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S.Y. Mensah

A.M. Dikandé

F.K.A. Allotey

and

N.G. Mensah

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S.Y. Mensah[1]
Department of Physics, Laser and Fibre Optics Centre,
University of Cape Coast, Cape Coast, Ghana
and
The Abdus Salam International Centre for Theoretical Physics, Trieste, Italy,

A.M. Dikandé[1,2]
Centre de Recherches en Physique du Solide, Départment de Physique,
Faculté des Sciences, Université de Sherbrooke,
J1K-2R1, Sherbrooke, Québec, Canada
and
The Abdus Salam International Centre for Theoretical Physics, Trieste, Italy,

F.K.A. Allotey
Atomic Energy Commission, Kwabenya, Ghana
and
The Abdus Salam International Centre for Theoretical Physics, Trieste, Italy

and

N.G. Mensah
Department of Mathematics, University of Cape Coast, Cape Coast, Ghana.

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[1]Regular Associate of the Abdus Salam ICTP.
[2]E-mail: amdikand@physique.usherb.ca

Abstract

Propagation of large-amplitude electromagnetic fields and their interactions with small-amplitude waves in finite superlattices are considered in the framework of the sine-Gordon theory. Finite-size effects result in modulating the large-amplitude fields to a lattice of kinked waves. This kink-lattice wave displays both a soliton feature and the particle property typical to nonlinear topological excitations. The interaction of the kink-lattice soliton with weak electromagnetic waves reveals an unusual number (exactly three) of bound states, which is attributed to the finite size of the propagation medium.

1 Introduction

Nonlinear fields and particularly those exhibiting solitary-wave solutions, are now of great interest in many physical problems [1]. Free-field solitons have been investigated extensively namely for quantum fields [2], but in condensed matter there are subsidiary requirements such as the knowledge of the soliton behaviours in the presence of various inherent objects as impurities, phonons and under bias from applied stresses [3,4]. The account of the soliton-phonon interactions has been quite enriching to the low-temperature statistical mechanics of kink condensate systems by providing the phase shift useful to construct a more accurate phenomenological free-energy density [5]. The basic idea involves the recognition that the appearance of the soliton excitation must be accompanied by consistent changes in the phonon density of states as well as on their shapes about the soliton. Moreover, the soliton stability in the phonons lattice can require an energy correction during which its symmetry invariance, broken by the interaction with phonons, is restored. In general, the breaking (or restoring) of the soliton translational invariance is attributed to its collision with specific modes of the phonon spectrum, with (or without) an energy cost. These specific modes thus belong to the discrete lattice phonon spectrum and are characteristic of the system. For the sine-Gordon (sG), for instance, only one bound state coinciding with the translational restoring has often been reported [5] while the ϕ^4 is usually claimed to possess exactly one additional non-zero frequency bound state [5,6]. However, it seems evident that these results are not general but depend instead on certain conditions. Indeed the ϕ^4 model was recently shown [7] to display five non-degenerate bound states in the finite-length limit.

Investigating soliton excitations and their scatterings with lattice phonons involving a definite system size is, by the way, interesting for evident reasons: not only the finite-length condition agrees well enough with the reality, but also by their "kink-lattice" features finite-length solitons are likely to reveal novel non-trivial properties quite distinct from those of the usual uncorrelated (or single) kink solitons [8-10]

In this work we will be concerned with the interactions of solitons with small -amplitude waves in finite-size superlattices. Solitonic phenomena in superlattices are generally considered in two different contexts, that is, to the structural and the electronic view points. In the first solitons are identified with atomic bond defects or dislocations, which has been widely studied [11,12] since the pioneering works of Frenkel-Kontorova and Frank-van der Merwe[13]. In the second, it is suspected that a strong Electromagnetic (EM) wave could make its propagation medium essentially nonlinear due to the self-action effect, in virtue of which it is modulated into a soliton EM field. Tetervov [14] remarked that if the superlattice is acted upon by a weak probing EM wave simultaneously with the soliton EM field, the scattering coefficients of the probing wave will depend on the soliton parameters. However, in our opinion, the conjectured soliton collapse after some characteristic collision time is rather speculative by the very intrinsic stability properties of topological solitons [1,2]. We base our argument on the well established sustainable role of bound states whose essential contribution is to restore the energy lost by the soliton during its interaction process with the weak probing EM wave.

The highlight result of the present study is that finite-size effects increase the number of such bound states and in turn enhance the stability of the soliton EM wave in the superlattice. In the next section (section 2) we obtain the basic nonlinear equation in terms of 1+1 dimensional(1D) sG equation resulting from the electronic dispersion law in the superlattice. In section 3, this equation is solved assuming a conduction in the lowest miniband and a finite-size superlattice. The interaction of the resulting "kink-lattice"soliton with EM waves is anlaysed and particular attention is paid to the bound states. Section 4 is devoted to some concluding remarks.

2 The sine-Gordon EM field equation of the superlattice

Proceeding as in [15], we assume that the characteristic length in which a significant change in the EM field is large enough compared with the de Broglie wavelength of the electrons or with the superlattice period. Therefore the electron current density can be written as

$$j = -e \sum_{p} f(p) v(p + \frac{e}{c} A(r,t)) \tag{1}$$

where $f(p)$ is the distribution function of the electron canonical momentum p , $v(p)$ is the electron velocity, e the electron charge and $A(r,t)$ the vector potential. The key physical parameter describing the electron distribution in the bands is the dispersion relation, for superlattices the following dispersion law is most often considered [14-16]:

$$\epsilon_\nu(p) = \frac{p_\perp^2}{2m} + \epsilon_\nu - \Delta_\nu \cos(\frac{p_z}{\hbar} d) \tag{2}$$

In eqn.(2), $p_\perp$ and p_z are the transverse and longitudinal(relative to the superlattice axis) components of the quasi momentum, respectively, Δ_ν is the half width of the ν^{th} allowed miniband,

$$\epsilon_\nu = \frac{\hbar^2}{2m} (\frac{\pi}{d_o})^2 \nu^2 \tag{3}$$

are the size-quantized levels in an isolated conduction film, $d = d_o + d_1$ (d_o is the width of the rectangular potentail wells and d_1 is the potential depth with a non zero quantum transparency) is the superlattice period.

We assume that electrons are confined to the lowest conduction miniband ($\nu = 1$) and omit the miniband indices. This is to say that the field does not induce transitions between the filled and empty minibands. We further assume that the characteristic time for change in the field is short compared with the mean free time of electrons τ. We therefore ignore the collision of electrons with the lattice. The electron velocity is given by

$$v_z(p) = \frac{\partial \epsilon(p)}{\partial p_z} = \frac{\Delta d}{\hbar} \sin(\frac{p_z d}{\hbar}) \tag{4}$$

Substituting eqn. (4) into eqn.(1) and making the following transformation $p_z \to p_{oz} + \frac{e}{c} A_z$, we obtain for the non-degenerate electron gas the following expression for the z component of the current density j,

$$j_z = j_o \sin(\frac{e}{\hbar c} A_z d) \tag{5}$$

where

$$j_o = -\frac{e\Delta d}{\hbar} \sum_p f(p) \cos p_{oz} d = -n \frac{e\Delta d}{\hbar} \frac{I_1(\Delta/kT)}{I_o(\Delta/kT)} \tag{6}$$

with n the conduction electron density and $I_k(x)$ the Bessel function of imaginary argument. We evoke the classical Maxwell equation for the vector potential, i.e.,

$$\Delta A - \frac{1}{v_o^2} \frac{\partial^2 A}{\partial t^2} = -\frac{4\pi}{v_o} j \tag{7}$$

and substitute j_z from (5) to obtain the nonlinear field equation:

$$\frac{\partial^2 \phi}{\partial t^2} - v_o^2 \Delta \phi + \omega_o^2 \sin \phi = 0 \tag{8}$$

where

$$\phi = \frac{e}{\hbar c} A_z d, \ \omega_o^2 = \frac{m \omega_p^2 d^2 \Delta I_1(\Delta/kT)}{\hbar^2 I_o(\Delta/kT)} \tag{9}$$

Here v_o is the EM velocity in the absence of electrons and $\omega_p^2 = \frac{4\pi n}{m} e^2$ is the square of the Langmuir frequency.

Equation (8) is quite frequent in the literature of nonlinear processes where it is called sine-Gordon equation. The most popular version is the 1D sG equation whose single and double soliton solutions, i.e. kink and breather, deeply influenced our understanding of various condensed matter phenomena among which are charge transfers in quasi-1D conductors[17,18], flow of flux quanta in Josephson Junctions[19] and also in superlattices[14-16].

The single-kink solution of the 1D sG equation is usually obtained in the form:

$$\phi(x,t) = 4\arctan\{\exp[\pm(\frac{x-ut}{l(u)})]\},\ l^2 = l_o^2\gamma^{-2},\ \gamma = (1-\frac{u^2}{v_o^2})^{-1/2},\ l_o^2 = \frac{v_o^2}{\omega_o^2} \quad (10)$$

This describes a smoothed-out function with limiting values $\phi_o \to \pm\pi$ as $x \to \pm\infty$. The quantity l refers to the spatial extension of the kink spreading, the dependence of l on the kink translation velocity u is suggestive of its "particle-like" features. The stability of sG kinks is well understood in terms of their collisions with small amplitude waves. For eqn. (10), one of the phonon modes excited by these collisions is a bound state[5]. This mode has zero frequency which ensures the preservation of the kink profile upon collision.

In the next section we reformulate this concept by involving finite-size effects.

3 Electromagnetic "kink-lattice" wave collisions with weak waves: bound states

We are looking for the topological soliton solution of eqn.(8) for an arbitrary, definite length of the propagation medium. With respect to the present content, this corresponds to assuming finite-size superlattice. We can access such solutions by demanding that $\phi_o \to \pm\pi$ as $x \to \pm\frac{L}{2}$. We find:

$$\phi^o(x,t) = 2\arccos[sn(\frac{x-ut}{kl}|k)], \quad 0 \leq k \leq 1 \tag{11}$$

where sn is the Jacobi Elliptic (snoidal) function of modulus k. In figure (1), (9) appears to form an EM kink-lattice wave. The periodicity of this kink-lattice soliton at the boundaries $x = \pm\frac{L}{2}$ gives[8]

$$L = 2klK(k). \tag{12}$$

Here K(k) is the Jacobi Elliptic Integral of the first kind. It is also worth mentioning the "particle-like" feature of eqn. (11), by virtue of which its topological energy possesses the classical velocity dependence law:

$$\epsilon^{(\nu)}(k) = \gamma\epsilon^o(k), \quad \epsilon^o(k) = \tfrac{4v_o^2}{kl_o}[2E(k) - (1-k^2)K(k)] \tag{13}$$

where E(k) is the Jacobi Elliptic Integral of the second kind.

Examining the stability of the kink-lattice wave eqn. (11) upon collisions with small-amplitude EM fields we rewrite the solution of eqn. (8) including the resulting soliton dressing field, i.e.,

$$\phi(x,t) = \phi^o(x) + \psi(x,t), \quad \psi(x,t) \sim \Psi(x)\exp^{-i\omega t} \tag{14}$$

This new soliton ansatz traduces a soliton dressed by a stationary EM field irrespective of the soliton dynamic property. Then we are led to:

$$-v_o^2\Psi_{xx} + \omega_o^2[2sn^2(\frac{x}{kl_o}) - 1]\Psi = \omega^2\Psi \tag{15}$$

Eqn. (15), which is of quantum-mechanical Schrödinger type, is transformed to a dimensionless form by setting:

$$X = \frac{x}{kl_o}, \quad \Omega = \frac{\omega^2}{\omega_o^2}, \quad h = k^2(1+\Omega^2) \tag{16}$$

We thereby obtain

$$\Psi_{XX} + [h - n(n+1)k^2 sn^2(X)]\Psi = 0, \quad n = 1 \tag{17}$$

So we arrive at an eigenvalue problem described by a first-order Lamé equation[20]. It is instructive to remark that the case $n = 2$, i.e. the second order Lamé equation corresponds to the ϕ^4 model. This last case has been discussed very recently[7]. Following the method developed in [7] we obtain exactly three bound states. They are listed as follows:

i) the zero frequency (the so called Goldstone translation) mode;

$$\omega_1 = 0, \ \Psi_1(x) = A_1(k)dn(\frac{x}{kl_o}) \tag{18}$$

ii) the first non-zero frequency bound state;

$$\omega_2 = \frac{\omega_o}{k}, \ \Psi_2(x) = A_2(k)sn(\frac{x}{kl_o}) \tag{19}$$

iii) the second non-zero frequency bound state;

$$\omega_3 = \frac{\sqrt{1-k^2}}{k}\omega_o, \ \Psi_3(x) = A_3(k)cn(\frac{x}{kl_o}) \tag{20}$$

These three modes form an orthonormal subset which is represented by the relation

$$\int_{-L/2}^{L/2} \Psi_\nu(x)\Psi_\mu(x)dx = \pi\delta(\nu,\mu), \quad \nu,\mu = 1,2,3 \tag{21}$$

For $\nu = \mu$, eqn. (21) becomes the normalization relations of the three modes, which enable us to determine $A_\mu(k)$. Thus,

$$A_1^2(k) = \frac{\pi}{2kl_oE(k)} \tag{22}$$

$$A_2^2(k) = \frac{\pi}{2kl_o[K(k) - E(k)]} \tag{23}$$

$$A_3^2(k) = \frac{\pi}{2k^3l_o[E(k) - (1-k^2)K(k)]} \tag{24}$$

The three bound states are sketched in figures $2a$, $2b$ and $2c$.
It is worthy to note that as $k \to 1$, $L \to \infty$ in eqn. (21) then $sn \to$ tanh, dn $\to$ sech and $cn \to$ sech such that the problem tends to the infinite-length sG case. Namely, the first and third bound states merge into the Goldstone mode while the second collapses.

4 Concluding Remarks

The propagating of large-amplitude EM fields in superlattices has been shown to be intimately dependent on the material size. Finite-size effects are manifested both on the shape modulation of the field and in its scattering properties with weak probing EM waves. In the first context, finite-size effects result in modulating the large EM field into a periodic kink lattice, the period of which is proportional to the soliton width, as well as to the length of the propagating medium. A fundamental implication of this dependence of the soliton shape on the length is the possibility of fitting a desired EM wave profile to the appropriate material size. The appearance of new bound states due to finite-size effects is significant enough and traduces a direct incidence of the material size on the soliton stability. These results are all of great interest from the physical point of view, namely they open a relevant new path in the electronic designs of the superlattice material toward which the manipulation of the EM wave amplitudes and shapes can facilitate a desired device perfomance.

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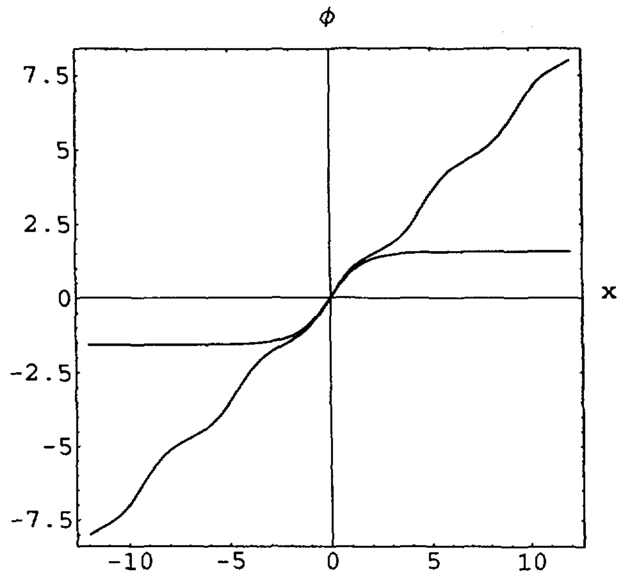

Figure 1: "kink-lattice" ($k \sim 0.8$) and single-kink($k \sim 1$) soliton solutions of the sine-Gordon equation.

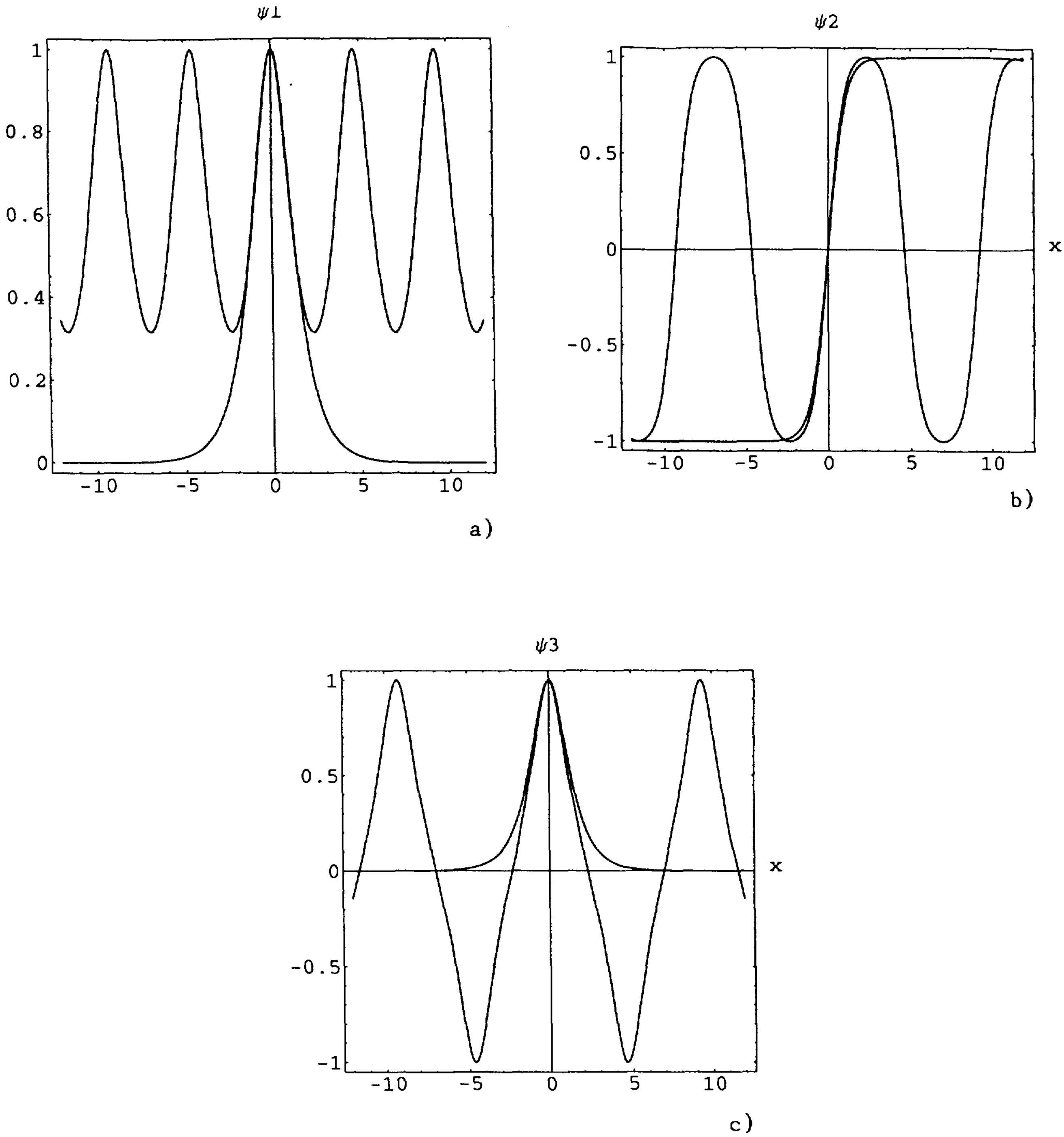

Figure 2: The three bound states of the finite-size sine-Gordon system: (2a) is the Goldstone mode, the two non-zero frequency modes (2b) and (2c) are the new bound states induced by finite-size effects.