The Journal of Developing Areas
Volume 50 No. 3 Summer 2016

LONG MEMORY AND ARFIMA MODELLING: THE CASE OF CPI INFLATION RATE IN GHANA

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ABSTRACT

In macroeconomic theory and economic policy, changes in the general price level or the rate of inflation plays an essential role. Hence, for example, one of the motives behind the adoption of Inflation Targeting policy (IT) by Ghana and the treaty espoused by the European Monetary Union, known as the Maastricht Treaty, was the convergence of inflation rates. On the other hand there is a controversy about which is the order of integration in the inflation rates, some authors arguing that this variable is stationary I(0) Whittle others saying it is nonstationary I(1). In this study we examine the CPI inflation rates of Ghana from a different perspective allowing for fractional degrees of differentiation. Thus, the methodology is based on long memory or long-range dependence processes, using fractional integration and employing techniques based on Whittle parametric and semiparametric methods and autoregressive fractionally integrated moving average (ARFIMA) models. Standard I(0)/I(1) methods were also employed. Our findings indicate that long memory exists in the CPI inflation rate of Ghana. After processing fractional differencing and determining the short memory components, the following two models, ARFIMA(3,0.427,1) and ARFIMA(2,0.499,1) were respectively specified to describe the pre and post introduction of IT policy in May 2007. Consequently, the CPI inflation rate of Ghana is fractionally integrated and mean reverting. Long memory in financial time series has important implications for the critical explanation of financial time series behaviour, as it could provide an opportunity to earn speculative profits in financial markets and cast disbelief on the correctness of the EMH. For instance, when price changes exhibit long memory or long-range dependence, asset pricing models based on the Efficient Market Hypothesis (EMH) may overestimate or underestimate investment risk. Furthermore, the presence of long memory in inflation rates can provide vital information about the likely impact of shocks (e.g. demand/supply) on the economy with respect to time. The results obtained in this study would be very useful in setting up monetary policies or consolidating previous policies such as IT in order to enhance economic growth. Moreover, estimation of long memory in inflation rates can serve as an evaluation tool to assess the performance of monetary policy under different dispensations. Lastly, the presence of long memory can assist in identifying inflationary pressures in the economy.

JEL Classifications: C14, C22, E31
Keywords: Consumer Price Index (CPI) inflation rates, Whittle method, Long memory, AFIMA model
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INTRODUCTION

In macroeconomic theory and economic policy, changes in the general price level or the rate of inflation plays an essential role. Hence, for example, one of the motives behind the adoption of Inflation Targeting policy (IT) by Ghana and the treaty espoused by the European Monetary Union, known as the Maastricht Treaty, was the convergence of inflation rates**.** Moreover, the last two decades of macro and financial economic research has resulted in a huge collection of important contributions in the area of long memory modelling, both from a theoretical and an empirical standpoint. From a theoretical perspective, considerable effort has been focussed in the areas of testing and estimation, and a few significant contributions include Granger (1980), Granger and Joyeux (1980), Hosking (1981), Geweke and Porter-Hudak (1983), Lo (1991), Sowell (1992a), Ding *et al.* (1993), Cheung and Diebold (1994), Robinson (1994; 1995a,b), etc. The empirical analysis of long memory models also has seen equally remarkable treatment, including studies by Diebold and Rudebusch (1989, 1991a,b), Hassler and Wolters (1995), Gil-Alana and Robinson (1997), Hyung and Franses (2001), Bos *et al*. (2002). Indeed, the considerable array of publications on the subject is not surprising, given the importance of long memory models in economics following the seminal contributions made by Clive W.J. Granger (see e.g. Granger (1980).

We have seen that a significant number of the analyses of financial time series and econometrics hinge on the assumption of an efficient market hypothesis (henceforth EMH), which in its weak form states that returns of variables such as inflation rates, exchange rates, interest rates, equity prices among others, are expected to be i.i.d. white noise. This means, they follow the martingale process, hence not predictable (Fama, 1970). Notwithstanding the countless number of research papers following the pioneering work of Nelson and Plosser (1982), differences still remain in the literature on the main question of whether or not the post-war inflation possesses a unit root. Even though there is substantial evidence backing the unit root process (e.g. Barsky, 1987; MacDonald and

[1]Prof. Gil-Alana gratefully acknowledges financial support from the Ministerio de Economía y Competitividad (SEJ2014-2017, ECO2014-55236).
Alexander Boateng is a Ph.D. student and faculty researcher. The authors would like to express their sincere gratitude to Statistics South Africa and the Bank of Ghana for providing the data, and also to the DST-NRF Centre of Excellence in Mathematical Statistical Sciences (Coe-MaSS) for supporting the student. Opinions expressed in the paper and the conclusions arrived at are those of the authors and are not necessarily to be attributed to Statistics South Africa, the Bank of Ghana and CoE-MaSS. Comments from the Editor and an anonymous referee are gratefully acknowledged.

Murphy, 1989; Ball and Cecchetti, 1990; Wickens and Tzavalis, 1992; and Kim 1993), Rose (1988) provided evidence of stationarity in inflation rates. Mixed evidence has been provided by Kirchgassner and Wolters (1993) whereas Brunner and Hess (1993) argued that the inflation rate was stationary before the 1960s, but that it possesses a unit root since then.

A probable resolution to this debate should not only be of academic interest, as nonstationarity in inflation would have dire consequences for central banks' ratification of inflationary shocks with its ripple effect on the macroeconomic policymakers' response to external pressures. A breakthrough to the challenge of this conflicting evidence was recently provided by modelling inflation rates through fractionally integrated processes. Using fractional differentiation and ARFIMA models, Baillie *et al*. (1996) examined a group of seven countries (UK, USA, Italy, France, Germany, Canada and Japan), and found strong evidence of long memory in the inflation rates with the exception of Japan. Comparable evidence of long memory in inflation rates of the USA, the UK, Germany, France and Italy is also provided by Hassler and Wolters (1995). Delgado and Robinson (1994) found evidence of long memory in the Spanish inflation rates. The clarification of this evidence put forward that inflation rates are mean-reverting processes and that inflationary shock will persist but ultimately dissipate.

A time series exhibits long memory when there is significant dependence between observations that are separated by a long period of time. Characteristics of a long memory time series are an autocorrelation function $\rho(k)$ that decays hyperbolically to zero and a spectral density function $f_x(.)$ that is unbounded in the neighborhood of zero frequency. Recent statistical literature has concerned itself with a study of long memory models which go beyond the presence of random walks and unit roots in univariate time series representations, and ARFIMA(p, d, q), processes are known to be capable of modelling long-run persistence. They were introduced by Granger and Joyeux (1980), who generalize Box-Jenkins models, when the order d of integration is allowed to be fractional.

Since modelling the inflation rate as a fractionally integrated process appears to improve our understanding of inflationary dynamics, this paper extends the existing long memory analysis on inflation rates of Ghana along two lines. First, it performs long memory analysis using Whittle methods developed by Künsch (1987) and Robinson (1995a) together with exact maximum likelihood (EML) developed by Sowell (1992a). Second, we model the long memory process using the ARFIMA model, in order to capture the short-and long-range effects of the inflation response to shocks (e.g. demand/supply shocks). Interestingly, what makes this research different from other research is the emphasis on the method of estimating the fractional differencing parameter and our quest to obtain an appropriate model to describe the inflationary dynamics of Ghana. This is because a clear appreciation of inflation behaviour will inform policymakers to consolidate the adoption of inflation targeting (IT) policy and/or adapt other policies to control inflation. We are motivated to examine the dynamics of inflation on the grounds of economic policy and also on the basis of the superficiality and gaps in the existing literature in exploring the issue of long memory especially in the Ghanaian context in terms of coverage of issues and methods of estimating the fractional integration parameter.

This paper makes a contribution to the existing literature on inflation in the following ways: (1) by establishing the existence or non-existence of long memory in the inflation of Ghana, and (2) knowing the properties of long memory in inflation may provide useful information to policymakers as well as to investors decisions on investment and risk management since Ghana is one of the two countries in Sub-Saharan Africa with IT policy.

LITERATURE REVIEW

In Ghana, some research has been done regarding modelling and forecasting inflation. However, the use of the fractionally integrated approach in describing inflation dynamics is very limited. For instance, Tweneboah *et al*. (2015) conducted a study on long memory behaviour of real interest rates in Ghana using ARFIMA and FIGARCH models and found them to exhibit an indistinguishable integration property. Alagidede *et al*. (2014) also conducted a study on a regional analysis of inflation dynamics in Ghana: persistence, causes and policy implications, using fractional integration. Notable evidence of asymmetries in the degree of persistence was found in both regional and sectorial areas of the economy. Omane-Adjepong *et al*. (2013) examined which was the best approach for short-term forecasting of Ghanaian inflation between seasonal-ARIMA and Holt-Winters. From their study, they concluded that Ghana's inflation could be described by seasonal-ARIMA process especially for short-term forecasting. In another study, Atta-Mensah and Bawumia (2003) presented a Vector-Error-Correction Forecasting Model (VECFM), based on broad money to forecast some selected Ghanaian macroeconomic variables such as money, growth, inflation, output growth, treasury-bill rate and exchange rate. Their results revealed that the VECFM model performed well around the turning points.

In Ghana, this will probably be the first paper developed to address the concept of long memory through a fractionally integrated approach and modelling the process using an ARFIMA model. It is evident that there is a serious lack of research in the area of long memory. The present study places itself in that context. The rest of the article is organised as follows: Section 3 describes the empirical framework. Section 4 briefly describes the data used. Section 5 presents the empirical results and Section 6 provides some concluding remarks.

EMPIRICAL METHODOLOGY

In this section we provide a brief definition and tests for long memory together with an ARFIMA model.

Definition of Long Memory

Long memory describes the correlation structure of a time series that displays temporal dependence between observations distant, discrete and far apart in time (Baillie, 1996). By contrast, if the correlation between observations becomes negligible during long lags, the series is said to exhibit short memory. To be more precise, with a long memory process, shocks to financial asset returns tend to decay at a hyperbolic rate, whereas in the of case short memory process, shocks tend to decay at an exponential rate. McLeod and Hipel (1978, p.492) have suggested that 'it is often assumed that recent values of time series possess more information with regard to present and future values than the values in the distant past'. They define a stationary process as having long memory such that its absolute autocorrelation function has an infinite sum. Accordingly, the autocorrelation function ρ_j at lag j is defined according to:

$$\lim_{n\to\infty} \sum_{j=-n}^{n} |\rho_j| = \infty \tag{1}$$

where n is equal to the number of observations. This definition is consistent with the process that does not have unit roots, but whose autocorrelation function does not decay too fast or rapidly. Also, a process has a long memory if there exists a real number $H \in (0.5,1)$ and a finite positive constant C such that the autocorrelation function $\rho(k)$ at lag k has the following rate of decays (Assaf, 2006):

$$\rho(k) = Ck^{2H-2} \text{ as } k \to \infty \tag{2}$$

The parameter Hwhich is referred to as the Hurst exponent (Hurst, 1951), is a numerical estimate that represents the degree of long memory properties in a series. The value of H can be interpreted as follows:

1. For $0 < H < 1$, the process is said to be long memory (long-range dependence or persistent). This indicates that larger values tend to be followed by larger values, smaller values tend to followed by smaller values.

2. For $0 < H < 0.5$, the process is said to be anti-persistent. This means that larger values will be followed by smaller values and vice versa.

3. For $H = 0.5$, the process is said to be a random walk.

As explained in Tsay (2002), Mills and Markellos (2008), Gil-Alana (2008), an example of long memory process is the fractional integrated process defined by:

$$(1-L)^d x_t = y_t; \quad 0.5 < d < 0.5 \tag{3}$$

where $\{y_t\}$ is a white noise process, L is the back shift operator and d is the fractional integration parameter. The fractional integrated parameter is related to the Hurst exponent as follows: $d = H - 0.5$. Again, from the perspective of fractional integration, when $0 < d < 0.5$, then the process is regarded as long memory, whereas $d \geq 0.5$ and $-0.5 < d < 0$ processes are regarded respectively as nonstationary and anti-persistent

Indeed, long memory or long range dependence property describes the high-order correlation structure of a series; where the series are characterized by irregular cyclical fluctuations. Mandelbrot (1977) describes long memory processes as having fractal dimensions, in the form of non-linear behaviour marked by distinct but non-periodic cyclical patterns and long-term dependence between distant observations. A variety of measures have been used to detect long memory in time series. For example, in the time domain, long memory is associated with a hyperbolically decaying autocovariance function. Equivalently, the presence of long memory is indicated by a spectral density function that approaches infinity near the zero frequency; in other words, such series display power at low frequencies (Lo, 1991; Disario *et al*., 2008). These concepts have led several authors to develop stochastic models that capture long memory behaviour, such as the fractionally-integrated $I(d)$ models introduced to economics and finance by Granger (1980), Granger and Joyeux (1980), and Hosking (1981). In particular, fractional integration theory asserts that the fractional difference parameter which indicates the order of integration, is not an integer value (0 or 1) but a fractional value (Baillie *et al*., 1996). Fractionally integrated processes are different from both stationary and unit-root processes in that they are persistent (i.e., they reflect long memory), but are also mean reverting and as a consequence provide a flexible alternative to standard $I(1)$ and $I(0)$ processes.

Long Memory Test

Long memory is an important empirical feature of many financial variables. The presence of long memory implies the existence of nonlinear forms of dependence between the first and the second moments, and hence the potential of time series predictability. Testing for long memory is an essential task since evidence of long memory will support the use of fractionally integrated ARIMA.

Whittle Estimator

The Whittle estimator, is often used to estimate the fractional differencing parameter d. One of the most promising of these is the local Whittle estimator initially proposed by Künsch (1987) and modified later by Robinson (1995b). This is obtained by minimizing the local Whittle log likelihood at Fourier frequencies close to zero, given by:

$$\Gamma(d) = -\frac{1}{2\pi m}\sum_{j=1}^{m}\frac{I(\omega_j)}{f(\omega_j;d)} - \frac{1}{2\pi m}\sum_{j=1}^{m} f(\omega_j;d) \tag{4}$$

where $f(\omega_j;d)$ is the spectral density (which is proportional to $(\omega_j)^{2d}$). As frequencies close to zero are used, we require that $m \to \infty$ and $\frac{1}{m} + \frac{m}{n} \to 0$ as $n \to \infty$. Taqqu and Teverovsky (1997) showed that $\hat{d}_w$ can be obtained by maximizing the following function:

$$\Gamma(d) = ln\left(\frac{1}{m}\sum_{j=1}^{m}\frac{I(\omega_j)}{I(\omega_j)^{-2d}}\right) - 2d\frac{1}{m}\sum_{j=1}^{m}\ln(\omega_j) \tag{5}$$

Indeed Robinson (1995b) showed that for the estimates of d obtained in this way, $(4m)^{\frac{1}{2}}\,(\hat{d}_w - d) \to N(0.1)$ for $-0.5 < d < 0.5$. The robustness of the standard, local and aggregate Whittle estimator was studied by Taqqu and Teverosky (1997) and it was found to perform well in finite samples.

Exact Maximum Likelihood Estimator

The exact Gaussian maximum likelihood objective function for the model ARFIMS model

$\phi(L)(1-L)^d X_t = \theta(L)\varepsilon_t,\ t \in Z$ is (when $-0.5 < d < 0.5$):

$$L_E(d, \phi, \theta, \sigma^2, \mu) = -\frac{T}{2}\ln|\Omega| - \frac{1}{2}(X-\mu)'\Omega^{-1}(Y-\mu l) \quad (6)$$

where $l = (1, \dots, 1)'$, $X = (x_1, \dots, x_T)$, ϕ and θ are the parameters of $\phi(L)$ and $\theta(L)$, μ is the mean of X, and Ω is the variance matrix of X, which is a complicated function of d and the remaining parameters of the model. Sowell (1992a) derived an efficient procedure for solving this function in terms of hypergeometric functions. Nevertheless, an important shortcoming is that the roots of the autoregressive polynomial cannot be multiple.
Collecting the parameters in the vector $\gamma = (d, \phi', \theta', \sigma^2, \mu)'$, the exact maximum likelihood (EML) estimator is obtained by maximizing the likelihood function (see Equation 6) with respect to γ. Sowell (1992a) showed that the EML estimator of d is a consistent and asymptotically normal, i.e.

$$\sqrt{T}(\hat{d}_{EML} - d) \rightarrow_d N(0(\frac{\pi^2}{6} - C)^{-1}) \quad (7)$$

where $C = 0$, when $p = q = 0$ and $c > 0$ otherwise. The variance of the EML may be obtained as $(1,1)'th$ element of the inverse of the matrix:

ARFIMA Model

The concept of fractional integration in literature was pioneered by Granger and Joyeux (1980) and Hosking (1981). The model, known as Autoregressive Fractionally Integrated Moving Average (ARFIMA), allows for increased flexibility as far as modelling low-frequency dynamics is concerned. The ARFIMA model is given as follows:

$$\phi(L)(1-L)^d y_t = \theta(L)\varepsilon_t, \varepsilon_t \sim iid(0, \sigma^2) \quad (8)$$

Where d is the fractional integration parameter, L is the lag operator and ε_t is white noise residual. Polynomial structures of Equation (8) including AR, $\phi(L)$ and MA, $\theta(L)$ lie outside the unit circle, satisfying the stationarity and invariability conditions. The fractional differencing lag operator $(1-L)^d$ is defined by the binomial expansion as follows:

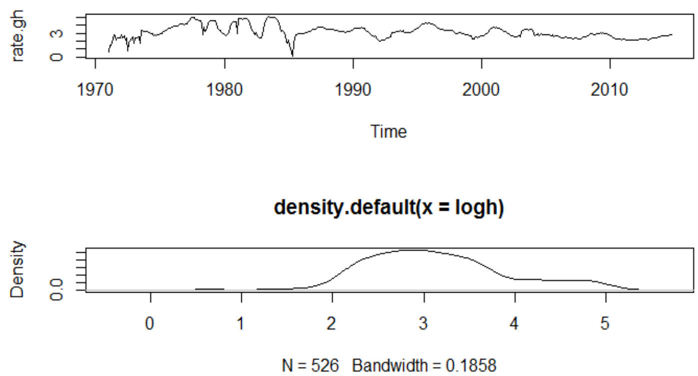
$$(1-L)^d = 1 - 2d + \frac{d(d-1)}{2!}L^2 - \frac{d(d-1)(d-2)}{3!}L^3 + \cdots, \quad (9)$$

ARFIMA process is nonstationary when $d \geq 0.5$. For $0 < d < 0.5$, the process is said to exhibit long memory. The process shows short memory when $d = 0$ and anti-persistence when $d < 0$.

DATA AND STOCHASTIC PROPERTIES

We perform the analysis on CPI inflation for Ghana (GHCPI). The data set obtained from the Bank of Ghana dated January 1971 to October 2014, totaling 526 observations was log transformed. A visual inspection of Figure 1 shows that Ghanaian inflation is skewed to the right and potentially subject to some structural breaks, especially between 1980 and 1990.

FIGURE 1. THE BEHAVIOUR OF CPI INFLATION RATES OF GHANA



Source: Diagram produced from R
Note: The data is obtained from Bank of Ghana (BoG)

The summarised statistics of the CPI inflation of Ghana is given in Table 1. The distribution of CPI inflation is fat tailed since kurtosis is greater than 3, as shown in Table 1. The coefficient of skewness is 0.292 which shows that the CPI inflation of Ghana is skewed to the right. This shows that the distribution is non-normal and leptokurtic. The Jarque-Bera (JB) test confirms these findings since it rejects normality. Results from the ARCH (18) test for conditional heteroscedasticity provide strong evidence of ARCH effects in the inflation rates of both countries. The presence of a significant non-zero autocorrelation can be seen in Table 1 with the Box-Pierce, Q-statistic coefficients of 2985.60.

TABLE 1. SUMMARY AND DESCRIPTIVE STATISTICS

Series	**Mean**	**Median**	**Std. Deviation**	**Skewness**
GHCPI	3.128	3.027	0.786	0.292
Series	**Kurtosis**	**J-B test**	**ARCH-LM (18)**	**Q (18)**
GHCPI	3.498	12.942 (0.000)	357.906 (0.000)	2985.6 (0.000)

***Note**: GHCPI indicates the CPI inflation rates of Ghana; J-B indicates J Jarque-Bera normality test; ARCH indicates LM conditional variance; Q(.) indicates Box-Pierce correlation test. In each case the null hypothesis is rejected. Source: Authors own calculation and BoG as data source.*

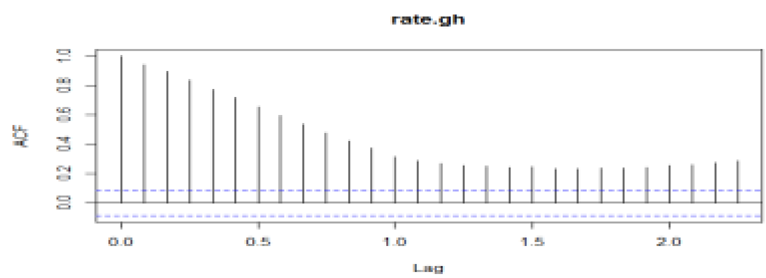
RESULTS AND INTERPRETATION

In this section, we discuss the sample results from the autocorrelation function analysis, tests for structural breaks and long memory. We shall also apply the ARFIMA model to the GHCPI after incorporating structural breaks borrowing the procedure proposed by Shimotsu (2006), which examines the null hypothesis against an alternative of structural change.

Autocorrelation Function

The distributional characteristics of GHCPI inflation presented in Figure 2 can be investigated further by analysing the behaviour of their autocorrelation functions. The autocorrelation function of GHCPI decreases slowly at a hyperbolic rate, an indication of long memory (or long-range dependence), which is also conformed to a fractionally integrated series (Haslett and Raftery, 1989; Gil-Alana, 2008).

FIGURE 1. THE BEHAVIOUR OF ACF FOR CPI INFLATION RATES OF GHANA



Stationarity Test

Before testing for long memory, CPI inflation of Ghana was subjected to a stationarity test using the Augmented Dickey-Fuller (ADF) (1979), Phillips- Perron (PP) (1988) and Kwiatkowski–Phillips–Schmidt–Shin (KPSS) (1992) tests in order to determine whether the series is level stationary or differenced stationary. As presented in the Appendix 1, the hypothesis of stationarity is rejected at the 5% and 10% significance level for the Ghanaian inflation rate under KPSS test.

Results of Long Memory Test

We apply the Whittle and exact maximum (EML) tests to GHCPI inflation. The results obtained are reported in Table 2. The hypothesis of no long memory is rejected under the Whittle method and exact maximum likelihood (EML), indicating the presence of long memory. We are guided by the algorithm of Hyndman-Khandakar (2008) in choosing our short memory parameters. It is also well known that a neglected structural break could lead to bias in estimating long memory parameters. The explanation given by Granger and Hyung (2004) is related to the fact that long memory may be the result of various kinds of misspecifications and/or the presence of structural breaks. In this scheme of things, a greater accumulation of misspecifications would naturally lead to greater spurious long memory. This paper takes the adoption of IT policy (in May 2007) in Ghana into consideration, in order to assess its significance in controlling inflation.[1]

TABLE 2. LONG MEMORY TEST FOR CPI INFLATION RATE IN GHANA

Test	1971M1 – 2007M5 (Before IT policy)	2007M6 – 2014M10 (After IT policy)
Whittle test	0.957 (0.078)	0.929 (0.175)
EML (Sowell, 1992)	0.999 (0.000)	1.0004 (0.000)

Note*: The null hypothesis of no long memory is rejected. Standard error are presented in parenthesis. The estimation of the long memory parameter was done using the first difference of GHCPI inflation.*

From Table 2, it is evident that GHCPI inflation possesses long memory and its highly persistent and non-mean reverting even after adopting IT policy in May 2007. Hence, the need to consolidate the IT policy and/ or adopt other monetary policies to control inflation.

Modelling GHCPI Inflation Rates with the ARFIMA Model

Next, we fit the ARFIMA model to the GHCPI inflation and the results are depicted in Tables 3 and 4 respectively, taken the introduction of the IT policy in 2007 into account.

TABLE 3. DIFFERENT ARMA SPECIFICATION FOR GHCPI INFLATION BETWEEN 1971M1-2007M5

ARMA	Log. Likelihood	AIC	SIC
(1, 0)	-66.968	139.936	152.176
(1, 2)	-41.200	92.400	112.800
(3, 1)	-39.151	90.302	114.781
(4, 2)	-39.074	94.149	126.788
(4, 1)	-39.083	92.167	120.726

Note*: ARMA(3,1) specification was selected based on the AIC.*

[1]Using the approach in Gil-Alana (2008) that combines fractional integration with structural breaks, we found little evidence of a break at that date. Nevertheless, we separate the sample in two subsamples according to the introduction of the IT policy in May 2007.

TABLE 4. PARAMETER ESTIMATES FOR ARFIMA(3, 0.427, 1) FOR 1971M1 – 2007M5

Parameter	Estimate	Std. Error	t-value	P(>│t│)
d	0.427	0.078	5.443	0.000
ϕ_1	-0.414	0.121	-3.437	0.000
ϕ_2	0.658	0.121	5.462	0.000
ϕ_3	0.299	0.121	2.480	0.013
θ_1	0.866	0.192	4.521	0.000

ARFIMA (3,0.427,1) model was specified to GHCPI inflation 1971M1 -2007M5 through the Whittle method of estimation. From the results in Table 4 and 6, the estimate of d and accompanying p-values for the null hypothesis of no long memory for GHCPI is 0.427 and 0.499 respectively for the period of 1971M1-2007M5 and 2007M6-2014M10. The estimation of d for GHCPI is significantly greater than zero, hence GHCPI inflation may contain long memory and are mean reverting. Essentially, the fractional integration parameter d specified by the ARFIMA model in both context, captures the long-run effects, and the ARMA (see Table 3 and 6) parameters capture the short-run effects. In fact, the short-run effects describe the behaviour of the fractionally differenced process$(1 - L)^d y_t$, through ARMA processes whereas the long-run effects describe the behaviour of the fractionally integrated y_t .

TABLE 5. DIFFERENT ARMA SPECIFICATION FOR GHCPI INFLATION BETWEEN 2007M6-2014M10

ARMA	Log. Likelihood	AIC	SIC
(1, 0)	-37.202	80.405	87.871
(0, 1)	-37.722	81.444	88.909
(1, 2)	-34.068	78.137	90.581
(2, 1)	-34.048	78.097	90.540
(3, 3)	-30.209	96.327	76.418

Note*: ARMA(2,1) specification was selected based on the AIC.*

TABLE 6. PARAMETER ESTIMATES FOR ARFIMA(2, 0.499, 1) FOR 2007M6-2014M10

Parameter	Estimate	Std. Error	t-value	P(>|t|)
d	0.499	0.174	2.872	0.004
ϕ_1	0.431	0.156	2.744	0.006
ϕ_2	0.206	0.157	1.315	0.188
θ_1	-0.012	0.106	-0.109	0.912

We have estimated different specifications previously described. Based on the AIC and the log likelihood (Log. Lik), an appropriate ARFIMA model was selected. AIC and log likelihood (Log. Lik) are reported in Table 3 and 5. The parameter estimates of these selected models are displayed in Table 4 and 6. The estimate of d is large and statistically significant, indicating the evidence of long memory in GHCPI inflation. Again, we can see that, most of the estimated AR terms in both models are small and statistically insignificant for GHCPI. This indicates that the fractional integration parameter d has accounted for all the dependencies in GHCPI. It is worth noting that the AR only affects the shape of the autocorrelation function in the short run, but the long-run dynamics is hardly affected.

Model Diagnostics

We perform residuals analysis on ARFIMA (3,0.427,1) and ARFIMA (2,0.499,1) in order to ensure white noise. We apply tests for normality, heteroscedasticity, ARCH LM and Box and Ljung tests. From Table 7 and 8, we observe that all the tests are statistically different from zero with the exception of normality.

TABLE 7. RESIDUAL DIAGNOSTICS FOR ARFIMA(3,0.427,1)

	Test type	Test statistic	P-value
Residual Corr.	Box-Jung test	5.946	0.819
Serial Corr.	Box-Pierce test	5.833	0.829
ARCH Effects	ARCH-LM	14.203	0.164
Normality	Jarque Bera test	3813.616	0.000

TABLE 8: RESIDUAL DIAGNOSTICS FOR ARFIMA(2,0.499,1)

	Test type	Test statistic	P-value
Residual Corr.	Box-Jung test	4.079	0.943
Serial Corr.	Box-Pierce test	3.762	0.957
ARCH Effects	ARCH-LM	3.957	0.949
Normality	Jarque Bera test	177-828	0.000

These results are consistent with previous studies. For instance, Hassler and Wolters (1995) investigated inflation rates of the US, the UK, France, Germany and Italy. They estimated ARFIMA models and identified "reliable" specifications. Table 6 of their work contains diagnostic tests for the ARFIMA models, and it turns out that normality of the residuals is rejected for all conventional significance levels for all models. In another study, Bos et al. (1999) estimated an ARFIMA model for US inflation with $d = 0.5$. Unfortunately, they did not test for normality of the residuals. Our results are largely in agreement with similar work on inflation rates. There are indications of long memory in inflation notwithstanding the rejection of normality of residuals (See Appendix 2), the ARFIMA models seem to offer a well specified model for inflation rates.

DISCUSSIONS AND CONCLUSIONS

In this paper, we investigated the presence of long memory dynamics in the CPI inflation rates of Ghana (GHCPI). Applying Whittle methods along with EML tests, we found evidence of long memory in the inflation rate. The ARFIMA(3,0.427,1) and ARFIMA(2,0.499,1) models were found to be the most appropriate for GHCPI inflation before and after the introduction of IT policy in 2007. Thus, the fractional integration parameter which is a measure of speed of adjustment of inflation to equilibrium state, were found to be 0.427 and 0.499 for both regimes respectively (i.e.1971M1-2007M5 and 2007M6-2014M6), indicating the presence of long memory. Clearly the impact of the 2007 cannot be underestimated since the fractional differencing parameter increases in terms of magnitude in the second regime (2007M6-2014M10). The complete results suggest that GHCPI inflation are not efficient and can be predicted given the past information. Understanding possible causes of long memory in GHCPI is beyond the scope of this paper, but it is an essential issue to be addressed. Largely, the significance of d obtained for GHCPI inflation accounted for the dependence in the series. Hence, the need to adopt more efficient policies including IT policy in order to steer the ever increasing inflation of to an equilibrium position.

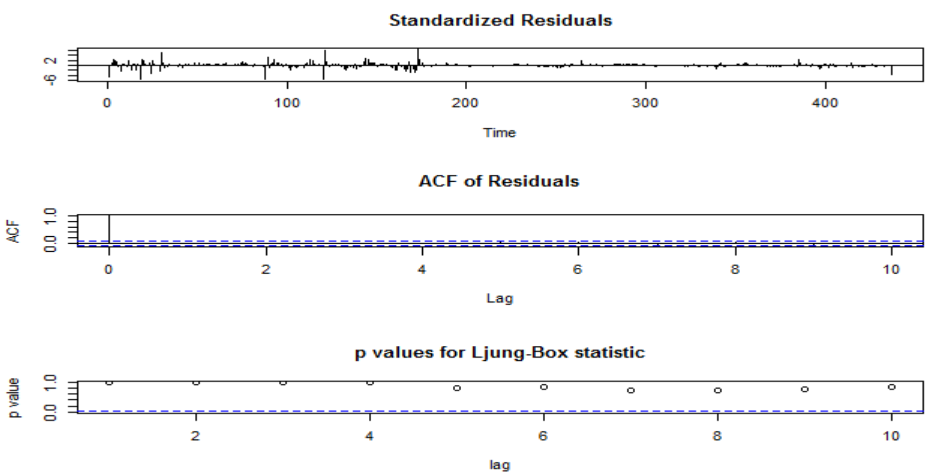
Long memory in financial time series has important implications for the critical explanation of financial time series behaviour, as it could provide an opportunity to earn speculative profits in financial markets and cast disbelief on the correctness of the EMH. For instance, when price changes exhibit long memory or long-range dependence between observations, asset pricing models based on the EMH may overestimate or underestimate investment risk. The error in the Value-at-Risk (VaR) evaluation of investment risk implies an incorrect evaluation of assets (Lo, 1991). In a nut shell, the study of long memory will provide an important guide when implementing linear pricing models and it allows the development of non-linear pricing models to be taken into consideration. Furthermore, the presence of long memory in inflation rates can provide vital information about the likely impact of shocks (e.g. demand/supply) on the economy with respect to time. This information can be useful for the purposes of setting out monetary policy or consolidating previous policies such as IT in order to enhance economic growth. Moreover, estimation of long memory in inflation rates can serve as an evaluation tool to assess the performance of monetary policy under different dispensations. Lastly, the presence of long memory can assist in identifying inflationary pressures in the economy. In the near future it is our intention to address the issue of volatility in the GHCPI inflation.

APPENDIX 1. STATIONARITY TEST

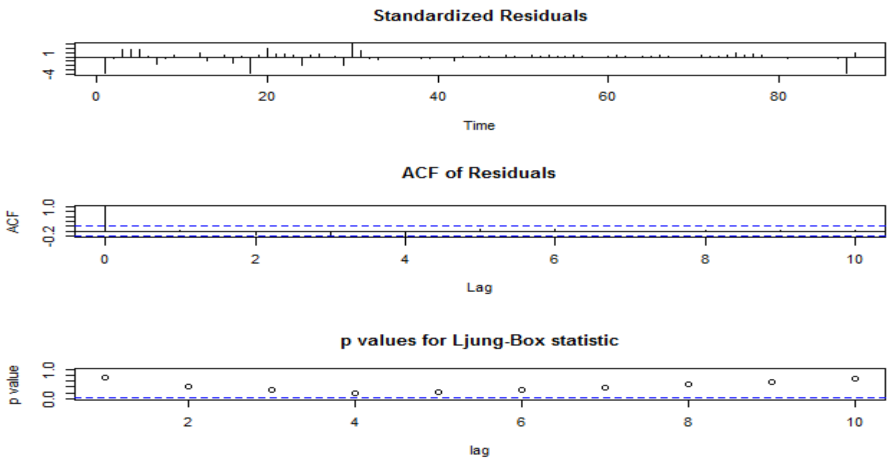
Series	ADF test		PP test		KPSS test	
	Cont.	Trend	Cont.	Trend	Cont.	Trend
GHCPI	-3.77** (-3.43)	-4.35** (-3.41)	-4.64** (-2.87)	-5.22** (-3.42)	0.74** (0.46)	0.15** (0.14)

Note*: Values in the parenthesis are the critical values at 5%, ** the hypothesis of stationarity is rejected*

APPENDIX 2A: RESIDUALS DIAGNOSTICS FOR ARFIMA(3,0.427,1)



APPENDIX 2B: RESIDUALS DIAGNOSTICS FOR ARFIMA(2,0.499,1)



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