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S.Y. Mensah
F.K.A. Allotey
N.G. Mensah
H. Akrobotu
and
G. Nkrumah
United Nations Educational Scientific and Cultural Organization
and
International Atomic Energy Agency
THE ABDUS SALAM INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

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S.Y. Mensah
Department of Physics, Laser and Fibre Optics Centre, University of Cape Coast,
Cape Coast, Ghana
and
The Abdus Salam International Centre for Theoretical Physics, Trieste, Italy,

F.K.A. Allotey
Institute of Mathematical Sciences, Accra, Ghana
and
The Abdus Salam International Centre for Theoretical Physics, Trieste, Italy,

N.G. Mensah
Department of Mathematics, University of Cape Coast,
Cape Coast, Ghana,

H. Akrobotu
Department of Physics, Laser and Fibre Optics Centre, University of Cape Coast,
Cape Coast, Ghana
and

G. Nkrumah
Department of Physics, University of Ghana, Legon, Accra, Ghana,

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Abstract

The dc acoustoelectric current $J^{ac}$ induced by the interaction of acoustic phonons with the conducting electrons is calculated. The calculation is done in the hypersound regime where $q\ell \gg 1$ (here $q$ is the acoustic phonon number and $\ell$ is the electron mean free path). The electric field consists of dc field $E_0$ and a low frequency alternating field $E_1$. A nonlinear dependence of $J^{ac}$ on $E_0$ and $E_1$ is observed. Depending on the direction of the dc field $E_0$ the acoustoelectric current can either be positive or negative. At $\omega_q = 10^{11}\text{sec}^{-1}$ and in the absence of $E_0$, $J^{ac}$ is very small and tends to zero at $E_1 = 1.3 \times 10^5 V_m - 1$. Further increase of $\omega_q$ from $10^{11}\text{sec}^{-1}$ sees a dramatic shoot "up" of $J^{ac}$. Lastly the acoustoelectric current in the presence of $E_0$ is modulated by $E_1$. 
1 Introduction

The acoustoelectric effect is the generation of dc electric current (the so-called acoustoelectric current) in a nonbiased device by a coherent acoustic wave or a flux of phonons. The study of acoustoelectric effect in bulk materials has received a lot of attention [1-5]. Recently there has been a growing interest in observing this effect in mesoscopic structures [6-8]. The interaction between surface acoustic waves (SAW) and mobile carriers in quantum wells is an important method to study the dynamic properties of two-dimensional (2D) systems. The SAW can trap carriers and induce acoustic charge transport as has been investigated in a number of systems in view of possible device applications [9]. Also, the SW-method was applied to study the quantum Hall effects [10-12], electron transport through a quantum point contact [13] and lateral nanostructures [14], the fractional quantum Hall effect [15], Fermi surfaces of composite Fermion around a half-filled Landan level [16] and commesurability effects caused by the lateral superlattice induced by a SAW [17]. It has also been noted that the transverse acoustoelectric voltage (TAV) is sensitive to the convex concentration in the semiconductor, thus it has been used to provide a characterization of electric properties of semiconductors [18]. Interface state density [19], junction depth [20] and carrier mobility [21] have been measured with this method.

In this paper we continue the work in [22, 23] where we studied the nonlinear acoustoelectric effect in a semiconductor superlattice. In that paper, the effect of constant electric field on the acoustoelectric current was considered and a nonlinear dependence was observed. Here, we study the presence of constant electric field and low frequency alternating field on the acoustoelectric current. The study of this effect is vital because of the complimentary role it may play in the understanding of the properties of the SL. A similar situation has been considered in a quantum constriction [8] where it was observed that transitions between the propagating and reflecting states can both decrease and increase transmission probability and thus lead both to negative or positive acoustic conductance.

In this work we also noted that the acoustic current is negative or positive depending on the direction of the constant electric field. The low frequency alternating field is modulating the acoustoelectric current.

This paper is organized as follows. In section 2 we outline the theory and conditions necessary to solve the problem. In section 3 we discuss the results and in section 4 we conclude.

2 Theory

Proceeding as in [22, 23], we shall consider the acoustic wave as a hypersound i.e., $q \ell \gg 1$. Under such circumstance the acoustic wave can be interpreted as monochromatic phonons having a $\delta$-function distribution

$$ N(k) = \frac{(2\pi)^3}{\hbar \omega_q} \delta(k - q) \quad \hbar = 1 $$

(1)
where \( k \) is the phonon wavevector, \( h \) is the Planck's constant divided by \( 2\pi \), \( \phi \) is the sound flux density and \( \omega_q \) and \( s \) are respectively the frequency and group velocity of sound wave, with the wavevector \( q \).

It is assumed that the sound wave and the applied electric field \( E(t) \) propagate along the \( z \) axis of the SL. The problem is solved in the quasi-classical case, i.e. \( 2\Delta > \tau^{-1}, eE_0d \ll 2\Delta, eEd \ll 2\Delta \) ([\( \tau \) is the relaxation time, \( d \) is the period of SL, \( 2\Delta \) is the width of the lowest energy miniband and \( e \) is the electron charge]). The density of the acoustoelectric current can then be written as [24]

\[
j_{ac} \propto \frac{2e}{(2\pi)^3} \int U_{ac}(p) d^3p
\]

Here \( \psi(p) \) is the solution of the Boltzmann kinetic equation in the absence of magnetic field; \( p \) is the electron momentum and

\[
U_{ac} = \frac{-2\pi\phi}{\omega_q s} |G_{p-q,p}|^2 [f(\epsilon_{p-q}) - f(\epsilon_p)] \delta(\epsilon_{p-q} - \epsilon_p + \omega_q)
+ |G_{p+q,p}|^2 [f(\epsilon_{p+q}) - f(\epsilon_p)] \delta(\epsilon_{p+q} - \epsilon_p - \omega_q)
\]

where \( G(p, q) \) is the matrix element of the electron-phonon interaction, \( f(\epsilon_p) \) is the distribution function and \( \epsilon(p) \) is the energy of electron. Introducing a new term \( p' = p - q \) and applying the principles of detailed balance, i.e.

\[
|G_{p'}|^2 = |G_{pp'}|
\]

we express eq.(2) as

\[
j_{ac} = -\frac{e\phi}{2\pi^2 s \omega_q} \int |G(p, q)|^2 [f(\epsilon_{p+q}) - f(\epsilon_p)] [\psi(p + q) - \psi(p)] \cdot \delta(\epsilon_{p+q} - \epsilon_p - \omega_q) d^3p
\]

where the vector \( \psi(p) \) is expressed in [25] as the mean free path \( \ell_i(p) \).

Thus the acoustocurrent in eq.(5) in the direction of the SL axis becomes

\[
j_{ac}^z = -\frac{e\phi}{2\pi^2 s \omega_q} \int |G(p, q)|^2 [f(\epsilon_{p+q}) - f(\epsilon_p)] [\ell_z(p + q) - \ell_z(p)] \cdot \delta(\epsilon_{p+q} - \epsilon_p - \omega_q) d^3p
\]

For \( qd \ll 1 \), \( G(p, q) \) is given as

\[
|G(p, q)|^2 = \frac{\Lambda^2 q^2}{2\sigma \omega_q}
\]

here \( \Lambda \) is the deformation potential constant, and \( \sigma \) is the density of the SL. As indicated in [22] in the \( \tau \) approximation and further, when \( \tau \) is taken to be constant, \( \ell_z \) is given as

\[
\ell_z = \tau v_z
\]

where

\[
v_z = \frac{\partial \epsilon}{\partial p_z}
\]

The most convincing argument in favour of this condition is in [26], where it is established experimentally that the relaxation \( \tau \) is a constant in \( GaAs/AlAs \) SL above 40K and is temperature independent.
Inserting eqs.(7) and (8) into eq.(6), we obtain the acoustoelectric current as:

\[
\mathbf{j}_z^{ac} = -\frac{e\phi|\Lambda|^2 q^2 \tau}{4\pi^2 s\omega_q^2} \int \left[ \frac{f(\varepsilon_{p+q}) - f(\varepsilon_p)}{\varepsilon_{p+q} - \varepsilon_p - \omega_q} \right] \left[ v_z(p+q) - v_z(p) \right] \cdot (\varepsilon_{p+q} - \varepsilon_p - \omega_q) d^3 p \tag{10}
\]

Applying this general expression for \( J_z^{ac} \) in SL we quote the result of [23] as:

\[
\mathbf{j}_z^{ac} = -\frac{e\phi|\Lambda|^2 q^2 \tau}{4\pi^2 s\omega_q} \int_0^\infty \frac{dt'}{\tau} \exp(-t'/\tau) \cdot \\
\left\{ \sinh \left( \frac{\omega_q}{2T} \cos(eEdt') \right) \sinh \left( \frac{\Delta}{T} \cos \left( \frac{qd}{2} \cos(eEdt') \right) \right) \sqrt{1 - b^2} \\
- \frac{\Delta}{T} \sqrt{1 - b^2} \sin(eEdt') \sin \left( \frac{qd}{2} \right) \cosh \left( \frac{\omega_q}{2T} \cos(eEdt') \right) \right\} \tag{11}
\]

3 Results and discussion

We considered an external electric field of the form \( E_0 + E_1 \sin \omega t \) where the ac field is very weak low frequency regime, i.e., \( \omega \tau \ll 1 \). Under such conditions we can replace the constant electric field \( E \) in eq.(11) with \( E_0 + E_1 \sin \omega t \) and average the result over time. As can be seen, this cannot be solved analytically hence we used numerical methods. For solutions when \( E_1 = 0 \) see [23]. We studied the behaviour of the acoustoelectric current when the amplitude of the slow varying ac field \( E_1 \) is kept constant. The behaviour is shown in Fig.1. It is noted that for \( \omega_q = 10^{11} \text{sec}^{-1}, \Delta = 1.6 \times 10^{-20} \text{J}, \cos(\frac{qd}{2}) = 0.8, \omega \tau = 10^{-1} \) and \( T = 300 \text{K} \). The acoustoelectric current rises, reaches a maximum, then falls off in a manner similar to that observed during a negative differential conductivity when \( E_0 \) is negative. On the other hand, when \( E_0 \) is positive the current decreases to a minimum and then rises. This can be attributed to the Bragg reflection at the band edge. At this frequency i.e., \( \omega_q = 10^{11} \text{sec}^{-1} \) the graph is symmetrical about the origin. As \( \omega_q \) increases the symmetry breaks down and at \( \omega_q = 10^{13} \text{sec}^{-1} \) we observed that the absolute value of the maximum peak \( |J_z^{ac}/J_0^{ac}|_{\text{max}} \) is greater than the absolute minimum value \( |J_z^{ac}/J_0^{ac}|_{\text{min}} \) (Fig.2). As can be seen in Fig.2a the ratio of \( |J_z^{ac}/J_0^{ac}|_{\text{max}}/|J_z^{ac}/J_0^{ac}|_{\text{min}} \) is about \( \sim 11 \) which is quite big. In the presence of ac field \( E_1 \) we observed that the peak values of \( J_z^{ac} \) decreases as \( E_1 \) increases. The \( E_1 \) field in this case is behaving as a modulator. A very interesting observation is noticed when \( \cos(\frac{qd}{2}) \) is negative i.e., \( |J_z^{ac}/J_0^{ac}|_{\text{min}} > |J_z^{ac}/J_0^{ac}|_{\text{max}} \) in other words there is an inversion. This occurs when \( \frac{3\pi}{d} > q > \frac{\pi}{d} \) (see Fig.2b).

We examined herein, the behaviour of the acoustoelectric current \( J_z^{ac} \) when the dc field \( E_0 \) is kept constant and the amplitude of the ac field \( E_1 \) is varied. In Fig.3a we observed for \( \omega_q = 10^{11} \text{sec}^{-1} \) and \( E_0 = 0 \), the acoustoelectric current \( J_z^{ac} \) is very small and tends to zero at \( \frac{eE_1}{\hbar} \). To get the numerical estimate of \( E_1 \) for a typical GaAs/AlGaAs SL, the following parameters are chosen: \( d = 100 \text{Å} \); \( \tau = 10^{-12} \text{sec} \). For these values we obtained \( E_1 \approx 1.3 \times 10^5 \text{V m}^{-1} \). We further observed that as \( E_0 \) is increased either in the direction of the
acoustic phonons or in the opposite direction $J^{ac}$ increases likewise. With increase in phonon frequency $\omega_q = 10^{13}\text{sec}^{-1}$, $J^{ac}$ increases rapidly even in the absence of $E_0$ (see Fig.3b).

It is worth noting that at about $E_0 = 2 \times 10^5 V_m^{-1}$ and above, $E_1$ ceases to modulate the acoustoelectric current.

Finally in Fig.4 we present the three dimensional graphs of $J_z^{ac}/J_0^{ac}$ against $E_0$ and $E_1$ to give a vivid picture of what is happening.

4 Conclusions

We have studied the acoustoelectric current in SL in the presence of a constant and low frequency alternating field and noted a strong nonlinear dependence of $J_z^{ac}$ on both $E_0$ and $E_1$. We observed that in the absence of dc field $E_0$, when $\omega_q = 10^{11}\text{sec}^{-1}$, $J_z^{ac}$ is very small and tends to zero at $E_1 = 1.3 \times 10^5 V_m^{-1}$. Further increase in $\omega_q$ to $10^{13}\text{sec}^{-1}$ shoots up the acoustoelectric current $J_z^{ac}$ dramatically. When $E_0$ is introduced $J_z^{ac}$ increases with the increase of $E_0$. Lastly we noted that $E_1$ modulates the acoustoelectric current in the presence of $E_0$ and at field value of $E_0 = 2 \times 10^5 V_m^{-1}$ this modulation ceases.

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References


Dependence of $J_z^{ac}/J_0^{ac}$ on $\frac{eE_0 d\tau}{h}$ for $\frac{eE_0 d\tau}{h} = 0$; $\frac{eE_1 d\tau}{h} = 0.5$; $\frac{eE_1 d\tau}{h} = 0.7$; $\frac{eE_1 d\tau}{h} = 1$.

For $\omega = 10^{11}$; $\Delta = 1.6 \times 10^{-20}$J; $\cos \left( \frac{qd}{2} \right) = 0.8$; $T = 300K$

Fig 1
Dependence of $J^\text{ac}_z / J^\text{ac}_0$ on $\frac{eE_0 d\tau}{h}$ for $\cos \left( \frac{qd}{2} \right) = 0.8$; $T = 300\,\text{K}$.

For $\omega_q = 10^{13}$; $\Delta = 1.6 \times 10^{-20}\,\text{J}$; $\cos \left( \frac{qd}{2} \right) = 0.8$; $T = 300\,\text{K}$.
Dependence of \( J_z^{ac} / J_0^{ac} \) on \( \frac{eE_1 d\tau}{\hbar} \) for \( \omega_q = 10^{13}; \Delta = 1.6 \times 10^{-20} J; \cos \left( \frac{q d}{\lambda} \right) = -0.8; T = 300K \)

For \( \omega_q = 10^{11}; \Delta = 1.6 \times 10^{-20} J; \cos \left( \frac{q d}{\lambda} \right) = -0.8; T = 300K \)
Dependence of $\frac{J_z^{ac}}{J_0^{ac}}$ on $\frac{eE_0 d\tau}{\hbar}$ and $\frac{eE d\tau}{\hbar}$ for $\cos(\phi/2) = 0.8$, $\omega_q = 10^{13} \text{s}^{-1}$, $\Lambda = 1.6 \times 10^{-20} J$, $T = 300K$.

Fig 4