UNIVERSITY OF CAPE COAST

STRUCTURAL EQUATION MODELING OF THE IMPACT OF TEACHER VARIABLES ON STUDENT ACADEMIC PERFORMANCE IN ELECTIVE MATHEMATICS OF SELECTED SENIOR HIGH SCHOOLS IN GHANA

JOSEPH KWEKU ANSAH

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BY

JOSEPH KWEKU ANSAH

Thesis submitted to the Department of Mathematics, School of Physical Sciences, College of Agriculture and Natural Sciences, University of Cape Coast, in partial fulfilment of the requirements for award of Doctor of Philosophy Degree in Mathematics

OCTOBER 2020

DECLARATION

Candidate's Declaration

I hereby declare that the thesis is the result of my own original research and that no part of it has been presented for another degree in this university or elsewhere.

Candidate's Signature……………………… Date……………………

Name: ………………………………………………………………………..

Supervisors' Declaration

We hereby declare that the preparation and presentation of the dissertation were supervised in accordance with the guidelines on supervision of thesis laid down in the University of Cape Coast.

Principal Supervisor's Signature…………………… Date……….……………

Name: ………………………………………………………………………..

Co-Supervisor's Signature…………………………. Date……….……………

Name: ……………………………………………………………………….

ABSTRACT

The purpose of the study was to investigate the effect of teacher variables such as teachers' qualification, teaching experience, teachers' knowledge of subject matter, and teachers' pedagogical content knowledge, on student performance in elective mathematics in Senior High Schools (SHS) in Ghana. The positivist philosophy and descriptive survey design were adopted for the study. A sample of 225 SHS elective mathematics teachers and 6,750 SHS elective mathematics students was selected from Greater Accra, Ashanti, Central, Western and Northern regions for the study through multi-stage sampling technique. Two questionnaires; one for teachers and the other for students were used for the data collection. The data was analyzed using Structural Equation Modeling. The study found a statistically significant predictive relationship among teacher prior education and training, teaching experience, teacher subject matter knowledge, teacher pedagogical content knowledge, and students' performance in elective mathematics. Teaching experience was found to moderate the relationship between teachers' subject matter knowledge and students' performance in elective mathematics while it played an insignificant moderating role in the relationship between pedagogical content knowledge and performance in elective mathematics. It was concluded that teacher variables: prior education and training, teaching experience, subject matter knowledge, and pedagogical content knowledge all play significant roles in the performance of students in elective mathematics. It was recommended that stakeholders in education should formulate policies that will ensure the hiring of professional and qualified elective mathematics teachers to facilitate effective teaching and learning in senior high schools. It was also recommended that all non-professional and unqualified elective mathematics teachers who are already engage, should be encouraged to persue post-graduate studies in mathematics education.

KEYWORDS

Teacher Subject Matter Knowledge

Elective Mathematics Performance

Teaching Experience

Pedagogical Content Knowledge

Teacher Prior Education and Training

Elective Mathematics Teachers

Elective Mathematics Students

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My wife, Joyce and my cousin, Dr Howard have both been there to urge me on when I felt like giving up at certain points and so deserve thanks from the bottom of my heart. To the heads, teachers and students of the institutions where I gathered my data, I say a big thank you for making this a success. Last but not least, I say thank you to Mr. Franck Quansah and Mr. Sylvester Tenkorang for their immense technical support.

DEDICATION

To my family.

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LIST OF ACRONYMS

EXP	Teaching Experience
SMK	Subject Matter Knowledge
PCK	Pedagogical Content Knowledge
TPET	Teacher Prior Education and Training
ACH	Academic Performance of Students in Elective Mathematics

CHAPTER ONE

INTRODUCTION

Educators are important role models for students and have a big impact in helping to shape, create, support and establish students' strengths, goals and knowledge. It is essential therefore to be aware of the effective characteristics that an educator brings into a learning environment (Ofem, Iyam, & Bassey, 2015).

For many years, educational scholars and policy makers have been concerned with characteristics that influence student performance. A study(Campbell, Coleman, Hobson, McPartland, Mood, Weinfeld &York, 1966)in the past suggested that, there were insignificant inputs by schools in determining students' performance, but now other studies (eg Wright, Horn & Sanders, 1997; Wilmot, 2009) have shown that, schools and, by extension, teachers, contribute significantly to the performance of students. The controversial question has been about which teacher variables are remarkable.

Some scholars (eg.Kosgei, Mise, Odera, & Ayugi, 2013) were of the view that teachers' subject matter knowledge has a strong positive relationship with student performance; Others (eg. Adedoyin, 2011; Borisade, 2011; Abe, 2014) claimed that teachers' subject matter knowledge was not the only variable but other variables such as teacher prior education and training, teacher experience, and teacher pedagogical content knowledge, all contribute to the performance of students.

Considering the poor performance of students in mathematics at both the Junior and Senior high schools in Ghana which has been a national concern, there is a need to investigate which teacher variables will improve the performance of students in mathematics. The study therefore aims at examining these variables in the Ghanaian context.

Background to the Study

Education reduces inequalities, can break the cycle of poverty, foster tolerance, reach gender equality, and empower people to live healthier lives and attain more productive livelihoods. Education is both a goal in itself and a means for attaining all the other Sustainable Development Goals (SDGs). It is not only an integral part of sustainable development, but also a key enabler for it (United Nations, UN, 2016)

Ghana has been a regional leader in the delivery of Education for All, reaching the education Millennium Development Goals (MDGs) well ahead of time. In 2016, net enrolment ratio reached 92% at the Primary level, and 50% at JHS level. Gender parity has been achieved at the kindergarten, primary, and JHS levels. Nonetheless, many students in Ghana do not benefit from a quality education, and girls are disproportionately disadvantaged, especially during the transition to senior secondary education. Often, the school environment is not conducive to learning: classes are overcrowded, water and sanitation facilities are lacking, and trained teachers are in short supply (United Nations, UN, 2016).

Sustainable Development Goal (SDG) 4 ensures inclusive and equitable quality education and promotes lifelong learning opportunities for all. One major indicator for the realisation of SDG 4 (Education Agenda 2030) is teacher quality. Teacher quality is widely recognised by policymakers, practitioners, and researchers alike to be the most powerful school-related influence on a student's academic performance (Gichuru & Ongus, 2016).Fenster (2014) maintains that a teacher who is highly effective improves both students' academic learning in the short-term and their long-term quality of life. The quality of the teacher in any school setting is claimed to be the most critical component for improving student performance and closing achievement gaps (Gichuru & Ongus, 2016).

Studies have shown that a lot of factors influence the academic performance of students; teacher quality and non-teacher quality such as school factors (e.g., Bidya, 2003; Kosgei, Mise, Odera, & Ayugi, 2013; Ofem, Iyam, & Bassey, 2015). Student performance may indeed be shaped by the extent of the use of creative teaching materials, quality of such materials, effective teaching techniques employed by the teachers, and resources available during the teaching and learning processes. Numerous perennial problems plaguing our educational system today are indeed the school system itself and the dearth of teacher quality (Bidya, 2003).

Wenglinshy (2002) opined that, teachers input influences professional development, professional development influences classroom practices, while classroom practices influence student performance. Suffice to say therefore, that all these factors one way or the other take their roots from both teacher quality and school factors. Hence, teacher quality determines the type of input which is said to have an impact onstudents' academic performance.

Indeed, teacher quality and school factors together with resources have been found to be the real variables that have been contributing major constraint to poor academic performance in mathematics (Borisade, 2011). The gross endemic poor academic performance of students in mathematics at public examinations could therefore be said to be a direct reflection of teacher quality and some inhibiting school factors such as learning environment, monitoring etc (Borisade, 2011).

Simplicio (2000) further reiterated that, enough motivation should be provided for teachers to enable them put in extra commitments in order to achieve the desired results.It is obvious that teacher quality cannot be underestimated and should be considered as a critical factor in the teaching of mathematics and mathematics related subjects.

Teaching experience and academic performance

One common indicator of teacher quality is the teaching experience expressed in terms of the number of years of teaching a teacher has in the classroom setting. Teacher experience has a significant effect on pupils' performance in primary schools and at upper secondary level. Experienced teachers have a richer background of experience to draw from and can contribute insight and ideas to the course of teaching and learning, are open to correction and are less dictatorial in classroom (Kosgei, Mise, Odera, & Ayugi, 2013).

Olaleye, (2011) posited that students taught by more experienced teachers achieve at a higher level, because their teachers have mastered the content and acquired classroom management skills to deal with different types of classroom problems. Furthermore, experienced teachers are considered to be able to concentrate on the most appropriate way to teach particular topics to students who differ in their abilities, prior knowledge and backgrounds (Wirth & Perkins, 2013).

Other studies have suggested that teacher experience effects may be evident for a longer period of time (Rivers & Sanders, 2002; Stronge, Ward, Tucker & Hindman, 2007; Lai, 2011). Lai (2011) stated that experience had a significant positive effect on elementary student achievement among teachers during their first seven years of teaching. Stronge, Ward, Tucker, and Hindman (2007) reveal that, at the high school level, students taught by teachers with more than nine years of experience had significantly higher test scores than students whose teachers had five to nine years of experience. Rivers & Sanders (2002) suggest that teachers' effectiveness increases dramatically each year during the first ten years of teaching". In the extreme case, Clotfelter, Ladd, and Vigdor (2007) found evidence of growing teacher effectiveness out to 20 or more years in their analyses of North Carolina teacher data, although more than half of the gains in teacher effectiveness occurred during the first few years of teaching.

Some other studies (eg. Hamishek & Rivkin, 2006; Borisade, 2011) suggested contrarily to the years of experience having impact on student performance. Hanushek and Rivkin in 2006, suggested that, in many instances, the correlation between years of experience and student performance is statistically weak and therefore cannot contribute to a strong assumption of the effect. If there is a positive correlation between the performance of students and teacher experience, then it is only because the more experienced teachers teach higher level classes with highly achieving students. Simply put, senior teachers can choose to teach in the better schools (Borisade, 2011). Borisade (2011) suggested that due to eldership within organizations and institutions, experienced teachers choose to teach higher-level classes with more highly achieving students. Due to these factors, it is difficult to conclude that teaching experience alone can impact student performance.

Several (eg Goldhaber & Anthony, 2003; Kaine & Staiger, 2008; Haber, Gorman, Gengaro & Ouellette, 2012) studies show that even though teaching experience impacts student performance, it levels off at some point. Harber, Gorman, Gengaro, Butisingh, Tsang, and Ouellette (2012) for instance, suggested "First year teachers have much lower performance on average than other teachers. After that, teacher performance improves markedly, peaking in the teacher's fourth year". A study by Kaine, Rockoff andStaiger (2008) concluded, "teachers make long strides in their first three years, with very little experience- related improvement after that". Goldhaber and Anthony (2003), in their paper titled, *Teacher Quality and Student Achievement*, suggested, teacher experience may predict teacher effectiveness, but there is very little evidence of this beyond the first couple of years of teaching. Page and Rosenthal (1990), as a result of the above stated, most successful schools employ the services of both experienced teachers and new teachers. While the new teachers bring fresh ideas and enthusiasm, the experienced teachers mentor the new teachers and provide stability to the school.

Although there has been inconclusiveness to majority of research surrounding teacher experience and student performance, a trend is to be regarded or observed. Many of the studies on teacher experience show that teaching experience has a positive impact on student performance, at least after some years before the impact levels off. Generally, teaching experience has been found as a significant predictor of academic performance.

Teaching qualification and academic performance

Academically qualified teachers refer to those who have academic training as a result of enrolment into educational institution and obtained qualifications such as HND, BSc, B.A, and M.A. and so on; while professionally qualified teachers are those who got professional training that gave them professional knowledge, skills, techniques, aptitudes as different from the general education (Edu & Kalu, 2012). They hold degrees like, B.Ed., B.Sc. Ed, B.A. Ed, and MEd and so on.

The teacher has been identified by psychologists as a principal character that can make or mar any successful learning within the formal system of education. According to Buddin and Zamarro (2009), teacher quality is a key element of student academic success. Obasi (2010) also asserts that the professional and academic training and qualification of the teacher can raise the prospects of a student's academic performance and attainments. This validates the truism that the quality of our schools cannot be better than the quality of the teachers we have.

Koedel and Betts (2007) found that specific teacher qualification had little effect on student achievement. George (2004) attributed poor achievement of students in mathematics to teacher qualification, inadequacy of materials as well as administrative factors. Buddin and Zamarro (2009), further argue that better educated or more skilled teachers (as measured by licensure exams) may inherently be better able to teach, but they may not consistently deliver their best performance in the classroom. Khurshid (2008) found that the performance of the students taught by untrained teachers with B.A. / BSc qualifications was better than the students taught by trained teachers with professional qualifications like BEd and M.Ed. Zuzovsky (2009) believed that teacher qualification was indeed positively associated with student achievement in mathematics and science. Several studies have found a significant relationship between teachers' qualification and specialized knowledge in mathematics and students' achievement in core mathematics and further mathematics (e.g., Yeboah-Appiagyei, Osei-tutu& Darkwa, 2014; Aliyu, Yashe, & Adeyeye, 2013; Owolabi & Adedayo, 2012).

In teaching mathematics, Abe (2014) opined that with an exception of holders of minimum of BSc in mathematics, many other teachers would be confronted with problem of teaching secondary school mathematics syllabus effectively. Hence, Abe (2014) argued that no one gives what he/she does not possess. He further said that no matter how good a course curriculum is, if we do not have well trained, qualified and motivated teachers, we may not achieve the desired goals.

There are conflicting reports on the relationship between teacher qualification and students' achievements in various subject areas. Goldhaber and Brewer (2000), for example, found a positive relationship between teacher qualification and students' mathematics achievement but no such relationship was found between teacher qualification and students' achievement in science.Zuzovsky (2009) found that while having a major in mathematics had no effect on student achievement in mathematics, having a substantial amount of under- or post-graduate coursework had a significant positive effect on students' achievement in physics but not in a subject like life sciences.

Teacher pedagogical content knowledge and academic performance

In a classroom, it is not only the knowledge acquired by the classroom teacher that matters. The process of imparting the knowledge to the learners by employing techniques and strategies that will make the least in the class in terms of understanding to get a clear picture of the lesson, is what matters. The failure of the teachers to devise this means of enriching the class is one of the reasons why the lesson of mathematics is ever becoming a subject of discussion among students and consequently leads them to poor result whenever they sit for an examination that has to do with mathematics. This problem emanates due to quite a number of reasons; one of which is pedagogical know-how (Tsafe, 2013).

Pedagogical content knowledge has variously been defined by scholars to mean different things depending upon the context of its usage. Pedagogical content knowledge is a strategy and style which allows the teacher to present his lesson in a stimulating way (Korau, 2010). If a teacher is able to present his lesson in such a way that learners appreciate and appeal strongly, it means the pedagogical content knowledge of the teacher is sound. By being pedagogically knowledgeable (Tsafe, 2013).

It is an undisputable fact that an adequate supply of mathematics teachers is an essential ingredient for mathematics teaching (Odili, 2006). Such teachers if supplied are supposed to be with substantial amount of knowledge that is capable of making them successful in the process of their delivery in the classroom. This was further noted by Adedoyin (2011) that one of the characteristics of good teachers is that they possess a substantial amount of specialized knowledge which is referred to as pedagogical content knowledge.

Pedagogical content knowledge is the knowledge of how to transform formal subject matter knowledge into meaningful learning outcomes for students and it also involves an understanding of a particular topic and how teachers explain the topic or concepts to make sense to the students in the classroom (Korau, 2011). The persistent failure and lack of proper understanding of mathematics by its learners have always been of interest to the scholars. Teachers' knowledge of subject matter as such continues to draw an increasing attention from policy makers in recent years all over the world, since more emphasis is given to highly qualified teachers (Crespo & Nicole, 2006).

According to Ball (2003), a teacher with good mathematical pedagogical content knowledge can break down mathematical knowledge into less polished and abstract forms, thereby, making it accessible to students who are at different cognitive levels.A teacher with good pedagogical content knowledge can unpack the mathematics into its discrete elements and can explain a concept or procedure at a level that includes the steps necessary for the students to make sense of reasoning (Korau, 2011).

Teachers with good mathematical pedagogical content knowledge understand where students may have trouble learning the subject and are able to represent mathematical concepts in a way that their students can comprehend its structure and avoid these difficulties (Zuzovsky, 2009). Since mathematics teachers' pedagogical content knowledge is essential in teaching process, pedagogical content knowledge as well as mathematical content knowledge are needed in order to construct mathematical concepts in students' mind (Adedoyin, 2011). Yusuf and Amali (2014) found a positive correlation between teachers' pedagogical skills and students' performance but there was no correlation between teachers' use of instructional materials and students' performance. It is obvious that teacher pedagogical content knowledgeplays a significant role in the academic performance of students.

Subject matter knowledge and academic performance

Studies have consistently shown that teacher quality whether measured by content, experience, training and credentials or general intellectual skills is strongly related to students' achievement (e.g., Adediwura & Bada, 2007; Darling-Hammond, 2000; Zuzovsky, 2009). Kimberly (2009) stated that teachers must be knowledgeable in their area of study. In truth, if a teacher is not enlightened in his/her subject, then any hope of effectiveness goes right out the window. Hence, effective teaching could be measured by the level of a teachers' subject matter competence which Mullens (1993) regarded as a prime predictor of students' learning.

It has been established that there is a high correlation between what teachers know and what they teach. Thus, the ability to teach effectively depends on the teachers' knowledge of the subject matter. Teachers are handicapped if they are unfamiliar with the body of knowledge taught and teachers' characteristics is subject specific. Adediwura and Bada (2007) stated in their study that nobody could teach what he does not understand or know. They went further to state that they (teachers) must thoroughly understand the content of what they teach. Teachers whose understanding of topic is thorough use clearer language, their discourse is more connected, and they provide better explanations than those whose background is weaker.

The way the students perceive the teaching in terms of their (teachers) knowledge of content of subject matter may significantly affect the students' academic performance (Darling-Hammond, 2000). Because of this, the teacher should therefore master the subject matter before teaching commences. Most teachers go into teaching without knowing what to teach. It is to be noted that pedagogical content knowledge is not exactly the same as knowledge of subject matter, they nevertheless are, intimately linked, because teachers' mastery and use of them in the classroom indicate the depth of their knowledge of subject matter (Fakeye, 2012).

Academic performance of SHS students in elective mathematics

The West Africa Examination Council (WAEC), for a number of decades has been the only body in Ghana especially for assessing performance of students at the end of Junior High School (JHS) as well as Senior High School (SHS) level. A number of concerns have been expressed by a large number of citizens on students' failure rate especially in mathematics. The government of Ghana is committed to ensuring the provision of high quality mathematics education since mathematics is a compulsory subject at all levels in pre-university education. Various attempts have been made in the past to improve the performance of mathematics in schools. In spite of government efforts, however, mathematics has not undergone much change in terms of how it is presented. This reflects consistently in low achievement levels in mathematics among students at the SHS (Agyei, 2010). It is regrettable, therefore, that in the contemporary times, many students struggle with mathematics and perform abysmally low in their final examinations in most jurisdictions (Obasi, 2010). According to the Chief Examiner's Report (2018), there was a decline in the performance of candidates as compared to that of the previous year. The Chief Examiner suggested that teachers should give in-depth tuition in areas of students' weaknesses by explaining thoroughly the relevant concepts, as candidates exhibit poor understanding of mathematical concepts and are unable to form the appropriate mathematical models which could be tackled with the requisite skills.

Performance in elective mathematics as well, is not too encouraging. Taking a look at the performance of students in elective mathematics in WASSCE from 2013 – 2017, the results show that more than 50% of the students had grades from D7 – F9, apart from 2016 where D7 – F9 was a little over 43%. For the years2014 and 2015, more than half of the students who took the elective mathematics examination failed, and 41.58% failed in 2017 (Table 1).

Table 1: WASSCE Results in Elective Mathematics

Year	Total Sat	A1-C6		D7-E8		F9	
		N	%	N	%	N	%
2013	134,993	63,182	46.80	41,126	30.47	30,310	22.45
2014	75,804	15,430	20.36	19,023	25.09	41,051	54.15
2015	87,314	19,437	22.26	20,510	23.49	45,022	51.56
2016	90,055	50,187	55.73	19,857	22.05	19,091	21.20
2017	92,477	32,158	34.77	21,436	23.18	38,456	41.58

Source: WAEC (2018)

Figure 1 shows no definite trend of results. It appears, however, that students' performance had not been encouraging.

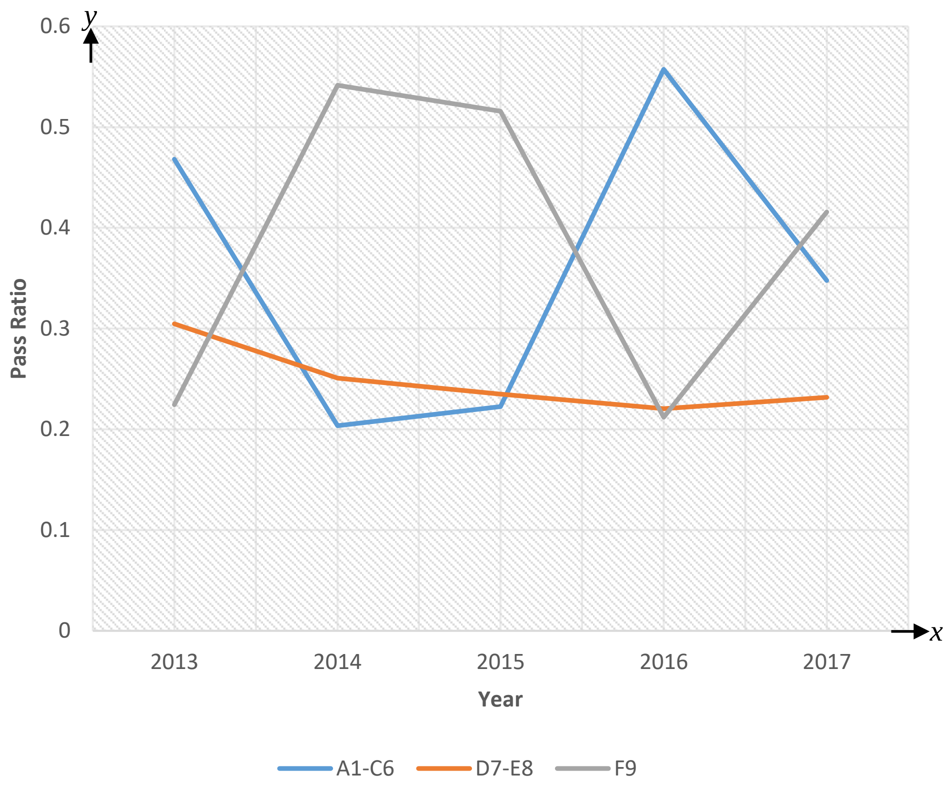

Figure 1: Performance Trends in Elective Mathematics

Statement of the Problem

The competence gain in the study of mathematics is widely used in all spheres of human life. Mathematics plays a key role in shaping how individuals deal with the various spheres of private, social, and civil life (Anthony & Washhaw, 2009). This justifies the compulsion of the study of the subject by all students who go through basic and secondary education in most countries. Mathematics is therefore a core subject at these levels of education in Ghana. It is regrettable, therefore, that in the contemporary times, many students struggle with mathematics and perform abysmally low in their final examinations in most jurisdictions (Obasi, 2010).

Considering the national outcry in Ghana over the poor performance of students in elective mathematics in national assessments, which prevents a bulk of senior high school graduates from pursuing mathematics related programs in the tertiary institutions, such as engineering, scientific research and medicine among others, which form the core of every country's development, it is obvious that this problem is critical, and requires investigation into which teacher variables would improve the performance of the student in mathematics. Taking advantage of the paper; Which Teachers make a Difference? Yael Duthilleul and Rebecca Allen (2005), specifically found that teacher effectiveness could be attributed to a combination of teacher training, subject matter competency and classroom practices. This study seeks to use mathematical modeling to do an investigation into some teacher variables such as teacher prior education and training, subject matter knowledge, pedagogical content knowledge, and teacher experience, and their effect onstudents' academic performance in elective mathematics in the senior high schools in Ghana.

A scan through the literature revealed that the factors affecting mathematics performance in Ghana have been largely investigated (e.g., Ankomah, 2015; Anthony& Washhaw, 2009; Enu, Osei, & Nkum, 2015; Mereku, 2003; Appiagyei, Joseph&Fentim, 2014). Majority of these studies, however, focused on the students' factors affecting mathematics performance. It was prevalent that students' attitude, interest, inadequate TLM, insufficient practice, home factors, and peer factors were found as the dominant factors affecting mathematics performance of students in SHS. It appears that few of the studies focused on teacher characteristics as predictors of mathematics performance, and specifically, elective mathematics in SHS. Although a study byAppiagyei et al., (2014) examined the effects of professional qualifications of teachers on academic performance of SHS students in the Tamale

Metropolis of Ghana, other characteristics such as experience, pedagogical content knowledge and subject matter knowledge of the teachers have not been explored in Ghana.

Assumptions of the Study

The study was guided by the following assumptions:

1. Teachers with longer length of experience in the teaching ofelective mathematics are more likely to have students who perform well in elective mathematics.
2. Students taught by teachers with prior education and training in mathematics education would perform better in elective mathematics.
3. Teachers with competence in subject matter knowledge greatly impact the learning of elective mathematics.
4. Elective mathematics students who are taught by teachers with excellent pedagogical skills would perform well academically.
5. With much experience, teachers who have excellent subject matter knowledgewould produce excellent elective mathematics students.
6. With much experience, teachers who have better pedagogical skills would produce excellent elective mathematics students.

Purpose of the Study

The purpose of the study is to investigate the effect of teacher variables such as teachers' qualification, teaching experience, teachers' knowledge of subject matter, and teachers' pedagogical content knowledge, on student academic performance in elective mathematics in Senior High Schools in Ghana.Specifically, the study sought to examine:

1. The relationship between teacher prior education and training, and performance of students in elective mathematics.
2. The relationship between teaching experience and performance of students in elective mathematics.
3. The relationship between teacher subject matter knowledge and performance of students in elective mathematics.
4. The relationship between teacher pedagogical content knowledge and performance of students in elective mathematics.
5. The role of experience in the relationship between teacher subject matter knowledge and performance of students in elective mathematics.
6. The role of experience in the relationship between teacher pedagogical content knowledge and performance of students in elective mathematics.

Objective of the Study

The obecjtive of the study is to sue mathematical modelling to do an investigation into teacher variables which would improve the academic performance of studnets in Elective Mathematics in Senior High Schools in Ghana.

Hypotheses Statement

In addressing the research problem, the study tested the following hypotheses:

1. H_0: There is no significantrelationship between teacher prior education and training, and performance of students in elective mathematics.

   H_1: There is a significant relationship between teacher prior education and training, and performance of students in elective mathematics.

2. H_0: There is no significantrelationship between teaching experience and performance of students in elective mathematics.

   H_1: There is a significant relationship between teaching experience and performance of students in elective mathematics.

3. H_0: There is no significantrelationship between teacher subject matter knowledge and performance of students in elective mathematics.

   H_1: There is a significant relationship between teacher subject matter knowledge and performance of students in elective mathematics.

4. H_0: There is no significantrelationship between teacher pedagogical content knowledge and performance of students in elective mathematics.

   H_1: There is a significant relationship between teacher pedagogical content knowledge and performance of students in elective mathematics.

5. H_0: Teaching experience does not significantly moderate the relationship between teacher subject matter knowledge and performance of students in elective mathematics.

   H_1: Teaching experience significantly moderates the relationship between teacher subject matter knowledge and performance of students in elective mathematics.

6. H_0: Teaching experience does not significantly moderate the relationship between teacher pedagogical content knowledge and performance of students in elective mathematics.

H_1: Teaching experience significantly moderates the relationship between teacher pedagogical content knowledge and performance of students in elective mathematics.

Significance of the Study

In terms of studying issues of teacher variables and student performance in elective mathematics, not much work has been done in Ghana. The study would provide empirical information on the teacher related factors that could predict student academic performance in elective mathematics. The study would also contribute to research, efforts geared towards finding a permanent solution to the problem of poor performance of students in elective mathematics. It would also serve as an eye opener to teachers and other stakeholders on which of the teacher related factors could predict students' academic performance in elective mathematics. This could help policy makers to establish strategies that could be adopted to enhance the quality of teacher workforce to improve the performance of mathematics in the senior high schools.

This study would be a valuable tool of reference to teachers who wish to undertake a self-evaluation in terms of their effectiveness and efficiency in teaching. This would help improve the quality of teaching leading to improved student performance. It is envisaged that a careful identification of the teachers'qualifications would provide the state education board with possible means of improving students' performance in the subject as it concentrates on the issues of teacher quality.

The study would also help elective mathematics teachers to see the need to improve their knowledge on the subject; by way of seminar and workshops or additional academic qualification so as to appropriately have more positive impact on their students. This brings to bear the necessity to improve, in fact constantly improve the academic capabilities of the teacher as this would in turn have impact on the social and scientific base of the nation. This study would provide the teacher and educational policy makers with information on the need for retraining so as to keep the teacher abreast with the latest and best practices in their area of specialization to give out the best in the dissemination of knowledge to the students.

Delimitation

Five cosmopolitan regions of Ghana (Central, Western, Northern, Greater Accra and Ashanti) were purposively sampled. Nine schools from each of the regions with 150 SHS 2 elective mathematics students and fiveSHS 2 elective mathematics teachers from each of the forty- five selected schools participated in the study. This included category of schools and their areas of location in each region. Gender was considered as three-male schools, three-female schools and three-mixed schools where possible, from each of the selected regions.

Limitations

A study of this nature revealed challenges which might affect the validity of the responses. Firstly, the nature of the design resulted in issues of validity and reliability. Survey designs capture brief moments in time just like taking a photograph of an on-going activity. This suggests that the validity of the findings in survey studies is time-bound and therefore, reduces with time passage. Secondly, the researcher did not have adequate control over certain extraneous variables which might have implications for the study. Because of the assessmenttest used, students who had taken private classes at home or school before, might influence the findings of this result. Although the researcher made efforts to exclude students who had private teachers at home or in school, this will be only possible at the honesty of the students.

Operational Definition of Terms

Subject Matter knowledge - Refers to the ability of the teacher to have mastery over content of elective mathematics.

Teacher Experience – Refers to the number of years of teaching a teacher has in the classroom setting.

Teacher Prior Education and Training- Refers to a certified professional in the subject area.

Pedagogical content knowledge –Refers to specialised knowledge for teaching.

Student Performance – Refers to students' scores on the elective mathematicstest.

Organisation of the Study

This research report is organised into five chapters. Chapter one, which is the introduction chapter, focused on the background to the study, the statement of the problem, purpose of the study, research questions and hypotheses, significance of the study, limitations,delimitations and operational definition of terms.Chapter two provided information on relevant and related literature in the area of teacher variables and academic performance.The literature review included the theory underlying the study, the conceptual base of the study and the empirical review.

Chapter three focused on the following areas: research paradigm research design, population, sampling procedure, data collection instrument, data collection procedures, and data processing and analysis. Chapter four dealt with how the collected data was analysed, as well as the interpretation of the results, thereof. The findings were also discussed. Chapter five brought the entire study to an end by making summaries, key findings, conclusions and recommendations as well as identifying an area for further studies.

CHAPTER TWO

LITERATURE REVIEW

Introduction

The purpose of the study was to investigate the effect of teacher variables such as teachers' qualification, teaching experience, teachers' knowledge of subject matter, and teachers' pedagogical content knowledge, on student academic performance in elective mathematics in senior high schools in Ghana.

Literature was reviewed under different sub-headings. These are the theoretical review, conceptual review, empirical review and conceptual framework.

Theoretical Review

Zone of Proximal Development (Vygotsky, 1930)

The theory of Zone of Proximal Development (ZPD) was propounded by Vygotsky in 1930. The theory states that social interaction precedes development and that consciousness and cognition arethe end product of socialization and social behaviour. The theory focuses on connections between people and social cultural context in which they act and interact in shared experience (Vygotsky, 1930).

The theory is founded on three major themes: (a) Social interaction plays a fundamental role in the process of cognitive development (e.g. mathematical ability), (b) The more knowledgeable others. This refers to anyone with higher ability level than the learner (e.g. the mathematics teachers, in the context of this study), (c) The zone of proximal development (ZPD). ZPD is a term for the range of tasks that are too difficult for children to master alone but can be learned with the guidance and assistance of more skilled person. Lower limit of ZPD is the level of skills reached by the child working independently. The upper limit of ZPD is the level of additional responsibility the child accepts with the assistance of more knowledgeable others (Vygotsky, 1930).

In his view of cognitive development, Vygotsky (1930) provided evidence that he accepted the idea that inherent developmental stages, similar to those detailed by Piaget (1970), are at the basis of learning. Vygotsky also combined those developmental beliefs with the Marxist view that humans ultimately develop their intellectual potential through the driving forces of their environmental exposure (Vygotsky, 1931). Piaget (1970) and Bruner (1985) shared similar constructivist perspectives with Vygotsky and agreed with his thoughts on the processes of child development.

Bandura's (1977) Social Learning Theory also complimented Vygotsky's Social Development Theory (1930). Vygotsky would have supported Bandura's statement,"learning would be exceedingly laborious, not to mention hazardous, if people had to rely solely on the effects of their own actions to inform them what to do" (Bandura, 1977, p. 22). Vygotsky (1930) specifically focused on the social-historical environment while developing his integrative theory. He recognized that traditional internal developmental cues, such as imitation and natural curiosity, lead to maturational growth, but theorized those internal forces alone could not direct an individual through more advanced thought processes.

Vygotsky (1935) began to study child behaviour in more depth and to look specifically at students" performance along with school instruction. He realized that conventional tests only evaluated the level of mastery of students performing in isolation. After observing students under directed instruction, Vygotsky discovered that students often performed at higher cognitive levels with modest assistance. In a particular reported instance, Vygotsky described a situation in which two students on the same individually academic performance level achieved at markedly higher levels when given assistance on a more advanced problem. This incident revealed to Vygotsky that future cognitive development is determined by not only the level of individual performance, but also by the performance the student is capable of with assistance.

In 1930, Vygotsky named this measured ability between a student's actual performance level and the student's potential level the ZPD. With the ZPD, Vygotsky proposed children were capable of combining their innate abilities and knowledge with social cues and interactions in order to further extend their cognitive growth. Vygotsky believed adult instruction assisted and often prompted student development further than what was obtained through self-discovery alone. For this reason, Vygotsky encouraged educators to develop further studies for determining the extent of interaction between intrinsic development and socio-cultural forces. Even though modern educators accept the view that an interaction between natural development and external factors affects the student learning process, an uncertainty remains as to the specific implications of certain types of interactions.

Vygotsky's ZPD concept highlighted the role of the teacher in increasing a child's learning potential. His observational study (1930) provided the evidence necessary for future researchers to explore instructional techniques. Vygotsky's (1935) example of an infant initially walking by holding on to an adult's hand, even though the infant could not yet walk alone, supported the theory of ZPD. The analogy of an adult guiding an infant's steps before the infant was developmentally ready allows educators to make similar assumptions for other cognitive and internal development connections. Therefore, the ZPD directly pertains to teacher-student interaction, and provides suitable justification for teachers initiating knowledge beyond students' current performance levels.

One can also apply the ZPD theory to today's educational settings, revealing that conceptual skills and abstract thought can be stimulated in students through teachers" instructional techniques. Once aware of the ZPD and how this takes place in the learning process, a teacher can choose to utilize an instructional practice that guides the learner into greater depths of knowledge rather than merely delegating or dispensing informational titbits to the learner. Teachers have the ability to prompt students to question new learning strategies that are just beginning to develop. A teacher's prompt to delve further into the learning will challenge student thinking and possibly extend the learning.

The importance of Vygotsky's (1930) theory of Zone of Proximal Development in explaining the basic idea underlying this study cannot be overemphasised. Vygotsky's (1930) description of social environment includes the school settings where students interacts with peers and most importantly their teachers. These teachers are more knowledgeable than the students, it must be said. The teachers, therefore, are present to assist the students to learn and understand concepts in their respective subjects. What this means is that if students are not performing well in any subject, say Elective Mathematics, then it can be mentioned that teachers to some extent contribute to this poor performance. It is of essence to state that the ability of teachers to effectively help these students depends on a number of teacher characteristics- educational qualification, experience, subject matter knowledge, and pedagogical skills. If there is a problem with any of these characteristics, it will probably hinder the ability of the teachers to assist in the cognitive development of the students in Elective Mathematics.

It is, therefore, not out of way that theory provides a strong argument for using appropriate models and concrete material to illustrate mathematical concepts and for actively involving students in learning process. Major instruction implication is that teachers should explain new information in terms of knowledge students already possess. A teacher should collaborate with their students in order to facilitate learning mathematics. Learning mathematics according to Reys, Lindquest, Lambdin, Smith and Marlyn (2001) involves the following principles: actively involving students built on their previous knowledge and use of resources learning process. Actively involving students in learning mathematics leads to retention of information for long term.

Theory of Instruction (Bruner, 1985)

The theory of instruction was developed by Bruner in 1985. Bruner (1966) compared mental growth to the rises and rests of a staircase. As the learner's knowledge is nurtured, the concepts eventually mature and advance to the next level of learning. Rather than focusing on a student's readiness to learn, Bruner (1966) believed that the staircase of learning potential was present for every learner, but environmental influences, including instructional tactics, had the ability to halt, slow, or advance the learning.

Bruner believed that learning is indeed sensitive by nature, but the learner is rather adaptive to a teacher's range of instructional approaches (Bruner, 1985). In the Theory of Instruction, Bruner (1966) confirmed Vygotsky's finding that a learner's growth and development were assisted by various external means. Bruner (1985) stated, any learner has a host of learning strategies at command, but it is the teacher who equips students with the procedures and sensibilities to lead them to their potential for obtaining knowledge. Conversely, if a teacher has the ability to lead students to their potential, a teacher may also fail to provide students with opportunities that build complex thought patterns. Bruner (1985) recognized that certain educational environments could potentially be responsible for irreversible learning deficits in students. Bruner (1966) warned that these potential learning deficits could be established prior to entering a formal educational system. Whether the instructor is a parent or a formal instructor, the relationship between instructor and student is significant and has long lasting effects.

Bruner (1966) stated that:

> Since this is a relation between one who possess something and one who does not, there is always a special problem of authority involved in the instructional situation. The regulation of this authority relationship affects the nature of the learning that occurs, the degree to which a learner develops an independent skill, the degree to which he is confident of his ability to perform on his own, and so on. The

> relations between one who instructs and one who is instructed is never indifferent in its effect upon learning. (p. 42).

With the current educational dilemma (debating the methods and practices best suited for leading students into the twenty-first century) one may find Bruner's wisdom even more relevant today. Bruner (1966) stated, we are entering a period of technological maturity in which education will require constant redefinition. He also explained, "The period ahead may involve such a rapid rate of change in specific technology that narrow skills will become obsolete" (Bruner, 1966, p. 32). Indeed, education has evolved greatly over the past few decades, and Bruner correctly acknowledged that without the mastery of simpler skills, more elaborated ones will become increasingly out of reach for students. Therefore, Bruner suggested that educators provide students with opportunities to share in dialogue, paraphrase their thoughts, and internalize their learning in order to reach higher skills. He also charged educators to facilitate the exploration of learning to encourage learning and problem-solving (Bruner, 1966).

In order to address the accelerated change within the educational system, Bruner proposed a theory of instruction:

> A theory of instruction is prescriptive in the sense that it sets forth rules concerning the most effective way of achieving knowledge or skill. By the same token, it provides a yardstick for criticizing or evaluating any particular way of teaching or learning. (Bruner, 1966, p. 40)

Bruner's (1966) Theory of Instruction is prescriptive, rather than descriptive, in the sense that instructional methods are prepared by educators based on how the material can best be learned by the student. Bruner's Theory of Instruction consists of four major principles that are practical for not only analysing instruction, but for determining the best method to lead a child toward learning.

First, Bruner declared that instruction should specify which educational experiences most effectively lead a learner toward learning. With various instructional methods producing a range of educational experiences for learners, teachers should be mindful of the means in which material is delivered to students in order to reap the most benefit from the experience. Second, instruction must identify how a body of knowledge should be organized so that learners can readily grasp the concept. When teaching a complex body of knowledge, teachers should examine the structure of the knowledge in order to provide students with smaller components of information at a given time. By breaking the body of knowledge down into more manageable bits of information, students build mental frameworks that enable them to obtain the whole body of knowledge.

Third, instruction must detail the manner and order in which the material should be disseminated for learning. For example, if a teacher was to prepare lessons pertaining to the Laws of Motion, the teacher would need to decide how to best introduce the material so students gain the most understanding. The teacher may choose to provide students with experimental motion experiences before mentioning the laws. The teacher may probe students to inquiry by questioning the relationship between forces and motion.

In contrast, the teacher might simply state the laws of motion, display the mathematical formulas for each, and follow up with discussion.

Bruner (1966) suggested that the sequences teachers choose to deliver material have an impact on how well students attain the overall body of knowledge. Bruner provided educators the flexibility to determine the pace and choice between extrinsic and intrinsic reward, once the material was released to students. Fourth, Bruner believed that instruction must utilize both types of reward, but that student learning progressed further with intrinsic rewards. Bruner's theory of instruction reminds educators that the purpose of teaching is not only to supply students with a prescribed body of knowledge, but also to supply them with the know-how to process knowledge.

A statement of caution pertaining to the role of the teacher in Bruner's Theory of Instruction prompted, in part, the rationale for this study. Bruner warned educators to avoid the problem of interfering with students' ability to take over their own role in the learning process (Bruner, 1966). Grasha (1996) also recognized the importance of teacher-student interactions in the learning process. Student learning remains contingent upon teacher-student encounters and the level of success of those two-way encounters. He said that students develop through changes in teacher and student perceptions of each other, their actions toward each other, and the give and take inherent in their encounters (Grasha, 1996).

Bruner explained that students can possibly develop a dependence on teacher assistance in their learning, and that teacher-student communication cycles can sometimes stall or even block the learning process (Bruner, 1966). Although Bruner acknowledged that learners were capable of coping and adjusting to various instructional techniques, certain teaching practices could actually hinder student development if the teacher's style is not preferred by the student.

Bruner's (1985) theory of Instruction is significant for this study. The theory explains one of the major predictors of the study which is pedagogical skills of the teacher. Being informed by Bruner's theory, students would perform very well when course contents and materials are presented in a way which is meaningful to the students. In other words, the kind of instructional strategies adopted by teachers (Mathematics) explains large variances in the students' performance. Therefore, if teachers have limited pedagogical skills in course content delivery, students would probably not perform well. The foundation of this study is the premise that students‟ mathematical achievement is contingent upon the style of teaching and teaching techniques utilized by the teacher during the learning process.

Mathematical Theory of Relational and Instrumental Understanding

In order to formulate a complete theoretical framework for this study, a need exists for a review of literature related to mathematical learning. Skemp (1987), a mathematician and psychologist, devoted much time to researching and analysing the thought processes which learners implement when they learn mathematics. After much research, Skemp (1987) became increasingly concerned with the discovery that many intelligent students couldn't "do" mathematics. Skemp sought to find the answer as to why otherwise intelligent students had difficulties learning mathematics. Through his research and personal investigations, Skemp (2006) developed and proposed the Mathematical Theory of Relational and Instrumental Understanding.

To best explain the Mathematical Theory of Relational and Instrumental Understanding, Skemp (2006) used the analogy of a person visiting a town for the first time. When unfamiliar with a town, an individual would attempt to learn how to get from point A to point B. The first successful route, regardless of time or distance, would become the fixed plan this individual continues to use to travel between points A and B. The noted references along the way direct the individual between these two points. With these references, the individual may turn right out of the front door, go straight past the church, and so on. After some time, the individual would begin to explore the town, with little to no intent of getting to or from any given point, but rather to create a mental map of the town. During this exploration, however, the individual makes connections to other routes that lead successfully to points A and B.

In contrast to the first scenario, if a person with a mental map of the town gets off track, the individual would be able to produce a variety of plans in order to reach points A and point B. Additionally, if this individual made a wrong turn in the derived path plan, the individual, still aware of locations on the mental map, could successfully adjust path routes without getting lost. In the first example, the fixed form of knowledge required to get from one point to another based on memorized references was what Skemp (2006) described as instrumental understanding. In the second example, the formation of an overall awareness of point A and point B as they appear on a mental map of the town, was what Skemp (2006) referred to as relational understanding.

Skemp's analogy of the person visiting a new town and the learning of mathematics arequite similar. A situation in which students learn what actions to perform using a step-by-stepmethod in order to reach an answer to a problem is an example of instrumental learning (Skemp, 2006); instrumental learning occurs many times using direct teaching pedagogy. A situation in which students can produce an unlimited set of plans in order to solve a problem successfully is an example of relational learning (Skemp, 2006); relational learning is more likely to occur with inquiry or discovery pedagogy.

Instrumental learning is what many educators identify as rote memorization. With instrumental learning, students simply memorize procedures and steps to reach an outcome. This process of instrumental learning utilizes less brain function than relational learning and provides little depth into the reasoning or true understanding of a mathematical problem (Skemp, 1987). Many species other than humans learn using this instrumental or memorization method of learning (Skemp, 1987). Skemp (1987) acknowledged that even the simplest of animal species can learn ordered steps to complete complicated tasks, but for the learner, instrumental learning is absent of any underlying principles or meaning as to the purpose for performing the task, or to what benefit there is in finding the end result.

Relational learning is one of the things that separate humans from all other species. With relational learning, individuals are able to manipulate abstract structures and formulate new conceptual ideas. The new individual structures of knowledge are referred to as schema (Skemp, 2006). Individual schemas are then stored as knowledge and utilized to construct further knowledge when introduced to other structures of information (Skemp, 1987). When schemas are joined to form a new structure of knowledge, they become a concept. According to Skemp (1987), the ability for learners to put new mathematical experiences together with previously learned mathematical schemas and concepts in order to make a mathematical application is the highest level learning strategy that learners possess. Skemp (1987) identified this highly developed mathematical learning strategy of utilizing schema and concepts as relational learning.

When considering whether one form of learning mathematics was more ideal than the other, Skemp (2006) once again compared these two forms of mathematical learning to the two processes described in learning how to maneuver around a new town. Could the individual in both scenarios get from point A to point B? The answer is "yes," but an educator must ask a more probing question: Which is better, instrumental or relational learning, for students to master mathematics and be able to build future concepts of knowledge? Skemp (1987) believed that learners utilizing relational learning rather than instrumental learning would progress more successfully in their mathematical development (Skemp, 1987).

Skemp issued four statements of learning virtues supporting the usage of relational learning versus instrumental learning. First, Skemp (2006) acknowledged that learners who used relational learning strategies became quickly independent in reaching their own schema of knowledge. Second, the ability for learners to build their own schema of knowledge was found to be intrinsically satisfying to the learners (Skemp, 2006). Skemp's third virtue of relational learning described that the first two virtues enhanced a learner's level of confidence and therefore, encouraged the learner to pursue additional schemas of knowledge. The final virtue for developing relational learning over instrumental learning was that learners would come to realize that their schemas are never complete (Skemp, 2006).

As relational learners developed schemas and concepts of knowledge, Skemp discovered that they became ever more aware of endless learning possibilities. With relational learners‟ gaining confidence in the third virtue and awareness of endless learning potential in the fourth, Skemp (2006) concluded that relational learning leads learners to self-reward; therefore, the learning process becomes meaningful and self-perpetuating. These four learning virtues of relational learning were not evidenced within the learners of instrumental learning.

On the contrary, Skemp (1987) found students who utilized instrumental learning while developing mathematical skills often reported boredom, disconnect from the content material, and a lack of purpose for performing the mathematical tasks (Skemp, 1987). Skemp's (2006) mathematical theory of relational understanding enhances the theoretical framework of this study by proposing the need for instructional practices that foster specific learning strategies so students can best develop mathematically.

The essence of Skemp's (2006) mathematical theory of relational and instrumental understanding serves as a foundation for this study. The theory speaks to the fact that students ought to be taught in a way that they become knowledge and as well be able to transfer the knowledge acquired to different setting (application, synthesis, evaluation). Teachers who have limited content knowledge and poor pedagogical skills would find it difficult to transmit knowledge to the students. This makes subject content matter and pedagogical skills essential elements when talking about issues of academic achievement.

Conceptual Review

Concept of Mathematical Learning

Before educators can make improvements in the teaching of mathematics, a true understanding of how students learn mathematics is crucial. Skemp (1987) attempted to answer this question for educators: What is mathematical understanding, and by what means can educators help to foster true mathematical understanding in their students? After much research, Skemp (1987) categorized two basic types of mathematical learning, which he named "habit learning" and "intelligent learning". Skemp (2006) formulated the Mathematical Theory of Relational and Instrumental Understanding based on these two basic categories of mathematics learning. In mathematics, the habit learning notion became known as instrumental learning, while intelligent learning was recognized as relational learning.

Despite his profound comprehension of student development with mathematical concepts, Skemp (1987) was baffled as to why his students were still struggling with mathematics. At the prompting of the following words by Hassler Whitney, Skemp sought to advance his personal knowledge and improve his teaching by delving into the psychology of learning and teaching mathematics:

> For several decades we have been seeing increasing failure in school mathematics education, in spite of intensive efforts in many directions to improve matters. It should be very clear that we are missing something fundamental about the schooling process. But we do not even seem to be sincerely interested in this; we push for „excellence" without regard for causes of failure or side effects of interventions; we

try to cure symptoms in place of finding the underlying disease, and we focus on the passing of tests instead of meaningful goals. (Whitney, 1985, p.127).

Skemp (2006) argued that educators would not be any more successful in the future than in the past if the factors that affected student learning, including the ways in which students were being taught, were not addressed. For this reason, Skemp devoted his efforts to understanding two fundamental areas: how children learn and teaching practices.

Understanding relational and instrumental learning alone was not enough for Skemp (1989) to solve his professional problem as a teacher. He noticed that mathematics was not the only subject poorly taught and misunderstood, but the difficulties in developing mathematical knowledge proved to stand out in learners. Skemp declared that educators could measure the intelligence of a learner at any given time, but the capacity of intelligence a learner possessed was not possible to measure in any one circumstance. With this view, Skemp (1989) argued "Learners of any age will not succeed in learning mathematics unless they are taught in ways which enable them to bring intelligence, rather than rote learning, into use for their learning of mathematics" (p.26).

As evidenced even more recently by Wiggins and McTighe (2008), they too recognized the differences in mathematical learning and how students perform. Wiggins and McTighe (2008) revealed that students typically perform well on mathematical assessments that require recall (instrumental), but do not perform well on material requiring analysis or application (relational). The continual trend of student underperformance on mathematical assessments stresses the point that teachers should provide students with instructional opportunities that develop understanding in the content before challenging students with the complexities of relational learning (Whitney, 1985).

Skemp (1989) provided a scenario of a learner memorizing a 10-digit phone number in one instance and a separate 10-digit sequence in another instance in order to explain the advantage of intelligence learning over habit learning. In the first example, he acknowledged that the memorization of phone numbers would provide a learner with little to no interest. Furthermore, Skemp recognized that being asked to memorize additional phone numbers would not only be difficult for the learner, but that the memorization of the first phone number provided the learner no assistance in memorizing the others. In the second scenario, however, the learner discovered a pattern to the sequence of 10 digits, and only had to learn the first number and the pattern in order to remember all 10 numbers. If asked to extend the sequence of numbers, the learner could easily utilize the pattern discovered from the original list and generate a lengthier list of numbers. In contrast to the first scenario where the learner relied on habit learning, the learner in the second scenario utilized intelligent learning to not only recreate the original list of numbers, but to extend the list indefinitely, which further developed his mathematical proficiency.

The 10-digit learning analogy exemplified that intelligent learning is highly adaptable in mathematics, whereas habit learning, no matter how persistent, has low adaptability in problem solving (Skemp, 1989). Although Skemp estimated that 95% of mathematical content requires intelligent learning, compared to 5% habit learning, he found that the majority of students learned mathematics by utilizing habit learning methods. Because a child can memorize so many rules, and the fact that the early years of mathematical content are number and order specific, a teacher may not realize the harm being done by habit learning until students reach obstacles in the middle grades.

All too often, Skemp (1989) found that the memorization methods became too complicated for the increasing mathematical content, and inevitably led students to frustration and a cessation in mathematical progress. When student progress slows or stops, educators may wonder why students have stalled in their learning or why some students struggle with certain mathematical content, while other students do not. Teachers unknowingly support habit learning by providing students with frequent explanations for solving problems and providing step-by-step instructions when they struggle with mathematical understanding (Fennema et al., 1996).

As a consequence, the way mathematics is taught in schools is often disconnected from the way children need to think about solving problems in real world scenarios (Carpenter, Fennema, Peterson, Chiang, & Loef, 1989). Skemp proposed that teachers could save students from mathematics failure by utilizing intelligent teaching. Skemp (1989) stated, “Intelligent teaching involves knowing which kind of learning to get children to use for different kinds of tasks, and how to get them to use it” (p.35).

When compared to other subjects being taught in the school environment, Skemp (1989) noted the lack of continuity in the learning of mathematics. From the onset of students learning to read, they learn that reading is beneficial for gaining information, providing entertainment, and expanding knowledge. For many reasons, both children and adults expect reading to enrich their minds and provide a lifetime of resources. In contrast, Skemp acknowledged that most students at the primary level first learned mathematics for the purpose of teacher or parent approval, and secondly for achieving an acceptable academic grade or reward.

To expound the issue further, one study reported that teachers related students' knowledge in mathematics to the students' performances on examinations (Peterson, Fennema, Carpenter, & Loef, 1989). Therefore, teachers' instruction in mathematics is oftentimes based falsely around misleading information about what students really know or understand mathematically. Skemp (1989) acknowledged purposes for learning mathematics were not only contradictory between the school environment and the real world, but prevented learners from experiencing the power and enjoyment of obtaining beneficial mathematical tools.

In addition to understanding the manner in which learners develop and obtain their mathematical knowledge, Miller, Stringfellow, Kaffar, Ferreira, and Mancl (2011) identified three modes of mathematical knowledge: conceptual, procedural, and declarative. Conceptual knowledge is the depth of understanding of mathematical operations, concepts, and meaning. Procedural knowledge is the step by step methodology utilized in solving mathematical problems. Declarative knowledge is the ability to memorize and recall mathematical facts (Miller et al., 2011). Miller et al recognized the importance of teaching strategies and suggested that teachers should specifically plan their instructional strategies so their teaching addresses all three modes of students' mathematical knowledge during the learning process.

With the current educational challenges regarding mathematics and increasing international demand for enhancing the mathematical knowledge of students, researchers are beginning to delve more into the environmental factors that surround students during their mathematics instruction (Papanastasiou, 2002; Powell & Kalina, 2009). Traditionally, teachers taught mathematics in a manner which promoted memorization, recall of facts, and computational operations.

The teaching of mathematics has not advanced much beyond the basic level of declarative knowledge. In contrast, modern technological advancement drives a greater need for complex conceptual innovation in mathematics (Powell & Kalina, 2009). Many of the current researchers find that educators need to claim greater instructional responsibility (Hansson, 2010) and devise means to become more effective in teaching mathematical constructs in order for students to truly develop conceptual mathematical knowledge (Desoete, Roeyers, & Buysse, 2001; Powell & Kalina, 2009). As societal expectations of students' mathematical knowledge evolve, so too should teaching strategies.

The Philosophy of Mathematics and Teaching

Mathematics is one of the important subjects taught in all schools throughout the world due to its relevance to other subject most especially in the development of science and technology. It is an integral part of life because it is needed by everyone for successful living. Mathematics is an indispensable tool in the study of sciences, humanities and technology. Its usefulness to man activities cannot be overemphasized. Man uses it directly or indirectly in everyday life or activities. It is a human invention, borne out of human in attempt to solve human problems (Kolawole & Oluwatayo, 2005). They also stated that the history of Mathematics reveals that Mathematical concepts such as counting, measuring, fractions, probability and others had their origin in problems faced by the scientists and Mathematicians of the past.

Oyedeji (2000) described Mathematics as a creative language, a tool and a process. Mathematics as the science of numbers and shapes, the process of calculating and using numbers. It can be defined as the science of numbers and shapes. According to Onoshakpokaiye (2006), Mathematics is an expression or graphical representation of what resides in the sub-conscious and also a mental activity. Mathematics is a branch of science that deals with shapes and numbers, Ezenweani (2006) described Mathematics as "the branch of knowledge that seeks to improve on human perception of himself and his immediate environment by using clear, logical precise and exact thinking processes". He also stated that Mathematics is autonomous science that springs up on define basis and develop in any direction based on the unfolding of knowledge. Lappan and Schman (1998) also sees Mathematics as a way of thinking and organizing ones expression. Agwagah (2005) stated that Mathematics involves thinking, modelling, conjecturing and describing all aspects of reasoning about situations.

The concept of teaching is better described than define because of different definitions. Various authors have defined teaching according to their own points of view, because of this; it has been very difficult to accept one as being the best. Teaching was originally the work of parents, elders and religious leaders (Kolawole & Oluwatayo, 2005). The duty of these people is to teach children how to be well-behaved and have respect, what to believe and how they can think positively to live a successful life. The Adults share their ideas and what they experience with the children who gradually acquired the knowledge. As a result of knowledge acquired by these young ones through traditional ways, they in turn carried out teaching. In our present day, teaching is no longer the responsibility of parents, elders and religious leaders as it used to be. It has become more sophisticated and is now the responsibility of trained or professional teachers (Onoshakpokaiye, 2006).

Many institutions have been opened to train people on how to teach. Ezenweani (2002) defined "teaching as the ability to guide one to gain knowledge in a classroom setting, the teacher does the work of guiding the learner through social interactions (student - teacher, student - student) to gain experiences that are worthwhile for social living in the larger society". Frazer (1990), defines teaching as an activity which is aimed at presenting certain learning content by giving instruction to someone to enable that person learns something from it. He also viewed teaching as an activity whereby the person being taught desires to benefit from it and wishes to acquire particular learning content.

According to Inomiesa (2010), teaching should be a process of probing by the teacher and discovery by the students. According to him, it should not be a process of regurgitating all the facts to the students rather it should be a process whereby the teacher guides the student in searching for a new facts and truths. Ebenezar (2009) stated that teaching strategies that one uses will undoubtedly affect one's philosophy of teaching. He went further stating that

Education is a cooperative enterprise that works well if the students are permitted to contribute to it when teacher listen and respond. The teacher should show interest to what they are teaching and do some explanations; the teacher should also show respect and concern for students and their learning.

Okpala (2006) stated that "one of the important median of realizing the educational objectives is the appropriate teaching method." According to him, the method adopted by the teacher either promote or hinder learning, it may increase mental activities which are the basis of social power. According to Popoola and Olarewaju (2006) when competent and qualified teachers are given the right training, then there will be quality and when one is self-motivated and responsible as teacher, most disciplinary problems are solved as he stimulates learning through cognitive and affective methods. Behind every successful lesson is a good teacher. Effective teaching implies productive, purposeful, result oriented, qualitative, meaningful and realistic teaching (Kolawole & Oluwatayo, 2005). The essence of being an effective teacher lies on what to do to foster student learning.

Teacher Knowledge in Mathematics

Teacher knowledge, including subject matter knowledge and pedagogical content knowledge, is the basis for teachers' instructional practices in their classroom. Subject matter knowledge has been defined as emphasizing knowledge and understanding of facts, concepts, and principles and the ways in which they are organized, as well as knowledge about the discipline (Even, 1993).

Valanides (2000) have found that insufficient subject matter knowledge among teachers led their students to develop misconceptions, misunderstandings, and misinterpretations regarding the subject matter during instruction. In addition, teachers who have higher subject matter knowledge have more favourable attitudes toward their teaching than those who lack or have limited subject matter knowledge (Barlow & Cates, 2006; Quinn, 1997). These findings could indicate that subject matter knowledge affects not only teachers' teaching practice but also their pedagogical content knowledge (Ozden, 2008). Researchers have determined that teachers' pedagogical content knowledge is highly related to students' academic achievement (Darling-Hammond, 2000). Therefore, teachers must develop and maintain sufficient and comprehensive subject matter knowledge for the betterment of their students' mathematics performance (Rizvi, 2004; Schmidt et al., 2009).

Pedagogical content knowledge differs from subject matter knowledge in the sense that it has a significant role in characterizing and identifying teachers' knowledge regarding their students' difficulty with subject matter and ability to connect mathematical ideas, use examples, provide explanations, and apply strategies when encountering mathematical concepts (Wilson, Floden, & Ferrini-Mundy, 2002). Researchers found that Pedagogical content knowledge factors such as teachers' pedagogical preparation, their use of routines in the classroom, and their degree content coverage influenced students' academic achievement growth (Cankoy, 2010; Rowan, Correnti, & Miller, 2002).

Some researchers have attempted to find relationships between teacher subject matter knowledge and students' mathematical academic achievement (eg. Wilson et al., 2002) and others have tried to connect pedagogical content knowledge to student learning (Ball, Hill, & Bass, 2005; Ball, Thames,

&Phelps, 2008). Schulman (1986) introduced the term pedagogical content knowledge and defined it as a special amalgam of content and pedagogy that is uniquely the province of teachers, their own special form of professional understanding. Since Shulman's (1986) demonstration of the importance of teacher knowledge, attention to teacher knowledge has shifted over time from an analysis of subject matter knowledge to pedagogical content knowledge (Ball et al, 2008). This shift to pedagogical content knowledge research has come with the development of various categorizations of pedagogical content knowledge.

The subscales of pedagogical content knowledge have been categorized differently depending on the researchers analysing them. In particular, since Shulman (1986) introduced pedagogical content knowledge, many researchers in mathematics education have described the subscales of pedagogical content knowledge by extending Shulman's (1986) concept. Lannin et al. (2013) categorized pedagogical content knowledge subscales for teaching mathematics as teachers' knowledge of the following: curriculum for mathematics, assessment for mathematics, instructional strategies for mathematics, and student understanding within mathematics.

In contrast, Hauk, Toney, Jackson, Nair, and Tsay (2014) considered pedagogical content knowledge to include knowledge of discourse, curricular thinking, anticipatory thinking, and implementation thinking. Hill, Ball, and Schilling (2008) provided a categorization of both subject matter knowledge and pedagogical content knowledge subscales, dividing the concept of subject matter knowledge into common content knowledge, knowledge at the mathematical horizon, and specialized content knowledge; and pedagogical content knowledge into knowledge of curriculum, knowledge of mathematical content and students, and knowledge of mathematical content and teaching (Ball et al., 2008).

Teachers' competency and knowledge in using and teaching problem posing are influential factors that can influence students' conceptual understanding in problem posing. Instructors should have the capacity to generate and reformulate problems in order to provide relevant activities for students' learning. It is critical for teachers to acquire problem posing experiences if they intend to provide new and different forms of learning experiences to their students (Crespo & Sinclair, 2008; Rowland, Huckstep, & Thwaites, 2003; Singer & Voica, 2012). Teachers were able to identify students' mathematical misconceptions through problem-posing proves as well as posed problems from the students (Koichu, Harel, & Manaster, 2013).

Teachers may benefit from problem-posing activities during their professional development experience to develop subject matter knowledge (Barlow & Cates, 2006). Recognizing teachers as critical agents in problem-posing activities for students is important (Silver & Voica, 2012). Thus, proper trainings for the teachers are critical. Despite of the importance of teacher's role, teachers had a narrow concept of problem posing. Since many teachers have not experienced problem posing when they were students or pre-service teachers, they tended to avoid problem posing or provide routinized exercises (Ball, 1990). Crespo (2003) indicated that teachers posed problems that were predictable, undemanding, ill formulated, and unsolvable when they extended a given problem. The majority of the posed problems focused on memorization and procedural understanding rather than on mathematical reasoning and conceptual understanding (Stein, Smith, Henningsen, & Silver, 2000). It is, therefore, important to broaden teacher knowledge in problem posing.

Philosophical arguments as well as common sense support the conviction that teachers' own subject matter knowledge influences their efforts to help students learn subject matter. Conant (2003) wrote that "if a teacher is largely ignorant or uniformed he can do much harm" (p. 93). When teachers possess inaccurate information or conceive of knowledge in narrow ways, they may pass on these ideas to their students. They may fail to challenge students' misconceptions; they may use texts uncritically or may alter them inappropriately. Subtly, teachers' conceptions of knowledge shape their practice--the kinds of questions they ask, the ideas they reinforce, the sorts of tasks they assign (Grossman, 2008).

Although early attempts to validate these ideas, to demonstrate empirically the role of teachers' subject matter knowledge, were unsuccessful (e.g., Begle, 2009), research on teaching and on teacher knowledge is revealing ways in which teachers' understandings affect their students' opportunities to learn (e.g., Grossman, 2008; Lampert, 2006; Leinhardt & Smith, 2005;Shroyer, 2001; Wilson, 2008; Wineburg and Wilson, 2008). This research is proving fruitful, in part, because of the researchers' conceptual work on dimensions of subject matter knowledge, work that is moving the field beyond the counting of course credits as a measure of teacher knowledge. Shulman's (2006) three categories of content knowledge--subject matter content knowledge, pedagogical content knowledge, and curricular content knowledge--are at the heart of much of the current inquiry. This paper focuses on the first, on what Shulman (2006) calls subject matter content knowledge.

What teachers need to know about the subject matter they teach extends beyond the specific topics of their curriculum. Shulman (2006) argues that "teachers must not only "teachers must not only be capable of defining for students the accepted truths in a domain. They must also be able to explain why a particular proposition is deemed warranted, why it is worth knowing, and how it relates to other propositions" (p. 9). This kind of understanding encompasses an understanding of the intellectual fabric and essence of the subject matter itself. For example, while English teachers need to know about particular authors and their works, about literary genres and styles, they also need to know about interpretation and criticism (Grossman, 2008). A history teacher needs detailed knowledge about events and people of the past but must also understand what history is: the nature of historical knowledge and what it means to find out or know something about the past. Scheffler (2003) writes that this kind of subject matter understanding strengthens the teacher's powers and, in so doing, heightens the possibilities of his art.

Lampert's (2006) own understanding of the substance of mathematics as well as its nature and epistemology shape what she is trying to help her students learn. When a student in her class asserted that .0089 is a negative number, for example, Lampert interpreted his claim as a conjecture whose validity could be judged by the classroom mathematical community rather than as a misconception that she should correct. Because she conceives of mathematics as a system of human thought rather than as a fixed body of procedures, she believes that students must have experience in developing and pursuing mathematical hunches and learning to make mathematical arguments for their ideas within the context of a discourse community. Orchestrating this in a fifth-grade classroom requires that the teacher draw simultaneously on her substantive understanding of mathematics--in this case, place value and decimal numeration--and her knowledge about the discourse, activities, and epistemology of mathematics. This knowledge of mathematics is necessary but not sufficient. Good teaching demands that teachers know a lot of other things--for example, about learning, about their students, and about the cultural, social, and political contexts within which they work (Lampert, 2006)

Review of Related Studies (Empirical Review)

Teaching Experience and Academic Performance

Gichuru and Ongus (2016) established the relationship between teacher quality and pupils' performance in mathematics in primary 6 national examinations in private schools in Gasabo District, Kigali City, Rwanda between the years 2012 and 2014, inclusive. The target population comprised of 1346 respondents out of which 75 were mathematics teachers, 31 head teachers from all the 31 private primary schools in Gasabo District, Rwanda and 1240 primary 6 pupils. The resulting sample size was 63 mathematics teachers, 28 head teachers and 302 pupils calculated from the total population using Yamane's simplified formula for determining a sample size. The sample size was selected using Multi stage Random Sampling technique. Five teacher qualities were studied: teacher qualification, teacher experience, communication skills, teaching practice and teacher preparedness. The study applied correlation comparative research design. Questionnaire and interview guides were used as data collection instruments. Data was analysed using frequencies, percentages, standard deviations, weighted means and Multiple Regression Analysis. Interview guide were analysed using content analysis.

Gichuru and Ongus (2016) noted that teachers with effective teaching practice register a higher student performance as opposed to teachers who resort to ineffective teaching practice. Additionally, teachers with more teaching experience impact pupils' performance than fresh graduates. Likewise, teachers' communication skills register a higher pupil's performance. There was improved performance in mathematics in those schools where the teachers were committed to their duties, had positive attitude towards mathematics, prepared well before going to teach, used plenty of teaching relevant resources, and engaged their pupils through evaluation and assessment.

Dial (2008) conducted a study to examine whether years of teaching experience and teacher's degree level have an effect on overall achievement of students on the communication arts and mathematics sections of the Missouri Assessment Program. Descriptive statistics and factorial ANOVA was used in this study. Data was analysed from both the communication arts and mathematics sections of the Missouri Assessment Program exam from the 2005-06 and 2006-07 school years. Whereas the overall results indicated that years of experience, as well as the interaction between years of experience and degree level, had an effect on student achievement in both communication arts and mathematics inconclusive results indicated teacher degree level alone had no effect on student achievement. The study recommended that further research could be continued using future test score data on the basis of the results.

Kimani, Kara and Njagi (2013) investigated the relationship between selected teachers' demographic characteristics and classroom instructional practices and students' academic achievement in selected secondary schools in Nyandarua County, Kenya. Participants in the study were drawn from one hundred and fifty- three teachers selected randomly from eighteen schools in three districts in the County. Based on their aggregate performance in Kenya Certificate of Secondary Education (KCSE) in the last three years, the schools were categorized as above average, average, and below average. Two schools per district were selected in each category. The researchers developed questionnaires to collect while linear regression and One-way ANOVA were used to test the relationship between the selected variables and performance in KCSE. The study revealed that teachers' age, gender, professional qualifications and teaching experience had significant relationship with academic achievement. In contrast, teachers' job group had significant and positive relationship with students' academic achievement in secondary schools.

Kosgei, Kirwa, Odhiambo, and Ayugi (2013) in their study in Nandi District, Kenya to establish the relationship between teacher quality and student achievement. The study was guided by Education Production Function theory (EPF) which connects student academic achievement to teacher characteristics. The population comprised of teachers of all 26 public secondary schools in Nandi District, Kenya.A causal comparative research design was applied in the study. A questionnaire was used for data collection which was analysed using descriptive and inferential statistical techniques. He observes that teacher experience has significant impact on students'performance but also notes that teacher qualification has no much relationship with student achievements.

Kimani, KaraandLucy (2013) investigated the relationship between selected teachers' demographic characteristics and classroom instructional practices and students' academic achievement in selected secondary schools in Nyandarua County. One hundred and fifty- three teachers selected randomly from eighteen schools in three districts in the County participated in the study. The schools were categorized as above average, average, and below average based on their aggregate performance in Kenya Certificate of Secondary Education (KCSE) in the last three years. In each category, two schools per district were selected. Data were collected using a questionnaire developed by the researchers. Linear regression and One-way ANOVA were used to test the relationship between the selected variables and performance in Kenya Certificate of Secondary Education (KCSE) at $p<.05$.

Kimani et al. (2013) found that teachers' age, gender, professional qualifications and teaching experience were not significantly related to academic achievement. Teachers' job group had significant and positive relationship with students' academic achievement in secondary schools. Teachers' weekly teaching workload, administration of students' classroom assignments, evaluation of students' Continuous Assessment Test (CATs) results, provision of individualized attention to weak students, time of completion of Form Four syllabus and setting performance targets for KCSE significantly affected students' academic achievement.

Ofem, IyamandBasses (2015) investigated the influence of teacher demographic variables on secondary School students' academic achievement in Home Economics in Calabar educational zone of Cross River State. Hypotheses were postulated to guide the study. Some relevant literatures were reviewed based on the two variables of the study. The study adopted a survey design. Simple random sampling technique was utilized to draw four hundred and twenty (420) respondents comprising of twenty (20) Home Economics teachers and four hundred (400) JSS III Students from the population. Two sets of instruments were used to elicit information from the sample. These instruments include: "Teacher demographic variables questionnaire" (T.D.V.Q) and "Home Economics Achievement Test" (HEAT). Kuder Richardson formula – 21 was used to establish the reliability coefficient of HEAT with an estimate of 0.77. T-Test was used in the data analysis. The results of data analysis showed that the two hypotheses were significant at 0.05 probability level. This means that educational qualifications and experience of the teacher influence significantly the students' academic achievement in Home Economics in the study area. Based on these findings some recommendations were made.

EwetanandEwetan (2015) studied the influence of teachers' teaching experience on the academic performance of public secondary school students in Mathematics and English Language in Ado-Odo/Ota and Ifo Local Government Areas in Ogun State. The study adopted descriptive research design. Study population comprised all the 31 Senior Secondary Schools in the selected two local government areas. A sample of 20 Schools was drawn from the population through the process of simple random sampling technique, made up of 14 schools in Ado-Odo/Ota, and 6 schools in Ifo Local Government Areas. An inventory schedule was the instrument used for data collection. 400 questionnaires, 20 questionnaires per school were administered. 388 (97%) questionnaires were returned. Their responses were analyzed through content analysis. The regression analysis and t-test were used to test hypotheses generated for the study at 0.05 alpha levels. Findings reveal that teachers' teaching experience has significantly influenced students' academic performance in Mathematics and English Language as measured by their performance in the SSC examinations and as perceived by the respondents. Schools having more teachers with above 10 years teaching experience achieved better results than schools having more teachers with 10 years and below teaching experience.

Akpo (2012) examined the impact of teacher-related variables on students" junior secondary certificate mathematics results in Namibia using questionnaire, multi-correlation and regression analysis and found that teacher educational qualifications, teaching experience, subject specialization, standards-based professional development, standard-based classroom activities, and classroom management beliefs are related to students' academic achievement in JSC Mathematics.

Chhinh and Tabata (2003) studiedthe effects of selected teacher factors on the Mathematics achievement of urban primary school pupils in the state of Cambodia, used questionnaires and achievement test to construct an index of academic performance. The results of the stepwise regression analysis revealed that teachers" economic status, their years of teaching experience and job satisfaction have statistically significant relationships with the achievement of the pupils whose economic status had been held constant. However, these three teachers" variables explain only about 20 percent of the variance in the pupil learning achievement. In a similar study on the impact of different teacher and class characteristics on third graders outcomes in Germany.

Yusuf andDada (2016), examined the impact of teachers' qualification and experience on students' performance in Colleges of Education in Kaduna State, Nigeria. Two Colleges of Education in Kaduna state were used for the study. A total of twenty (20) teachers and one hundred (100) students were randomly selected from the two Colleges of Education. The data was collected using questionnaire and students' test scores. Data was analyzed using frequency counts, percentages and t-test. The results revealed that a significant difference existed in the performance of students taught English language by professional and experienced teachers. The study recommended among others, that only qualified and experienced teachers should be allowed to teach in Colleges of Education in Kaduna state, Nigeria. All nonprofessional and unqualified teachers should be encouraged to pursue their post graduate studies (such as Post Graduate Diploma in Education, Master's and Doctoral degrees in Education) on a part-time or full-time basis. This will help to improve the quality of their teaching and consequently improve the performance of students and the quality of teacher education in Nigeria.

Teacher Pedagogical Skills and Academic Performance

OdundoandGung (2013) studiedtheeffects of application of instructional methods on learner achievement in Business Studies in Secondary Schools n Kenya. Primary data was obtained from 288 form four business studies students across the country. A mixture of probability and non-probability sampling procedures were used to select students and teachers for inclusion in the study. Bivariate analysis obtained cross-tabulations with Chi square ($\chi 2$) and one-way Analysis of Variance (ANOVA) for significance tests; while multivariate analysis obtained β coefficients, Exp(β), -2LL statistic and significance tests. The study found that takeaway assignments accounted for the largest proportion of variance in improved student performance (9.1%), brainstorming (8.8%), group discussions (8.3%), dictation (7.9%), lectures (6.3%) and chalkboard notes (5.9%) thus giving prominence to constructivist approach

Ganyaupfu (2013) investigated the differential effectiveness of teaching methods on students' academic performance. A sample of 109 undergraduate students from the College's Department of Economic and Business Sciences was used for the study. Using the inferential statistics course, students' assessment test scores were derived from the internal class test prepared by the lecturer. The differential effectiveness of the three teaching methods on student academic performance was analysed using the General Linear Model based univariate ANOVA technique. The $F (2, 106) = 10.125$; $p < 0.05$) and the Tukey HSD post-hoc results indicate significant differences on the effectiveness of the three teaching methods. The mean scores results demonstrate that teacher-student interactive method was the most effective teaching method, followed by student-centered method while the teacher-centered approach was the least effective teaching method.

Arranges, Seabra and lagers (2007), used a sample of more than 100 students, this study revealed that students perceived learning depends directly on their interest, pedagogical affect, and their learning performance and indirectly on the students-instructor interaction, the instructors'responsiveness, course organization, the instructors likeability/concern, and the students learning performance. Like ability/concern indirectly affects student interest by influencing learning performance. The results yield recommendations for schools, department heads, and university administrators.

RaheemandAmali (2013) examined teachers' pedagogical skills and use of instructional materials as a correlate of students' performance in social studies in Yenagoa Metropolis, Bayelsa State. Descriptive survey research of the correlational type was adopted for the study. 701 Junior Secondary School III students within Yenagoa Metropolis were randomly selected for the study. A researchers' designed Questionnaire on Teachers' Characteristics (QTC) and an adopted Social Studies Performance Test (SSPT) from the Bayelsa State Basic Junior School Certificate Examination 2008, 2009 and 2010 past questions were the instruments used in collecting data for the study. The Questionnaire on Teachers' Characteristics (QTC) and an adopted Social Studies Performance Test (SSPT) was pilot-tested using test-retest and split half methods respectively. A reliability coefficient of 0.72 was obtained for the QTC while 0.74 reliability coefficient was obtained for SSPT. Pearson's Product Moment Correlation was used to analyze the data at a significance level of 0.05 through SPSS for Windows version 17.

RaheemandAmali (2013) found a positive correlation between teachers' pedagogical skills and students' performance in Social Studies while there was no correlation between teachers' use of instructional materials and students' performance in Social Studies. The study therefore recommended thatgovernment at all levels with a meaningful and purposive collaboration with the organized private sector should periodically provide windows of opportunities like learned workshops, seminars and in-service trainings for social studies teachers to equip and improve their knowledge of teaching.

Enzi (2017), examined information about teachers' effectiveness at the hiring stage is particularly scarce despite its importance for personnel decisions. Using the German setting of teacher training, Enzi investigated the relationship of teachers' pre-service cognitive and pedagogical skills as measured by two state examinations and the high-school GPA on later effectiveness. Enzi applied standard value-added models to rich German student-achievement panel data and found that being in the top quartile in these skill domains is linked with significantly higher teacher effectiveness. Better teacher skills are associated with a more efficient way of classroom management.

Teacher Qualification and Academic Performance

Hill, Rowan and Loewenberg (2005) conducted a study to explore whether and how teachers' mathematical knowledge for teaching contributes to gains in students' mathematics achievement. A linear mixed-model methodology was used in which first and third graders' mathematical achievement gains over a year were nested within teachers, who in turn were nested within schools. It was concluded that teachers' mathematical knowledge was significantly related to student achievement gains in both first and third grades after controlling for key student- and teacher-level covariates. Based on the research findings, it was recommended that policy initiatives should be designed to improve students' mathematics achievement by improving teachers' mathematical knowledge.

Darling-Hammond, Holtzman, Gatlin, and Heilig (2005) examinedwhether certified teachers are generally more effective than those who have not met the testing and training requirements for certification and also linkedstudents' characteristics and achievement with data about their teachers' certification status, experience, and degree levels from 1995-2002. Data was collected to ascertain the effectiveness of Teach for America (TFA) recruits from selected universities who receive a short-term training before they begin teaching compared to experienced certified teachers. A series of regression analyses focusing on the 4th and 5th grade students' achievement gains on six different reading and mathematics tests was done over a six-year period. It was found out that certified teachers consistently produce stronger student achievement gains than do uncertified teacher. Additionally, the study concluded that teachers' effectiveness strongly related to the preparation the teachers had received for teaching.

Kane, Rockoff, and Staiger (2008), studied the relationship between teacher certification and student performance, Kane, Rockoff, and Staiger used six years of panel data on students and teachers to evaluate the effectiveness of recently hired teachers in the New York Public schools. Upon classification and analysis of teachers based on their certification status in their year of hire, research findings revealed that Certified, uncertified, international, and alternative certified (AC) teachers differ along a number of observable dimensions. For instance, the fraction of teachers who are black or Hispanic was found to be lower among regularly certified teachers and TFA corps members (about 20%) than among Teaching Fellows (30%) uncertified teachers (49%), or international teachers (48%). Consequently, there is more likelihood that certified teachers and international recruits will have graduate education than other groups. Based on the research findings, it was revealed that on average, the initial certification status of a teacher has small impacts on student test performance. There were large and persistent differences among teachers with the same experience and certification status. Such evidence suggested that classroom performance during the first two years is a predictive basis of a teacher's future effectiveness.

Akinsolu (2010) carried out a descriptive study using post-hoc datasetin Nigeria to examine the number of qualified teachers and its relationship to students' academic performance in public secondary schools. Twenty-one (21) public secondary schools, one in each Local Government Areas (LGA) from a population of thirty-one (31) LGA in the Osun State, Nigeria were sampled. The Senior School Certificate Examination results from 2000/01 to 2004/05 data were analyzed using ANOVA and Spearman rank correlation coefficient to test the three operational hypotheses. Findings of this study revealed that teachers' qualifications, experience and teacher–student ratio were significantly related to students' academic performance. The researcher recommended that the findings could be used to guide planners about the need for hiring qualified teachers for effective teaching and learning in secondary schools in Nigeria.

Yara and Otieno (2010) studiedthe effect of teaching/learning resources on academic performance in secondary school mathematics in Bondo district of Kenya. A descriptive survey design was adopted with a total of 405 senior four students as the population of the study. A random selection was applied on two hundred and forty- two (242) students drawn from nine schools out of 24 schools in the three divisions of Bondo districts. Using multiple regression analysis to analyse data, it was found that there was a positive correlation among the eight independent variables and the dependent. The study recommends that review of curriculum, in-servicing of trained teachers, recruiting more competent teachers, motivation of learners, improved government support to education, good teaching methods, improved students-book ratio and better remuneration of teachers are factors that the government and all stakeholders should pay more attention to in order to improve performance in mathematics.

Baumert et al (2010) investigated the importance of teachers' content knowledge and pedagogical content knowledge for high-quality instruction and student progress in secondary-level mathematics. The study conducted in Germany over a period of one year sampled Grade 10 classes and their mathematics teachers. Multilevel structural equation models revealed that there was a substantial positive effect of pedagogical content knowledge on students' learning gains that was mediated by the provision of cognitive activation and individual learning support.

Makewa, Role, Too and Kiplagat (2012) investigated teacher-related factors associated with performance in mathematics in public day primary schools in Nandi Central district, Kenya. A total of seventy-four (74) mathematics teachers participated in the study. Sampling techniques used to obtain the samples for the study included: stratified, random, and purposive. A questionnaire was used to collect data which had been validated and subjected to a pilot study to establish its reliability. Descriptive statistics and inferential statistic (t-test) were used to analyse the data. Based on the findings of the study, a majority of mathematics teachers in Nandi Central district public day primary schools were found to be trained with a teaching experience of between 11–20 years. An average rating was given on the mathematics teachers' use of learning resources, teaching methodology, teacher preparation, commitment, and assessment and evaluation. Moreover, teachers in high performing schools rated the attitudes toward mathematics, teaching methodology, commitment, preparation, and use of learning resources, evaluation and assessment higher than their counterparts in the low performing schools. It was recommended from the study that future research should link research on teacher preparation with teacher induction with professional development.

Kasiisa and Tamale (2013) studied the impact of teacher's qualification on the performance of Primary social studies in Eastern Uganda. A cross-sectional survey design was adopted with a sample size of 128 Senior Primary Schools social studies teachers. The research findings revealed that students taught by teachers with higher qualifications performed better than those taught by teachers with lower qualifications. Based on the research findings, it was recommended that experienced teachers with professional qualifications should teach Social Studies.

Abe (2014) studied the effect of teachers' qualification on students' performance in mathematics. Three hundred students were randomly selected from ten schools out of sixteen schools on purpose in Ikere Local Government Area of Ekiti State. The criterion for the selection of mathematics teachers was based on teacher qualification. T-test statistic was used to test the three hypotheses in the study. According to the results, there was a significant difference in the performances of students taught by professional teachers. The difference was also registered between students taught by NCE (Nigeria Certificate in Education) teachers and B.Sc Ed. Teachers and also between B.Sc teachers and B.Sc Ed. teachers. In its recommendation the study suggested that only qualified mathematics teachers should be allowed to teach mathematics at the secondary school level. Furthermore, the study recommended that holders of lesser qualifications such as Nigeria Certificate in Education (NCE) be allowed to proceed in their education either through part-time or study leave. In the same vein the study recommended that teachers without teaching qualification should be advised to pursue their Post Graduate Diploma in Education (PGDE). This may improve their teaching method in order to improve the performance of students in mathematics.

Mosha (2014) investigated the factors affecting students' performance in English language subject in Zanzibar's Secondary Schools using Bloom's (1982) model of evaluation as a framework in the study. This study investigated the factors affecting students' performance in English language subject in Zanzibar Secondary Schools. The study employed qualitative and quantitative approaches. Data were collected using interviews, classroom observation, questionnaire and documentary review.

Mosha (2014) revealed that students were highly motivated to learn English for future expectations such as local and international communication, academic advancement and employment prospects. However, students' performance was affected by shortage of English teachers and absence of teaching and learning materials. The findings revealed that due to incompetence from untrained and un-qualified teachers, cases of unprofessional malpractices such as skipping topics deemed difficult were prevalent. In addition to this a host of other factors such as infrequent use of English language at school and home, large class size, teachers' responsibilities, poor conducive teaching and learning environment in the classrooms, limited home support environment and poverty had negative influence on proficiency in English. Part of the study's recommendation is the need to offer in-service teacher training to equip teachers of English with competent skills in the subject.

Akiba, M., Le Tendre, and Scribner (2003) collected data from 46 countries on Trends in International Mathematics and Science Study. The results showed an opportunity gap in students' access to qualified teachers between students of high and low socioeconomic status (SES) was among the largest in the world against the backdrop of a similarity between the national level of teacher quality in the United States and the international average. Cross-national analyses revealed that the countries with better teacher quality produced higher mathematics achievement. However, larger opportunity gaps in access to qualified teachers did not predict larger achievement gaps between high-SES and low-SES students cross-nationally. The analyses provide empirical, cross-national evidence of the importance of investing in teacher quality for improving national achievement. The study recommended that National policies and practices related to improving teacher quality appear to be a promising area for future research to identify how other countries have achieved both excellence and equity in student achievement.

Macauand Abere (2015) investigated performance in Science, Mathematics and Technology (SMT) subjects among students in Kitui County, Kenya has perpetually been unsatisfactory. The aim of this study was to look into the extent to which teacher qualification influenced students' academic performance in SMT subjects. The study applied ex-post-facto survey research design. Random sampling was used to select eight secondary schools in Kitui County. It included eight head teachers, 40 teachers of SMT subjects and 600 candidates who sat for the Kenya Certificate of Secondary Education (KCSE) in the year 2012. Data were collected using questionnaire and document analysis. It was analyzed using descriptive and inferential statistical tools. The study found that there was no significant difference in means between teacher qualification and students' performance in Science, Mathematics and Technology(SMT) subjects at form four level (1,37)=0.017, P>0.05.

Macauand Abere (2015) further revealed that majority of the teachers of Science, Mathematics and Technology (SMT) subjects were trained graduates, most of them had attended in-service or refresher courses which resulted in slight improvement in the students' performance in Science, Mathematics and Technology (SMT) subjects. Recommendation is made for organization of more regular in-service and refresher training of Science, Mathematics and Technology (SMT) subject teachers to enable them embrace and conform to the emerging technologies in pedagogy.

Kosgei, Miser, Oder, and Ayugi (2013) established the relationship between teacher characteristics and students' academic achievement. The study was guided by Education Production Function theory (EPF) which connects student academic achievement to teacher characteristics. The study was conducted in Nandi District, Kenya and the target population comprised of teachers of all 26 public secondary schools. The study applied a causal comparative research design. A questionnaire was used for data collection. Data was analyzed using descriptive and inferential statistical techniques. The study findings suggest that there was no significant relationship between teacher qualification and student academic achievement.

Borisade (2011) aimed at investigating the effects of teacher qualities and school factors on the academic performance of secondary school students in mathematics. A purposive random sampling technique was used to select 150 teachers from 75 secondary schools in Ekiti State. May/June, 2011 WAEC result and a constructed questionnaire were used to obtain information on these factors. Three hypotheses were raised and tested at 0.05 level of significant using a t-test statistical analysis. The results showed a no significant different in the academic performance of students in mathematics from either urban or rural areas. And from those schools established before 1980 and those after 1980; the results were the same for those taught by mathematics teacher specialists and the non-specialist. These results showed that students in the rural areas performed likely as those in urban areas, and those taught by the non-specialist were at par with those taught by specialist and that year of establishment of a school did not have much effect on the students' academic performance

Omo (2011) maintainedthat the quality of teachers is strongly correlated to students' performance. The study presented evidence on teacher quality impact on student achievement with a sample of 400 students and 200 teachers from 40 purposively selected secondary schools in Ibadan metropolis in Nigeria. The schools were classified into four categories including: public elite schools; public non-elite schools; private elite schools and private non-elite schools. A composite measure of the quality of the teachers covering qualification, experience, patience, creativity, and communication skills was utilized. The students' performance was measured by their scores in the two compulsory subjects of English Language and Mathematics in the general school leaving certificate examination. Descriptive and inferential statistical analysis was used to analyse the data. The results revealed that the observed variations in the students' performance across the four categories of the schools were significantly explained by the differences in the quality of the teachers. The study concluded that the quality (qualification, experience, patience, creativity and communication skills) of teachers matters for student performance in schools.

Sharifirad et al. (2012) conducted a study on the knowledge, attitude and performance of academic members of School of Health, Isfahan University of Medical Sciences with regard to effective communication skills. A descriptive–analytical study was used to analyse data, where all academic members of the School of Public Health, Isfahan University of Medical Sciences, were studied during the second academic semester of 2006-2007. The data were collected by a valid and reliable three-part questionnaire including knowledge (8 questions and maximum score of 8), attitude (31 questions and maximum score of 155) and observational communication skills checklist (20 questions and maximum score of 20). The study found that the mean knowledge score of studied people in terms of communicational skills, attitude and performance were 4.1 out of 8, 114.4 out of 155 and 16.3 out of 20, respectively. It was concluded that though the information of the participants of the study in terms of communication skills was not sufficient, there seemed to have a positive attitude and relatively acceptable performance in communication skills.

Farooq andShahzadi (2006), compared the effectiveness of teaching of professionally trained and untrained teachers and the effect of students' gender on secondary school students' achievement in mathematics. Data were collected from four public and private boys and girls high schools' record. Four hundred secondary school graduates (Two hundred boys and two hundred girls) taught by trained and untrained teachers of mathematics were selected conveniently. The results of the study supported the fact that the students taught by trained teachers showed better results in mathematics and gender has no significant effect on achievement in mathematics

Darling-Hammond, L. (1999), investigated students' test results in reading and mathematics. In the study 44 states with 65 000 teachers were included. The data comprised several variables indicating teacher competence, such as certification and experience. A number of other variables were included in the study such as education policy, demographics, student characteristics and school characteristics. Controlling for student background, teacher certificate and subject matter knowledge were shown to correlate with students' test results and to have great explanatory power

Ochieng, KiplagatandNyongesa (2016), emphasizedon mathematics performance examined nationally by the Kenya National Examinations Council (KNEC) hence necessitating the current study. As a matter of fact, students' performance in mathematics has attracted attention from various stakeholders and further raised interest in teachers' qualification in delivering the subject matter. This study ought to establish the influence of teacher competence on mathematics performance in KCSE examinations among public secondary schools in Nyatike Sub-county, Kenya. Teacher competence was conceptualized in terms of teacher educational qualification, teacher training and teacher experience and mathematics performance as the dependent variable. The study was based on three models namely; economical approach, stakeholder theory of management and humanistic-progressive model. The study used a descriptive research design with a survey method and targeted 50 public secondary schools. The researcher used structured questionnaires to collect data from the sampled respondents. The collected questionnaires were then analysed using SPSS version 20. Both linear regression analysis and Pearson correlation analysis were used and the results presented in tables. The study findings revealed positive correlations between; teacher educational qualifications with mathematics performance, teacher training with mathematics performance, and teacher experience with mathematics performance. However, it was also noted thatsome teachers holding diplomas from reputable colleges such as Kenya Science University College perform better than those with undergraduate degrees from universities.

Appiagyei, JosephandFentim (2014) examined the effects of professional qualifications of financial accounting teachers on academic performance of SHS financial accounting students in the Tamale Metropolis of Ghana. Twenty- nine (29) teachers from the seven (7) senior high schools in the metropolis were selected for the study. It was a descriptive study where questionnaire was used to collect the data. The study revealed that financial accounting students perform academically better in financial accounting when they are taught by professional financial accounting teachers. This implies that teachers who possess sound professional training and qualification are wellequipped with the requisite competencies that enable them to promote effective teaching and learning. The study also recommended that, the government through the Ministry of Education should consider reinstating retired financial accounting teachers who are still energetic. As it is often said, “experience is the best teacher,” reengaging the service of retired financial accounting teachers would help to harness their rich experience on the teaching of the subject.

Aliyu, Yashe, and Adeyeye (2013), examined the effects of teachers’ qualifications on performance in further mathematics among secondary school students in Kaduna state. By purposive sampling, 12 senior secondary schools were selected from four inspectorate divisions in the state namely Anchau, Kaduna, Kafanchan and Zaria which participated in this study. In the second stage, a random sample of 160 further mathematics students were finally selected across the four divisions. Two instruments: Teacher Self-Assessment Test (TSAT) with reliability index of 0.87 and a 30-item four option multiple choice Further Mathematics Achievement Test (FMAT) constructed by the researchers (with cronbach’s alpha of 0.87 and item difficulty of $0.40 < p < 0.82$) were administered. Two research questions and one hypothesis were formulated to guide the study. The Analysis of Variance (ANOVA) revealed that significant difference exists between students’ performance on account of their teachers’ qualifications.

Owolabi (2012), examined the effect of teacher’s qualification on the performance of Senior Secondary School students in Physics. The purpose was to determine whether the status of the teacher has any impact on the performance of the students in Physics. The survey type of descriptive research design was adopted. The sample for the study consisted of 100 Senior Secondary Schools Physics students in Ekiti State and the teachers that prepared and presented the students in each school for 2009/2010 West African School Certificate Examination. The year's result summary for each school was collated with the bio-data of their respective Physics teachers. Four hypotheses were postulated and tested at 0.05 significance level. The data collated were analysed using inferential statistics. The results revealed that students taught by teachers with higher qualifications performed better than those taught by teachers with lower qualifications. It was also showed that students performed better in physics when taught by professional teachers. The result also showed that teacher's gender has no effect on their ability to impact knowledge on the students, much as he/she is a skilled teacher in that field of study. However, the experience of the teacher is significant at impacting the students' academic performance in Physics. Based on the findings, it was recommended that experienced teachers with professional qualifications in higher level should teach Physics at the certificate class.

Abe (2014), examined the effect of teachers' qualification on students' performance in mathematics. Three hundred students were randomly selected from ten schools that were purposively selected from sixteen secondary schools in Ikere Local Government Area of Ekiti State and used as sampled for the study. The qualification of the teachers was used as the criteria for selection of mathematics teachers. The three hypotheses in the study were tested using t-test statistic. The results showed that a significant difference existed in the performances of students taught by professional teachers and non-professional teachers, between students taught by NCE teachers and B.Sc Ed. Teachers and also between B.Sc teachers and B.Sc Ed. teachers at $P < 0.05$. The study recommended that, only qualified mathematics teachers should be allowed to teach mathematics at the secondary school level. While the holders of Nigeria Certificate in Education (NCE) should be allowed to proceed in their education either through part-time or study leave likewise teachers without teaching qualification should be advice to pursue their Post Graduate Diploma in Education (PGDE). This may improve their teaching method in order to improve the performance of students in mathematics.

KolaandSunday (2015), reviewed the controversy surrounding the teachers' qualifications and its influence on students' academic achievement. The paper measured teachers' qualification using seven indicators which are: formal education, experience, subject matter knowledge, pedagogy studies, duration of training, certificate/licensing and professional development. The paper reviewed different opinions on the relationship between these indicators and students' academic achievement. Though, it seems there is no consensus among the authors as regard the subject. However, there is a common opinion that subject matter knowledge, pedagogy studies, professional development and years of experience are imperative and positively correlated with students' academic achievement. As a new dimension to the argument, this paper conceptualized teacher qualification into two categories. The article posits that one of the categories, teacher's personal quality is more important than certification. The paper suggested some recommendations.

Unanma, Abugu, Dike, and Umeobika (2013), examined the relationship between Teacher's academic qualifications and academic achievement of Senior Secondary School Students in Chemistry. The area for the study was Owerri West LGA. A case study of four secondary schools formed the research design. Teacher's academic qualifications and the SS I student's third term result formed the data which was analysed using simple percentage and Pearson Correlation. Three research questions were answered and the findings of the research reviewed that there is a positive relationship between the teacher's academic qualifications and students' academic achievement. Since teachers' professional qualification influences students' academic achievement in Chemistry, the government and all stakeholders in education sector should endeavour to implement its policy on basic education for all and thus, create an enlightened society in which every Chemistry teacher would be educated enough to have a positive influence on their Chemistry students for better achievement in the subject.

Dele-Rotimi andOyinlana, (2014), adopted an ex-post-factor and survey research designs in which there were no treatment and manipulation of the subject. Instead, it involves the collation of data from the records. The study aims at examining the effect of Teachers' Qualification (TQ) on academic performance of Junior Secondary School Students in mathematics. The target population consisted of all Junior Secondary School Students and mathematics teachers in Ikere Local Government Area of Ekiti State. Random sampling technique is used to select six secondary schools out of which 100 students were sampled. Three hypotheses were analysed using correlation matrix and t-test statistics at the 0.05 level of significance. The results show a significant relationship between JSS 1, JSS 2 and JSS 3 mathematics performance. At the same time, significant correlation between Teachers' Qualification (TQ) and Junior Secondary School Students' performance in mathematics. Based on the results of the findings, it is recommended that government should always enforce professionalism in teaching. Employment of professionally trained, competent and qualified teachers should be ensured by the government, while in-service training should be conducted to no-professional teachers who also assist in teaching.

Akinsolu (2010), examined the number of qualified teachers and its relationship to students' academic performance in public secondary schools in a sample of Local Government Areas (LGA) of Osun State. This descriptive study used a post-hoc dataset. An instrument titled "Quantity and Quality of Teachers and Students' Academic Performance" (QQTSAP) was used for the study. Twenty-one (21) public secondary schools, one in each LGA from a population of thirty-one (31) LGA in the State, were sampled. The Senior School Certificate Examination results from 2000/01 to 2004/05 were used to analyse students' academic performance and reflected some concerns in the school system. The data were analysed using ANOVA and Spearman rank correlation coefficient to test the three operational hypotheses. Findings of this study showed teachers' qualifications, experience and teacher–student ratio were significantly related to students' academic performance. These findings can be used to guide planners about the need for qualified teachers to facilitate effective teaching and learning in secondary schools in Nigeria.

Machingambi, Oyedele, Chikwature and Oyedele (2018), reviewed the influence of teachers' qualification on students' performance in 'A'-level

Science subjects at selected secondary schools in the Mutare District, Manicaland Province in Zimbabwe. The researchers were spurred into investigating this area because of the misgivings of various presuppositions as regards to poor performance in ‘A’-level Science subjects. A qualitative research design was used in this study. Quota and purposive sampling techniques were used to select the schools and participants. The sample included twelve ‘A’-level ScienceTeachers, three heads of department and three school heads. Semi-structured interviews, focus group discussions, document analysis and observation were used in collecting the study data. These data were analysed and discussed using the thematic approach based on the research questions. The major findings of this research were that teaching qualification has a great influence on students’ performance in ‘A’-level Science subjects, as students taught by a qualified teacher performed better in the public examinations than those taught by unqualified teacher.

Machingambi et al. (2018) also found that practical lessons which are key components of Science subjects were been shunned by unqualified teachers. In addition, it emerged that most teachers were not using scientific methods in teaching Science subjects and this was negatively affecting their students’ performance in Science subjects. Moreover, findings from this research study indicated that subject matter knowledge, pedagogy studies, professional development and years of experience were imperative and positively influencing students’ academic achievements. The conclusions of this study were that teachers’ qualification is crucial in any educational system and it is important to recruit qualified teachers in Science subjects. As a result, it is important to ensure that each teacher is academically and professionally qualified to teach their assigned subjects. This study therefore recommends that school heads should work closely with the district schools' inspectors to ensure the recruitment and deployment of academically and professionally qualified teachers. It also recommends the review of the teacher training system in Zimbabwe, in order to adequately prepare teachers to meet the current trends in education.

Abuseji (2007), constructed and tested a model for providing a causal explanation of secondary school achievements in chemistry in terms of student variables – gender, study habit, mathematical ability and teacher's variables – gender, age, qualification and years of experience. An ex-post facto design was adopted for the study. The population was made up of all senior secondary school year two (SSII) students and their teachers in Epe and Ibeju-Lekki local government areas of Lagos state, Nigeria. However, six and four schools were used in the two local government areas respectively. Four sets of instrument were used; these were, (i) Personal Data Questionnaire for Teachers (PDQT) (ii) Study Habit Inventory (SHI) (iii) Mathematical Ability Test (MAT) and (iv) Chemistry Achievement Test (CAT). The results showed that 7.60% of the variability in students' achievement in chemistry (X8) was accounted for by all the sevenpredictor variables when taken together. It was also revealed that only four of the variables-teacher age (X1), teacher gender (X2), qualification (X3) and experience (X4) had direct causal effect on student's achievement in chemistry (X8). Recommendations based on the importance of these variables were then highlighted.

Iheonunekwu, Anyatonwu, andOkoro (2014), examined the number of qualified teachers and its relationship to students' academic performance in

Nigerian public secondary schools. This study was carried out in Abia State of Nigeria; the study is descriptive study survey which adopted an ex-post-facto research design. Four research questions were posed and four hypotheses were formulated and tested at 0.05 level of significance. An instrument titled "Quantity and Quality of Teachers and Students' Academic Performance" (QQTSAP) was used for the study. Seventeen (17) public secondary schools, one in each LGA in Abia State were used for the study. The Senior School Certificate Examination results from 2005/2006 to 2009/2010 were used to analyse students' academic performance. The data were analysed using ANOVA and Spearman rank correlation coefficient to test the three operational hypotheses. Findings of this study showed that teacher's qualifications, experience and teacher–student ratio were significantly related to students' academic performance. These findings can be used as a guide to educational planners on the need to have qualified teachers to facilitate effective teaching and learning in public secondary schools in Nigeria

Brown (2018) studied teacher qualification characteristics appear to be related to students' achievement in mathematics. This qualitative study incorporated the Grounded Theory (GT) approach which generated a substantive theory that explained how five Teacher Qualification Characteristics were related to Grade 12 students' 2016 CSEC Mathematics achievement in the British Virgin Islands. There were 33 participants involved in semi-structured individual and focus group interviews. The data revealed that teachers' competence and content knowledge dictate that teachers should have at least a Bachelor's Degree in Mathematics. Teachers' professional certification allowed them to help their students transfer their mathematical knowledge.

Brown (2018) revealed that the more years of teaching experience a teacher has, the more likely their students will be successful at CSEC mathematics. Professional development sessions should be subject-specific and relevant. Preparing students for life, fostering communication for understanding and making mathematics simple are hallmarks in the academic coaching experiences. This qualitative study yielded 17 themes. These themes are evidence that saturation was reached that allowed for the development of the GT—Mathematical Mastery Maximization is not vacuous; it hinges on five teacher qualification characteristics.

Subject Matter Knowledge and Academic Performance

Jin, Skemer, Green, and Herget (2007) made an attempt in a paper to estimate the effects of features of teachers' preparation on teachers' value added to student test score performance in math and English Language Arts. Results indicate variation across preparation programs in the average effectiveness of the teachers supplied to New York City schools. In particular, teachers in their first year appear to be beneficiaries of preparation directly linked to practice. The most likely explanation to this scenario is the "freshness" of knowledge and general professional alertness that allows for application of the theoretical knowledge from college to the working environment. The underlying revelation is that newly appointed teachers tend to prepare lessons meticulously in comparison to older counterparts. In return, it results into effective delivery of knowledge to an equally enthusiastic group of learners.

Makewa, Role, Too, and Kiplagat (2012), investigated teacher-related factors associated with performance in mathematics in public day primary schools in Nandi Central district, Kenya. Seventy-four (74) mathematics teachers participated in the study. Samples for the study were collected using, stratified, random, and purposive sampling techniques. Data collection was done using questionnaire which had been validated and subjected to a pilot study to establish its reliability. Each subscale of the questionnaire yielded a Cronbach's alpha reliability coefficient of 0.60 and higher. The study employed the descriptive statistics and inferential statistic (t-test) to analyse gathered data. The results from the study revealed that the majority of mathematics teachers in Nandi Central district public day primary schools were trained with a teaching experience of between 11– 20 years. However, they gave an average rating on the mathematics teachers' use of learning resources, teaching methodology, teacher preparation, commitment, and assessment and evaluation. In addition, teachers in high performing schools rated the attitudes toward mathematics, teaching methodology, commitment, preparation, and use of learning resources, evaluation and assessment higher than their counterparts in the low performing schools.

Fakeye (2012), investigated the extent to which teachers 'qualification and subject mastery could predict students' achievement in English language among senior secondary students in Ibarapa Division of Oyo state. The study adopted a descriptive research design of survey type to provide answers to four research questions. The study covered twenty (20) senior secondary schools randomly sampled. In each of the schools, a total number of fifty (50) senior secondary II students were selected to participate in the study making a total of one thousand (1000) S.S.II students in all. All the S.S. II English language teachers in the selected schools also participated in the study. Subject Mastery(r=.74) Questionnaire and English Language Achievement Test(r=.72) were used in data collection. Data collected were analyzed using frequency counts and simple percentage. Multiple regression analysis was also used for data analysis. All research questions were answered at 0.05 level of significance. The findings of this study showed that: Teachers' teaching qualification has a significant relative contribution to students' academic achievement in English language.

Muzenda (2013), analysed the effect of lecturers' competences on Students' academic performance among higher education and training students. A sample of 115students was selected and used for the study using simple random sampling procedure. A structured questionnaire was used to gather data on students' level of agreement on the extent to which distinct variables measuring lecturers' determine their academic performance. The data collected using the survey instrument was processed and analysed using SPSS statistical package. The scale reliability Cronbach's alpha of 0.822 and the sampling adequacy Keiser-MeyerOlkin of 0.769; with a total declared variance of 66.519 percent were obtained from the analysis. Four hypotheses were tested using Stepwise regression approach. Results indicate that subject knowledge, teaching skills, lecturer attendance and lecturer attitude have significant positive influence on students' academic performance.

Olaitan (2018), investigated how perceived teachers' competence and perceived classroom environment affect academic performance. Two hundred and sixty randomly selected final year National Diploma (ND II) students of the Federal Polytechnic, Ilaro, Ogun State, Nigeria were involved in this study. They were given two questionnaires that took approximately, fifty minutes to complete. The study was conducted in a classroom environment during regular school hours. After collecting information from the students through questionnaires, their comprehensive Grade Point Average (GPA) in previous year were also collected. This Grade Point Average (GPA) data were then compared to the scores obtained from the questionnaires. This study concludes that there was a statistically significant relationship between teachers' competence and Grade Point Average (GPA), and that there is a statistically significant interaction between classroom environment and students' academic achievement. Consequently, it was recommended that governments and school authorities must embark on regular training programmes and encourage teachers advance their academic and professional qualifications so that they would be more competent. Also, stakeholders in education should formulate policies that will ensure the appointment of qualified and competent teachers as well as the adequate provision of academic facilities in all schools as such would result in higher academic achievement.

Salami (2014), studied the effect on pre-service teachers' subject matter knowledge in a Primary mathematics methods course. Past studies on activity-based have focused more on the effectiveness of such strategies on pupils'/students' performance but did not sufficiently determine the effect of the strategy on the teachers' subject matter knowledge (SMK), hence this study. Pre-test post-test control group quasi-experimental research design was adopted and the participants were 215 preservice primary mathematics teachers in two colleges of education in South-western Nigeria. Primary Mathematics Achievement Test (0.81) and two instructional guides were the research instruments used. Hands-on/Mind-on activity based strategy enhanced pre-service primary mathematics teachers' subject matter knowledge more than the conventional strategy. This study has shown the instructional strategy that can be used to develop the capacity of pre-service primary mathematics teachers in the area of subject matter knowledge in a way to ensure sustainable effective mathematics learning. It was recommended that lecturers of primary school mathematics methodology courses in the colleges of education should be encouraged to acquire and utilise hands-on/mind-on activity-based strategy.

PeerzadaandJabeen (2014) investigated the impact of teacher's subject matter knowledge and their behaviour on the student's performance. Qualified and well behaved teachers help to boost learner's confidence and involvement in learning activities. In Pakistan, the quality of school level education is continuously deteriorating. It may be because of the reason that most of the private schools, due to their financial constraints, hire less qualified and untrained teachers. The data was collected through a 5 points Likert scale questionnaire. Responses were obtained from the students of secondary and matric classes of three different schools. Data was analysed using various statistical techniques, (factor analysis, regression analysis, mean comparison).

PeerzadaandJabeen (2014) revealed a significant relation between the student performance, teacher subject matter knowledge and teacher's behaviour. Besides this, it was found that the achievement level of students of high income area is better than that of low income area. Furthermore, the subject matter knowledge and behaviour of the teachers of high income area schools were found to be better than those of low income area. This study suggests that the subject matter knowledge and behaviour of the teachers should be given prime consideration while hiring them. In-service teachers' training programs must also be held to keep teachers updated in the subject. Personality development and motivational trainings should also be held to help teachers in improving their behaviour. Besides this the perks and privileges should also be raised for the teachers to attract knowledgeable and well behaved people to the teaching profession.

Campbell et al. (2014) conducted a quantitative study to investigate the relationship between teachers' mathematical content and pedagogical knowledge, teachers' perceptions, and student achievement. They found significant statistical relation between teachers' content knowledge of mathematics and students' achievement in both the upper elementary and middle classes. They further found that in upper elementary grades other teacher and students factor were also involved besides teacher content matter knowledge. They also found that with each standard deviation increase in teacher content matter knowledge, the expected mathematics scores of their students increased by 22% of an SD. Subject matter knowledge is an essential factor of student learning. For an effective teacher it is important that they should be knowledgeable about the ideas they teach. It is evident that students, taught by the teachers with the proper subject matter knowledge on a given concept, performed well than those students whose teachers were lacking subject matter knowledge. A teacher without expertise, may teach the concept erroneously, and students may develop the same malicious information as their teacher has. Efficacy of middle school science teachers may so depend on command over all the concepts they teach than with the extent of their understanding in any particular topic.

Metzler and Woessmann (2012) studied the impact of the teacher's subject knowledge on student's achievement, using data of primary school from Peru, containing test scores in two subjects not only for each student, but also for each teacher, data from a total of over 12,000 students, in nearly 900 randomly sampled primary schools, was obtained. The study revealed a significant effect of teacher's subject knowledge on student achievement. It was observed that a one-standard-deviation increase in teacher subject knowledge resulted 10 percent of a standard deviation increase in student's achievement. It was suggested that at the time of the teacher recruitment, subject knowledge should essentially be kept in consideration. Attention to teacher subject knowledge must also be considered for teacher training courses, and teacher's reward schemes.

Mullens et al. (1996) studiedqualitatively the effect of training and subject matter knowledge on teaching effectiveness. They examined the four types of variables for teaching effectiveness: educational achievement of a teacher, participation in training programs, teacher's subject matter knowledge and teacher's instructional approaches. For students' achievement, the data was collected by the difference of pre-test and post-test of 1043 third-grade students was analysed. The study showed that students learn more when their teacher has full command over the subject. The academic training of teachers and years of teaching experience were not directly linked with the student's learning. It was proposed that teacher's subject matter knowledge should highly be considered while hiring them. It was also stated that to make the training programs more effective, the contents and the structure of the training programs may also be re-designed.

Hill, Rowan, and Ball (2005) studied the effects of teacher's mathematical knowledge on student's achievement. A three-level hierarchical model in which mathematical gains of 1190 first graders and 1773 third graders over a year with the data of 334 and 365 teachers respectively of 115 schools were collected. It was revealed that the teacher's mathematical knowledge had a high effect on students' gain in both grades. It was recommended that for the increase in students' performance in mathematics the upgrading in teachers' content matter knowledge in the subject was vital.

Akbari and Allvar (2010) studied relation of teaching style, teachers' sense of efficacy, and teacher's reflectivity to students' achievement gains in an English-language teaching (ELT) context. The results of the study showed a high correlation between teacher reflectivity and student achievement outcomes, as the reflective teachers focused on student learning and were committed to help students to succeed. These teachers intended to discover and draft strategies to help children learn. Their reflection was stimulated by their zealous commitment to help children to learn. Actually, this was an enthusiastic want of reflective teachers to convert problematic classroom environment into opportunities for students to learn and grow.

Kunter, Kaufmann, Baumert, Richter, Voss, and Hachfeld (2013), investigated teachers' pedagogical content knowledge, professional beliefs, work-related motivation, and self-regulation as aspects of their professional competence. Specifically, it examines how these aspects impact instruction and, in turn, student outcomes. In a nationally representative sample of 194 German secondary school mathematics classes, multiple measures were used to assess teacher competence, instructional quality, and students' achievement and motivation. The effect of teachers' professional competence on student outcomes was estimated in a 1-year repeated-measures design. Two-level structural equation models revealed positive effects of teachers' pedagogical content knowledge, enthusiasm for teaching, and self-regulatory skills on instructional quality, which in turn affected student outcomes. In contrast, teachers' general academic ability did not affect their instruction. The multidimensional model of teachers' professional competence introduced in this article seems suited to stimulate further research on the personal indicators of teacher quality.

Lee, Capuano, and Capuano, (2018), investigated teachers' subject matter knowledge (SMK), knowledge of content and teaching (KCT), and knowledge of content and students (KCS) in terms of problem-posing. A qualitative study design and inductive analysis were used to gather and interpret data from interviews conducted with four mathematics teachers. Results indicated that participants had SMK of problem posing, but their actual problem-posing results did not reflect their SMK well. In terms of KCS and KCT, teachers were aware of the importance of problem posing for students' mathematical development but felt that there were several significant factors impeding the effective incorporation of problem posing within their classes. These findings underscore the importance of professional development for teacher pedagogical knowledge in problem posing.

Obot (2012), studiedthe background of perceived decrease in students' interest in the learning of Social Studies Education, the research on Influence of Teachers' Competence in Subject Matter on Students' Interest in the Learning of Social Studies Education in Akwa Ibom State, Nigeria was aimed at determining the contributions of students' perception of teachers' competence in subject matter on students' interest in learning with particular attention to Social Studies Education. The ex-post facto research design was adopted for the study. Null hypothesis was formulated to guide the study. The data for the work were collected with the help of Influence of Teachers' competence in subject matter on students' interest in the learning of social studies questionnaire (ITCSMSILSQ) as the working instrument. The stratified random sampling technique and the simple random sampling technique were adopted for the choice of samples from the population which consisted of all Social Studies Education Students in Akwa Ibom State. The face and content validity were established and the alpha Cronbach coefficient was used to establish the reliability of the instrument. The collected data was then coded and analyzed using the One-way analysis of variance (ANOVA). The hypothesis was calculated at F-value of 10.30 to be significant at 0.05 level with a critical value of 3.00. The Fishers'LSD Multiple Comparison Analysis was used to determine the group means that brought the significance. All showed that students with moderate and high level perception of teachers' characteristics experienced more interest in Social Studies Education than students with low level perception. Based on the above result, a conclusion was drawn that teachers' competence in subject matter has much influence on students' interest in learning with particular reference to Social Studies

Education. Some recommendations made included the need to ensure proper and effective teacher education in Nigeria.

Conceptual Framework

After the thorough review of literature, a conceptual framework was developed to guide the study. It was obvious from the review that teacher variablessuch as subject matter knowledge, teacher experience, pedagogical content knowledge, teacher prior education and traininghave significant impact on the academic performance of students. Again, it appears that teaching experience can moderatethe relationship between teacher subject matter knowledge as well as pedagogical content knowledge and the performance of student in elective mathematics.Figure 2 provides the details.

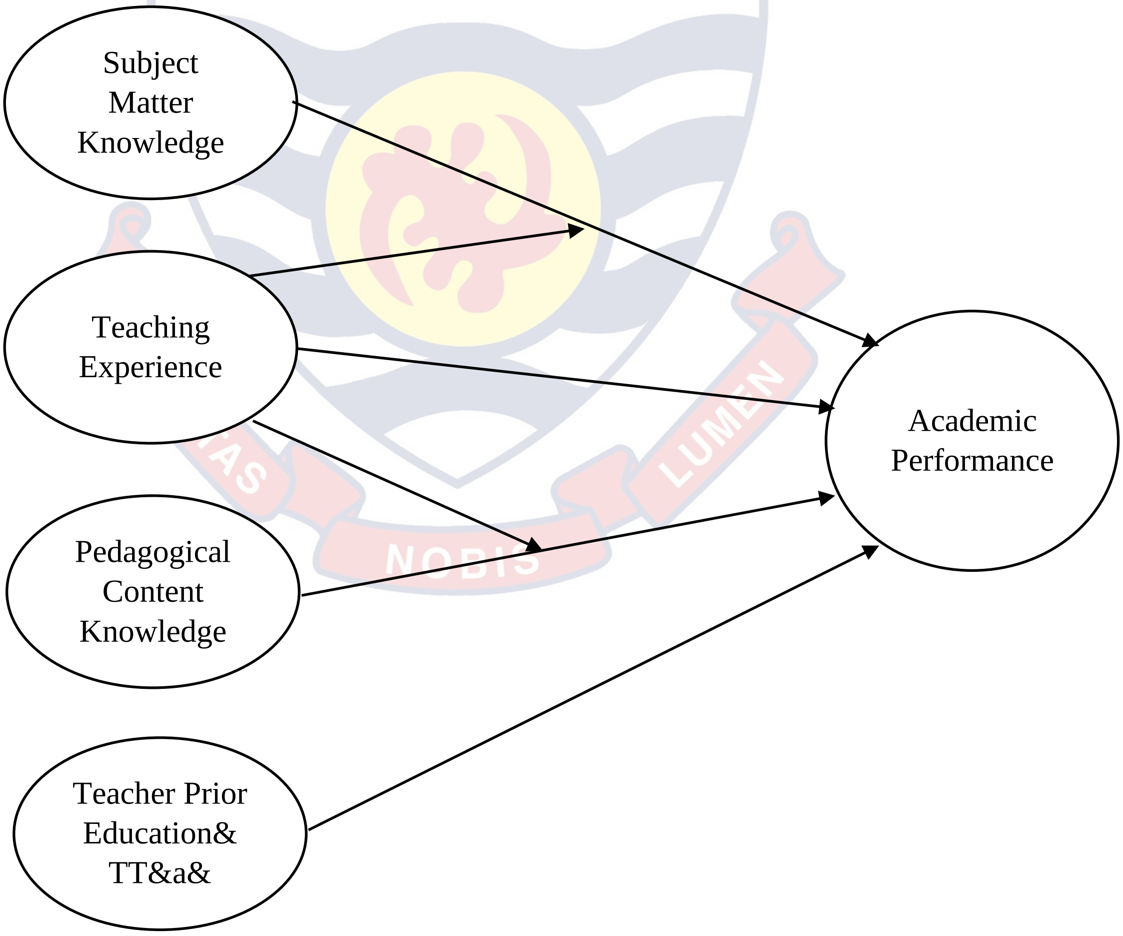

Figure 2: Conceptual Framework Showing the Relationship between Teacher variables and Academic Performance

Steps in Sem Analysis

Structural equation modeling is a statistical approach to testing hypotheses about the relationship among observed and latent variables. It is very flexible and include many other techniques ie. multiple regression, confirmatory factor analysis, path analysis and Anova.

The basic building blocks of all **SEM** analysis follow a logical sequence of steps or processes: model specification, parameter estimation, and assessment of fit. These basic building blocks are absolutely essential to all **SEM** models.

Model Specification

Model specification is the development of a theoretical model, using all of the available applicable theory, research and information. Before any data collection or analysis, a specific model that should be confirmed with variance-covariance data is specified by the researcher. In other words, available information is used to decide which variables to include and not to include in the theoretical model, and how the variables are related. Model specification involves determining every relationship and parameter in the model that is of interest to the researcher.

A given model is said to be properly specified when the true population model is considered predictable with the implied theoretical model being tested, that is, the sample covariance matrix **S** is sufficiently reproduced by the implied theoretical model. The researcher's aim is to determine the best possible model that generates the sample covariance matrix.

Parameter Estimation

Before the estimation of parameters in **SEM**, it is critical that the researcher settle the identification problem. In the identification problem, the following question is asked: can unique set of parameter estimates be found on the basis of the sample data contained in the sample covariance matrix **S** and the theoretical model implied by the population covariance matrix ∑?

For instance the theoretical model might suggest that **x + y = 8**, and yet it may be that no unique solution for **x** and **y** exists (**x = 6, y = 2 or x = 5, y = 3** etc). This implies that there is the possibility that the data fit more than one implied theoretical model equally well, since there are not enough constraints on the model and the data to obtain unique estimates of **x** and **y**. To solve this problem, we need to impose some constraints such as, fix **x = 1**, then **y** would have to be **7**. In this case, we have solved the identification problem by imposing one constraint.

Each potential parameter in a model must be specified to be either a free parameter, a fixed parameter, or a constrained parameter. Once the model is specified and the parameter specifications are indicated, the parameters are combined to form one and only one ∑ (model-implied variance – covariance matrix).

In structural equation model, there are different methods for estimating the parameters, that is, estimates of the population parameters. We want to obtain estimates for each of the parameters specified in the model that produce the implied ∑, such that the parameter values yield a matrix as close as possible to **S**, our sample co-variance matrix of the observed variables. When the difference between the elements in the matrix **S** and that of the matrix ∑ equal zero (**S** – ∑ = **0**), then one has a perfect model fit to the data. The parameter estimates can be done by SEM software, iterative process, and also using the first principles.

Using First Principles to Estimate Parameters in the Structural Equation Model

Let **y** be the target variable and $\mathbf{x_1, x_2, \ldots x_n}$ be the set of input variables. Then **y** as a linear combination of input variables is

$$\mathbf{y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \ldots + \beta_n x_n} \qquad (1)$$

Here **β** represents the parameters given to the input variables, which is unknown. **y** cannot be accurately calculated as a linear combination of input variables. Using a linear combination of input variables, we can calculate a value $(\mathbf{y^1})$ as close to **y** as possible. The difference between **y** and $\mathbf{y^1}$ is the error. We therefore estimate **β** which will help to calculate $\mathbf{y^1}$with minimum error. Equation **(1)** can be written in matrix form (from linear algebra) as

$$\mathbf{y = x \,.\, \beta} \qquad (2)$$

Here **y** is the list of all values of target variables, **x** is the input variables matrix where each column represents a variable, and **β** is the list of all parameters for input variables in **x**. **"."** Denotes matrix multiplication (for the purpose of readability).

Equation **(2)** has a solution only if **y** can be expressed as a linear combination of input variables. We therefore estimate $\mathbf{y^1}$, a list of values as close to **y** as possible.

$$\mathbf{y^1 = x \,.\, \beta} \qquad (3)$$

Now the error **E** is defined as

$$\mathbf{E = y - y^1} \qquad (4)$$

In linear algebra, one fundamental rule for error is, error is orthogonal to the input variable space.

$$\mathbf{xt \,.\, E = 0} \qquad (5)$$

where **xt** is transpose of matrix **x.**

Substituting equation **(4)** in equation **(5**), we get

$$xt.(y - y^1) = 0$$

$$xt.y^1 = xt.y$$

$$xt.x.\beta = xt.y \text{ (using equation 3)}$$

Multiplying both sides by $(xt.x)^{-1}$ which is the inverse of matrix **(xt . x),** we get

$$\beta = (xt.x)^{-1}.xt.y \qquad (6)$$

Equation **(6)** is the final list of parameters corresponding to all input variables.

Assessment of Fit

Fit refers to how much the predicted covariances (or correlations) differ from the observed covariances.

After obtaining the parameter estimates for a specified **SEM** model, the researcher should determine how well the data fit the model. Specifically, to what magnitude is the theoretical model supported by the obtained sample data. This is done by examining the fit of individual parameters of the model. Three main features of the individual parameters can be considered. One feature is whether a free parameter is significantly different from zero. Standard errors for each estimate are computed once parameter estimates are obtained. A ratio of the parameter estimate to the estimated standard error can be formed as a critical value. If the critical value exceeds the expected value at a specified **α** level (eg, **1.96** for a two – tailed test at the **0.05** level), then that parameter is significantly different from zero. The parameter estimate, standard error and critical value are generally provided in the computer output for a model. A second feature is whether the sign of the parameter agrees with what is expected from the theoretical model. The third feature is that parameter estimates should make sense, that is, they should be within an expected range of values.

If the fit of the implied theoretical model is not as strong as one would like, then the next step is to modify the model and subsequently evaluate the new modified model.

Chapter Summary

Literature was reviewed under different sub-headings. These are the theoretical review, conceptual review, empirical review and conceptual framework. Three main theories were reviewed under the theoretical review. These were theZone of Proximal Development (Vygotsky, 1930), Theory of Instruction (Bruner, 1985) and the Mathematical Theory of Relational and Instrumental Understanding (Skemp, 2006).

All the three theories talked about teacher–student relationship. According to the theories, teachers are more knowledgeable than students and as such, teachers are present to assist students to learn and understand concepts in their respective subjects. What this means is that if students are not performing well in any subject say elective mathematics, then it can be mentioned that teachers to some extent, contribute to this poor performance. It is of essence to state that the ability of teachers to effectively help these students depends on a number of teacher variables – professional qualification, experience, subject matter knowledge, and pedagogical skills. If there is a problem with any of these variables, it will probably hinder the ability of the teacher to assist in the congnitive development of the studnets in Elective Mathematics.

In the conceptual review, it was revealed that the way mathematics is taught in schools is often disconnected from the way students need to think about solving problems in real world scenarios (Carpenter, 1989). Skemp (1989) proposed that teachers could save students from mathematics failure by utilizing intelligent teaching.

According to Inomiesa (2010), intelligent teaching should be a process of probing by the teacher and discovery by the students. According to him, it should not be a process of spewing all the facts to the students, rather it should be a process whereby the teacher guides the student in searching for new facts and truths.

The empirical studies were reviewed under the following sub-headings: Teaching Experience and Academic Performance, Teacher Pedagogical skills and Academic Performance, Teacher Qualification and Academic Performance, Subject Matter Knowledge and Academic Performance.

After the thorough review of literature, a conceptual framework was developed to guide the study. It was obvious from the review that teacher variables such as subject matter knowledge, teacher experience, pedagogical content knowledge, teacher prior education and training, all have significant impact on the academic performance of students in elective mathematics.

In chatper 3, the study addresses the topics of research design, population sample, hypotheses, research variables, instrumentation, data collection procedures, and statistical analysis as related to this study.

CHAPTER THREE

RESEARCH METHODS

Introduction

The purpose of the study was to investigate the effect of teacher variables such as teachers' qualification, teaching experience, teachers' knowledge of subject matter, and teachers' pedagogical content knowledge, on student academic performance in elective mathematics in senior high schools (SHS) in Ghana. This chapter provides information on the research methodology employed in the conduct of this study. The chapter, specifically, presents a step by step procedure in gathering valid and reliable information as well as how the data is analysed with the aim of achieving the overall objective of the study. The chapter is organised into the following sections: research paradigm, research design, population, sampling procedure, data collection instrument, data collection procedures, and data processing and analysis.

Research Paradigm

The study employed the positivist philosophy in its enquiry. Positivism rich tradition comes from a long history. This is generally regarded as a "scientific" approach to methods that are highly organized, measurable and taken by the scientific unit involved in researching the behavior of the natural world. Denscombe (2012) describes how the positivists have the purpose of social research to discover patterns and regularities in the social world, applying the type of scientific methods used with such good results in the natural sciences.

Positivists use more quantitative methods, such as structured questionnaires, social survey and public statistics, their excellent reliability and representativeness. Positivism in research, tends to look at relationships and correlations, between two or more variables which is known as comparative methods (Creswell, 2012). Since the study seeks to exmaine the relationshp between teacher variables and academic performance of students in elective mathematics, positivism which uses quantitative methods was employed.

Research Design

Bless and Higson-Smith (2000) acknowledge that every research work requires a research design that is carefully tailored to meet the exact needs of the researcher as well as the problem. In this case, a descriptive survey was employed in order to accomplish the purpose of the study. With descriptive survey design, data is collected directly from a population or sample. According to Leedy and Ormrod (2005), survey research involves acquiring information about one or more groups, perhaps about their characteristics, opinions, attitudes or previous experiences by asking them questions and tabulating their answers.

Creswell (2012) noted that using survey as a research design allows the researcher to select a sample of interest and administer questionnaire, and or conduct interview to collect information about a phenomenon. These authors further acknowledged the fact that surveys are frequently used in educational research to describe attitudes, beliefs, opinions and other forms of information because accurate information can be obtained from a large number of people with a small sample.

The descriptive survey was used for this study because it was convenient to administer instrument to respondents individually to collect data to describe a phenomenon. In survey research, high reliability is easy to obtain by presenting all subjects with a standardized stimulus and that greatly eliminates observer subjectivity.

Population

Gorard (2001) opines that population is a group usually of individuals from which a sample can be selected to generate results of a study. This particular study covered Senior High Schools in Ghana. Specifically, the study was targeted to SHS in five regions (Central, Western, Northern, Greater Accra, and Ashanti Regions). The population for this study comprised two different groups of people. SHSelective mathematics teachers and their corresponding elective mathematics students were the subjects for the study.

Sampling Procedure

The sample of the study was 6,750 SHS elective mathematics students and 225 SHS elective mathematics teachers from 45SHSfrom five regions of Ghana (Greater Accra, Ashanti, Central, Western and Northern).A multistage sampling technique was employed in this study. The data was to be analyzed using SEM, which has a high data requirement of at least 200 varaibles. A decision was made to use 225 teachers; 45 teachers from each of the five regions. When contact was made with the various academic heads of the senior high schools about the minimum number of teachers teaching elective mathematics, the number was four. Since majority of the schools had at least five teachers teaching SHS elective mathematics, purposive sampling technique was employed to select 45 senior high schools which had at least five teachers teaching elective mathematics, with nine from each of the five reigons. The simple random sampling technique was then used to select five SHS elective mathematics teachers from each of the 45 senior high schools. For each of the selected teachers, 30 of their elective mathematics students were sampled randomly.

Data Collection Instruments

The instruments for this study included two questionnaires; one for teachers and the other for students. The students' questionnaire was grouped into two parts; students' background and students' assessment questions. Student background was measured utilizing six questions: the father's level of education, the mother's level of education, mathematics books and other resources from home, extra tuition, number of students in class, and assignments given by their teachers. The teachers' questionnaire on the other hand was grouped into four parts: I, II, III and IV, representing the variables: Teacher Prior Educaiton and Training, Teacher experience, teacher subject matter knowledge, and Pedagogical content knowledge of the teacher, respectively. Part I contained five items and Part II contained ten items.

Majority of the items on Parts III and IV was adapted from the Knowledge of Algebra for Teaching (KAT) project at Michigan State University (MSU). Most of the contexts and wording of the questions in the KAT instrument were changed to reflect Ghanaian contexts as part of the adaptations. For instance, the prices of items were changed to reflect market values in Ghana as the dollar in some questions was replaced with the cedi. In addition, variations in names of words used were also changed to reflect the right contexts in Ghana. A word like "pants" for example was replaced by "trousers" as it is commonly called in Ghana.

Parts III and IV of the teachers' questionnaire contained 20 items in all; sixteen multiple-choice and four free response questions about the knowledge of SHS elective mathematics curriculum and pedagogical content knowledge of the teacher. The students'assessment on the other hand, comprisednine multiple-choice and one free response questions based on SHS elective mathematics curriculum. To permit the performance of students and teachers to be measured on the same scale, all the ten items on the students'assessment were the same as the ten items on Part III of the teachers' instrument. The remaining ten items on Part IV werebased on the pedagogical content knowledge of the teacher.

Data Collection Procedure

Contact was made with heads and / or the assistant heads of the senior high schools, the elective mathematics teachers, as well as the elective mathematics students who participated in the study. At the meetings with the teachers, the purpose of the study and the voluntary nature of their participation were discussed; giving them opportunity to decide, without any form of coercion, whether to participate or not to participate in the study. In all, 45 out of the 75 heads of senior high schools who agreed to allow the study to be conducted in their schools were purposively sampled for the reason as explained under sampling procedure, to obtain the number of elective mathematics teachers needed for the study.

As was agreed in each of the participating schools, the instruments were administered after breakfast on Saturday, so as not to disrupt the normal weekday classes. In each of the participating schools, students were brought together in the assembly hall where they were given 60 minutes to complete the instruments under the supervision of research assistants. While students were completing the instrument, the teachers who agreed to participate were also brought together in a classroom to complete the teachers' instrument at a sitting, lasting not more than 70 minutes.

Data Processing and Analysis

The data was processed using Smartpls SEM (Partial Least Squares Structural Equation Modeling).**Smartpls**is a software with graphical user interface for variance-based structural equation modeling (SEM) using the partial least squares (PLS) path modellingmethod (Wong, 2013; Hair, Hult, Ringle, & Gudergan, 2017). Besides estimating path models with latent variables using the PLS-SEM algorithm, the software computes standard results assessment criteria (e.g., for the reflective and formative measurement models, the structural model, and the goodness of fit) and it supports additional statistical analyses (e.g., confirmatory tetrad analysis, importance-performance map analysis, segmentation, multigroup) (Hair, Sarstedt, Ringle, & Gudergan, 2017).

PLS is a soft modeling approach to SEM with no assumptions about data distribution (Wong, 2010). Thus, PLS-SEM becomes a good alternative to CB-SEM when these situations (sample size is small and applications have little available theory) are encountered (Hwang, Malhotra, Kim, Tomiuk, & Hong, 2010; Wong, 2010):

Due to the fact that the study employed a smaller sample size (225) as compared with C – B SEM which requires a lot more sample size, and again, the distribution of the data was not known as compared with C – B CEM, that needs data to be normally distributed, it is thought that using PLS – SEM was a better choice.

Derivation of Ordinary Least Square Estimator (Emerick, 2011)

The Smartpls SEM uses OLS regression for the estimation of parameters. It does this by estimating multiple OLS regression based on the predictors and the criterion. Multiple OLS regression, requires an estimation for a critical set of parameters (a regression constant and one regression coefficient for each independent variable in the model) that minimize the sum of the squared errors of prediction (SSE).

The classified OLS regression model can be expressed as follows:

$$\boldsymbol{y_i = a_o + a_i x_i + e_i, \quad i = 1, 2, \ldots n} \quad \textbf{(7)}$$

where y_i is the score on the DV, a_o is the regression constant, a_i is the regression coefficients for the effects x_i, and e_i is the error made in predicting $y_i$from x_i.

Now,

$$\mathbf{S.S.E} = \sum_{i=1}^{n} \mathbf{e}_i^2 \quad (\mathbf{8})$$

We specifically find the values of a_o and a_i that minimize the quantity in equation (2) above

From equation (1)

$$\boldsymbol{e_i = y_i - a_o - a_i x_i} \quad \textbf{(9)}$$

Substituting equation (9) back into equation (8), we get

$$\boldsymbol{S.S.E} = \sum_{i=1}^{n} (\boldsymbol{y_i - a_o - a_i x_i})^2 \quad \textbf{(10)}$$

We set up the minimization problem that is the starting point for deriving the formula for the OLS intercept and the coefficients. That is

$$\min_{a_o\, a_i} \sum_{i=1}^{n} (y_i - a_o - a_i x_i)^2$$

We now take the partial derivatives of SSE with respect to $a_o and\ a_i$, set these derivatives to zero, and solve for a_o and a_i. Note that n is the sample size for the data.

Now from equation (10), taking partial derivatives of SSE with respect to a_o, we get

$$\frac{\partial SSE}{\partial a_o} = \sum_{i=1}^{n} -2(y_i - a_o - a_i x_i)$$

$$= -2\sum_{i=1}^{n} y_i + 2\sum_{i=1}^{n} a_0 + 2a_i \sum_{i=1}^{n} x_i \qquad (11)$$

Setting the derivative in equation (11) to zero, we get

$$-\sum_{i=1}^{n} y_i + \sum_{i=1}^{n} a_0 + a_i \sum_{i=1}^{n} x_i = 0$$

Which implies

$$na_o + a_i \sum_{i=1}^{n} x_i = \sum_{i=1}^{n} y_i \qquad (12)$$

Substituting

$\sum_{i=1}^{n} x_i = n\bar{x}\ and \sum_{i=1}^{n} y_i = n\bar{y}$ into equation (12), we get

$na_o = n\bar{y} - na_i\bar{x}$ which implies

$$a_o = \bar{y} - a_i\bar{x} \tag{13}$$

Taking the partial derivative of SSE with respect to a_i,we get

$$\frac{\partial SSE}{\partial a_i} = \sum_{i=1}^{n} -2x_i(y_i - a_o - a_i x_i)$$

$$= -2\sum_{i=1}^{n} x_i y_i + 2a_o\sum_{i=1}^{n} x_i + 2a_i\sum_{i=1}^{n} x_i^{\,2} \tag{14}$$

Setting the derivative in equation (14) to zero, we get

$$-\sum_{i=1}^{n} x_i y_i + a_o\sum_{i=1}^{n} x_i + a_i\sum_{i=1}^{n} x_i^2 = 0 \quad \text{which implies}$$

$$\sum_{i=1}^{n} x_i y_i - a_o\sum_{i=1}^{n} x_i - a_i\sum_{i=1}^{n} x_i^2 = 0$$

$$\sum_{i=1}^{n} x_i y_i - a_o n\bar{x} - a_i\sum_{i=1}^{n} x_i^2 = 0 \tag{15}$$

Substituting $a_o = \bar{y} - a_i\bar{x}$into equation (15), we get

$$\sum_{i=1}^{n} x_i y_i - n\bar{x}(\bar{y} - a_i\bar{x}) - a_i\sum_{i=1}^{n} x_i^2 = 0$$

$$\sum_{i=1}^{n} x_i y_i - n\bar{x}\bar{y} + na_i\bar{x}^2 - a_i\sum_{i=1}^{n} x_i^2 = 0$$

$$a_i\sum_{i=1}^{n} x_i^2 - na_i\bar{x}^2 = \sum_{i=1}^{n} x_i y_i - n\bar{x}\bar{y} \quad \text{which implies}$$

$$\frac{\sum_{i=1}^{n} x_i y_i - n\bar{x}\bar{y}}{\sum_{i=1}^{n} x_i^2 - n\bar{x}^2} \tag{16}$$

$$\text{but} \sum_{i=1}^{n} x_i y_i - n\bar{x}\bar{y} = \sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})$$

$$and \sum_{i=1}^{n} x_i^2 - n\bar{x}^2 = \sum_{i=1}^{n} (x_i - \bar{x})^2$$

Therefore equation (16) becomes

$$a_i = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{n}(x_i - \bar{x})^2}$$

Note

The second partial derivatives of SSE with respect to a_o and a_i are respectively 2n and $2nx_i^2$. Since all these values are necessarily positive, we can be sure that the values of a_o and a_i that satisfy the equation generated by setting each partial derivatives to zero refer to minimum rather that maximum values of SSE.

Structural Equation Modeling

The data was analyzed using Structural Equation Modeling. Structural Equation Modeling is a statistical approach to testing hypotheses about the relationships among observed and latent variables (Hoyle, 1995). Observed variables are called indicator variables or manifest variables. Latent variables are denoted as unobserved variables or factors. Examples of latent variables in education are mathematics ability and intelligence and in psychology are depression and self-confidence. The latent variables cannot be measured directly. Researchers must define the latent variable in terms of observed variable to represent it.

SEM is a very strong analysis technique which consists of multivariate statistical technique whose working areas are theoretic structures that are represented by latent variables. It scrutinizes the suitability of the estimated covariance matrix, which is composed, according to the theoretical structure, of the covariance matrix of the observed data.

SEM is also a methodology that takes a confirmatory (i.e. hypothesis-testing) approach to the analysis of a theory relating to some phenomenon. Byrne (2001) compared SEM against other multivariate techniques and listed four unique features of SEM:

1. SEM takes a confirmatory approach to data analysis by specifying the relationships among variables a priori. By comparison, other multivariate techniques are descriptive by nature (e.g. exploratory factor analysis) so that hypothesis testing is rather difficult to do.
2. SEM provides explicit estimates of error variance parameters. Other multivariate techniques are not capable of either assessing or correcting for measurement error. For example, a regression analysis ignores the potential error in all the independent (explanatory) variables included in a model and this raises the possibility of incorrect conclusions due to misleading regression estimates.
3. SEM procedures incorporate both unobserved (i.e. latent) and observed variables. Other multivariate techniques are based on observed measurements only.
4. SEM is capable of modelling multivariate relations, and estimating direct and indirect effects of variables under study.

Due to the reasons explained above, and since the methods used in explaining students' performances are generally limited to correlation, regression and factor analysis, it is thought that working with SEM, which is an analysis method that can measure the stated variables more comprehensively, significantly and reliably, will contribute to the literature. However, one disadvantage of SEM is its high data requirement of at least 200 variables.

Steps in Structural Equation Modeling

The Structural Equation Modeling (SEM) application comprises five steps (Bollen & Long, 1993), although they vary slightly from researcher to researcher. They are (a) model specification, (b) model identification, (c) parameter/model estimation, (d) model fit, and (e) model re-specification. The aforementioned steps are explained to have an idea about SEM.

Model Specification

First, model specification is concerned with formulating a model based on a theory and/or previous studies in the field. Relationships between variables – both latent and observed – need to be made explicit, so that it becomes clear which variables are related to each other, and whether they are independent or dependent variables. Such relationships can often be conceptualized and communicated well through diagrams. This has been shown in Figure 2.

Rectangles represent observed variables (e.g., test items, responses to questionnaire items), and ovals indicate unobserved variables. Unobserved variables are also called factors, latent variables, constructs, or traits. In the case of this study, the unobserved variables include teaching experience, pedagogical content knowledge, subject matter knowledge, teacher prior education and training, and academic performance.

Circles indicate measurement errors or residuals. Measurement errors are hypothesized when a latent variable affects observed variables, or one latent variable affects another latent variable. Observed and latent variables that receive one-way arrows are usually modelled with a measurement error. A one-headed arrow indicates a hypothesized one-way direction, whereas a two-headed arrow indicates a correlation between two variables. The variables that release one-way arrows are independent variables (also called exogenous variables), and those that receive arrows are dependent variables (also called endogenous variables). For this study, the four predictors (e.g., teaching experience, pedagogical content knowledge, subject matter knowledge, and teacher prior education and training) were the exogenous variables and the criterion (academic performance) was endogenous variable.

Additionally, SEM models often comprise two subsets of models: a measurement model and a structured model. A measurement model relates observed variables to latent variables, or, defines more broadly, and specifies how the theory in question is operationalized as latent variables along with observed variables. A structured model relates constructs to one another and represents the theory specifying how these constructs are related to one another. The study was, however, limited to the structured model.

Model Identification

At this stage, the concern is whether a unique value for each free parameter can be obtained from the observed data. This is dependent on the choice of the model and the specification of fixed, constrained and free parameters. Schumacker and Lomax (2004) indicated that three identification types are possible. If all the parameters are determined with just enough information, then the model is 'just identified'. If there is more than enough information, with more than one way of estimating a parameter, then the model is 'over identified'.

If one or more parameters may not be determined due to lack of information, the model is 'under identified'. This situation causes the positive degree of freedom. Models need to be over identified in order to be estimated and in order to test hypotheses about the relationships among variables. A researcher has to ensure that the elements in the correlation matrix (i.e. the off-diagonal values) that is derived from the observed variables are more than the number of parameters to be estimated. If the difference between the number of elements in the correlation matrix and the number of parameters to be estimated is a positive figure (called the degree of freedom), the model is over-identified. The following formula is used to compute the number of elements in a correlation matrix:

$$[p (p + 1)]/2$$

where p represents the number of observed(measured) variables. When the degree of freedom is zero, the model is just-identified. On the other hand, if there are negative degrees of freedom, the model is under-identified and parameter estimation is not possible. Of the goals in using SEM, an important one is to find the most parsimonious model to represent the interrelationships among variables that accurately reflects the associations observed in the data. Therefore, a large degree of freedom implies a more parsimonious model. Usually, model specification and identification precede data collection. Before proceeding to model estimation, the researcher has to deal with issues relating to sample size and data screening.

Parameter/Model Estimation

In estimation, the goal is to produce a $\Sigma(\theta)$ (estimated model-implied covariance matrix) that resembles S (estimated sample covariance matrix) of the observed indicators, with the residual matrix ($S - \Sigma(\theta)$) being as little as possible. When $S - \Sigma(\theta) = 0$, then $\chi 2$ becomes zero, and a perfect model is obtained for the data. Model estimation involves determining the value of the unknown parameters and the error associated with the estimated value. As in regression, both unstandardized and standardized parameter values and coefficients are estimated. The unstandardized coefficient is analogous to a Beta weight in regression and dividing the unstandardized coefficient by the standard error produces a z value, analogous to the t value associated with each Beta weight in regression. The standardized coefficient is analogous to β in regression.

In the estimation process, a fitting function or estimation procedure is used to obtain estimates of the parameters in θ to minimize the difference between S and $\Sigma(\theta)$. Apart from the Maximum Likelihood Estimation (MLE), other estimation procedures are reported in the literature, including unweighted least squares (ULS), weighted least squares (WLS), generalized least squares (GLS), and asymptotic distribution free (ADF) methods.

In choosing the estimation method to use, one decides whether the data are normally distributed or not. For example, the ULS estimates have no distributional assumptions and are scale dependent. In other words, the scale of all the observed variables should be the same in order for the estimates to be consistent. On the other hand, the Maximum Likelihood and Generalised Least Square methods assume multivariate normality although they are not scale dependent.

When the normality assumption is violated, Yuan and Bentler (1998) recommend the use of an ADF method such as the WLS estimator that does not assume normality. However, the ADF estimator requires very large samples (i.e., n = 500 or more) to generate accurate estimates (Yuan & Bentler, 1998). In contrast, simple models estimated with MLE require a sample size as small as 200 for accurate estimates. Since the sample size for this study was more than 200, and the normality assumption was validated, the OLS estimation was used.

Model Fit

The main goal of model fitting is to determine how well the data fit the model. Specifically, the researcher wishes to compare the predicted model covariance (from the specified model) with the sample covariance matrix (from the obtained data). On how to determine the statistical significance of a theoretical model, Schumacker and Lomax (2004) suggested three criteria.

The first is a non-statistical significance of the chi-square test. A non-statistically significant chi-square value indicates that sample covariance matrix and the model-implied covariance matrix are similar. Secondly, the statistical significance of each parameter estimates for the paths in the model. These are known as critical values and computed by dividing the unstandardized parameter estimates by their respective standard errors. If the critical values or t values are more than 1.96, they are significant at the .05 level. Thirdly, one should consider the magnitude and direction of the parameter estimates to ensure that they are consistent with the substantive theory.

Researchers are recommended to report various fit indices in their research (Hoyle, 1995, Martens, 2005). Overall, researchers agree that fit indices fall into three categories: absolute fit (or model fit), model comparison (or comparative fit), and parsimonious fit (Kelloway, 1998; Mueller & Hancock, 2004; Schumacker & Lomax, 2004).

Absolute fit indices measure how well the specified model reproduces the data. They provide an assessment of how well a researcher's theory fits the sample data (Hair et al., 2006). The main absolute fit index is the $\chi 2$ (chi-square) which tests for the extent of misspecification. As such, a significant $\chi 2$ suggests that the model does not fit the sample data. In contrast, a non-significant $\chi 2$ is indicative of a model that fits the data well. In other word, we want the p-value attached to the $\chi 2$ to be non-significant in order to accept the null hypothesis that there is no significant difference between the model-implied and observed variances and covariances.

However, the $\chi 2$ has been found to be too sensitive to sample size increases such that the probability level tends to be significant. The $\chi 2$ also tends to be greater when the number of observed variables increases. Consequently, a nonsignificant p-level is uncommon, although the model may be a close fit to the observed data. For this reason, the $\chi 2$ cannot be used as a sole indicator of model fit in SEM.

Three other commonly used absolute fit indices are described below. The Goodness-of-Fit index (GFI) assesses the relative amount of the observed variances and covariances explained by the model. It is analogous to the R^2 in regression analysis. For a good fit, the recommended value should be GFI > 0.95 (1 being a perfect fit). An adjusted goodness-of-fit index (AGFI) takes into account differing degree of model complexity and adjusts the GFI by a ratio of the degrees of freedom used in a model to the total degrees of freedom. The standardized root mean square residual (SRMR) is an indication of the extent of error resulting from the estimation of the specified model. On the other hand, the amount of error or residual illustrates how accurate the model is hence lower SRMR values (<.05) represents a better model fit. The root- mean square error of approximation (RMSEA) corrects the tendency of the χ 2 to reject models with large sample size or number of variables. Like SRMR, a lower RMSEA (<.05) value indicates a good fit and it is often reported with a confidence level at 95% level to account for sampling errors associated with the estimated RMSEA.

In comparative fitting, the hypothesized model is assessed on whether it is better than a competing model and the latter is often a baseline model (also known as a null model), one that assumes that all observed variables is uncorrelated. A widely used index example is the Comparative Fit Index (CFI) which indicates the relative lack of fit of a specified model versus the baseline model. It is normed and varies from 0 to 1, with higher values representing better fit. The CFI is widely used because of its strengths, including its relative insensitivity to model complexity. A value> .95 for CFI is associated with a good model. Another comparative fit index is the Tucker-Lewis Index (TLI), also called the Bentler-Bonnet NNFI (nonnormed fit index) by Bentler and Bonnet (1980) is used to compare a proposed model to the null model. Since the TLI is not normed, its values can fall below 0 or above 1. Typically, models with a good fit have values that approach 1.0.

Parsimonious indices assess the discrepancy between the observed and implied covariance matrix while taking into account a model's complexity. A simple model with fewer estimated parameters will always get a parsimony fit. This is because although adding additional parameters (thus increasing the complexity of a model) will always improve the fit of a model but it may not improve the fit enough to justify the added complexity. The parsimonious indices are computed using the parsimony ratio (PR), which is calculated as the ratio of degrees of freedom used by the model to the total degrees of freedom available (Marsh, Balla, & McDonald, 1988). An example of parsimony fit indices is the parsimony comparative-of-fit index (PCFI), which adjust the CFI using the PR. The PCFI values of a model range from 0 to 1 and is often used in conjunction with the PCFI of another model (e.g. null model). Because the AGFI and RMSEA adjust for model complexity, they may be also used as indicators of model parsimony.

Model Modification

If the fit of the model is not good, hypotheses can be adjusted and the model retested. This step is often called re-specification (Schumacker & Lomax, 2004). In modifying the model, a researcher either adds or removes parameters to improve the fit. Additionally, parameters could be changed from fixed to free or from free to fixed. However, these must be done carefully since adjusting a model after initial testing increases the chance of making a Type I error. At all times, any changes made should be supported by theory. To assist researchers in the process of model modification, most SEM software such as AMOS compute the modification indices (MI) for each parameter. Also called the Lagrange Multiplier (LM) Index or the Wald Test, these MI report the change in the $\chi 2$ value when parameters are adjusted. The LM indicates the extent to which addition of free parameters increases model fitness while the Wald Test asks whether deletion of free parameters increases model fitness. The LM and Wald Test follow the logic of forward and backward stepwise regression respectively.

Martens (2005) noted that model modifications generally result in a better fitting model. Hence researchers are cautioned that extensive modifications may results in data-driven models that may not be generalizable across samples (e.g., Chou & Bentler, 1990; Green, Thompson, & Babyak, 1998). This problem is likely to occur when researchers (a) use small samples, (b) do not limit modifications to those that are theoretically acceptable, and (c) severely mis-specify the initial model (Green et al., 1998).

Great care must be taken to ensure that models are modified within the limitations of the relevant theory. If a Wald test indicated that the researcher should remove the freely estimated parameter from perceived ease of use (PEU) to perceived usefulness (PU), the researcher should not apply that modification, because the suggested relationship between PEU and PU has been empirically tested and well documented. Ideally, model modifications suggested by the Wald or Lagrange Multiplier tests should be tested on a separate sample (i.e. cross-validation). However, given the large samples required and the cost of collecting data for cross-validation, it is common to split an original sample into two halves, one for the original model and the other for validation purposes. If the use of another sample is not possible, extreme caution should be exercised when modifying and interpreting modified models.

Chapter Summary

This chapter described the research methodology and procedure used in this research study. The smartpls SEM (partial least squares structural equation modeling) was the statistical approach used to determine the effect of teacher variables (SMK, EXP, TPET and PCK) on student Elective Mathematics performance in Senior High Schools. In chapter four, the results of this study are presented.

CHAPTER FOUR

RESULTS AND DISCUSSION

Introduction

For the objective, the study seeks to use mathematical modeling to do an investigation into which teacher variables would improve the academic performance of students in elective mathematics in Senior High Schools in Ghana. The previous chapter focused on the methodology. About 225 elective mathematics teachers were sampled and the instrument was administered to them. About 6,750 elective mathematics students were also assessed to examine their performances in elective mathematics. For each of the 225 teachers, 30 of their students were assessed. This chapter presents the results of the study.

Hypotheses Testing

The study was guided by six hypotheses. These hypotheses were tested in three different models.

First Model

In this model, the first four hypotheses were tested. These hypotheses were tested together to identify the contributions of each independent variable to the prediction of the performance of students in elective mathematics.

1. H_0: There is no significant relationship between teacher prior education and training, and performance of students in elective mathematics.
2. H_0: There is no significant relationship between teaching experience and performance of students in elective mathematics.
3. H_0: There is no significant relationship between teacher subject matter knowledge and performance of students in elective mathematics.

4. H_0: There is no significant relationship between teacher pedagogical content knowledge and performance of students in elective mathematics.

Observed, endogenous variable

- Academic Performance in Elective Mathematics (ACH)

Observed, exogenous variables

- Subject Matter Knowledge (SMK)
- Pedagogical Content Knowledge (PCK)
- Teacher Prior Education and Training (TPET)
- Teaching Experience (EXP)

Unobserved, exogenous variables

- Measurement error of the observed, endogenous variable (d1)

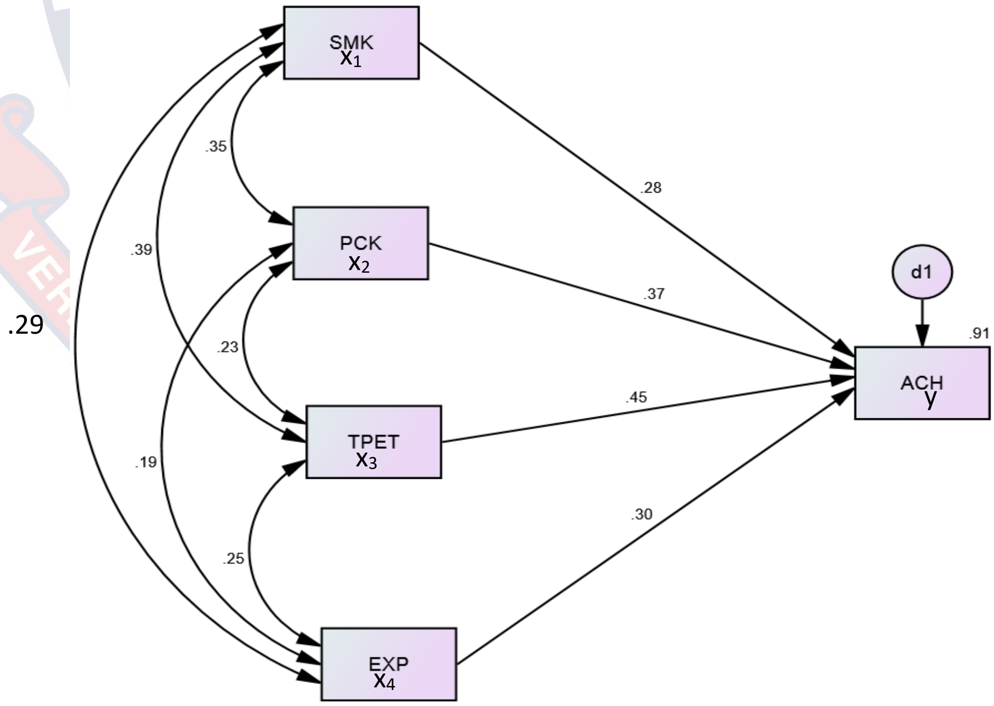

Figure 3: Prediction of SMK, PCK, TPET, and EXP to Performance in Elective Mathematics

Source: Field Survey

The model in figure 3 is represented as

$$y = 0.28x_1 + 0.37x_2 + 0.45x_3 + 0.30x_4 + d_1 \quad (17)$$

Figure 3 shows the predictive relationship between the independent variables and the dependent variable (academic performance). It can be seen that all the independent variables are related to the dependent variable.

Specifically, TPET(x_3) with a coefficient value of 0.45 is shown to be the highest predictor of academic performance (equation 17). The standardised values in figure 3 are confirmed in Tables 2 and 3. Table 2 shows the covariance and correlations among the independent variables.

Table 2: Covariance and Correlations

			Covariance				Correlations
			Estimate	S.E.	C.R.	P	
SMK	<-->	PCK	1.570	0.317	4.956	0.000	0.351
SMK	<-->	TPET	2.553	0.474	5.389	0.000	0.386
SMK	<-->	EXP	1.223	0.293	4.176	0.000	0.291
PCK	<-->	TPET	1.596	0.485	3.292	0.000	0.225
PCK	<-->	EXP	0.860	0.306	2.810	0.005	0.191
TPET	<-->	EXP	1.643	0.458	3.585	0.000	0.247

Source: Field Survey

Table 2 shows how the independent variables are related to each other. It is clear from the table that a positive relationship exists among the variables. Specifically, the correlation between Subject Matter Knowledge (SMK) and Teacher Prior Education and Training (TPET) is the strongest with co-efficient ($r=0.386$, $p<.05$).

However, the correlation between Pedagogical Content Knowledge (PCK) and Teaching Experience (EXP) is the weakest with co-efficient (r=0.191, p<.05). Overall, it can be seen in table 2 that all the p-values are significant in that they are all less than the .05 significant level. In theory, the p-value of the independent variable is indicative of whether the independent variable has statistically significant predictability. A p-value is significant when it is less than 0.05, which signifies the model is fit for the data and therefore the null hypothesis can be rejected. A non-significant p-value implies the hypothesis can be accepted.

Table 3 shows the predictive relationship between the teacher variables and academic performance in elective mathematics.

Table 3: Prediction of Teacher variables to Academic Performance in Elective Mathematics

			B	S.E.	Beta	C.R.	P	Lower CI	Upper CI
ACH	<---	SMK	1.487	0.121	0.283	12.291	0.000	1.151	1.779
ACH	<---	PCK	1.810	0.106	0.368	17.140	0.000	1.526	2.118
ACH	<---	TPET	1.488	0.073	0.448	20.393	0.000	1.359	1.592
ACH	<---	EXP	1.572	0.111	0.301	14.227	0.000	1.385	1.734

R^2 - 0.91 Source: Field Survey

Table 3 shows that all the teacher variables predict academic performance of students. This is because all the P-values are less than .05 significant level. Thus, subject matter knowledge (SMK), pedagogical content knowledge (PCK), teacher prior education and training (TPET) and teaching experience (EXP) are predictors of students' performance in elective mathematics.

The standardised scores/beta values show that teacher prior education and training is the highest predictor of academic performance (0.448). In terms of order of predictiveness, it is seen that pedagogical content knowledge (0.368), teaching experience (0.301) and subject matter knowledge (0.283) follow after teacher prior education and training. These values are also seen in the model presented in Figure 3. Based on this, it can be seen that all the four hypotheses tested in the first model obtained significant results.

It can also be seen from table 3 that the model recorded an R^2 value of -0.91. This implies that teacher prior education and training, experience, subject matter knowledge, and pedagogical content knowledge of teachers, explained about 91% of the variances in students' performance in elective mathematics. Thus, a greater portion of students' performance in elective mathematics is determined by teacher variables.

Table 4 shows the model fit indices of the first model showing the predictive relationship between teacher variables and the academic performance of students in elective mathematics. It can be seen in Table 4 that the main absolute fit index, the $\chi 2$ (chi-square) which tests for the extent of misspecification was non-significant. A significant model would have implied that the model does not fit the sample data.

However, since the $\chi 2$ was non-significant (0.633), there is no significant difference between the model-implied and observed variances and covariances. In this sense, the model fits the sample data.

Table 4: Model Fit Indices for the First Model

Fit Indices	Cut-off	Model in Figure 3
Chi-square (χ2)	Non-significant	0.633, non-significant
Goodness-of-Fit (GFI)	<0.95	0.534
Adjusted Goodness-of-Fit (AGFI)	<0.95	0.356
M Standardized Root Mean Residual (SRMR)	<0.08	0.035
Root Mean Square Error of Approximation (RMSEA)	<0.07	0.029
Comparative Fit Index (CFI)	>0.95	0.997
Tucker-Lewis Index (TLI)	>0.95	0.995

Source: Field Survey

Again, it can be seen in Table 4 that the Goodness-of-Fit index (GFI), adjusted goodness-of-fit index (AGFI), standardized root mean square residual (SRMR), root- mean square error of approximation (RMSEA), Comparative Fit Index (CFI) and Tucker-Lewis Index (TLI) figures all show that the model in Figure 3 is fitting for the study.

Overall, Table 4 shows that the model is fitting for the study. The model indices fitting the predictors (i.e., prior education and training, experience, subject matter knowledge, and pedagogical content knowledge) to the criterion was found acceptable, indicating that teacher variables play important role in the performance of elective mathematics.

Based on the results in Figure 3 and tables 2 and 3, all the four hypotheses that were tested in the first model that there is no significant relationship between teacher variables and academic performance, are rejected. On the basis of the results, it is concluded that there is a significant relationship between the independent variables (teacher prior education and training, teaching experience, teacher subject matter knowledge and teacher pedagogical content knowledge) and the dependent variable (academic performance).

Second Model (Moderation)

In the second model, the fifth hypothesis which aimed at finding out the extent to which teaching experience moderates the relationship between teacher subject matter knowledge and performance of students was tested.

H_0: Teaching experience does not significantly moderate the relationship between teacher subject matter knowledge and performance of students in elective mathematics.

Observed, endogenous variable

- Academic Performance in Elective Mathematics (ACH)

Observed, exogenous variables

- Subject Matter Knowledge (SMK)
- Teaching Experience (EXP)
- Moderator: Product of SMK & EXP (SMK_EXP)

Unobserved, exogenous variables

- Measurement error of the observed, endogenous variable (d2)

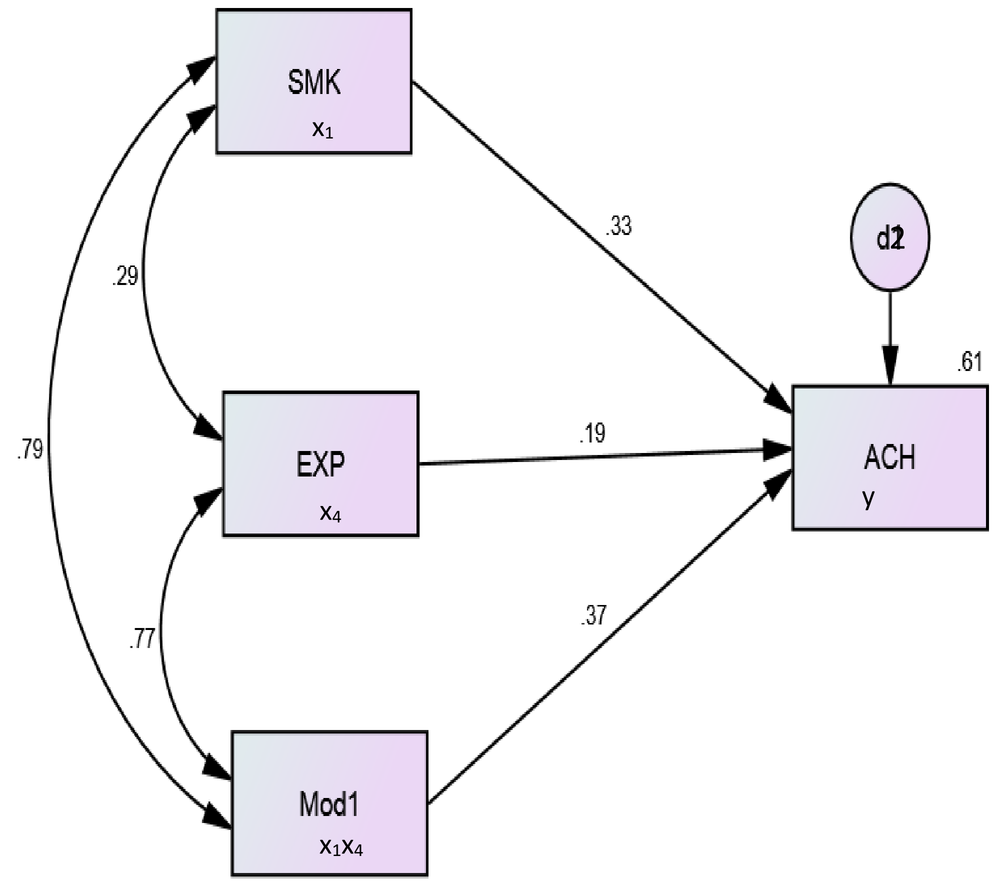

Figure 4: Moderating Role of Experience in the Relationship between Subject Matter Knowledge and Academic Performance in Elective Mathematics

Source: Field Survey

The model in figure 4 is represented as

$$y = 0.33x_1 + 0.19x_4 + 0.37x_1 x_4 + d_2 \qquad (18)$$

Figure 4 shows the moderating role of teaching experience (EXP) in the relationship between Subject Matter Knowledge (SMK) and Academic Performance (ACH) in elective mathematics. It can be seen that teaching experience moderates the relationship between subject matter knowledge and academic performance. Specifically, it can be seen that the interaction term

Mod1(X_1X_4) moderates the relationship since it predicts academic performance higher with standardised beta score of 0.37 (Equation 18).

Table 5 presents the results as shown in Figure 4 in a tabular form.

Table 5: Moderating Role of Experience in the Relationship between Subject Matter Knowledge and Academic Performance in Elective Mathematics

			B	S.E.	Beta	C.R.	P
ACH	<---	SMK	1.741	0.603	0.331	2.888	0.004
ACH	<---	EXP	0.969	0.583	0.186	1.663	0.096
ACH	<---	SMK_EXP	0.200	0.094	0.366	2.114	0.034

R^2- 0.609 Source: Field Survey

Table 5 shows clearly that teaching experience moderates the relationship between subject matter knowledge and academic performance in elective mathematics. The interaction term SMK_EXP predicts performance (0.366) significantly ($p<.05$). The p-value of 0.034 (significant) is less than the 0.05 significant level. Thus, it can be concluded that teaching experience was found to moderate the relationship between teachers' subject matter knowledge and students' performance in elective mathematics.

Again, it can be seen from table 5 that the model recorded an R^2 value of -0.609. This implies that the interaction term explained about 61% of the relationship between subject matter knowledge and academic performance. Therefore, teaching experience is found to significantly moderate the relationship between subject matter knowledge and academic performance of students in elective mathematics. In addition, since the correlation values are positive, it can be indicated that the effect of teachers' subject matter knowledge on students' elective mathematics performance was found to be high for teachers with experience.

Table 6 presents the model fit indices for the model in Figure 4.

Table 6: Model Fit Indices for the Second Model

Fit Indices	Cut-off	Model in Figure 4
Chi-square (χ2)	Non-significant	0.001, non-significant
Goodness-of-Fit (GFI)	<0.95	0.429
Adjusted Goodness-of-Fit (AGFI)	<0.95	0.257
Standardized Root Mean Residual (SRMR)	<0.08	0.048
Root Mean Square Error of Approximation (RMSEA)	<0.07	0.008
Comparative Fit Index (CFI)	>0.95	0.999
Tucker-Lewis Index (TLI)	>0.95	0.997

Source: Field Survey

In Table 6, the model fit indices are shown. It can be seen that model in figure 4 is fit for the study since the main absolute fit index, the χ2 (chi-square) was found to be non-significant. The χ2 being non-significant implies that the mode fits the data well. All the other indices, Goodness-of-Fit index (GFI), adjusted goodness-of-fit index (AGFI), standardized root mean square residual (SRMR), root- mean square error of approximation (RMSEA), Comparative Fit Index (CFI) and Tucker-Lewis Index (TLI) show that the model in Figure 4 is fitting for the study. Overall, Table 6 shows that the model is fitting for the study. The implication is that teaching experience significantly moderates the relationship between subject matter knowledge and the academic performance of students in elective mathematics.

The results in Figure 4 and Tables 5 and 6 show that teaching experience significantly moderates the relationship between subject matter knowledge and the academic performance of students in elective mathematics. Therefore, the null hypothesis that teaching experience does not significantly moderate the relationship between teacher subject matter knowledge and performance of students in elective mathematics is rejected. On the basis of this, it is concluded that teaching experience can moderate the relationship between subject matter knowledge and the academic performance of students in elective mathematics.

Third Model

The sixth hypothesis which aimed at establishing the extent to which teaching experience moderate the relationship between teacher pedagogical content knowledge and the performance of students in elective mathematics was tested in the third model.

H_0: Teaching experience does not significantly moderate the relationship between teacher pedagogical content knowledge and performance of students in elective mathematics.

Observed, endogenous variable

- Academic Performance in Elective Mathematics (ACH)

Observed, exogenous variables

- Pedagogical Content Knowledge (PCK)
- Teaching Experience (EXP)
- Moderator: Product of PCK & EXP (PCK_EXP)

Unobserved, exogenous variables

- Measurement error of the observed, endogenous variable (d3)

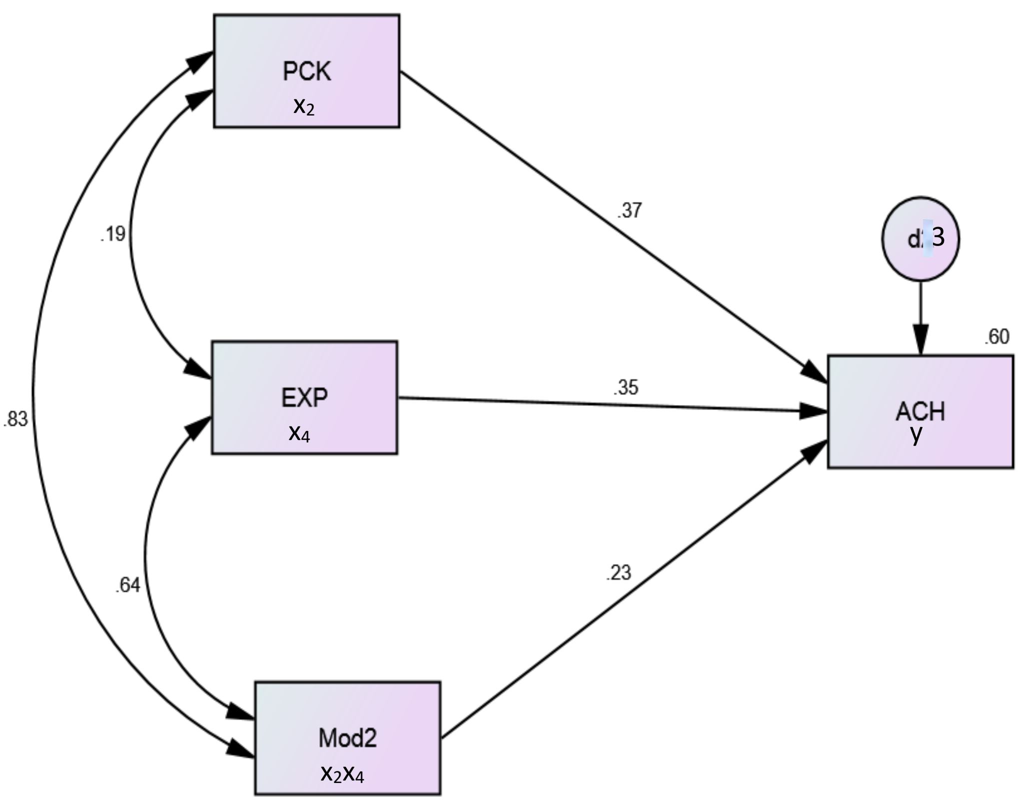

Figure 5: Moderating role of experience in the relationship between pedagogical content knowledge and academic performance in elective mathematics

Source: Field Survey

The model in figure 5 is represented as

$$y = 0.37x_2 + 0.35x_4 + 0.23x_2x_4 + d_3 \qquad (19)$$

Figure 5 shows the moderating role of teaching experience in the relationship between pedagogical content knowledge and academic performance in elective mathematics. It can be seen that considered individually, pedagogical content knowledge and teaching experience predict academic performance higher. Therefore, it can be inferred that the interaction term Mod2 (X_2X_4) with a standardised beta score of 0.23, plays an insignificant role in the relationship between pedagogical content knowledge and academic performance in elective mathematics (Equation 19).

Table 7 presents the results in Figure 5 in a tabular form.

Table 7: Moderating Role of Experience in the Relationship between Pedagogical Content Knowledge and Academic Performance in Elective Mathematics

			B	S.E.	Beta	C.R.	P
ACH	<---	PCK	1.826	0.599	0.372	3.045	0.002
ACH	<---	EXP	1.825	0.459	0.350	3.974	0.000
ACH	<---	PCK_EXP	0.129	0.089	0.226	1.458	0.145

R^2- 0.601 Source: Field Survey

Table 7 shows the moderating role of teaching experience in the relationship between pedagogical content knowledge and academic performance of students in elective mathematics. It can be seen that interaction term (PCK_EXP) is responsible for 60% variation in the model, however this variation is non-significant (p=0.145) which is greater than the 0.05 significant level set for the study. Thus, it can be concluded that teaching experience plays an insignificant moderating role in the relationship between pedagogical content knowledge and academic performance in elective mathematics.

The results for the test for the model fitness are shown in Table 8.

Table 8: Model Fit Indices for the Third Model

Fit Indices	Cut-off	Model in Figure 5
Chi-square ($\chi 2$)	Non-significant	0.996, non-significant
Goodness-of-Fit (GFI)	<0.95	0.453
Adjusted Goodness-of-Fit (AGFI)	<0.95	0.089
Standardized Root Mean Residual (SRMR)	<0.08	0.005
Root Mean Square Error of Approximation (RMSEA)	<0.07	0.067
Comparative Fit Index (CFI)	>0.95	0.998
Tucker-Lewis Index (TLI)	>0.95	0.997

Source: Field Survey

Table 8 shows that the main absolute fit index, the **$\chi 2$** (chi-square) which tests for the extent of misspecification is non-significant. A non-significnat chi-square implies that the model fits the data well. In other words, the null hypothesis that there is no significant difference between the model-implied and observed variances and covariances can be accepted. Therefore, the model shown in Figure 5 is acceptable for the study. This is also confirmed in the other indices shown in the table.

Overall, it can be inferred from the results in Figure 5 and Tables 7 and 8 that teaching experience plays an insignificant role in the relationship between pedagogical content knowledge and academic performance of students in elective mathematics. On this basis, the null hypothesis that teaching experience does not significantly moderate the relationship between teacher pedagogical content knowledge and performance of students in elective mathematics is accepted. Thus, it is concluded that teaching experience does not moderate the relationship between teacher pedagogical content knowledge and performance of students.

Discussion

Discussion of the results is done in this section according to the main objectives of the study.

Relationship between teacher prior education and training and performance of students

The results of the study revealed a statistically significant relationship between teacher prior education and training and students' performance in elective mathematics. Specifically, among the teacher variables, the study found that teacher prior education and training had the highest contribution to the prediction of students' performance in elective mathematics. This implies that teacher prior education and training is the highest predictor of students' performance in elective mathematics. In teaching elective mathematics, the training and education of teachers is important. When teachers are properly or highly trained, they are more likely to be of much help to students in their studies.

During training, teachers are prepared in the best ways possible to teach the various concepts in elective mathematics. Therefore, if teachers have had quality prior education and training they are more likely to be effective in their teaching of elective mathematics. This explains why teacher prior education and training was found to be significantly related to the academic performance of students in elective mathematics.

The findings of the study confirm the findings of Farooq andShahzadi (2006) who compared the effectiveness of teaching of professionally trained and untrained teachers on students' performance in mathematics and found that the students taught by trained teachers showed better results in mathematics. Their findings emphasised the importance of training in the academic performance of students. In a similar vein, Darling-Hammond, Holtzman, Gatlin, and Heilig (2005) examined whether certified teachers were generally more effective than those who have not met the testing and training requirements for certification. They found that certified teachers consistently produced stronger student achievement gains than uncertified teachers. Based on their findings, Darling-Hammond et al. (2005) concluded that teachers' effectiveness strongly related to the preparation the teachers had received for teaching.

In another study, Brown (2018) examined teacher qualification characteristics and how it was related to students' achievement in mathematics. Brown revealed that teachers' professional certification allowed them to help their students transfer their mathematical knowledge. Thus, training and prior education of teachers influenced how they taught their students and ultimately improved their performance.

The similarity between the findings of the study and the findings of the other studies implies that teacher training and education significantly influences students' academic performance regardless of the environment. Teacher education and training is therefore important in improving students' performance.

Relationship between teaching experience and performance of students

The study found again in the first model that teaching experience was a significant predictor of students' performance in elective mathematics. This finding could probably be attributed to the fact that the more a teacher teaches a particular subject, the better the teacher gets in teaching the subject. In most activities, the more you keep engaging in the said activity, the better you get at doing it. This speaks to confirm the popular saying that practice makes perfect. In this sense, the current study's finding that teaching experience significantly predicts performance in elective mathematics is understandable. Teaching experience improves the teaching methods and performance of teachers which ultimately improves the performance of the students they teach.

Findings similar to the findings of the current study have been made in other studies (e.g Chhinh & Tabata, 2003; Akpo, 2012; Gichuru & Ongus, 2016). For instance, Chhinh and Tabata (2003) studied the effects of selected teacher factors on the mathematics achievement of urban primary school pupils in the state of Cambodia and found that teachers' years of teaching experience had a statistically significant relationship with the achievement of the pupils. They argued that the variances in the performances of pupils from difffernt schools could be attributed to the variances in the teaching experiences of the teachers.

Gichuru and Ongus (2016) also revealed that teachers with more teaching experience impact pupils' performance than fresh graduates. Their study showed specifically that there was improved performance in mathematics in those schools where the teachers had more years of teaching experience. When teachers are experienced, they are likely to know their flaws and identify better and effective ways of teaching.

Akpo (2012) also examined the impact of teacher-related variables on students' junior secondary certificate (JSC) mathematics results in Namibia using questionnaire, multi-correlation and regression analysis and found that teaching experience is related to students' academic achievement in JSC mathematics. By implication, teaching experience can improve the academic performance of students in mathematics.

Inferring from all the findings of the current study and the other studies, teaching experience can be recognised as significantly related to the academic performance of students. Thus, it has been argued that schools having more teachers with above 10 years teaching experience achieved better results than schools having more teachers with 10 years and below teaching experience (Ewetan&Ewetan, 2015).

Relationship between teacher subject matter knowledge and performance of students

The study revealed that teacher subject matter knowledge was significantly related to the performance of students in elecrive mathermatics. Thus, it was established that among the teacher variables that predict students' academic performance is the knowledge of teachers about the subject matter. This finding is understandable because if the teacher does not have knowledge concerning the concepts, he cannot deliver effectively. This is of much importance in elective mathematics because without adequate knowledge of the concepts being taught, the students cannot get a fuller understanding of the concepts and ultimately their performance will be affected negatively. It is therefore necessary for elective mathematics teachers to be fully abreast with and knowledgeable about all the concepts that students need to learn.

Other previous studies (e.g Hill, Rowan & Loewenberg, 2005; Muzenda, 2013; Peerzada & Jabeen, 2014) have highlighted the importance of subject matter knowledge in teaching and learning. For instance, Muzenda (2013) analysed the effect of lecturers' competences on Students' academic performance among higher education and training students, and found that subject matter knowledge was among the elements that have significant positive influence on students' academic performance. In a similar vein, PeerzadaandJabeen (2014) revealed a significant relation between the student performance and teacher subject matter knowledge. The researchers suggested that the subject matter knowledge should be given prime consideration while hiring teachers. This was to ensure that the performance of students was influenced positively.

Further, Hill, Rowan and Loewenberg (2005) conducted a study to explore whether and how teachers' mathematical knowledge contributes to gains in students' mathematics achievement. They concluded that teachers' mathematical knowledge was significantly related to student performance in both first and third grades after controlling for key student- and teacher-level covariates. This finding also implied that subject matter knowledge of teachers can improve the performance of students.

Valanides (2000) found that insufficient subject matter knowledge among teachers led their students developing misconceptions, misunderstandings, and misinterpretations regarding the subject matter during instruction. However, teachers who have higher subject matter knowledge have more favourable attitudes toward their teaching than those who lack or have limited subject matter knowledge (Barlow & Cates, 2006; Quinn, 1997). These findings could indicate that subject matter knowledge affects not only teachers' teaching practice but also their pedagogical content knowledge (Ozden, 2008).

All the previous studies highlighted along with the finding in the current study make it clear that the amount of knowledge teachers have about the subject matter determines to a large extent the performance of their students.

Relationship between teacher pedagogical content knowledge and performance of students

The findings of the study showed that the pedagogical content knowledge of teachers was significantly related to the academic performance of students. Pedagogical content knowledge differs from subject matter knowledge in the sense that it has a significant role in characterizing and identifying teachers' knowledge regarding their students' difficulty with subject matter. It also covers the ability to connect mathematical ideas, use examples, provide explanations, and apply strategies when encountering mathematical concepts (Wilson, Floden, & Ferrini-Mundy, 2002). Researchers found that pedagogical content knowledge factors such as teachers' pedagogical preparation, their use of routines in the classroom, and their degree content coverage influence students' academic achievement growth (Cankoy, 2010; Rowan, Correnti, & Miller, 2002).

The current study's finding that pedagogical content knowledge is a predictor of academic performance supports the findings of other researchers.

RaheemandAmali (2013) found a positive correlation between teachers' pedagogical skills and students' performance in Social Studies and recommended that workshops, seminars and in-service trainings should be organised for teachers to equip and improve their knowledge of teaching. Ganyaupfu (2013) also investigated the differential effectiveness of teaching methods on students' academic performance and found that the pedagogical content knowledge and skills used by teachers affected the academic performance of students.

Darling-Hammond (2000) opined that teachers' pedagogical content knowledge is highly related to students' academic achievement. Researchers (Rizvi, 2004; Schmidt et al., 2009) suggested that teachers must develop and maintain sufficient and comprehensive pedagogical content knowledge for the betterment of their students' mathematics performance. Wilson, Floden and Ferrini-Mundy (2002) posited a positive correlation between pedagogical content knowledge and students' mathematics performance.

When teachers have sufficient pedagogical content knowledge, they are more likely to influence students' conceptual understanding in mathematical problems. This probably explains why pedagogical content knowledge was found to be influential in the academic performance of students in elective mathematics.

Overall, the first model confirmed that teacher variables such as teaching experience, prior education and training, subject matter knowledge and pedagogical content knowledge influenced academic performance of students in elective mathematics. This confirms Vygotsky's (1930) theory of Zone of Proximal Development which explains that students need some form of assistance to be able to improve their knowledge level. Teachers are more knowledgeable than the students and as such should assist the students to learn and understand concepts in their respective subjects. This implies that teachers could be responsible for the performance of their students in elective mathematics. The specific teacher variables that can help students improve their level of knowledge are educational qualification, experience, subject matter knowledge, and pedagogical content knowledge. All these variables work together to ensure better performance of students in elective mathematics.

Role of teaching experience in moderating the relationship between teacher subject matter knowledge and performance of students

The results of the study showed that teaching experience significantly moderated the relationship between subject matter knowledge and the performance of students in elective mathematics. This result implied that teachers with a lot of experience and are knowledgeable in their respective subjects can contribute to high performance among students. Therefore, teaching experience and subject matter knowledge work together to produce highly performing students. This finding could be due to the fact that teachers who have taught for a long period of time are more likely to have sufficient knowledge about the subject that they teach. Thus, there is a mutual relationship between the two teacher variables which ultimately leads to high academic performance.

In the teaching of elective mathematics, both subject matter knowledge and teaching experience come to play. An inexperienced teacher who is highly knowledgeable in his subject matter may not impact his students significantly compared to an experience teacher with sufficient knowledge in his subject matter.

The finding of the current study confirms the findings of Dial (2008) who examined whether years of teaching experience and teacher's level of knowledge have an effect on overall achievement of students on the communication arts and mathematics sections of the Missouri Assessment Program. Dial found that the years of experience as well as the interaction between years of experience and level of knowledge, had an effect on student achievement in both communication arts and mathematics. Similarly, the finding of the current study supports the finding of Ofem, IyamandBasses (2015) who investigated the influence of teacher demographic variables on secondary school students' academic achievement in Home Economics in Calabar educational zone of Cross River State. They found that experience of the teacher together with the knowledge of the teacher acquired through training influenced significantly students' academic achievement.

The findings of the studies of Dial (2008) and Ofem et al. (2015) together with the findings of the current study have confirmed that teaching experience can significantly moderate the relationship between subject matter knowledge and academic performance of students. Therefore, teachers who have high subject matter knowledge and more years of teaching experience are likely to have performing students in elective mathematics. Thus the two teacher variables together ensure better performance of students in elective mathematics.

Role of teaching experience in moderating the relationship between teacher pedagogical content knowledge and performance of students

Finally, the study found that teaching experience plays an insignificant role in the relationship between pedagogical content knowledge and academic performance of students in elective mathematics. The role of teaching experience as a moderator in the relationship between pedagogical content knowledge and academic performance is therefore not established. In other words, teaching experience plays no role in how pedagogical content knowledge influences academic performance. In more practical terms, it can be inferred from the finding that if a teacher has high pedagogical content knowledge, it does not matter the teaching experience, he can improve the performance of his students.

The finding that teaching experience does not moderate the relationship between pedagogical skills and academic performance, confirms a study by Kimani, Kara and Njagi (2013) who investigated the relationship between selected teachers' demographic characteristics and classroom instructional practices and students' academic achievement in selected secondary schools in Nyandarua County, Kenya. Kimani et al. (2013) found that teachers' teaching experience and skills together were not significantly related to academic achievement.

The finding of the current study however contradicts the findings of EwetanandEwetan (2015) who studied the influence of teachers' teaching experience on the academic performance of public secondary school students in mathematics and English language in Ado-Odo/Ota and Ifo Local Government Areas in Ogun State. They revealed that teachers' teaching experience and their skills significantly influenced students' academic performance in mathematics and English Language as measured by their performance in the senior secondary school examinations and as perceived by the respondents. Salami (2014) also revealed that the instructional strategy of teachers and their experience can ensure sustainable effective mathematics learning. The contradiction between the finding of the current study and the other studies could be due to the fact that, a teacher with enough pedagogical content knowledge may have crossed the threshold and thus, experience may not be necessary to ensure effective delivery to influence performance in elective mathematics.

Chapter Summary

This chapter presented the results for all the three models used in testing the six hypotheses. The results revealed that all the teacher variables (SMK, EXP, PCK, TPET) have significant impact on student academic performance in elective mathematics. The result also showed that teaching experience significantly moderated the relationship between subject matter knowledge and the performance of students in elective mathematics but played an insignificant moderating role in the relationship between pedagogical content knowledge and performance of students in elective mathematics.

CHAPTER FIVE

SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

Overveiw

This chapter presents the summary, conclusions and recommendations. The conclusions and recommendations are based on the findings of the study. It is the desire of the investigator that the findings, conclusions, and recommendations of this study will guide policy direction and inform practices for all stakeholders in this field.

Summary

The purpose of the study was to investigate the effect of teacher variables such as teachers' qualification, teaching experience, teachers' knowledge of subject matter, and teachers' pedagogical content knowledge, on student performance in elective mathematics in Senior High Schools in Ghana. Specifically, the study sought to examine:

1. relationship between teacher prior education and training, and performance of students in elective mathematics.
2. relationship between teaching experience and performance of students in elective mathematics.
3. relationship between teacher subject matter knowledge and performance of students in elective mathematics.
4. relationship between teacher pedagogical content knowledge and performance of students in elective mathematics.
5. the role of experience in the relationship between teacher subject matter knowledge and performance of students in elective mathematics.

6. the role of experience in the relationship between teacher pedagogical content knowledge and performance of students in elective mathematics.

The study employed the positivist philosophy in its enquiry. Positivism rich tradition comes from a long history. The positivist tradition highlights on significance of quantitative research and attempts to obtain an overview of the trends, patterns rather than individuals such as the relationship between individual and social class. The study employed the quantitative strategy with a deductive approach. The quantitative strategy is utilized because the investigator was interested in finding out relationships and patterns among variables of interest in the study. A descriptive survey was employed in order to accomplish the purpose of the study. The descriptive survey was used for this study because it was convenient to administer instrument to respondents individually to collect data to describe a phenomenon.

This particular study covered Senior High Schools in Ghana. Specifically, the study was targeted to SHS in five regions (Central, Western, Northern, Greater Accra, and Ashanti Region). The population for this study comprised of two different groups of people: SHS elective mathematics teachers and their corresponding elective mathematics students.There were 225 SHS elective mathematics teachers and 6,750 SHS elective mathematics students for the study. The multi-stage sampling technique was employed in this study. The instruments for this study included two questionnaires; one for teachers and the other for students. The data was processed using Structural Equation Modeling to test all the six hypotheses.

The results revealed a statistically significant relationship among teacher prior education and training, teaching experience, teacher subject matter knowledge, teacher pedagogical content knowledge, and students' performance in elective mathematics. The study found that teacher prior education and training, teaching experience, teacher subject matter knowledge, and teacher pedagogical content knowledge were significant predictors of students' performance in elective mathematics, explaining about 81% of the variances in students' performance in elective mathematics. However, teachers' prior education and training had the highest contribution to students' performance in elective mathematics. This was followed by pedagogical content knowledge, teacher experience and subject matter knowledge in that order. The model indices fitting the predictors (i.e., prior education and training, experience, subject matter knowledge, and pedagogical content knowledge) to the criterion was found acceptable, indicating that teacher variables play important role in the performance of elective mathematics.

Experience was found to moderate the relationship between teachers' subject matter knowledge and students' performance in elective mathematics. It was found that the effect of teachers' subject matter knowledge on students' elective mathematics performance was significant for teachers with experience. The results again revealed that experience plays insignificant moderating role in the relationship between pedagogical content knowledge and academic performance in elective mathematics.

Conclusions

From the study, it can be concluded that teacher variables such as teacher prior education and training, teaching experience, teacher subject matter knowledge, and teacher pedagogical content knowledge, all play significant role in the performance of students in elective mathematics. However, the prior education and training of teachers as well as pedagogical content knowledge should be paramount if teacher variables are to be considered. This speaks to the fact that if teachers are trained in education and in a specialised mathematics field, they help students to attain mastery of what needs to be learnt.

When prior education and training is coupled with pedagogical content knowledge, students perform very well in elective mathematics. It is not surprising that subject matter knowledge was the least predictor of students' performance in elective mathematics. This is because having the content knowledge of the subject as a teacher does not guarantee effective teaching and learning. Teaching of elective mathematics, I must say, goes beyond content knowledge of the teacher. It is found in this study that the teachers background in mathematics education/training and great pedagogical skills, are significant for students to perform very well in elective mathematics. It must be said, however, that a teacher with several years of teaching experience (in elective mathematics) would largely impact students' performance in elective mathematics if the teacher has enough content knowledge.

Recommendations

Based on the findings of the study, the following recommendations were made:

1. Stakeholders in education should formulate policies that will ensure the hiring of professional and qualified elective mathematics teachers

to facilitate effective teaching and learning in public senior high schools in Ghana that would result in higher academic performance.

2. All non-professional and unqualified elective mathematics teachers who are already engaged, should be encouraged to pursue post graduate studies (such as PGDE, master's in education etc) on a part-time or full-time basis. This would help to improve the quality of their teaching and consequently improve the performance of students in elective mathematics in Ghana.
3. All stakeholders; Ministry of Education, Ghana Education Service, education planners, teacher unions, etc should periodically provide windows of opportunities like learned workshops, seminars and in-service training for elective mathematics teachers to adopt appropriate pedagogical skills in teaching the subject.
4. When hiring teachers with excellent subject matter knowledge, consideration should be given to those with experience.
5. Enough motivation should be given to knowledgeable in-service elective mathematics teachers to remain in the classroom for longer periods, gaining the much needed experience to improve the performance of Elective Mathematics in the Senior High Schools.
6. Further research on this study could be expanded in the development of a mathematical model, to examine the role of teacher motivation in the relationship between an experienced knowledgeable mathematics teacher and the academic performance of students inelective mathematics.

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APPENDICES

APPENDIX A: TEACHER ASSESSMENT QUESTIONNAIRE

Instructions

This instrument is grouped into four parts: I, II, III and IV representing variables: Teacher Prior Education and Training, Teacher Experience, Subject Matter Knowledge of the Teacher and Pedagogical Content Knowledge of the Teacher respectively.

The instrument contains five (5) questions on Part I and ten (10) questions on Part II. Please check all that apply by circling the answers to Parts I and II.

There are however eight (8) multiple-choice and two (2) free-response questions on each of Parts III and IV. Each multi-choice question has only one right answer. Please circle the correct answer for the multiple-choice questions and write all your responses to the free-response questions.

PART I

TEACHER PRIOR EDUCATION AND TRAINING

1. What was your bachelor's degree in?
    - Mathematics
    - Mathematics Education
    - Other (please specify) ……………………………………………

      ………………………………………………………………………
2. What was your minor/second teaching subject in college/university?
    - Mathematics
    - Mathematics Education

      Other (please specify) ……………………………………………

      ………………………………………………………………………

3. If you have a master's degree, in what area was it?

   - Mathematics
   - Mathematics Education
   - Other (please specify): ……………………………………………

     …………………………………………………………………

     I do not have a master's degree

4. Which of the following types of courses have you taken? Check all that apply

   Mathematics Courses:

   - Calculus
   - Linear Algebra (e.g. vector spaces, matrices, dimensions, eigenvalues, eigenvectors)
   - Abstract Algebra (e.g. group theory, field theory, ring theory, structuring integers ideals)
   - Advanced Geometry and/or Topology
   - Real and/or Complex Analysis
   - Number Theory and/or Discrete Mathematics
   - Differential Equations and/or Multivariate Calculus

   Mathematics Education Courses:

   - Methods of Teaching Mathematics (planning mathematics, lessons, using curriculum materials and manipulation, organising and delivery Mathematics lessons etc.)
   - Curriculum studies in Mathematics
   - Introductory Course in Philosophy of Mathematics (Nature of Mathematics, types of understanding)

- Problem Solving
- Psychology of learning Mathematics (how students learn, common students' errors of misconceptions, cognitive process etc)
- Assessment in Mathematics education (developing and using test and other assessments)

5. Indicate the type of certification you have
   - B. Ed (Maths)
   - Diploma in Education
   - Postgraduate Diploma/Certificate in Education

   Other (please specify): ……………………………………………
   ………………………………………………………………………….

PART II

TEACHER EXPERIENCE

6. Which areas are you currently teaching?

- Urban area
- Rural area

Other (please specify):

……………………………………………………………………………………

……………………………………………………………………………………

7. Which category of students do you teach?

Check all that apply:

- Science
- Arts
- Home Economics
- Business

Other (please specify)

……………………………………………………………………………………

……………………………………………………………………………………

8. Indicate how many years, if any, you have taught each of the following Mathematics courses at the SHS level?

	Core Mathematics	**Elective Mathematics**
1 – 2 years		
3 – 6 years		
7 – 10 years		
More than 10 years		
Have not taught it		

9. Which school type do you teach currently?

- Single Sex Male
- Single Sex Female
- Co-educational

10. How many periods of actual teaching do you have in a typical school week?

- ❑ Less than 24
- ❑ 24
- ❑ 25
- ❑ 26
- ❑ 27
- ❑ 28
- ❑ More than 28

11. On the average, how often do you meet or talk individually with the students'

parents to discuss his/her progress in Maths?

A. Once a week

B. Once or twice a month

C. Two times a year

D. Never

12. What would you do if a student begins to fall behind in Mathematics?

Check all that apply:

- I wait to see if performance improves with maturation
- I spend more time working mathematics individually with that student
- I have other students work on mathematics with the student having difficulty
- I assign homework to help the student catch up

13. How would you describe the performance of elective mathematics students in your class?

A. Most are above average

B. Most are average

C. Most are below average

D. Mathematics level varies greatly

14. Which of the following approaches do you use when teaching Elective Mathematics?

Check all that apply:

- Teaching the whole class as a group
- Teaching in small groups
- Students working in pairs or groups to solve problems
- A student working alone on problems
- Using available local materials
- Relating everyday life situations as much as possible

15. Which area have you been teaching in the last five years?

Urban area

Rural area

Other (please specify):

……………………………………………………………..………………………………

……………………………………………………………………………………………..

PART III

SUBJECT MATER KNOWLEDGE OF TEACHER

16. At a storewide sale, shirts cost ¢80 each and a pair of trousers cost ¢120 each. If S is the number of shirts and P is the number of trousers bought, which of the following is a meaning for the expression 80S + 120P?

A. The number of shirts and trousers bought

B. The cost of 80 shirts and 120 trousers

C. The cost of P shirts and S trousers

D. The cost of S shirts and P trousers

UCSMP Advanced Algebra Test

17. If $f(x) = ax^3 + bx^2 + cx + d$, what is the slope of the line tangent to this curve at

$x = 2$?

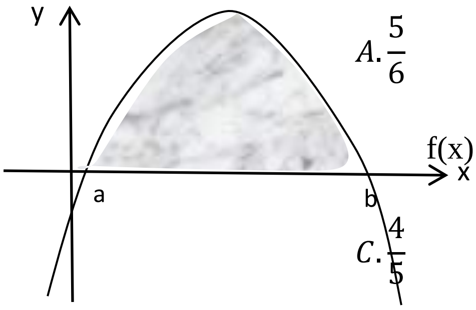
A. 8a + 4b + 2c

B. 8a + 4b + 2c + d

C. 12a + 4b + c

D. 12a + 4b + c + d

18. Find the area under the curve below in square units



$f(x) = -x^2 + 5x - 4$

$A. \frac{5}{6}$

$B. \frac{2}{3}$

$C. \frac{4}{5}$

$D. \frac{9}{2}$

19. How many real solutions does the following equation have?

$\sqrt{x-2} = \sqrt{1-x}$

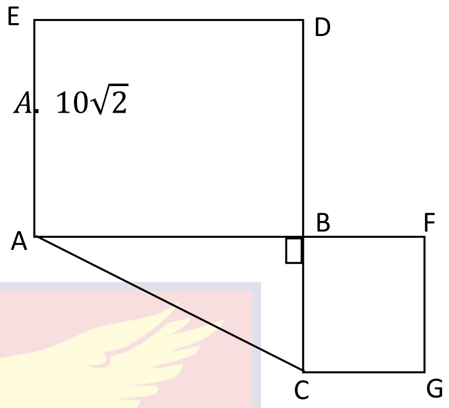
A. none

B. one

C. two

D. infinitely many

20. In the figure below, ABC is a right-angled triangle. ABDE is a square of area 200 square inches and BCGF is a square of area 100 square inches. What is the length, in inches of AC?



A. $10\sqrt{2}$

B. 300

C. $10\sqrt{3}$

D. 15

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21. Given the table to the right, determine f(g(3))

A. 4

B. -1

C. 1

D. 5

x	f(x)	g(x)
-2	0	5
-1	6	3
0	4	2
1	-1	1
2	3	-1
3	-2	0

22. A manufacturing company makes closed metal containers each with a capacity of $2000\pi \text{cm}^3$. If the radius of the cylinder is r cm, find the minimum area of the metal sheet required.

A. 1885.2cm^2

B. 942.5cm^2

C. 1000cm^2

D. 642.3cm^2

Mathematics for Senior High School (Akrong Series) (2016)

23. Mr. Ansah and Mr. Boateng work together. Mr. Ansah's salary is GH¢200.00 a year and he has an annual increment of GH¢20.00. Mr. Boateng is paid at first at the rate of GH¢80.00 a year and has an increment of GH¢8.00 every half-year. At the end of how many years will Mr. Boateng receive more salary than Mr. Ansah?

A. 6 years
B. 5 years
C. 6.5 years
D. 5.5years

Effective Elective Mathematics for Senior High School (MAG) (2009)

24. Solve the inequality: $x^2 + x - 20 > 0$ in two essentially different ways.

Method 1

Method 2

25. Let $A(x_1, y_1)$ be a point under a reflection in the line $y = mx + c$ and let A^1 (a, b) be the image of the point A. Show that:

(a) $mb + a = x_1 + my_1$

(b) $b - ma - 2c = mx_1 - y_1$

PEDAGOGICAL CONTENT KNOWLEDGE OF TEACHER

26. Which of the following (taken by itself) would give substantial help to a student

who wants to expand $(x + y + z)^2$?

i. See what happens in an example, such as $(3 + 4 + 5)^2$

ii. Use $(x + y + z)^2$ $[(x + y) + z\}^2$ and the expansion of $(a + b)^2$

iii. Use the geometric model shown below:

	X	y	Z
X	x^2	xy	xz
Y	Xy	y^2	yz
Z	xz	yz	z^2

A. iii only

B. i and ii only

C. ii and iii only

D. i, ii and iii

27. Which of the following can be represented by areas of rectangles?

i. The equivalence of fractions and percentages, e.g. $\frac{3}{5} = 60\%$

ii. The distributive property of multiplication over addition. For all real numbers **a, b** and **c** we have $\mathbf{a(b + c) = ab + ac}$

iii. The expansion of the square of a binomial $\mathbf{(a + b)^2 = a^2 + 2ab + b^2}$

A. ii only

B. i and ii only

C. ii and iii only

D. i, ii and iii

28. The statement '*For all whole numbers, if to the product of two consecutive whole numbers we add the larger number, the result is equal to the square of the larger number*' can be expressed symbolically as: for all whole number n.

A. $n^2 + 1 = n(n - 1) + n + 1$

B. $(n + 1)^2 = n^2 + 2n + 1$

C. $n^2 = n(n - 1) + n$

D. $(n + 1) n = n^2 + n$

29. Every *number to the power zero is one, says a student, therefore* $0^0 = 1$*'.*

Which of the following is true about this?

A. $0^0 = 0$ and not equal to 1

B. The theorem does not include zero

C. 0^0 is indeterminate because it has multiple possible answers and not 1, the agreed upon answer

D. Zero cannot be an exponent

30. Which of the following is a valid conclusion to the statement "*If a student is a high school band member, then the student is a good musician*"?

A. All good musicians are high school band members

B. A student is a high school band member

C. All students are good musicians

D. All high school band members are good musicians

31. In a first year elective mathematics class, which of the following is NOT an appropriate way to introduce the concept of slope of a line?

A. Toss a ball in the air and use a motion detector to graph its trajectory

B. Apply the formula slope $\frac{rise}{run}$ to several points in the plane

C. Discuss the meaning of m in the graphs of several equations of the form y = mx + b

D. Talk about the rate of change of a graph of a line on an interval.

32. Consider the statement below:

For all a, b □s, if ab = 0, then either a = 0 or b = 0

For which of the following sets is the above statement true?

i. The set of real number

ii. The set of complex numbers

iii. The set of 2 x 2 matrices with real number entries

A. i only

B. ii only

C. iii only

D. i and ii only

33. Which of the following situations can be modeled using an exponential function?

i. The height h of a ball t seconds after it is thrown into the air
ii. The amount of money A in a jar after w weeks, if each week $d are put in the jar
iii. The value v of a car after t years if it depreciates d% per year

A. i only
B. ii only
C. iii only
D. ii and iii only

34. Notice that $\frac{1}{2} = \frac{5}{10}$ and $\frac{1+2}{2} = \frac{3}{2}$ and $\frac{5+10}{10} = \frac{15}{10} = \frac{3}{2}$. Is it true in general that,

if $\frac{a}{b} = \frac{c}{d}$, then $\frac{a+b}{b} = \frac{c+d}{d}$ for all real numbers a, b, c, d, $\neq 0$?

Justify your answer

35. A student asked his teacher, "*Is the square root of 16 four and negative four*?"

The teacher answered, "*You are right*".

Explain whether you agree or disagree with the teacher

[Write your solution]

APPENDIX B: STUDENTS ASSESSMENT QUESTIONNAIRE

Instructions

This instrument is made up of two parts: I and II representing Students Background Questionnaire and Students Assessment Questions respectively. Feel free to answer Part I questions as they are.

Part II section of the instrument contains nine (9) multiple-choice and one (1) free-response questions about knowledge of elective mathematics. Each multiple choice-question has only one right answer. Please circle the correct answer for the multiple-choice questions, and write all your responses to the free-response question.

If you need more space to write, please use the back of one of the pages.

PART I

BACKGROUND QUESTIONNAIRE FOR STUDENTS

1. What is the highest level of education that your mother (or female guardian) has completed?

   ……………………………………………………………………….…..

   …………………………………………………………………………

2. What is the highest level of education that your father (or male guardian) has completed?

   …………………………………………………………………………

   …………………………………………………………………………

3. Do you have elective Mathematics books and learning resources from home?
   - Yes
   - No

   If yes, list them:

   …………………………………………………………………………

   ……………………………………………………………………………

4. Do your parents pay for extra tuition for you?
   - Yes
   - No

   If yes, how many hours per week do you have this?

…………………………………

5. How many times in a week does your teacher give you assignment/exercises?
   - Not at all
   - Once
   - Two times

   Other (please specify):

   ……………………………………………………………………………

   ……………………………………………………………………………

6. Does your teacher mark the assignments/exercises?
   - Yes
   - No

   If yes, does he/she give you the opportunity to discuss the mistakes if any? ……….…………………………………………………………..

…………………………………………………………………………………..

7. Does your teacher motivate or encourage you to like elective Mathematics?
   - Yes
   - No

   If yes, how does he/she do that?

   ……………………………………………………..……………………….

   …………………………………………………………………………..……

PART II

STUDENTS' ASSESSMENT QUESTIONS

1. At a storewide sale, shirts cost ¢80 each and a pair of trousers cost ¢120 each. If S is the number of shirts and P is the number of trousers bought, which of the following is a meaning for the expression 80S + 120P?

A. The number of shirts and trousers bought

B. The cost of 80 shirts and 120 trousers

C. The cost of P shirts and S trousers

D. The cost of S shirts and P trousers

UCSMP Advanced Algebra Test

2. Mr. Ansah and Mr. Boateng work together. Mr. Ansah's salary is GH¢200.00 a year and he has an annual increment of GH¢20.00. Mr. Boateng is paid at first at the rate of GH¢80.00 a year and has an increment of GH¢8.00 every half-year. At the end of how many years will Mr. Boateng receive more salary than Mr. Ansah?

   E. 6 years
   F. 5 years
   G. 6.5 years
   H. 5.5years

Effective Elective Mathematics for Senior High School (MAG) (2009)

3. If $f(x) = ax^3 + bx^2 + cx + d$, what is the slope of the line tangent to this curve at

   $x = 2$?

   E. $8a + 4b + 2c$

   F. $8a + 4b + 2c + d$

   G. $12a + 4b + c$

   H. $12a + 4b + c + d$

4. How many real solutions does the following equation have?

$\sqrt{x-2} = \sqrt{1-x}$

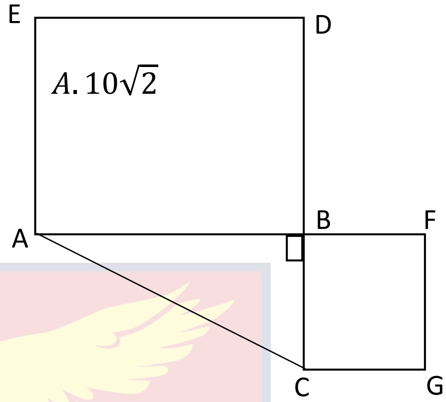
A. none

E. one

F. two

G. infinitely many

5. In the figure below, ABC is a right-angled triangle. ABDE is a square of area 200 square inches and BCGF is a square of area 100 square inches. What is the length, in inches of AC?



E. 300

F. $10\sqrt{3}$

G. 15

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6. Given the table to the right, determine f(g(3))

x	f(x)	g(x)
-2	0	5
-1	6	3
0	4	2
1	-1	1
2	3	-1
3	-2	0

E. 4

F. -1

G. 1

H. 5

7. A manufacturing company makes closed metal containers each with a capacity of $2000\pi \text{cm}^3$. If the radius of the cylinder is r cm, find the minimum area of the metal sheet required.

E. 1885.2cm^2

F. 942.5cm^2

G. 1000cm^2

H. 642.3cm^2

8. Which of the following situations can be modelled using an exponential function?
   i. The height h of a ball t seconds after it is thrown into the air
   ii. The amount of money A in a jar after w weeks, if each week $d are put in the jar
   iii. The value v of a car after t years if it depreciates d% per year

   A. i only

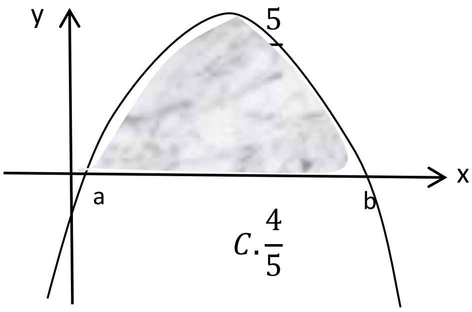
   B. ii only

   C. iii only

   D. ii and iii only

9. Find the area under the curve below in square units

$B. \frac{2}{3}$



$f(x) = -x^2 + 5x - 4$

$C. \frac{4}{5}$

$D. \frac{9}{2}$

10. Solve the inequality: $x^2 + x - 20 > 0$ in two essentially different ways.

Method 1

Method 2