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GALLIUM NITRIDE (GaN) SEMICONDUCTOR IN THE
PRESENCE OF HIGH-FREQUENCY ELECTRIC FIELD**

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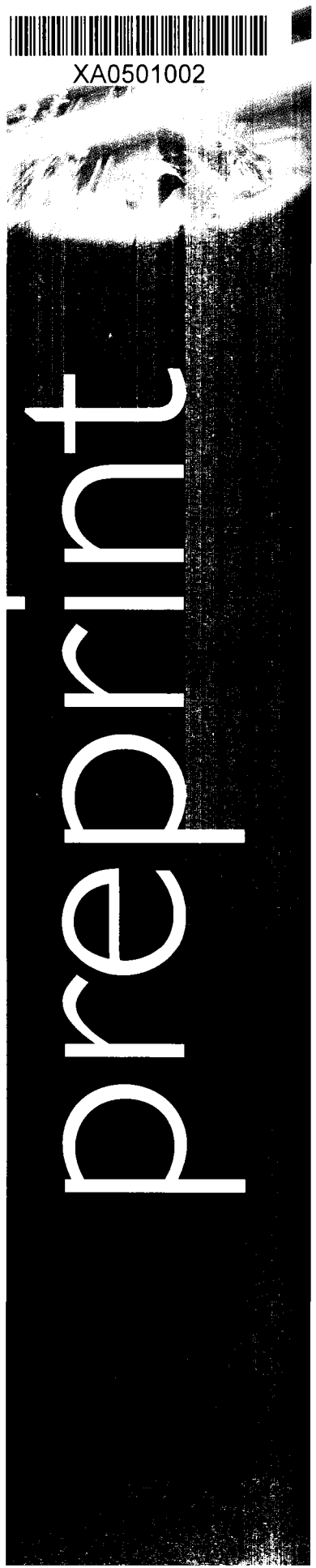
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**PROPAGATION OF ULTRASONIC WAVES IN BULK GALLIUM NITRIDE (GaN)
SEMICONDUCTOR IN THE PRESENCE OF HIGH-FREQUENCY ELECTRIC FIELD**

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Abstract

The propagation of ultrasound is studied in bulk GaN semiconductor in the presence of a strong, ac field oscillating at a frequency much higher than that of the ultrasound. Analytical expressions have been obtained for the attenuation coefficient (α) and the renormalized velocity (v) of the acoustic wave. It is shown that the dependencies of the ultrasonic absorption coefficient of the conduction electrons and the renormalized sound velocity on the field amplitude and the sound frequency have an oscillatory character which can be used to determine the effective mass and mobility of the material. The threshold field $E_{\min} = 3.3 \times 10^2 \text{ V/cm}$ needed to observe the oscillation is two orders smaller than that needed in the case of CdS.

1. INTRODUCTION

The study of the propagation behaviour of the high-frequency elastic waves in solids is well established effective means for examining certain physical properties of materials. Recent developments both in techniques of measurements and in understanding of the mechanisms of energy loss have brought forth results which clearly demonstrate the capabilities of ultrasonic methods in the study of fundamental physical properties of solid materials. It is very important in such studies to measure both attenuation and velocity of the propagating ultrasonic wave. Such measurements permit one to study the influence on the propagation behaviour of any such property of the solid that is sufficiently well coupled to the lattice; for example (1) Electron-phonon interaction (2) Thermo-elastic or heating effect (3) Magneto-elastic loss effect in ferromagnetic material (4) Phonon-phonon interaction (5) Acoustoelectric effect, etc.

In this paper, we shall study the acoustoelectric effect in a piezoelectric semiconductor. This effect arises from the interaction between the strong electric field associated with acoustic waves and electrons and holes drifting under the influence of an external field. This acoustic effect was first discussed by Parmenter [1], who predicted its occurrence in metals. It was later shown by Weinreich [2], however, that the effect should not be present in semiconductors with charge carriers of only one sign. It has, however, been established by Holstein[3] that it is possible but only for semiconductors with complicated band structures such as multi-valley band having well-defined maxima in the energy surface. In 1961, Hutson et al [4] reported the observation of amplification of acoustic signal under externally applied electric field when electric drift velocity exceeds that of sound velocity. Ever since there has been a lot of experimental and theoretical works on this subject [5 - 9]. More recent works are showing very interesting results [10 - 15]. We are, therefore, revisiting a paper written by Epshtein [16] on the propagation of ultrasound in semiconductors under the influence of a high frequency electric field, elaborating on the calculations and applying the results on bulk gallium Nitride (GaN). This work has been motivated by a paper written by Abdelraheen et al [17]. In their paper, emphasis was laid on why GaN should be studied and in our opinion it is worthwhile.

2. PROPAGATION EQUATIONS

Following the approach used in [16] we consider situations where the conditions $\Omega \gg \omega_c \equiv \omega_p^2 \tau$ and $\omega_p \tau \ll 1$ are fulfilled (Ω is the ac frequency, ω_p is the electronic plasma frequency and τ is the average electronic relaxation time). This means that the electric field penetrates the semiconductor both for $\Omega\tau \ll 1$ and for $\Omega\tau \gg 1$ [18]. We limit ourselves to the case in which the sound wavelength is much larger than the electronic mean free path, so

that Maxwell's electromagnetic field theory description of the electron-sound wave interaction is applicable.

We further consider the situation where longitudinal sound wave and the electric field are propagating along the main crystal axis. The complete system of equations describing the propagation of sound waves in the semiconductor has the form:

$$\frac{\partial^2 u}{\partial t^2} - v_0 \frac{\partial^2 u}{\partial x^2} = \frac{\beta}{\rho} \frac{\partial E}{\partial x}, \quad (1)$$

$$\varepsilon \frac{\partial E}{\partial x} - 4\pi\beta \frac{\partial^2 u}{\partial x^2} = 4\pi e(n - n_0), \quad (2)$$

$$\frac{\partial n}{\partial t} + \frac{\partial(nv)}{\partial x} = 0, \quad (3)$$

$$\frac{\partial v}{\partial t} + v \frac{\partial v}{\partial x} = \frac{e}{m} E - \frac{v}{\tau} - \frac{kT}{mn} \frac{\partial n}{\partial x}, \quad (4)$$

where u is the sound wave displacement, \mathbf{E} is the electric field, n is the electron concentration, n_0 is its equilibrium value, v is the average electron velocity caused by the electric field and the sound wave, v_0 is the non-renormalized sound velocity, ρ is the crystal density, β is the piezoelectric constant and ε is the dielectric constant of the lattice.

The total electric field \mathbf{E} consists of the external high-frequency electric field $\mathbf{E}(t) = E_0 e^{i\Omega t}$, and the electric field caused by the sound wave ($\partial \rightarrow +0$ is a parameter of the adiabatic switching-in of the external field at $t = -\infty$). In the absence of a sound wave we have

$$\left. \begin{aligned} n &= n_0 \\ v &= \bar{v}(t) \equiv \frac{eE_0}{m} \frac{e^{i\Omega t} \tau}{1 + \Omega^2 \tau^2} (\sin \Omega t - \Omega \tau \cos \Omega t). \end{aligned} \right\} \quad (5)$$

In the presence of a weak sound we seek solutions to the system eqns. (1) to (4) in the form:

$$\begin{aligned} E(x, t) &= \vec{E}(t) + e^{iqx} E_1(t), \\ v(x, t) &= \vec{v}(t) + e^{iqx} v_1(t), \\ n(x, t) &= n_0 + e^{iqx} n_1(t), \\ u(x, t) &= e^{iqx} u_1(t). \end{aligned} \quad (6)$$

Here we have linearized by retaining only terms with subscript 1. Eliminating E_1 and v_1 with the aid of eqns (2) and (3) we have:

$$\frac{\partial^2 u}{\partial t^2} + v_0^2 q^2 (1 + \chi) u = \frac{4\pi e \beta}{\epsilon \rho} e^{iqx} n_1(t) \quad (7)$$

Expressing x as

$$x = \tilde{x}(t) = \left[\int_{-\infty}^t \tilde{v}(t') dt' \right] \quad (8)$$

we have,

$$\frac{\partial^2 u}{\partial t^2} + v_0^2 q^2 (1 + \chi) u = \frac{4\pi e \beta}{\epsilon \rho} n_1(t) \exp \left[iq \int_{-\infty}^t \tilde{v}(t') dt' \right] \quad (9)$$

which can be expressed as

$$\frac{\partial^2 u}{\partial t^2} + v_0^2 q^2 (1 + \chi) u = \frac{4\pi e \beta}{\epsilon \rho} n_1(t) \exp \left[iq \int_{-\infty}^t \frac{e E_0}{m} \frac{e^{dt'} \tau}{1 + \Omega^2 \tau^2} (\sin \Omega t' - \Omega \tau \cos \Omega t') dt' \right] \quad (10)$$

and finally assumes the form

$$\frac{\partial^2 u}{\partial t^2} + v_0^2 q^2 (1 + \chi) u = \frac{4\pi e \beta}{\epsilon \rho} n_1(t) \exp \left[\frac{ie E_0 q}{m \Omega^2} \frac{\Omega \tau}{\sqrt{1 + \Omega^2 \tau^2}} \sin \left(\Omega t + \tan^{-1} \frac{1}{\Omega \tau} \right) \right] \quad (11)$$

Using the well- known expansion

$$\exp(iz \sin \varphi) = \sum_{p=-\infty}^{\infty} J_p(z) e^{ip\varphi}, \quad (12)$$

where $J_p(z)$ is the p -th order Bessel function, eqn (11) becomes

$$\frac{\partial^2 u}{\partial t^2} + v_0^2 q^2 (1 + \chi) u = \frac{4\pi e \beta}{\epsilon \rho} v \sum_{p=-\infty}^{\infty} J_p(a) \exp \left[ip \left(\Omega t + \arctg \frac{1}{\Omega \tau} \right) \right] \quad (13)$$

where $\chi = \frac{4\pi \beta^2}{\epsilon \rho}$ is the electromechanical coupling constant for the material under study.

Similarly, using eqns (3) and (4) and proceeding as above, neglecting second and higher order terms, we obtain

$$\frac{\partial^2 v}{\partial t^2} + \frac{1}{\tau} \frac{\partial v}{\partial t} + \frac{kT}{m} (q^2 + \kappa^2) v = \frac{4\pi e \beta}{\epsilon m} n_0 q^2 u \exp \left[iq \int_{-\infty}^{\infty} \frac{e E_0}{m} \frac{e^{dt'} \tau}{1 + \Omega^2 \tau^2} (\sin \Omega t' - \Omega \tau \cos \Omega t') dt' \right].$$

Finally, we express it in the form

$$\frac{\partial^2 \nu}{\partial t^2} + \frac{1}{\tau} \frac{\partial \nu}{\partial t} + \frac{kT}{m} (q^2 + \kappa^2) \nu = \frac{4\pi e \beta n_0 q^2}{\epsilon m} u \sum_{p=-\infty}^{\infty} J_p(a) \exp \left[ip \left(\Omega t + \arctg \frac{1}{\Omega \tau} + \pi \right) \right] \quad (14)$$

where,

$$\kappa = \left(\frac{4\pi^2 n_0}{\epsilon k T} \right)^{\frac{1}{2}} \quad (15)$$

is the screening radius and

$$a = \frac{e E_0 q}{m \Omega^2} \frac{\Omega \tau}{\sqrt{1 + \Omega^2 \tau^2}}. \quad (16)$$

is the argument of the Bessel function.

3. DISPERSION EQUATIONS

We now average eqns (13) and (14) over the period of the high-frequency field. Since we are considering only waves with frequencies much smaller than the field frequency, it is sufficient to replace ν and u by their averages, and to retain only terms with $p = 0$ on the right sides of the equations.

Eqns (13) and (14) then become

$$\frac{d^2 \bar{\nu}}{dt^2} + \frac{1}{\tau} \frac{d \bar{\nu}}{dt} + \frac{kT}{m} (q^2 + \kappa^2) \bar{\nu} = \frac{4\pi e \beta n_0 q^2}{\epsilon m} \bar{u} J_0(a) \quad (17)$$

$$\frac{d^2 \bar{u}}{dt^2} + s_0^2 q^2 (1 + \chi) \bar{u} = \frac{4\pi e \beta}{\epsilon \rho} \bar{\nu} J_0(a) \quad (18)$$

Assuming the average quantities to be proportional to $\exp(-i\omega t)$ for ($\omega \ll \Omega$) eqns (17) and (18) become

$$-\omega^2 \bar{\nu} - \frac{i\omega \bar{\nu}}{\tau} + \frac{kT}{m} (q^2 + \kappa^2) \bar{\nu} = \frac{4\pi e n_0 q^2}{\epsilon m} \bar{u} J_0(a) \quad (19)$$

$$-\omega^2 \bar{u} + \omega_0 (1 + \chi) \bar{u} = \frac{4\pi e \beta}{\epsilon \beta} \bar{\nu} J_0(a) \quad (20)$$

From eqns (19) and (20) we express $\bar{\nu}$ in terms of \bar{u} as

$$\bar{v} = -\frac{4\pi e \beta n_0 q^2 J_0(a)}{\omega^2 + i\frac{\omega}{\tau} - \frac{kT}{m}(q^2 + \kappa^2)} \quad (21)$$

Substituting eqn (21) into eqn (20) and making use of the fact that $\chi = \frac{4\pi\beta^2}{\epsilon\rho}$ we obtain

$$[\omega^2 - \omega_0^2(1 + \chi)] \left\{ \omega^2 + i\frac{\omega}{\tau} - \frac{kT}{m}(q^2 + \kappa^2) \right\} = \frac{4\pi e^2 n_0 \omega_0^2 \chi J_0^2(a)}{\epsilon m} \quad (22)$$

This further reduces to

$$[\omega^2 - \omega_0^2(1 + \chi)] \left\{ \omega^2 \tau + i\omega - \left(\frac{\omega_0^2 \kappa T}{m v_0^2} + \omega_p^2 \tau \right) \right\} = i\chi \omega_p^2 \tau \omega_0^2 J_0^2(a) \quad (23)$$

and after some manipulation becomes

$$[\omega^2 - \omega_0^2(1 + \chi)] \left\{ \omega + i \left(\omega_c + \frac{\omega_0^2}{\omega_D} \right) \right\} = -i\chi \omega_c \omega_0^2 J_0^2(a) \quad (24)$$

here $\omega_0 = v_0 q$, $\omega_c = \omega_p^2 \tau$ and $\omega_D = \frac{m v_0^2}{kT} \frac{1}{\tau}$.

Taking into consideration the fact that $\chi \ll 1$, we obtain for the sound velocity, v , and the coefficient of its absorption by electrons, α , from the expression

$$\omega = \omega_0 \left\{ 1 + \frac{\chi}{2} \left[1 - \frac{i\frac{\omega_c}{\omega_0} J_0^2(a) + J_0^2(a) \frac{\omega_c}{\omega_0} \left(\frac{\omega_c}{\omega_0} + \frac{\omega_0}{\omega_D} \right)}{1 + \left(\frac{\omega_c}{\omega_0} + \frac{\omega_0}{\omega_D} \right)^2} \right] \right\} \quad (25)$$

That is

$$v = \frac{\text{Re } \omega}{q} = v_0 \left\{ 1 + \frac{\chi}{2} \left[1 - J_0^2(a) \frac{\frac{\omega_c}{\omega_0} \left(\frac{\omega_c}{\omega_0} + \frac{\omega_0}{\omega_D} \right)}{1 + \left(\frac{\omega_c}{\omega_0} + \frac{\omega_0}{\omega_D} \right)^2} \right] \right\} \quad (26)$$

where $v_0 = \frac{\omega_0}{q}$.

Also, the attenuation coefficient is given by the expression

$$\alpha = \frac{2I_m \omega}{v_0} = \frac{\chi \frac{\omega_c}{v_0} J_0^2(a)}{1 + \left(\frac{\omega_c + \omega_0}{\omega_0 + \omega_D}\right)^2} = \chi \frac{\omega_c}{v_0} J_0^2(a) \left[1 + \frac{\omega_c^2}{\omega_0^2} \left(1 + \frac{\omega_0^2}{\omega_c \omega_D} \right)^2 \right]^{-1} \quad (27)$$

4. RESULTS AND DISCUSSION

In order to calculate the attenuation and velocity of the propagating ultrasonic wave in GaN, the electromechanical coupling coefficient χ has to be evaluated using the parameters of Ridley [19, 20], O'Clock and Duffy [21], and Shimada et al [22] who have tabulated the electric and elastic constants for GaN.

The effective piezoelectric constants for the wurtzite structure for LA and TA modes are [23]

$$e_L = (2e_{15} + e_{31}) \sin^2 \theta \cos \theta + e_{33} \cos^3 \theta, \quad (28)$$

$$e_T = (e_{33} - e_{15} - e_{31}) \cos^2 \theta \sin \theta + e_{15} \sin^3 \theta \quad (29)$$

where θ is the angle between the direction of propagation and the c-axis.

The electromechanical coupling coefficient is [19, 20]

$$\chi_{av}^2 = \frac{\langle e_L^2 \rangle}{\epsilon c_L} + \frac{\langle e_T^2 \rangle}{\epsilon c_T} \quad (30)$$

where c_L and c_T are the angular averages of elastic constants describing the propagation of LA and TA waves, respectively [23] and $\langle e_L^2 \rangle$, $\langle e_T^2 \rangle$ are the spherical averages of piezoelectric constants e_L and e_T respectively, which are

$$\langle e_L^2 \rangle = \frac{1}{7} e_{33}^2 + \frac{4}{35} e_{33} (e_{31} + 2e_{15}) + \frac{8}{105} (e_{31} + 2e_{15})^2 \quad (31)$$

$$\langle e_T^2 \rangle = \frac{2}{35} (e_{33} - e_{15} - e_{31})^2 + \frac{16}{105} e_{15} (e_{33} - e_{15} - e_{31}) + \frac{16}{35} e_{15}^2 \quad (32)$$

Table 1

Piezoelectric parameters for GaN

$\frac{\epsilon}{\epsilon_0}$	χ_{av}^2	$e_{15}(C/m^2)$	$e_{31}(C/m^2)$	$e_{33}(C/m^2)$	$c_L(10^{11} N/m^2)$	$c_T(10^{11} N/m^2)$
9.3	0.019	-0.49	-0.33	0.73	2.65	4.42

Using the value of χ in Table 1 we analysed our results, i.e. eqns (26) and (27). It is observed that under high frequency electric field, see figs. (1) and (2), the contributions made by electrons to the sound absorption coefficient and to the renormalization of the sound velocity are affected by a factor of $J_0^2(a)$ which made it oscillatory. The points of zero absorption associated with the roots of the Bessel function can be used to determine the mobility and effective mass of the electrons, fig. (2). For large values of the argument a , the Bessel function tends to an asymptotic value giving the expression for the absorption α to be periodic. Measurement of the oscillation amplitude can be used to separate the lattice and the electronic contributions to sound absorption [16]. The argument of the Bessel function

$$a = \frac{eE_0q}{m\Omega^2} \frac{\Omega\tau}{\sqrt{1+\Omega^2\tau^2}} \text{ can be rewritten as } a = \frac{v_D}{v_0} \frac{\omega_0}{\Omega} \frac{1}{\sqrt{1+\Omega^2\tau^2}}, \text{ where } v_D = \frac{e\tau E_0}{m} \text{ is the drift}$$

velocity. For $\Omega\tau \ll 1$ and $\omega_0 \ll \Omega$, a differs significantly from unity for $v_D \gg v_0$. To observe the oscillations in GaN, we used the recent data in [17] which predicted electron mobilities in the range of $\mu = 1500 \text{ cm}^2/Vs$ for $n > 5 \times 10^8 \text{ cm}^{-3}$ in bulk GaN at room temperature and calculated the threshold field E_{mn} of the high frequency field to be $3.3 \times 10^2 \text{ V/cm}$ which is two orders smaller than CdS as observed in [16].

Further analysis of the results shows that (see figs (3) and (4)), at low frequencies, i.e. $\omega_c \gg \omega_0$ the sound absorption tends to zero while the normalised velocity approaches unity especially when a is very small. As a increases we observe a shift from unity in the normalised velocity and at high frequencies, i.e. $\omega_c \ll \omega_0$, the normalised velocity becomes independent of a (see fig. 4) while the normalised absorption increases rapidly (fig. 3). Analysing fig. 5, we noted that the graph rises sharply assumes a maximum value and then drops off. To understand this phenomenon we did some analytical calculations which showed

that the maximum occurs whenever $\frac{\omega_c}{\omega_0} = \sqrt{1 + \left(\frac{\omega_0}{\omega_D}\right)^2}$. When $\omega_0 \ll \omega_D$ the maximum occurs

around unity and when $\omega_D \ll \omega_0$ then the maximum occurs at $\omega_0 = (\omega_c \omega_D)^{\frac{1}{2}}$. Conspicuous in

this expression is the diffusing frequency ω_D which we know is a major factor in the bunching of electrons. When ω_D is small, electrons bunch and facilitate the electron lattice interaction which enhances absorption. On the other hand, when ω_D is big the electrons smear out preventing bunching so absorption decreases. This explanation corroborates the observation that as ω_D decreases the sound absorption increases.

Finally, as can be seen from eqn (23), when the high frequency field is switched off the expression becomes equivalent to the absorption of sound in the absence of electric field.

In conclusion, the propagation of sound in a high-frequency electric field in bulk GaN semiconductor has been studied by analytical methods and the results analysed graphically. It has been shown that the contributions made by electrons to the sound absorption coefficient and to the renormalization of the sound velocity are affected by a factor of $J_0^2(a)$ and is oscillatory. The points of zero absorption associated with the roots of the Bessel function can be used to determine the mobility and effective mass of the electrons. The threshold field $E_{\min} = 3.3 \times 10^2 \text{ V/cm}$ needed to observe this oscillation in GaN is two orders smaller than that in CdS.

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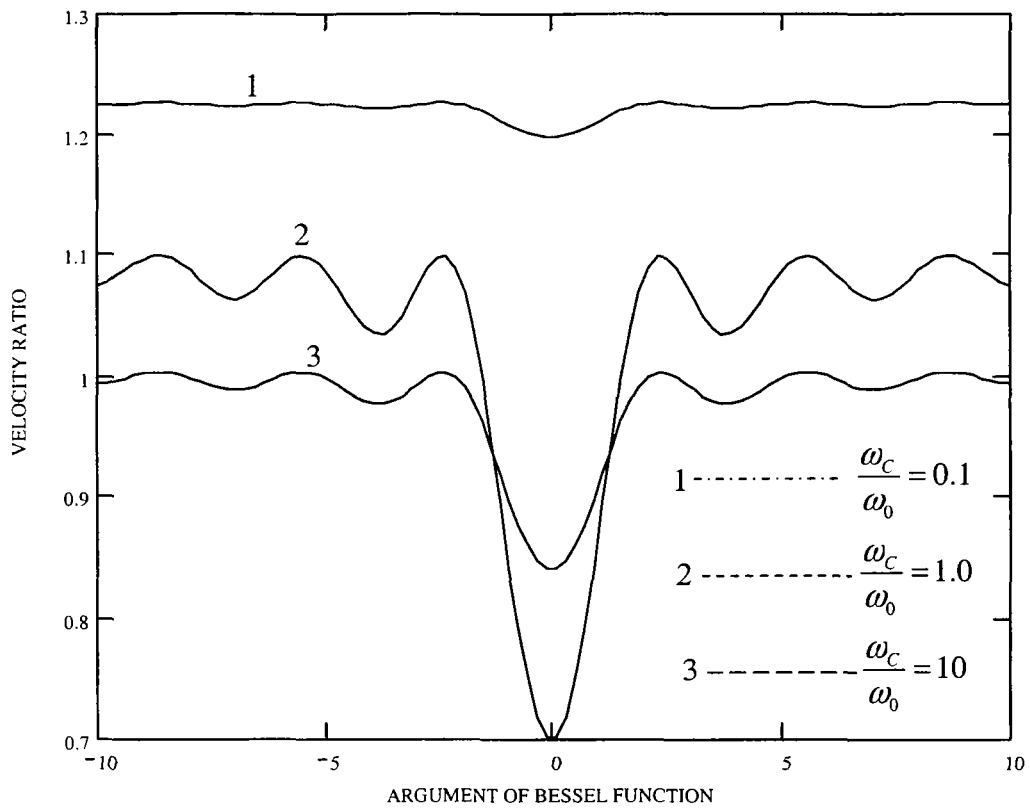


Fig 1: Velocity ratio $\frac{v}{v_0}$ with argument (a) of Bessel function for $\frac{\omega_0}{\omega_D} = 1.0$

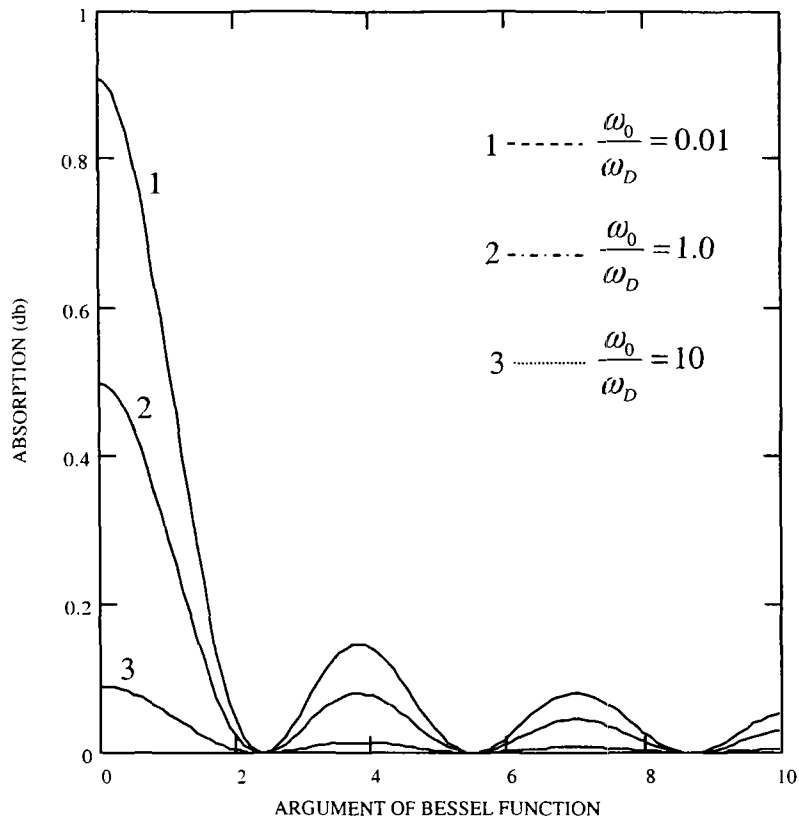


Fig 2: Variation of absorption (α) with argument (a) of the Bessel function for $\frac{\omega_c}{\omega_0} = 0.1$

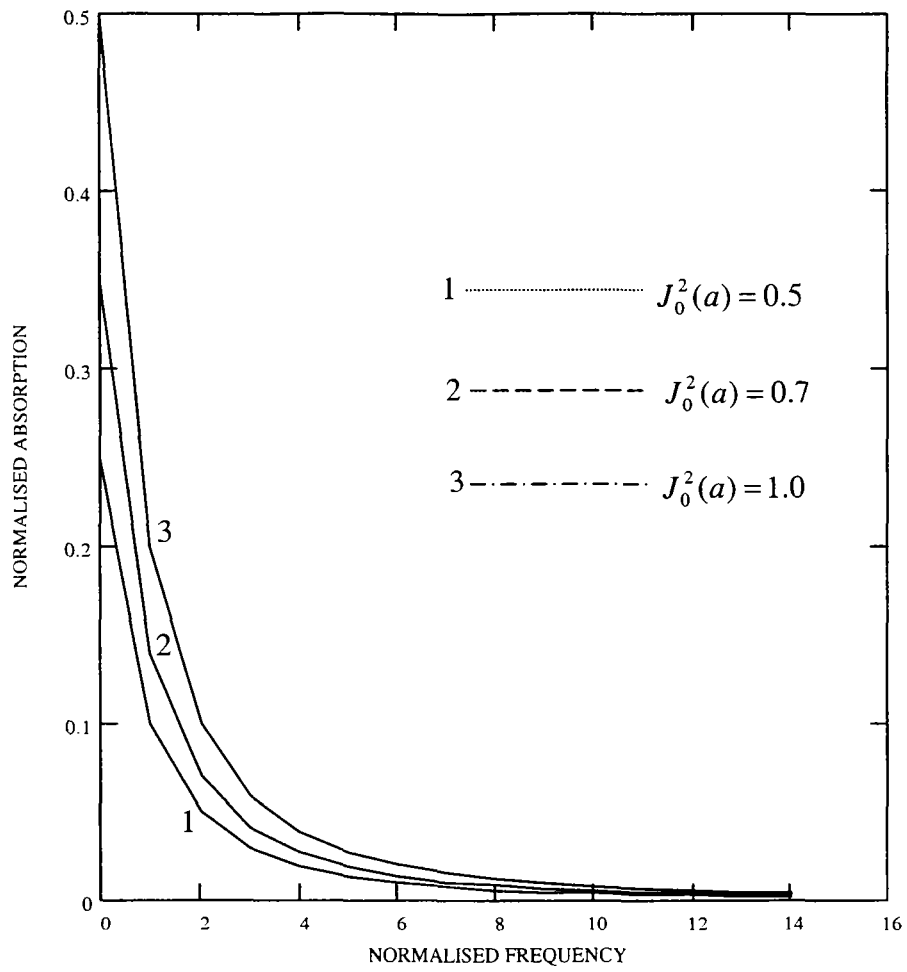


Fig 3: Normalised absorption $\frac{\alpha}{\alpha_0}$ versus normalised frequency $\frac{\omega_c}{\omega_0}$ for values of $J_0^2(a)$.

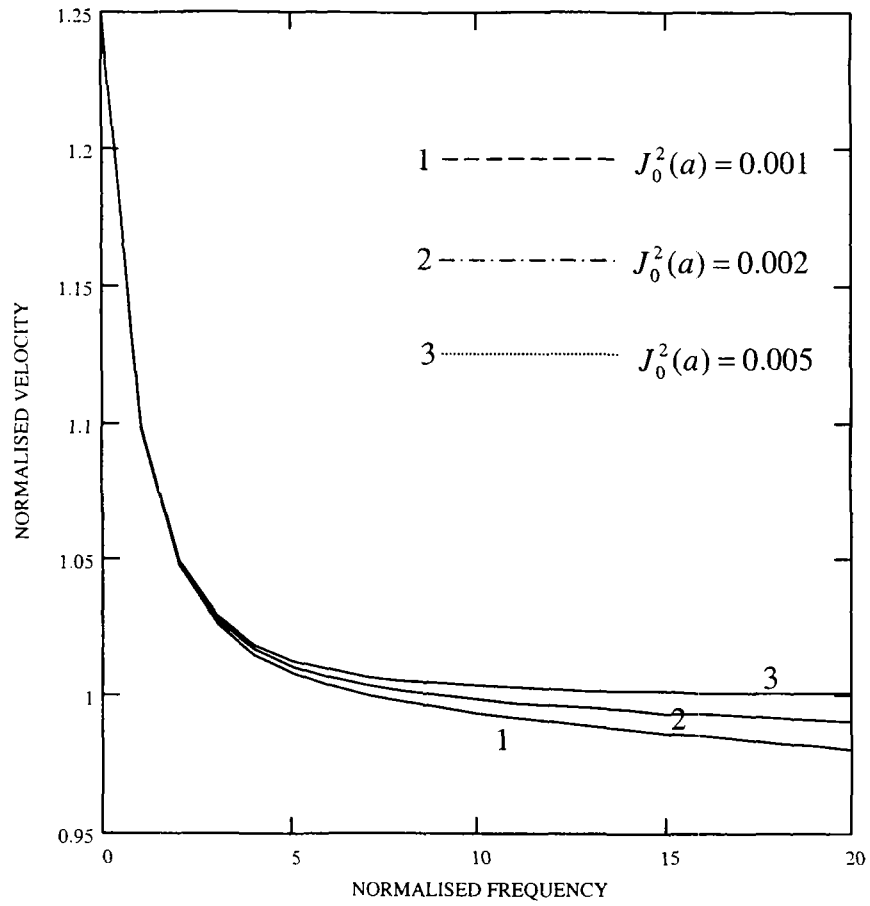


Fig 4: Normalised velocity $\frac{v}{v_0}$ versus normalised frequency $\frac{\omega_c}{\omega_0}$ for values of $J_0^2(a)$.

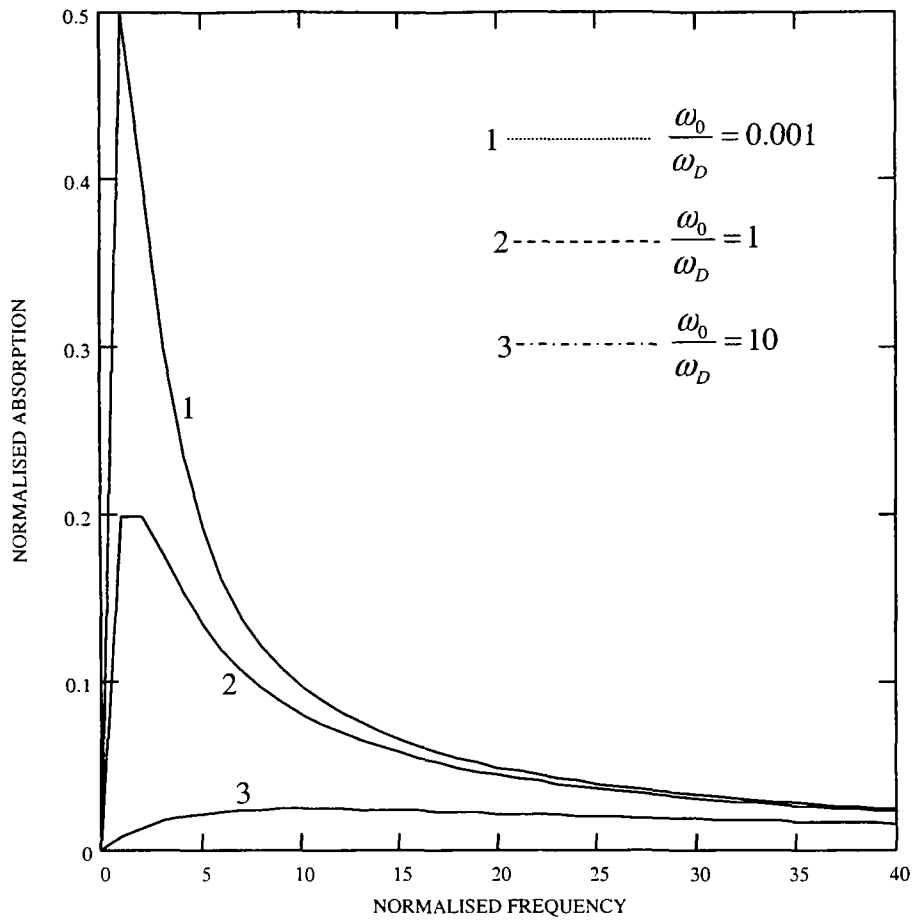


Fig 5: Normalised absorption $\frac{\alpha}{\alpha_0}$ versus normalised frequency $\frac{\omega_C}{\omega_0}$ for values of $\frac{\omega_0}{\omega_D}$.