} R \\ S \end{array} & \begin{pmatrix} 0.0069 & 0.9931 \\ 0.9275 & 0.0725 \end{pmatrix} \end{array}$$

$$\begin{bmatrix} \pi_1 & \pi_2 \end{bmatrix} = \begin{bmatrix} \pi_1 & \pi_2 \end{bmatrix} \begin{pmatrix} 0.0069 & 0.9931 \\ 0.9275 & 0.0725 \end{pmatrix}$$

$$\pi_1 = 0.0069\ \pi_1 + 0.9275\ \pi_2 \qquad (1)$$

$$\pi_2 = 0.9931\, \pi_1 + 0.0752\ \pi_2 \qquad (2)$$

$$\pi_1 + \pi_2 = 1 \qquad (3)$$

Steady state probabilities are shown below

State	Probability
1	0.483
2	0.517

$$P = \begin{array}{c c} & \begin{array}{cc} R & S \end{array} \\ \begin{array}{c} R \\ S \end{array} & \begin{pmatrix} 0.483 & 0.517 \\ 0.483 & 0.517 \end{pmatrix} \end{array}$$

There is a probability of 0.483 that it will be raining tomorrow given that it is rainy today. There is a probability of 0.517 that if will sunny tomorrow given that it is sunny today.

Two-state steady state probabilities of Tamale weather (month of January) using management science application software.

```
MARKOV PROCESSES
****************

YOU HAVE INPUT THE FOLLOWING TRANSITION PROBABILITIES:
******************************************************

  START     TRANSITION TO STATE
  STATE       1       2
  *****     *****   *****

    1       0.006   0.994

    2       0.914   0.086


THE STEADY STATE PROBABILITIES ARE AS FOLLOWS:
**********************************************

            STATE         PROBABILITY
            *****         ***********

              1              0.479

              2              0.521
```

Steady state probability for two-state chain of Kumasi weather (month of January) using management science application software

```
MARKOV PROCESSES
****************


YOU HAVE INPUT THE FOLLOWING TRANSITION PROBABILITIES:
******************************************************

  START     TRANSITION TO STATE
  STATE       1      2
  *****     *****  *****

    1       0.026  0.974

    2       0.916  0.085


THE STEADY STATE PROBABILITIES ARE AS FOLLOWS:
**********************************************

            STATE        PROBABILITY
            *****        ***********

              1             0.485

              2             0.515
```

Two-state steady state probabilities of Accra weather (month of January) using management science application software

```
MARKOV PROCESSES
****************


YOU HAVE INPUT THE FOLLOWING TRANSITION PROBABILITIES:
******************************************************

  START    TRANSITION TO STATE
  STATE      1      2
  *****    *****  *****

    1      0.007  0.993

    2      0.928  0.073


THE STEADY STATE PROBABILITIES ARE AS FOLLOWS:
**********************************************

           STATE        PROBABILITY
           *****        ***********

             1             0.483

             2             0.517
```

Four-state steady state probabilities of Tamale weather using the management science software

```
MARKOV PROCESSES
****************

YOU HAVE INPUT THE FOLLOWING TRANSITION PROBABILITIES:
*******************************************************

  START        TRANSITION TO STATE
  STATE      1      2      3      4
  *****    *****  *****  *****  *****

    1      0.580  0.420  0.000  0.000

    2      0.000  0.000  0.480  0.520

    3      0.530  0.470  0.000  0.000

    4      0.000  0.000  0.250  0.750


THE STEADY STATE PROBABILITIES ARE AS FOLLOWS:
***********************************************

           STATE        PROBABILITY
           *****        ***********

             1             0.236

             2             0.187

             3             0.187

             4             0.389
```

Four-state steady state probabilities of Kumasi weather using the management science software

```
MARKOV PROCESSES
****************


YOU HAVE INPUT THE FOLLOWING TRANSITION PROBABILITIES:
******************************************************

  START          TRANSITION TO STATE
  STATE       1      2      3      4
  *****     *****  *****  *****  *****

    1       0.670  0.330  0.000  0.000

    2       0.000  0.000  0.400  0.600

    3       0.650  0.350  0.000  0.000

    4       0.000  0.000  0.450  0.550


THE STEADY STATE PROBABILITIES ARE AS FOLLOWS:
**********************************************

            STATE        PROBABILITY
            *****        ***********

              1             0.371

              2             0.189

              3             0.189

              4             0.251
```

Four-state steady state probabilities of Accra weather using the management science software

```
MARKOV PROCESSES
****************

YOU HAVE INPUT THE FOLLOWING TRANSITION PROBABILITIES:
******************************************************

  START        TRANSITION TO STATE
  STATE      1       2       3       4
  *****    *****   *****   *****   *****

    1      0.410   0.590   0.000   0.000

    2      0.000   0.000   0.430   0.570

    3      0.420   0.580   0.000   0.000

    4      0.000   0.000   0.340   0.660


THE STEADY STATE PROBABILITIES ARE AS FOLLOWS:
**********************************************

           STATE        PROBABILITY
           *****        ***********

             1             0.162

             2             0.228

             3             0.228

             4             0.382
```

Appendix D

Mean First Passage Times of Four-State of Tamale

Below shows the equations that would be used to compute the mean passage times for a four-state chain

$M_{12} = 1 + 0.58M_{12}$

$M_{13} = 1 + 0.58M_{13} + 0.42M_{23}$

$M_{14} = 1 + 0.58M_{14} + 0.42M_{24}$

$M_{21} = 1 + 0.58M_{31} + 0.42M_{41}$

$M_{23} = 1 + 0.42M_{43}$

$M_{24} = 1 + 0.58M_{34}$

$M_{31} = 1 + 0.47M_{21}$

$M_{32} = 1 + 0.53M_{12}$

$M_{34} = 1 + 0.53M_{14} + 0.47M_{24}$

$M_{41} = 1 + 0.25M_{31} + 0.75M_{41}$

$M_{42} = 1 + 0.25M_{32} + 0.75M_{42}$

$M_{43} = 1 + 0.75M_{43}$

Also

$$M_{11} = \frac{1}{0.236} \qquad M_{22} = \frac{1}{0.187} \qquad M_{33} = \frac{1}{0.187} \qquad M_{44} = \frac{1}{0.390}$$

By solving the matrix above as system of equations using MATLAB software, we have

>> A=[0.42 0 0 0 0 0 0 0 0 0 0 0;0 0.42 0 0 -0.42 0 0 0 0 0 0 0;0 0 0.42 0 0 -0.42 0 0 0 0 0 0;0 0 0 1 0 0 -0.58 0 0 -0.42 0 0;0 0 0 0 1 0 0 0 0 0 0 -0.42;0 0 0 0 0 1 0 0 -0.58 0 0 0;0 0 0 -0.47 0 0 1 0 0 0 0 0;-0.53 0 0 0 0 0 0 1 0 0 0 0;0 0 -0.53 0 0 -0.47 0 0 1 0 0

0;0 0 0 0 0 -0.25 0 0 0.25 0 0;0 0 0 0 0 0 -0.25 0 0 0.25 0;0 0 0 0 0 0 0 0 0 0 0.25];

>> b=[1;1;1;1;1;1;1;1;1;1;1;1];

>> M=A\b

M = 2.3810, 5.0610, 7.8855, 6.943, 2.6800, 5.504, 4.2634, 2.2619, 7.7664, 8.2634, 6.2619, 4.0000

Mean First Passage Times of Four-State of Kumasi

Below shows the equations that would be used to compute the mean passage times for a four-state chain

$$M_{12} = 1 + 0.67M_{12}$$

$$M_{13} = 1 + 0.67M_{13} + 0.33M_{23}$$

$$M_{14} = 1 + 0.67M_{14} + 0.33M_{24}$$

$$M_{21} = 1 + 0.40M_{31} + 0.60M_{41}$$

$$M_{23} = 1 + 0.60M_{43}$$

$$M_{24} = 1 + 0.40M_{34}$$

$$M_{31} = 1 + 0.35M_{21}$$

$$M_{32} = 1 + 0.65M_{12}$$

$$M_{34} = 1 + 0.65M_{14} + 0.35M_{24}$$

$$M_{41} = 1 + 0.45M_{31} + 0.55M_{41}$$

$$M_{42} = 1 + 0.45M_{32} + 0.55M_{42}$$

$$M_{43} = 1 + 0.45M_{43}$$

Also

$$M_{11} = \frac{1}{0.371} \qquad M_{22} = \frac{1}{0.189} \qquad M_{33} = \frac{1}{0.189} \qquad M_{44} = \frac{1}{0.251}$$

By solving the above as system of equations using MATLAB software, we have

```
>> C=[0.33 0 0 0 0 0 0 0 0 0 0 0;0 0.33 0 0 -0.33 0 0 0 0 0 0 0;0 0 0.33 0 0 -0.33 0 0 0 0 0 0;0 0 0 1 0 0 -0.4 0 0 -0.6 0 0;0 0 0 0 1 0 0 0 0 0 0 -0.6;0 0 0 0 0 1 0 0 -0.4 0 0 0;0 0 0 -0.35 0 0 1 0 0 0 0 0;-0.65 0 0 0 0 0 0 1 0 0 0 0;0 0 -0.65 0 0 -0.35 0 0 1 0 0 0;0 0 0 0 0 0 -0.45 0 0 0.45 0 0;0 0 0 0 0 0 0 -0.45 0 0 0.45 0;0 0 0 0 0 0 0 0 0 0 0 0.55];
>> d=[1;1;1;1;1;1;1;1;1;1;1;1];
>> M=C\d
```

M = 3.0303, 5.1212, 6.6768, 5.1282, 2.0909, 3.6465, 2.7949, 2.9697, 6.6162, 5.0171, 5.1919, 1.8182

First Passage Times of Mean Four-State of Accra

Below shows the equations that would be used to compute the mean passage times for a four-state chain

$$M_{12} = 1 + 0.41M_{12}$$

$$M_{13} = 1 + 0.41M_{13} + 0.59M_{23}$$

$$M_{14} = 1 + 0.41M_{14} + 0.59M_{24}$$

$$M_{21} = 1 + 0.43M_{31} + 0.47M_{41}$$

$$M_{23} = 1 + 0.57M_{43}$$

$$M_{24} = 1 + 0.43M_{34}$$

$$M_{31} = 1 + 0.58M_{21}$$

$$M_{32} = 1 + 0.42M_{12}$$

$$M_{34} = 1 + 0.42M_{14} + 0.58M_{24}$$

$$M_{41} = 1 + 0.34M_{31} + 0.66M_{41}$$

$$M_{42} = 1 + 0.34M_{32} + 0.66M_{42}$$

$$M_{43} = 1 + 0.66M_{43}$$

Also

$$M_{11} = \frac{1}{0.162} \qquad M_{22} = \frac{1}{0.228} \qquad M_{33} = \frac{1}{0.228} \qquad M_{44} = \frac{1}{0.382}$$

By solving the above matrix as system of equations using MATLAB software, we have

```
>> E=[0.59 0 0 0 0 0 0 0 0 0 0 0;0 0.59 0 0 -0.59 0 0 0 0 0 0 0;0 0 0.59 0 0 -0.59 0 0 0 0 0 0;0 0 0 1 0 0 -0.43 0 0 -0.57 0 0;0 0 0 0 1 0 0 0 0 0 0 -0.59;0 0 0 0 0 1 0 0 -0.43 0 0 0;0 0 0 -0.58 0 0 1 0 0 0 0 0;-0.42 0 0 0 0 0 0 1 0 0 0 0;0 0 -0.42 0 0 -0.58 0 0 1 0 0 0;0 0 0 0 0 -0.34 0 0 0.34 0 0;0 0 0 0 0 0 -0.34 0 0 0.34 0;0 0 0 0 0 0 0 0 0 0 0 0.34];
>> f=[1;1;1;1;1;1;1;1;1;1;1;1];
>> M=E\f
```

M =1.6949, 4.4302, 4.7407, 8.7535, 2.7353, 3.0458, 6.0770, 1.7119, 4.7577, 9.0182, 4.6530, 2.9412