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TWO RESULTS IN LEŚNIEWSKI'S MEREOLOGY

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In section **1** we prove that a certain characterization of class can be proved without the aid of auxiliary definitions. In section **2** we show that the main results in [1] still hold in the weakened system constructed by replacing the original definition of class by the characterization given in section **1**.[1] In what follows we assume that the reader is acquainted with the Ontological Preliminaries in [1].

1. In the proofs given in [3] of

$$[Aa] \therefore A \,\varepsilon\, \mathbf{Kl}(a) \,.\!\equiv\!: A \,\varepsilon\, A : [B] : a \subset \mathbf{el}(B) \,.\!\equiv\!.\, A \,\varepsilon\, \mathbf{el}(B)^2$$

and

$$[AB] \therefore A \,\varepsilon\, \mathbf{el}(B) \,.\!\equiv\!: A \,\varepsilon\, A : [D] : D \,\varepsilon\, \mathbf{el}(A) \,.\!\supset\!.\, [\exists F] \,.\, F \,\varepsilon\, \mathbf{el}(D) \,.\, F \,\varepsilon\, \mathbf{el}(B)$$

Leśniewski's definition of set plays an important role. Sobociński has asked whether this definition is creative with respect to the above two theorems, or, if not, whether some auxiliary definition is required. The answer is no. The proof follows.

An axiom system for mereology, denoted $\mathcal{M}$, is given by *A1* through *A6* with *D1*. (This is not an independent axiom set. (*A1* and *A2* are derivable from the rest).)

A1 $[A] : A \,\varepsilon\, A \,.\!\supset\!.\, A \,\varepsilon\, \mathbf{el}(A)$

A2 $[AB] : A \,\varepsilon\, \mathbf{el}(B) \,.\, B \,\varepsilon\, \mathbf{el}(A) \,.\!\supset\!.\, A = B$

A3 $[ABC] : A \,\varepsilon\, \mathbf{el}(B) \,.\, B \,\varepsilon\, \mathbf{el}(C) \,.\!\supset\!.\, A \,\varepsilon\, \mathbf{el}(C)$

A4 $[AB] : A \,\varepsilon\, \mathbf{el}(B) \,.\!\supset\!.\, B \,\varepsilon\, B$

D1 $[Aa] \therefore A \,\varepsilon\, \mathbf{Kl}(a) \,.\!\equiv\!: A \,\varepsilon\, A : [D] : D \,\varepsilon\, a \,.\!\supset\!.\, D \,\varepsilon\, \mathbf{el}(A) : [D] : D \,\varepsilon\, \mathbf{el}(A) \,.\!\supset\!.\, [\exists EF] \,.$
$E \,\varepsilon\, a \,.\, F \,\varepsilon\, \mathbf{el}(D) \,.\, F \,\varepsilon\, \mathbf{el}(E)$

A5 $[ABa] : A \,\varepsilon\, \mathbf{Kl}(a) \,.\, B \,\varepsilon\, \mathbf{Kl}(a) \,.\!\supset\!.\, A = B$

A6 $[Aa] : A \,\varepsilon\, a \,.\!\supset\!.\, [\exists B] \,.\, B \,\varepsilon\, \mathbf{Kl}(a)$

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1. These two results have been included in the same paper because, though they are very different, their proofs are related.

2. The theorem in [3] actually used $\sqsubset$; Lejewski remarked that $\sqsubset$ could be replaced by $\subset$.

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A7	$[ABDa] : A \varepsilon \mathbf{Kl}(a) . A \varepsilon \mathbf{el}(B) . D \varepsilon a . \supset . D \varepsilon \mathbf{el}(B)$	
PR	$[ABDa] : \mathrm{Hp}(3) . \supset .$	
4)	$D \varepsilon \mathbf{el}(A) .$	$[D1; 1; 3]$
	$D \varepsilon \mathbf{el}(B)$	$[A3: 4; 2]$
A8	$[Aa] \therefore A \varepsilon \mathbf{Kl}(a) . \supset : [B] : A \varepsilon \mathbf{el}(B) . \supset . a \subset \mathbf{el}(B)$	$[A7]$
A9	$[A] : A \varepsilon A . \supset . A \varepsilon \mathbf{Kl}(A)$	$[D1, a/A, E/A, F/D; A1]$
A10	$[AB] \therefore A \varepsilon \mathbf{el}(B) : [D] : D \varepsilon \mathbf{el}(B) . \supset . [\exists F] . F \varepsilon \mathbf{el}(D) . F \varepsilon \mathbf{el}(A) : \supset . A = B$	
PR	$[AB] \therefore \mathrm{Hp}(2) . \supset :$	
3)	$B \varepsilon B :$	$[A4; 1]$
4)	$[D] : D \varepsilon A . \supset . D \varepsilon \mathbf{el}(B) :$	$[1]$
5)	$[D] : D \varepsilon \mathbf{el}(B) . \supset . A \varepsilon A . [\exists F] . F \varepsilon \mathbf{el}(D) . F \varepsilon \mathbf{el}(A) :$	$[1; 2]$
6)	$[D] : D \varepsilon \mathbf{el}(B) . \supset . [\exists EF] . E \varepsilon A . F \varepsilon \mathbf{el}(D) . F \varepsilon \mathbf{el}(E) :$	$[5]$
7)	$B \varepsilon \mathbf{Kl}(A) .$	$[D1; 3; 4; 6]$
8)	$A \varepsilon \mathbf{Kl}(A) .$	$[A9; 1]$
	$A = B$	$[A5; 7; 8]$
A11	$[AB] :: A \varepsilon \mathbf{el}(B) . \sim(A = B) . \supset \therefore [\exists D] \therefore D \varepsilon \mathbf{el}(B) : [F] : F \varepsilon \mathbf{el}(D) . \supset .$	
	$\sim(F \varepsilon \mathbf{el}(A))$	$[A10]$
A12	$[AB] : A \varepsilon A . B \varepsilon \mathbf{Kl}(A \cup B) . \supset . A \varepsilon \mathbf{el}(B)$	$[D1]$
A13	$[Aa] : A \varepsilon \mathbf{Kl}(a) . \supset . [\exists B] . B \varepsilon a$	
PR	$[Aa] \therefore \mathrm{Hp}(1) . \supset :$	
2)	$A \varepsilon \mathbf{el}(A) :$	$[A1; 1]$
3)	$[D] : D \varepsilon \mathbf{el}(A) . \supset . [\exists E] . E \varepsilon a :$	$[D1; 1]$
	$[\exists B] . B \varepsilon a$	$[3; 2]$
A14	$[ABa] : A \varepsilon \mathbf{Kl}(a) . \sim(B \varepsilon B) . \supset . [\exists D] . D \varepsilon a . \sim(D \varepsilon \mathbf{el}(B))$	
PR	$[ABa] \mathrm{Hp}(2) . \supset .$	
3)	$\mathbf{el}(B) \circ \Lambda .$	$[A4; 2]$
4)	$[\exists D] . D \varepsilon a .$	$[A13; 1]$
	$[\exists D] . D \varepsilon a . \sim(D \varepsilon \mathbf{el}(B))$	$[4; 3]$
A15	$[ABa] : A \varepsilon \mathbf{Kl}(a) . \sim(A \varepsilon \mathbf{el}(B)) . B \varepsilon B . \supset . [\exists D] . D \varepsilon a . \sim(D \varepsilon \mathbf{el}(B))$	
PR	$[ABa] \because\because \mathrm{Hp}(3) . \supset ::$	
	$[\exists C] ::$	
4)	$\quad C \varepsilon \mathbf{Kl}(A \cup B) .$	$[A6; 1]$
5)	$\quad B \varepsilon \mathbf{el}(C) .$	$[D1; 4; 3]$
6)	$\quad \sim(B = C) \because$	$[A12; 1; 2; 4]$
	$\quad [\exists D] \because$	
7)	$\qquad D \varepsilon \mathbf{el}(C) :$	$[A11; 5; 6]$
8)	$\qquad [F] : F \varepsilon \mathbf{el}(D) . \supset . \sim(F \varepsilon \mathbf{el}(B)) ::$	
	$\qquad [\exists GH] ::$	
9)	$\qquad\quad G \varepsilon A \cup B .$	$[D1; 4; 7]$
10)	$\qquad\quad H \varepsilon \mathbf{el}(D) .$	
11)	$\qquad\quad H \varepsilon \mathbf{el}(G) :$	
12)	$\qquad\quad G \varepsilon B . \supset . [\exists F] . F \varepsilon \mathbf{el}(D) . F \varepsilon \mathbf{el}(B) :$	$[3; 10; 11]$
13)	$\qquad\quad \sim(G \varepsilon B) .$	$[12; 8]$
14)	$\qquad\quad G \varepsilon A .$	$[9; 13]$
15)	$\qquad\quad H \varepsilon \mathbf{el}(A) \therefore$	$[11; 14; 1]$
	$\qquad\quad [\exists KL] \therefore$	

16) $K \varepsilon a\,.$
17) $L \varepsilon \mathbf{el}(H)\,.$ [$D1$; 1; 15]
18) $L \varepsilon \mathbf{el}(K)\,.$
19) $L \varepsilon \mathbf{el}(D)\,.$ [$A3$; 17; 10]
20) $\sim L \varepsilon \mathbf{el}(B) :$ [8; 19]
21) $K \varepsilon \mathbf{el}(B) .\supset. L \varepsilon \mathbf{el}(B) :$ [$A3$; 18]
22) $\sim(K \varepsilon \mathbf{el}(B))\,.$ [20; 21]
$[\exists D] . D \varepsilon a . \sim(D \varepsilon \mathbf{el}(B))$ [16; 22]

A16 $[ABa] : A \varepsilon \mathbf{Kl}(a) . \sim(A \varepsilon \mathbf{el}(B)) .\supset. [\exists D] . D \varepsilon a . \sim(D \varepsilon \mathbf{el}(B))$ [$A15$; $A11$]

A17 $[Aa] \therefore A \varepsilon \mathbf{Kl}(a) .\supset: [B] : a \subset \mathbf{el}(B) .\supset. A \varepsilon \mathbf{el}(B)$ [$A16$]

A18 $[Aa] \therefore A \varepsilon \mathbf{Kl}(a) .\supset: [B] : a \subset \mathbf{el}(B) .\equiv. A \varepsilon \mathbf{el}(B)$ [$A17$; $A8$]

A19 $[AB] \therefore A \varepsilon A : [C] : A \varepsilon \mathbf{el}(C) .\equiv. B \varepsilon \mathbf{el}(C) :\supset. A = B$

PR $[AB] \therefore \mathrm{Hp}(2) :\supset.$
3) $A \varepsilon \mathbf{el}(A)\,.$ [$A1$; 1]
4) $B \varepsilon \mathbf{el}(A)\,.$ [2; 3]
5) $B \varepsilon \mathbf{el}(B)\,.$ [$A1$; 4]
6) $A \varepsilon \mathbf{el}(B)\,.$ [2; 5]
$A = B$ [$A2$; 6; 4]

A20 $[Aa] \therefore [B] : a \subset \mathbf{el}(B) .\equiv. A \varepsilon \mathbf{el}(B) :\supset. A \varepsilon \mathbf{Kl}(a)$

PR $[Aa] \because\because \mathrm{Hp}(1) :\supset::$
2) $a \subset \mathbf{el}(\wedge) .\equiv. A \varepsilon \mathbf{el}(\wedge) :$ [1]
3) $a \subset \wedge .\equiv. A \varepsilon \wedge :$ [$A4$; 2]
4) $\sim(a \subset \wedge)\,.$ [3]
5) $[\exists D] . D \varepsilon a \therefore$ [4]
$[\exists C] \therefore$
6) $C \varepsilon \mathbf{Kl}(a) :$ [$A6$; 5]
7) $[B] : a \subset \mathbf{el}(B) .\equiv. C \varepsilon \mathbf{el}(B) :$ [$A18$; 6]
8) $[B] : A \varepsilon \mathbf{el}(B) .\equiv. C \varepsilon \mathbf{el}(B) :$ [1; 7]
9) $A = C\,.$ [$A19$; 6; 8]
$A \varepsilon \mathbf{Kl}(a)$ [6; 9]

A21 $[Aa] \therefore A \varepsilon \mathbf{Kl}(a) .\equiv: [B] : a \subset \mathbf{el}(B) .\equiv. A \varepsilon \mathbf{el}(B)$ [$A18$; $A20$]

A22 $[Aa] \therefore A \varepsilon \mathbf{Kl}(a) .\equiv: A \varepsilon A : [B] : a \subset \mathbf{el}(B) .\equiv. A \varepsilon \mathbf{el}(B)$ [$A21$]

We now proceed to the characterization of element.

A24 $[AB] \therefore A \varepsilon A . \sim(B \varepsilon B) .\supset: [\exists D] : D \varepsilon \mathbf{el}(A) : [F] : F \varepsilon \mathbf{el}(D) .\supset. \sim(F \varepsilon \mathbf{el}(B))$

PR $[AB] \therefore \mathrm{Hp}(2) .\supset:$
3) $A \varepsilon \mathbf{el}(A)\,.$ [$A1$; 1]
4) $[F] . \sim(F \varepsilon \mathbf{el}(B)) :$ [$A4$; 2]
5) $[F] : F \varepsilon \mathbf{el}(A) .\supset. \sim(F \varepsilon \mathbf{el}(B)) :$ [4]
$[\exists D] : D \varepsilon \mathbf{el}(A) : [F] : F \varepsilon \mathbf{el}(D) .\supset. \sim(F \varepsilon \mathbf{el}(B))$ [3; 5]

A25 $[AB] \therefore A \varepsilon A . \sim(A \varepsilon \mathbf{el}(B)) . B \varepsilon B .\supset: [\exists D] : D \varepsilon \mathbf{el}(A) : [F] : F \varepsilon \mathbf{el}(D) .\supset. \sim(F \varepsilon \mathbf{el}(B))$

PR $[AB] :: \mathrm{Hp}(3) .\supset:\cdot:$

(Steps 4 through 15 are identical with *A15*)

16) $[F] : F \varepsilon \mathbf{el}(H) .\supset. \sim(F \varepsilon \mathbf{el}(B)) :$ [$A3$; 10; 8]
$[\exists D] : D \varepsilon \mathbf{el}(A) : [F] : F \varepsilon \mathbf{el}(D) .\supset. \sim(F \varepsilon \mathbf{el}(B))$ [15; 16]

A26 $[AB] \therefore A \varepsilon A . \sim(A \varepsilon \mathbf{el}(B)) . \supset : [\exists D] : D \varepsilon \mathbf{el}(A) : [F] : F \varepsilon \mathbf{el}(D) . \supset . \sim(F \varepsilon \mathbf{el}(B))$ [A24; A25]

A27 $[AB] \therefore A \varepsilon \mathbf{el}(B) . \supset : A \varepsilon A : [D] : D \varepsilon \mathbf{el}(A) . \supset . [\exists F] . F \varepsilon \mathbf{el}(D) . F \varepsilon \mathbf{el}(B)$ [A3; A1]

A28 $[AB] \therefore A \varepsilon \mathbf{el}(B) . \equiv : A \varepsilon A : [D] : D \varepsilon \mathbf{el}(A) . \supset . [\exists F] . F \varepsilon \mathbf{el}(D) . F \varepsilon \mathbf{el}(B)$ [A27; A26]

2. We now construct the system $\mathcal{L}$ by replacing *D1* in $\mathcal{M}$ with *A22*. $\mathcal{L}$ is weaker than $\mathcal{M}$, *cf.* [2], but the main theorems of section 1 of [1] still hold, provided we alter the definition of set. However, the meta-theorem in section 2 of [1] breaks down completely. ($\mathcal{L}$ is not an independent axiom set; *L2* and *L5* are derivable from the rest.)

L1 $[A] : A \varepsilon A . \supset . A \varepsilon \mathbf{el}(A)$

L2 $[AB] : A \varepsilon \mathbf{el}(B) . B \varepsilon \mathbf{el}(A) . \supset . A = B$

L3 $[ABC] : A \varepsilon \mathbf{el}(B) . B \varepsilon \mathbf{el}(C) . \supset . A \varepsilon \mathbf{el}(C)$

L4 $[AB] : A \varepsilon \mathbf{el}(B) . \supset . B \varepsilon B$

DL1 $[Aa] \therefore A \varepsilon \mathbf{Kl}(a) . \equiv : A \varepsilon A : [B] : a \subset \mathbf{el}(B) . \equiv . A \varepsilon \mathbf{el}(B)$

L5 $[ABa] : A \varepsilon \mathbf{Kl}(a) . B \varepsilon \mathbf{Kl}(a) . \supset . A = B$

L6 $[Aa] : A \varepsilon a . \supset . [\exists B] . B \varepsilon \mathbf{Kl}(a)$

First we shall prove in $\mathcal{L}$ some basic properties whose original proofs given in $\mathcal{M}$ are not valid in $\mathcal{L}$.

L7 $[A] : A \varepsilon A . \supset . A \varepsilon \mathbf{Kl}(A)$ [DL1]

L8 $[Aa] : A \varepsilon \mathbf{Kl}(a) . \supset . a \subset \mathbf{el}(A)$ [DL1; L1]

L9 $[AB] : A \varepsilon \mathbf{el}(B) . \supset . B \varepsilon \mathbf{Kl}(A \cup B)$

PR $[AB] \therefore \mathrm{Hp}(1) . \supset :$

2) $B \varepsilon B :$ [L4; 1]

3) $[C] : B \varepsilon \mathbf{el}(C) . \supset . A \varepsilon \mathbf{el}(C) :$ [L3; 1]

4) $[C] : B \varepsilon \mathbf{el}(C) . \supset . A \cup B \subset \mathbf{el}(C) :$ [3]

5) $[C] : A \cup B \subset \mathbf{el}(C) . \supset . B \varepsilon \mathbf{el}(C) :$ [2]

6) $[C] : A \cup B \subset \mathbf{el}(C) . \equiv . B \varepsilon \mathbf{el}(C) :$ [4; 5]

$B \varepsilon \mathbf{Kl}(A \cup B)$ [DL1; 2; 6]

L10 $[AB] : A \varepsilon \mathbf{el}(B) . \equiv . A \varepsilon A . B \varepsilon \mathbf{Kl}(A \cup B)$ [L9; L8]

L11 $[Aa] : A \varepsilon \mathbf{Kl}(a) . \supset . [\exists B] . B \varepsilon a$

PR $[Aa] \therefore \mathrm{Hp}(1) . \supset :$

2) $[B] : a \subset \mathbf{el}(B) . \equiv . A \varepsilon \mathbf{el}(B) :$ [DL1; 1]

3) $a \subset \mathbf{el}(\Lambda) . \equiv . A \varepsilon \mathbf{el}(\Lambda) :$ [2]

4) $a \subset \Lambda . \equiv . A \varepsilon \Lambda :$ [L4; 3]

5) $\sim(a \subset \Lambda) .$ [4]

$[\exists B] . B \varepsilon a$ [5]

L12 $[a] : !\{a\} . \equiv . !\{\mathbf{Kl}(a)\}$ [L6; L11]

L13 $\Lambda \circ \mathbf{Kl}(\Lambda)$ [L12]

L14 $[a] \therefore \sim(a \circ \Lambda) . \supset : [B] : a \subset \mathbf{el}(B) . \equiv . \mathbf{Kl}(a) \subset \mathbf{el}(B)$

PR $[a] \therefore \mathrm{Hp}(1) . \supset :$

2) $[\exists A] . A \varepsilon \mathbf{Kl}(a) .$ [L6; 1]

3) $\mathbf{Kl}(a) \varepsilon \mathbf{Kl}(a) :$ [L5; 2]

4) $[B] : a \subset \mathbf{el}(B) . \equiv . \mathbf{Kl}(a) \varepsilon \mathbf{el}(B) :$ [DL1; 3]

$[B] : a \subset \mathbf{el}(B) . \equiv . \mathbf{Kl}(a) \subset \mathbf{el}(B)$ [3; 4]

L15 $[aB]:a \subset \mathsf{el}(B) .\equiv. \mathsf{Kl}(a) \subset \mathsf{el}(B)$ [*L13*; *L14*]

L16 $[A]: \rightharpoonup \{A\} .\supset. A \subset \mathsf{el}(A)$ [*L1*]

L17 $[a] . a \subset \mathsf{el}(\mathsf{Kl}(a))$ [*L15*, $B/\mathsf{Kl}(a)$; *L5*; *L16*]

L18 $[ab]: a \subset b .\supset. \mathsf{Kl}(a) \subset \mathsf{el}(\mathsf{Kl}(b))$ [*L17*, a/b; *L15*, $B/\mathsf{Kl}(b)$]

We now give a definition of set which differs from Leśniewski's.

DL2 $[Aa]:A \varepsilon \mathsf{st}(a) .\equiv. A \varepsilon A . [\exists b] . b \subset a . A \varepsilon \mathsf{Kl}(b)$

L19 $[Aa]:A \varepsilon \mathsf{st}(a) .\equiv. [\exists b] . b \subset a . A \varepsilon \mathsf{Kl}(b)$ [*DL2*]

L20 $[Aa]:A \varepsilon \mathsf{st}(a) .\supset. A \varepsilon \mathsf{Kl}(a \cap \mathsf{el}(A))$

PR $[Aa]: \mathrm{Hp}(1) .\supset.$

$[\exists b] .$

2) $b \subset a .$ } [*L19*; 1]

3) $A \varepsilon \mathsf{Kl}(b) .$ }

4) $b \subset \mathsf{el}(A) .$ [*L8*; 3]

5) $b \subset a \cap \mathsf{el}(A) .$ [2; 4]

6) $\mathsf{Kl}(b) \subset \mathsf{el}(\mathsf{Kl}(a \cap \mathsf{el}(A))) .$ [*L18*; 5]

7) $A \varepsilon \mathsf{el}(\mathsf{Kl}(a \cap \mathsf{el}(A))) .$ [3; 6]

8) $\mathsf{Kl}(a \cap \mathsf{el}(A)) \varepsilon \mathsf{Kl}(a \cap \mathsf{el}(A)) .$ [*L4*; 7]

9) $\mathsf{Kl}(a \cap \mathsf{el}(A)) \varepsilon \mathsf{el}(A) .$ [*L15*, $a/a \cap \mathsf{el}(A)$, B/A; 8]

$A \varepsilon \mathsf{Kl}(a \cap \mathsf{el}(A))$ [*L2*; 7; 9]

L21 $[Aa]:A \varepsilon \mathsf{st}(a) .\equiv. A \varepsilon \mathsf{Kl}(a \cap \mathsf{el}(A))$ [*L20*; *L19*]

L22 $[Aa]:A \varepsilon \mathsf{Kl}(a) .\supset. A \varepsilon \mathsf{st}(a)$ [*L19*; b/a]

L23 $[ab]: a \subset b .\supset. \mathsf{st}(a) \subset \mathsf{st}(b)$ [*L19*]

L24 $[Aa]:A \varepsilon \mathsf{Kl}(a) .\equiv. A \varepsilon \mathsf{st}(a) . a \subset \mathsf{el}(A)$ [*L22*; *L8*; *L21*]

L25 $[Aa]:A \varepsilon a .\supset. A \varepsilon \mathsf{st}(a)$ [*L19*; *L7*]

L26 $[Aa]:A \varepsilon \mathsf{st}(a) .\supset. A \varepsilon \mathsf{el}(\mathsf{Kl}(a))$ [*L21*; *L18*, $a/a \cap \mathsf{el}(A)$, b/a]

Although *L31* is not needed in the present line of proof, we prove it here since it shows that set still possesses one of the important properties of the collective class in this weaker system.

L27 $[a] . \mathsf{st}(a) \subset \mathsf{st}(\mathsf{st}(a))$ [*L23*, $b/\mathsf{st}(a)$; *L25*]

L28 $[ADa]: D \varepsilon \mathsf{st}(a) \cap \mathsf{el}(A) .\supset. D \varepsilon \mathsf{el}(\mathsf{Kl}(a \cap \mathsf{el}(A)))$

PR $[ADa]: \mathrm{Hp}(1) .\supset.$

2) $D \varepsilon \mathsf{st}(a) .$ } [1]

3) $D \varepsilon \mathsf{el}(A) .$ }

4) $D \varepsilon \mathsf{Kl}(a \cap \mathsf{el}(D)) .$ [*L21*; 2]

5) $a \cap \mathsf{el}(D) \subset a \cap \mathsf{el}(A) .$ [*L3*; 3]

$D \varepsilon \mathsf{el}(\mathsf{Kl}(a \cap \mathsf{el}(A))) .$ [*L18*; 5; 4]

L29 $[Aa] . \mathsf{Kl}(a \cap \mathsf{el}(A)) \subset \mathsf{el}(\mathsf{Kl}(\mathsf{st}(a) \cap \mathsf{el}(A)))$ [*L18*; *L25*]

L30 $[Aa]:A \varepsilon \mathsf{st}(\mathsf{st}(a)) .\supset. A \varepsilon \mathsf{st}(a)$

PR $[Aa]: \mathrm{Hp}(1) .\supset.$

2) $A \varepsilon \mathsf{Kl}(\mathsf{st}(a) \cap \mathsf{el}(A)) .$ [*L21*; 1]

3) $A \varepsilon \mathsf{el}(\mathsf{Kl}(a \cap \mathsf{el}(A))) .$ [*L15*; *L28*; 2]

4) $\mathsf{Kl}(a \cap \mathsf{el}(A)) \varepsilon \mathsf{Kl}(a \cap \mathsf{el}(A)) .$ [*L4*; 3]

5) $\mathsf{Kl}(a \cap \mathsf{el}(A)) \varepsilon \mathsf{el}(A) .$ [*L15*; 4]

6) $A \varepsilon \mathsf{Kl}(a \cap \mathsf{el}(A)) .$ [*L2*; 3; 5]

$A \varepsilon \mathsf{st}(a)$ [*L21*; 6]

L31 $[a] . \mathsf{st}(a) \circ \mathsf{st}(\mathsf{st}(a))$ [*L27*; *L30*]

Next we show that the definition of set given here is equivalent in mereology to Leśniewski's definition.

A29 $[Aa] \therefore A \,\varepsilon\, \mathbf{st}(a) .\equiv: A \,\varepsilon\, A : [D] : D \,\varepsilon\, \mathbf{el}(A) .\supset. [\exists EF] . E \,\varepsilon\, a . E \,\varepsilon\, \mathbf{el}(A) . F \,\varepsilon\, \mathbf{el}(D) . F \,\varepsilon\, \mathbf{el}(E)$

PR $[Aa] \therefore$

1) $A \,\varepsilon\, \mathbf{st}(a) .\equiv. A \,\varepsilon\, \mathbf{Kl}(a \cap \mathbf{el}(A))$ [*L21*]

2) $\equiv: A \,\varepsilon\, A . a \cap \mathbf{el}(A) \subset \mathbf{el}(A) : [D] : D \,\varepsilon\, \mathbf{el}(A) .\supset. [\exists EF] . E \,\varepsilon\, a \cap \mathbf{el}(A) . F \,\varepsilon\, \mathbf{el}(D) . F \,\varepsilon\, \mathbf{el}(E)$ [*D1*; 1]

3) $\equiv: A \,\varepsilon\, A : [D] : D \,\varepsilon\, \mathbf{el}(A) .\supset. [\exists EF] . E \,\varepsilon\, a . E \,\varepsilon\, \mathbf{el}(A) . F \,\varepsilon\, \mathbf{el}(D) . F \,\varepsilon\, \mathbf{el}(E)$ [2]

Of the theorems in section 1 of [1], *T9*, *T10*, *T12*, *T20*, *T21* are not valid in $\mathcal{L}$. The proofs of two theorems must be altered slightly. In *T29* replace 4) by

4a) $B \,\varepsilon\, \mathbf{Kl}((a - A) \cap \mathbf{el}(B))$. [*L21*; 2]
$[\exists C]$.

4b) $C \,\varepsilon\, (a - A) \cap \mathbf{el}(B)$. [*L11*; 4a]

The reason for 6), 7) and 8) becomes [4b]. In *T43* replace 6) and 11) respectively by

6) $A \,\varepsilon\, \mathbf{Kl}(b \cap \mathbf{el}(A))$. [*L21*; 5]

11) $E \,\varepsilon\, \mathbf{Kl}(a \cap \mathbf{el}(E))$. [*L21*; 10]

The reason for 7) and 8) becomes [*L11*; 6] and the reason for 12) and 13) becomes [*L11*; 11]. Now except for the trivial properties of outside used in *T5*, all the reasons that occur in the rest of the theorems have been proved in $\mathcal{L}$.

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