UNIVERSITY OF CAPE COAST

# PRE-SERVICE MATHEMATICS TEACHERS' KNOWLEDGE FOR TEACHING SENIOR HIGH SCHOOL ALGEBRA

GIDEON ENTSIE

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BY

GIDEON ENTSIE

Thesis submitted to the Department of Mathematics and Information and Communication Technology Education of the Faculty of Science Technology Education, College of Education Studies, University of Cape Coast, in partial fulfilment of the requirements for the award of Master of Philosophy degree in Mathematics Education

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# DECLARATION

**Candidate's Declaration**

I hereby declare that this thesis is the results of my own original research and that no part of it has been presented for another degree in this university or elsewhere.

Candidate's Signature: ................................. Date: .................................

Name: Gideon Entsie (ET/MDP/18/0025)

**Supervisor's Declaration**

I hereby declare that the preparation and presentation of the thesis were supervised in accordance with the guidelines on supervision of thesis laid down by the University of Cape Coast.

Supervisor's Signature: ........................................ Date: ...........................

Name: Dr. Forster D. Ntow

## ABSTRACT

Algebra being the foundation of Mathematics, senior high school students' weakness in algebra exhibited in their summative assessment over the years has been a subject of concern to Ghanaians. To overcome students' weakness in algebra, it is important the factors that characterise preservice teachers' knowledge for teaching algebra and their knowledge in algebra are clearly understood since literature establishes the fact that students' performance in Mathematics is a function of teachers' knowledge. Therefore, using the Expanded Knowledge of Algebra for Teaching framework, this study explored the dominant factors that characterize preservice teachers' knowledge for teaching high school algebra and their knowledge in algebra. Using the cross-sectional survey, 164 preservice Mathematics teachers were sampled using the cluster sampling technique. A Principal Component Analysis of exploratory factor analysis coupled with Parallel Analysis revealed that preservice Mathematics teachers' knowledge for teaching is characterised by three factors: School Algebra knowledge, School Algebra Teaching knowledge and Pedagogical Content Knowledge in teaching algebra. Also, descriptive statistics showed preservice Mathematics teachers' knowledge of algebra for teaching is limited. It was recommended that training universities should mount courses that will strengthen preservice teachers' knowledge in algebra and preservice teachers are to be mentored when they are freshly absorbed into the Ghana Education Service.

## KEY WORDS

Advanced Algebra Knowledge

Knowledge of Algebra for Teaching (KAT)

Off-Campus Teaching Practice

Pre-service teachers

School Algebra Knowledge

Teaching Knowledge

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# DEDICATION

To Mr. and Mrs. Yeboah

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# CHAPTER ONE

# INTRODUCTION

There is a worldwide consensus on the fact that students' academic achievement is a true representation of their teachers' expertise in the subject. In view of this, many attempts have been made to conceptualise teachers' knowledge for teaching. Earlier conceptualisations of teachers' knowledge for teaching have presented teachers' knowledge as domain neutral construct. Recent conceptualisations of teachers' knowledge for teaching stemmed out from the argument that the knowledge needed to teach different subject matters from the same discipline of studies would be different from one another and hence the need for reconceptualisation of teachers' knowledge as a domain specific construct.

The work of the Knowledge of Algebra for Teaching (KAT) project team brought to light a framework for conceptualising teachers' knowledge in the domain of Algebra. It can be hypothesized that their decision for focusing on Algebra is its versatility in Mathematics and other Mathematics related disciplines of study. Algebra permeates throughout Mathematics and, thus, proficiency in Algebra is a determinant to success in Mathematics.

This study seeks to further investigate the KAT framework in Ghana and explore other factors, if any, that characterize the knowledge of prospective teachers.

## Background to the study

Following the report of Coleman (1968) on *the concept of equality of educational opportunity*, which asserted that family environment is the key factor that affects students' achievement from grade one to grade twelve, many attempts have been made to research into the factors that affect students' achievement because they found the findings of Coleman (1968) to be alarming. Among the numerous researches conducted to find out the factors that have influence on students' academic performance, it was discovered that pupils' socioeconomic background, teachers' certification, teachers' method of instruction, the effective usage of educational materials in teaching, teachers' knowledge of the subject matter and students' level of motivation also affect students' academic performance (Enu, Agyman, & Nkum, 2015; Farooq, Chaudhry, Shafiq, & Berhanu, 2011; Hill, Rowan, & Ball, 2005; Isack, 2015; Mji & Makgato, 2006). This suggests that family background of a student can have influence on students' academic achievement but it cannot be the sole factor that affect learners' achievement. This factor coupled with other factors which are connected to the teacher and the school determine the pupils' academic success.

In the early eighties, numerous researches focused on finding the relationship between the teachers' subject matter knowledge and student achievement. Such studies have shown inconsistent relationships between teachers' knowledge of the subject matter and students' achievement (Andrews, Blackmon, & Mackey, 1980; Begle, 1979; Druva & Anderson, 1983; Haney, Madaus, & Kreitzer, 1987). Reviewing the National Longitudinal Study of Mathematical ability, Begle (1979) discovered a strong link between the number of credits a teacher had in Mathematics method courses and students' achievement. Contrary to Begle's (1979) finding from his review, Haney, Madaus, and Kreitzer (1987) found insignificant association between teachers' verbal test score and students' verbal achievement.

These discrepancies in results concerning the relationship between teachers' knowledge and students' academic performance can be associated with the proxy approaches in measuring teachers' knowledge. In Begle's (1979) analysis, students' achievement was correlated with the number of credits a teacher obtained in Mathematics method courses instead of the content knowledge teachers need to possess and the manner to present them in order to facilitate understanding in students.

These proxy ways of measuring teachers' knowledge can be as a result of the different ways of conceptualising teacher knowledge. Thompson (1984), in his case study of three junior high school Mathematics teachers, asserted that the teacher's view, belief and preference for Mathematics and the teaching of Mathematics play a vital role in shaping the teacher's instruction in Mathematics lessons. This implies that the mathematical knowledge teachers bring to bear in their teaching could be influenced by their beliefs, views and preference for Mathematics.

Leinhardt and Smith (1985) conceptualised teachers' knowledge under two strands of knowledge: Subject Matter Knowledge (SMK) and Lesson Structure Knowledge (LSK). SMK includes the relational understanding of the content that the teacher teaches, the algorithmic operations involved, and the connections of such operations with other algorithmic procedures, as well as the understanding of students' errors and curriculum presentations (Leinhardt & Smith, 1985). Lesson Structure Knowledge (LSK) comprises teachers' ingenuity to plan and deliver lesson smoothly and explain concepts clearly to students.

In later conceptualisation of teachers' knowledge, Shulman (1987) conceptualized teachers' knowledge under seven themes: "content knowledge; general pedagogical knowledge . . .; curriculum knowledge . . .; pedagogical content knowledge [PCK] . . .; knowledge of learners and their characteristics; knowledge of education contexts . . . and knowledge of educational ends, purposes and values . . . ." ( p. 8). The most important component of Shulman's (1986) conceptualisation which got the attention of most educationists and researchers is the PCK. In his submission, Shulman (1986) asserted that, PCK distinguishes a trained teacher from one who is an expert in the subject matter and another who is expert in pedagogy. In his explanation to PCK as a knowledge type as far as teaching is concern, Shulman (1986) argued that

> The basic to distinguishing the knowledge base of teaching lies in the middle of content and pedagogy, in the capability of a teacher to transmute the content knowledge he or she possesses into forms that are pedagogically powerful and hitherto adaptive to the discrepancies in capability and background presented by the students. (p.15)

Thus, PCK is a composition of content knowledge and pedagogical knowledge which depends on the teachers' ingenuity to blend these two knowledge types into another form which is efficient with reference to enhancing students' understanding.

Ma (1999) (cited in Howe (1999)) propounded Profound Understanding of Fundamental Mathematics (PUFM). The PUFM, as discussed, involves proficiency in the subject matter of Mathematics and the knowledge of communicating them to students. Ma's (1999) Conceptualisation of PUFM can be likened to Shulman's (1986) Conceptualisation of PCK since in both cases, it is extremely important to consider the numerous sorts of representations, explanation of concepts of the subject and illustration.

Over the years, Shulman's (1986) concept of Pedagogical Content Knowledge (PCK) has found itself in the curriculum of teacher training institutions and has also formed the conceptual framework of many educational researches. One major critique against Shulman's (1986) PCK is that, PCK has been presented as a domain neutral construct. It can be argued that even within the sciences, the knowledge base needed to teach Mathematics is different from the knowledge base needed to teach Physics. Similar argument can be made about the different knowledge bases needed to teach different domains [example Algebra, Geometry and Calculus] in Mathematics. I will support this argument with Byrne's (1983) assertion which states that

> It is surely plausible to suggest that insofar as a teacher's knowledge provides basis for his or her effectiveness, the most relevant knowledge will be that which concerns the particular topic being taught and the relevant pedagogical strategies for teaching it to the particular types of pupil to whom it will be taught. If the teacher is to teach fractions, then it is knowledge of fractions and perhaps of the closely associated topics which is of major importance…. Similarly,

knowledge of teaching strategies relevant to teaching fractions will be important. (p. 14)

Within the first decade of the twenty-first century, two major projects in Mathematics have brought to light the Conceptualisation of teachers' knowledge as domain specific constructs. In their conceptualisation, Hill, Ball, and Schilling (2008) unpacked three knowledge types each under Subject Matter Knowledge and Pedagogical Content Knowledge. Under Subject Matter Knowledge, they propounded Common Content Knowledge (CCK), Specialized Content Knowledge (SCK) and Knowledge at the Mathematical Horizon. CCK is the type of knowledge that can be likened to the subject matter knowledge in Shulman's (1986) conceptualisation. In their explanation, CCK is the knowledge of Mathematics that is common to everyone irrespective of their profession. SCK which is the key component in Ball's conceptualisation includes the ability to accurately present mathematical ideas, explain rules and procedures and examine unusual solution methods to problem (Ball, Hill, & Bass, 2005). In the other strand of knowledge, Hill et al. (2008) propounded Knowledge of Content and Student (KCS), Knowledge of Content and Teaching (KCT) and knowledge of curriculum to be the component of Pedagogical Content Knowledge suggested by Shulman (1986).

Another conceptualisation of teachers' knowledge which has been domain specific is that which resulted from the KAT studies. From a comprehensive review of literature on algebra teaching and teacher knowledge, as well as an analysis of data acquired from interviews and video recordings of teaching, McCrory, Floden, Ferrini-Mundy, Reckase, and Senk (2012) conceptualised teachers' knowledge for teaching under three knowledge strands: Knowledge of School Algebra (SK), Advanced Knowledge of Mathematics (AK) and Teaching Knowledge (TK). In their explanation to each of these knowledge types, School Algebra knowledge is the algebraic content teachers teach. In context, school knowledge is the algebraic content stipulated in the high school Mathematics curriculum. Advanced Knowledge is mathematical content often found at the college level that provides teachers with insights into algebra knowledge beyond what school algebra can provide. Teaching knowledge, basically, does not entail the pedagogy in teaching algebra. McCrory et al. (2012) characterised TK as the mathematical knowledge "that is intuitively for teaching" (p. 598).

Earlier attempt to validate the KAT framework of knowledge for teaching in Ghana did not fully corroborate the framework. In the study, Wilmot (2016) used two hundred and nine teachers comprising one hundred and eighty nine prospective Mathematics teachers and twenty in-service Mathematics teachers. Graph from the scree plot for the factor analysis revealed seven factors that can be retained instead of the eight factors with eigenvalues greater than one that could have been retained by the use of the Kaiser criterion for retaining factors. Further examination of the nature of item loadings on each of the extracted seven factors reveal that only two of the extracted factors can be labelled as factors. These factors are the Teaching and Advanced knowledge types. The results of this study imply that the Ghanaian Mathematics teachers' knowledge for teaching algebra is characterised by the Algebra Teaching knowledge and Advanced Knowledge of Algebra and not School Knowledge of Algebra which is knowledge of algebraic content as stipulated in the high school Mathematics curriculum. Two of the five unlabelled components exhibited cross item loading with two items from each of the KAT framework's three forms of knowledge. Wilmot (2016) proposed that the three knowledge kinds from the KAT framework can interlock to create four different types of knowledge at the interlocking locations based on this discovery. Figure 1 shows the proposed expanded KAT framework by Wilmot (2016).

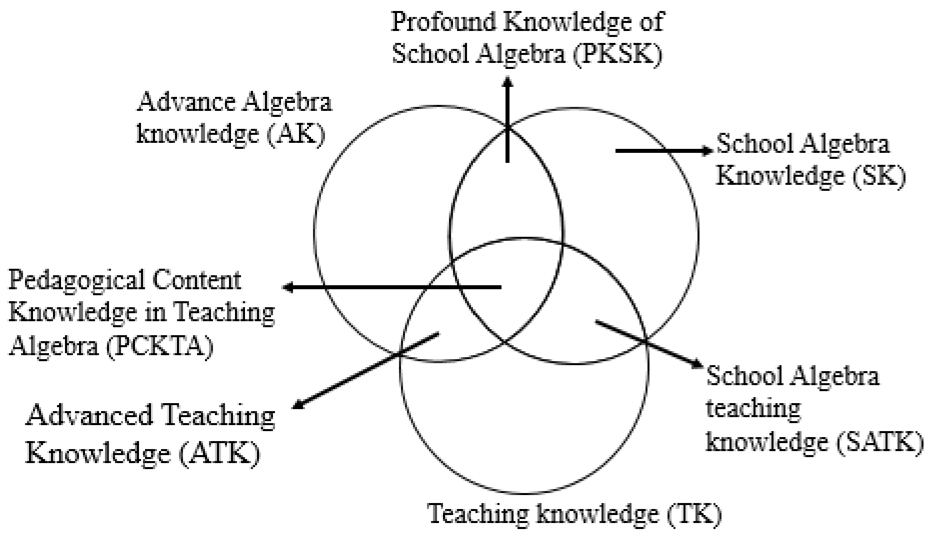



**Figure 1: The expanded KAT Framework (Wilmot, 2016; Wilmot et al., 2018)**

An attempt to validate the proposed expanded framework, Wilmot, Yarkwa, and Abreh (2018) used two hundred and fifty-two senior high school Mathematics teachers. Again, following the recommendation of Wilmot (2016) concerning the instrument, Wilmot et al. (2018) formulated multiple choice questions out of the open ended questions in the first adapted instrument. The number of items on the new adapted instrument were increased to ensure that items covered wide range of high school algebra content (SK), university Mathematics content (AK) and knowledge of teaching (TK). At the end, the new adapted version of the KAT instrument had eighty multiple choice items unlike the first adapted instrument which had twenty items comprising seventeen multiple choice items and three open ended items. Results from Wilmot et al's (2018) work fully confirmed the Expanded KAT framework proposed by Wilmot (2016).

Although a remarkable recommendation was made by Wilmot (2016) regarding the number of items on the adapted KAT instrument and such recommendation led to the attainment of the proposed expanded KAT framework (see Wilmot, Yarkwa & Abreh, 2018) , the results of his initial study and with the greatest number of the participant being pre-service Mathematics teachers leave an unanswered question in mind. Why is it that the knowledge of Ghanaian mathematics teachers for teaching algebra is not characterised by high school algebraic content? Is it possible that the tertiary Mathematics teacher preparation programme does not provide them with the necessary expertise to control high school algebra? It is for this reason that following the recommendation of Wilmot (2016), this research work aims to identify the most important elements that characterise the algebraic knowledge of pre-service teachers and to assess this knowledge. Hence the adoption of the Expanded KAT framework for this study.

**Field experience and its impact on pre-service teachers' knowledge**

Employees' year of experience is a relevant factor for consideration in every profession. The basic assumption is that experience gained over time strengthens one's knowledge, skills and productivity. In view of this, years of experience has been one major factor that influences the formulation of occupational policies such as promotions and benefit packages (Rice, 2010).

In the field of teaching, research has proven that teachers with some years of teaching experiences are more effective, in terms of their students' achievement, than their colleagues with no teaching experience. (Clotfelter, Ladd, & Vigdor, 2007; Harris & Sass, 2011; Kane, Rockoff, & Staiger, 2008). However, the results from these studies differs from one another. Harris and Sass (2011) reported that the more experience teacher turns out to be more effective in teaching elementary school Mathematics and reading and middle school Mathematics. Kane et al. (2008) also found out that the teachers' effectiveness is high in the first two years of teaching while Clotfelter et al's (2007) findings claim that teachers' effectiveness rise at the early few years of teaching.

Despite the divergence results of teaching experience on teachers' effectiveness, we cannot overlook its impact on teaching. As part of teacher training programme, teacher training institutions provide prospective teachers with the opportunity to embark on field experience. This field experience is to expose them to the teaching field in order to equip them with some classroom experience before they are posted to the classroom.

Research has proven the positive impact of field experience on prospective teachers (Cheong, 2010; Flores, 2015; Philipp et al., 2007; Potthoff et al., 2000; Yılmaz & Çavaş, 2008). These studies, on the other hand, have looked at how field experience affects primary school teachers' belief and their ability to teach. The work of Philipp et al. (2007) has been one among these researches which investigated the effect of field experience on elementary prospective teachers' mathematical content knowledge in addition to their belief. It is for this reason that this study seeks to investigate the difference in level of knowledge for teaching high school algebra among prospective teachers who have experienced off campus teaching practice and their counterparts who have not experienced such field experience.

**The nature of Mathematics curriculum in Ghana**

The Ghanaian high school Mathematics is divided into two courses: core Mathematics and elective Mathematics which are integrative in nature. Hence, each curriculum encompasses the content areas in Mathematics that students are to experience at the high school. The core Mathematics is a compulsory course for all high school students and it is geared toward equipping students with the competency and skills that they will deploy in solving daily problems and in their vocations (Ministry of Education, 2010). The Elective Mathematics, which is an optional Mathematics course, provides students with deeper mathematical knowledge, skills and competency which form the foundation to the demands of further studies in Mathematics oriented programmes at the tertiary level of education. In each case, the content is structured to cover a period of three years.

The content of elective Mathematics curriculum is categorized under the following areas: "algebra, coordinate geometry, vectors and mechanics, logic, trigonometry, calculus, matrices and transformation and statistics and probability" (Ministry of Education, 2010b, p.ii). In the case of the core Mathematics, the content areas are "numbers and numeration, plane geometry, mensuration, algebra, statistics and probability, trigonometry, vectors and transformation in a plane and problem solving" (Ministry of Education, 2010a, p. iii). In these two Mathematics curricular, Problem solving is not an independent topic but rather it is encouraged it runs through all topics. Thus, the teacher is encouraged to incorporate problems that will elicit students' mathematical thinking instead of recall of algorithms. These content areas are further divided into topics and subtopics with their associated specific objectives. Although some content areas run through both curricular, it has to be made clear that the demands in them are not the same. For instance, a topic like binary operation is found in both core and elective Mathematics. While substitution of real numbers into definitions of binary operations and further simplifying to show the properties of operations like commutativity is allowed in Core Mathematics, algebraic proving of such properties is required in Elective Mathematics.

To suit the nature of the Mathematics curricular in Ghana, teacher training institutions train prospective high school teachers to be capable to handle both Mathematics curricula and every content in them. Thus, it behoves the prospective Mathematics teachers to know and be capable to handle high school Mathematics contents of which algebra is a key content area because of its pervasiveness throughout the content.

High school students' abysmal performance in Mathematics at the West Africa Senior School Certificate Examination (WASSCE) has been of much concern to various stakeholders in the nation. Algebra as a content area in the high school Mathematics runs through all other content area in the Mathematics curricula. Due to the pervasive nature of algebra in the Mathematics curricula, it can be argued that high school students' poor performance in the WASSCE can be attributed to students' lack of competency to handle algebra which has a bearing on teachers' knowledge in algebra. It is for this reason that as part of investigating the extent to which the expanded KAT framework can be corroborated, this study seeks to assess prospective teachers' level of knowledge in Algebra. Are the prospective teachers of Mathematics well equipped with knowledge of algebra to teach algebra at the high schools in Ghana?

**Statement of the problem**

The importance of Mathematics in the development of every nation cannot be looked down upon. Every educational system relies heavily on Mathematics for it serves as the impetus that drives most subjects such as Physics, Chemistry, Economics, Statistics and Accounting. There is a growing interest for integrating Mathematics with other disciplines of study such as Biology and Medicine due to its ability to provide "insight into biological and biomedical phenomena with the aid of advance computational power" (Siddig, 2015). The economic development of every nation is attributed to the establishment of industries. Applied Mathematics such as data analysis, computer sciences, differential equations and geometric modelling are essential to the development of the economy in this technological world. Thus, Mathematics is indispensable to the economic growth of every country.

The versatility of Mathematics can be attributed to one of its domains, algebra, which permeate throughout Mathematics and its application to other disciplines of study. According to Grønmo (2018), algebra is the language of Mathematics and ones competency in algebra is essential to his success in Mathematics. Algebra is also relevant to other fields of study such as Physics, Chemistry, Economics, computing, engineering, just to mention a few, which culminate into economic development of a country. It can be argued that it is for this reason that algebra runs through Mathematics curricula across grade levels in some countries such as Ghana.

Despite the numerous importance of Mathematics to the nation, students' Mathematics performance have been stunted and abysmal in the past decades. Results from the 2015 edition of the Trends in International Mathematics and Science Study (TIMSS) which is an international assessment reveal that 48.84% of participating countries fall within the low benchmark, 39.53% of participating countries fall within the intermediate benchmark, 11.63% fall within the high benchmark with no placement in the advance benchmark (Mullis, Martin, Foy, & Hooper, 2016).

The abysmal performance in Mathematics is not limited to the international front but can be said about our nation, Ghana. Ghana in 2003 participated in TIMSS for the first time in order to find out how the Ghanaian eighth graders' [JSS 2 currently known as JHS 2 students] level of performance in Mathematics and Science cannot be compared with the international level. According to Anamuah-Mensah and Mereku (2005) students were assessed from five content areas (Number, Measurement, Data, Geometry and Algebra) of Mathematics. From their examination of the data from the TIMSS 2003 edition of which Ghana's eighth graders participated, it was revealed that Algebra, Geometry and Measurement were the difficult content areas for Ghanaians' eighth graders.

Students' abysmal performance in algebra permeates across grade levels of education and countries. Jupri and Drijvers (2016) used the perspective of mathematization and the reorganization of mathematical systems to identify the difficulties in algebra among Indonesian twelve- and thirteen-year-old students. From their findings, they identified formulation of mathematical model as the main difficulty of the Indonesian twelve- and thirteen-year-old students. This was evidenced by students' errors in formulating mathematical equations. Zuya (2017) found out that Nigerians' final year undergraduate students' conceptual knowledge in algebra is weaker as compared with their procedural knowledge in algebra.

Ghanaian high school students cannot be exempted from these weaknesses in algebra. Over the years, the Chief Examiner's reports of the West Africa Examinations Council (2012; 2014; 2015; 2016; 2017) have highlighted the weakness that high school students exhibit in their summative assessment, West Africa Senior School Certificate Examination (WASSCE). Specific difficulties in algebra that students have demonstrated over the years are the inability to write word problems in algebraic representation, solving of simultaneous equations, differentiating functions using the first principle of calculus, manipulation of trigonometric identities and its applications in solving equations involving trigonometry, simplifying and solving equations involving fractions, solving logarithmic equations and the use of algebra in computing probabilities. The following are two sample questions in which students exhibited their weakness in algebra.

Solve $(log_2 \mathrm{n})^2 + log_2 \mathrm{n}^3 = 10.$

**Figure 2: Sample question from 2014 WASSCE, Elective Mathematics**

According to the Chief Examiner, WAEC (2014) "was a popular question, however, performance was poor" (p. 14). Candidates were expected to simplify $\log_2 n^2$ into the form $3 \log_2 n$ and substitute $\log_2 n$ with any variable say $m$ in the equation. The resulting equation after simplifying and substituting should be $m^2 + 3m = 10$ which students now have to solve either by completing of square or by factorization. From the examiner's report, some students in attempt to simplify the individual terms in the equation simplified $(\log_2 n)^2$ to be $2\log_2 n$ which is algebraically wrong.

In other instances, candidates were given word problem of which they were expected to translate it into algebraic equation and finally solve. Candidates showed less competency in translating word problem in algebraic terms. See sample question in Figure 3.

**Three times the age of Felicia is four more than the age of Asare. In three years, the sum of their ages will be 30 years. Find their present ages.**

**Figure 3: Sample question from 2016 WASSCE, Core Mathematics**

The examiner reported that most students did not factor the phrase "in three years" in their translation to algebraic equation.

Having established that mathematical achievement of student is a function of the teachers' knowledge (Hill et al., 2005), it can be inferred that the abysmal performance of Ghanaian students in Algebra may be due to their teachers' knowledge of Algebra. A comparative analysis of teachers' knowledge in Algebra between Ghanaian teachers and United States (US) teachers by Wilmot (2015) indicated that Ghanaian high school Mathematics teachers' knowledge in Algebra (M=0.4086 SD=0.0778; M=0.4022 SD=0.0681) cannot be compared with their US counterparts (M=0.5509 SD=0.0325; M=0.5450 SD=0.0316) on the two instruments respectively. Although factors that might have accounted for this difference in performance among these two groups of high school Mathematics teachers were discussed by Wilmot (2015); Anamuah-Mensah, Mereku, and Ampiah (2008) observed that the content of Mathematics and Science taught at the eighth grade level in Ghana is superficial. The superficial nature of these contents can be viewed from two perspectives. The first of these is the curriculum at this level. It could be that the content of Mathematics and Science in the eighth-grade curriculum is superficial. Also, it can be that the curriculum has profound content of Mathematics and Science but the teachers' knowledge in these disciplines are rather shallow which resulted in superficial content delivery.

A detailed review of literature reveals that in attempt to address the abysmal performance of students in Mathematics as far as algebra is concerned, not much has been done on assessing prospective high school teachers' knowledge of algebra for teaching and Ghana is not exempted. It is for this reason that this work seeks to assess the level of algebraic knowledge that prospective high school Mathematics teachers possess for teaching algebra.

Also, earlier attempt made in conceptualising teachers' knowledge has presented teachers' knowledge as a domain neutral construct which in turn makes it difficult to measure accurately its component. As a result, many attempts to assess teachers' expertise have relied on proxies such as the number of university courses they have taken and the sort of degree they hold (Darling-Hammond, 2000) and advance coursework done by teachers (Monk, 1994). The work of the KAT research team conceptualised teacher knowledge under three constructs: high school and advanced algebraic content knowledge and Teaching knowledge. They further postulated that the interlocking regions of these knowledge types are fussy (Ferrini-Mundy, Floden, McCrory, Burrill & Sandow, 2005; McCrory et al., 2012).

It has to be acknowledged that further studies on the KAT framework by Wilmot et al. (2018) corroborated the expanded KAT framework in the domain of Algebra using in-service high school Mathematics teachers. This present study seeks to explore the factors that characterise the knowledge of tertiary Mathematics students who are undergoing training to become high school teachers.

**Purpose of the Study**

The purpose of the study is to investigate the underlying factors that explain pre-service teachers' knowledge for teaching high school algebra, measure the level of knowledge they possess for teaching and to investigate the impact of field-teaching experience on their knowledge.

**Research Questions**

The study is expected to answer the following questions:

1. What are the dominant factors that characterise the knowledge of tertiary Mathematics student for teaching senior high school Algebra?
2. How knowledgeable are tertiary Mathematics students to teach high school algebra?

**Research Hypothesis**

1. There is no significant difference in teachers' knowledge for teaching Algebra between pre-service teachers with Off-Campus Teaching Practice (Off-CTP) experience and their counterparts without Off-CTP experience.

## Significance of the Study

To begin with, understanding the nature of characterisation of prospective Mathematics teachers' knowledge for teaching will help the training institutions to also understand the state of the Mathematics education programme which will also serve as a guide for reconstruction of the programme.

Again, since research has shown that the teachers' knowledge for teaching a subject matter is a key influencer of students' achievement in the subject matter, the results from this study will help unveil the level of knowledge in algebra pre-service teachers possess as at the time they are about to be deployed to the senior high schools teach Mathematics. This, again, will form as a basis for reformation in the Mathematics education programme to ensure pre-service teachers are well equipped with both the relevant knowledge of content and how to deliver these contents. This will go a long way to improve student performance in Mathematics.

Furthermore, the study will form the basis of policy making in the Ghana Education Service with regards to the needed on-the-job training to be given to pre-service teachers at the early stages of their recruitment into the service since the results from this study will help unravel the nature and level of knowledge of algebra for teaching the Ghanaian pre-service teachers possess. At the senior high school level, the findings of this study may inform school administrators on the needed support such as mentorship to be provided to these teachers when they are deployed to the school either for internship or as permanent teachers.

**Delimitation**

Basically, this study sought to identify the major factors that explain the knowledge of prospective teachers for teaching high school algebra and the level of algebraic knowledge they possess. The following are the delimitations to the study.

The scope of content of Mathematics that this study focused on is algebra. The concentration was focused on algebra because algebra forms the basis for Mathematics manipulations in all other domains of Mathematics.

Also, the tertiary Mathematics students used in this study were Level 400 and Level 300 students reading Mathematics education programme. These cohort of pre-service teacher were used because at the time of the study, they have covered enough Mathematics content courses and methodology courses which are needed to teach high school Mathematics. Therefore, the study did not involve the participation of Level 100 and Level 200 Mathematics education students. The counterparts of Mathematics education students who, at the time of the study, were reading Bachelor of Science in Mathematics were also not included because they have not taken any Mathematics method courses which can give them the knowledge on how to teach Mathematics even though they had taken the required Mathematics content courses.

Geographically, the study involved tertiary Mathematics students in the central region who were undergoing training to become teachers. The concentration was shifted to this region because, it has two major universities that train majority of the high school Mathematics teachers. Even though there existed other universities that train Mathematics teachers for second cycle education in the other regions of Ghana, these universities were not involved because at the time of the study, these universities do not have pre-service Mathematics teachers up to Level 300 and Level 400. Hence their exclusion from the study.

**Limitations**

The major challenging factor that affected the findings from the study is the small number of tertiary Mathematics students who participated in the study. The low turnout was due to the outbreak of the COVID-19 pandemic which imposed restriction on movement into some schools and total closure of some schools.

**Definition of terms**

1. Pre-service teachers: This is operationally used to refer to Level 300 and Level 400 students of Mathematics education programme. These students have taken enough Mathematics content courses and Mathematics methodology courses. Therefore, it is assumed they possess adequate knowledge to teach high school Mathematics
2. Off-Campus Teaching Practice: This term was used to describe the one-semester mandatory internship program that pre-service teachers must do during their last year of studies at the university. During this period, these teachers select a senior high school of their choice and go to teach Mathematics under the supervision of a faculty member from the university and an assigned mentor in the school.

**Organization of the study**

This study is structured under five chapters. Chapter One, captioned Introduction, gives the Background to the study, Statement of the problem, Purpose of the study, Research questions and hypothesis, Significance of the study, Delimitation, Limitation and Definition of terms.

Chapter Two gives account of the critical review of related literature to the study. Key headings discussed in this chapter include overview of early research on teachers' knowledge, conceptualisation of teachers' knowledge, pre-service teachers' knowledge for teaching high school Mathematics, the need to focus on algebra, the impact of field teaching experience on pre-service teachers' knowledge and the conceptual framework that underpins the study.

The third chapter, Research methods, describes the methodological process that was followed to conduct the study whiles Chapter Four provides the results and the discussion of the results in line with literature. Chapter Five gives the summary of the entire study, conclusions to the findings and recommendations with regard to the findings of the study.

# CHAPTER TWO

# LITERATURE REVIEW

When attention of research on factors relating to teachers that contribute to good student performance was shifted from formulation of standard classroom practices of teachers to focusing on the mental faculty [knowledge] of the teacher, different researchers have conceptualised knowledge of teachers in different ways. The conceptualisations from Shulman (1986) to Ma (1999) (in Howe, 1999) on knowledge of teachers for teaching have heavily relied on qualitative data to describe the type of knowledge teachers must possess. One ground-breaking conceptualisation by the KAT research team conceptualised algebraic knowledge of teachers of high school Mathematics under three themes – knowledge of second cycle algebraic content, Advanced Mathematics content and the teaching of Mathematics (McCrory et al., 2012). They presented teachers' knowledge as measurable construct and developed an instrument to measure it accordingly. A recent conceptualisation of high school Mathematics teachers' knowledge, the expanded KAT conceptualisation by Wilmot et al. (2018), which is an expansion of the original KAT conceptualisation evolved out of the assumption that the various knowledge types in the original KAT conceptualisation can blend to form yet another complex knowledge types. This study is aimed at exploring the knowledge types that characterise the Algebra teaching knowledge of tertiary Mathematics students who are receiving training to become teachers by using the Expand KAT framework.

This chapter provides a review of relevant literature under six main headings: the overview of early research on teachers' knowledge, the different conceptualisation of teachers' knowledge, knowledge of prospective teachers of mathematics, the need to focus on algebra, the impact of field teaching experience on pre-service teachers' knowledge and finally the conceptual framework underpinning the study.

**Overview of early research on teachers' knowledge**

Literature has shown that early researches on teachers' knowledge started at the early part of the twentieth century in the form of process-product researches (Brophy & Good, 1984; Gage, 1978). The process-product research paradigm was geared towards finding a link between teachers' classroom practices and students' performance. In view of this, researchers like Gage (1978) and his associates formulated a number of "teacher should" statements which concentrated on specific teacher behaviours with the aim that these behaviours when practised in the classrooms will translate into good students' performance.

The process-product research paradigm received a number of criticisms (Gage & Needels, 1989; Solomon, 1979). The criticisms were categorized under four major areas: methodology, conceptualisation, interpretation-application and productivity. Methodology-wise, the process-product research paradigm was criticized for having implausible correlation between teachers' behaviours and students' achievement at relatively different times and in different subject matter. Criticism on the conceptualisation was mainly on setting standard teachers' classroom behviours while neglecting the teachers' intention [purpose] for a particular lesson. The remaining criticisms were with regard to the predictive power associated with the paradigm and the use of resulting research findings to formulate rules to teaching.

In the light of these criticisms, a new research design evolved out of the process-product research paradigm (Berliner, 1979; Peterson & Swing, 1982). In his Beginning Teacher Evaluation Study, for instance, Berliner (1979) brought in a new variable, Academic Learning Time (ALT) as a means of his modification to the this paradigm. According to Berliner (1979), ALT is the time students are engaged with a teacher-given task within a particular instructional period. He argues that if students are always engaged with easy tasks, the students' academic performance will not improve to any appreciable extent. In the same way, if students are engaged with more difficult task, they will not get time to master other skills, concepts and algorithms which will culminate to good performance. Berliner (1979) further argues that the ALT is essential because it serves as a direct link between teachers' behaviour and students' performance and also serves as an indicator to students' learning. Berliner's (1979) ALT failed to 1) indicate the nature of knowledge teachers must possess in order to judiciously judge the difficulty level of tasks given to students and 2) indicate when it will be appropriate for a teacher to move to a new concept.

When researchers like Peterson and Swing (1982); Putnam (1987) came into the scene of research in this direction, they argued that it will be very prudent to bring the mental faculty of the teacher to the centre of research. Putnam (1987) asserted that teachers, particularly experienced teachers, have knowledge of past students which forms a model of the student being taught in their mind. He further argued that it is this model of students possessed by experienced teachers that affords them the ability to decide whether to move to new task, provide more tasks on a kind of problem and to probe students' responses and activities for understanding. To these researchers, bringing the teachers' mental faculty to the centre of the research will afford researchers the ability to rightly study how teachers transform their knowledge to teaching practices.

It can be inferred that researchers like Shulman and those who came afterward who concentrated on conceptualising teachers' knowledge were moved by this new line of research.

**Conceptualisation of teachers' knowledge**

Efforts to research into teachers' knowledge gained much attention within the twentieth century. The impetus to research in this area of education stemmed out from a debate that ensued after the publication of the results from the study the concept of equality of educational opportunities by Coleman (1968). The finding revealed that only one-tenth of variance in students' performance could be explained by factors associated with schools such as the teacher and that family background characteristics accounts for greater variance in students' achievement. This finding made the integrity of schools questionable.

Amidst the debate that ensued after the findings of Coleman (1968), a number of researches were conducted to investigate whether or not factors associated with school are influencing factors to the students' achievement. Studies conducted within that period of time and beyond have revealed that school factors such as teachers' certification, teachers' instructional methods and effective use of instructional materials affect students' academic achievement (Enu et al., 2015; Farooq et al., 2011; Mji & Makgato, 2006). In addition to these findings, recent prominent studies have revealed that teachers' subject matter knowledge is a prime influencer of teachers' instructional activities and consequently students' achievement on the subject matter (Hill et al., 2005; RAND Mathematics Study Panel, 2003)

However, the aim of the these studies was not only to debunk the findings of Coleman (1968) but also to establish the fact that teachers possess a particular kind of knowledge that is peculiar to the teaching profession. The debate continues as to which knowledge is more important and more peculiar to teaching.

Attempts to research into the special knowledge that is peculiar to teaching brought a number of researchers into the research scene. Paramount among these researchers whose finding caught the attention of educationists is Shulman (1987). Shulman (1987) conceptualised teachers' knowledge under seven knowledge strands. These seven knowledge types are "content knowledge; general pedagogical knowledge . . .; curriculum knowledge . . .; pedagogical content knowledge . . .; knowledge of learners and their characteristics; knowledge of education contexts . . . and knowledge of educational ends, purposes and values . . . ." ( p. 8).

Key among the knowledge strands conceptualised by Shulman (1987) is PCK. Shulman (1987) argues that PCK is the knowledge that makes teachers' profession different from an expert in the content area. Shulman (1986) identified two key components of PCK namely (1) knowledge of instructional strategies and representation and (2) knowledge of students' misconceptions. Thus, PCK is the knowledge of the teacher that gives them the ability to transform their knowledge of the subject matter into powerful illustrations and presentations that are efficient and effective in making students understand the subject matter.

It has to be acknowledged that prior to Shulman's (1987) study, other studies also contributed to the conceptualisation of teachers' knowledge (Leinhardt & Smith, 1985; Thompson, 1984). Thompson (1984), for instance, asserted that the teacher's instruction in Mathematics is shaped by his belief, view and preferences for Mathematics. Thus, the teacher's knowledge for teaching Mathematics is prejudiced by his beliefs, views and preferences. Leinhardt and Smith (1985) also put forward Subject Matter Knowledge (SMK) and Lesson Structure Knowledge (LSK). SMK is the knowledge of the subject matter and its associated concepts and algorithms while LSK comprises the planning and organization of lesson and effective and efficient way of delivering the lesson.

Shulman's (1987) conceptualisation of PCK was a ground-breaking theory for explaining teachers' unique knowledge for teaching. PCK has formed the basis of measuring the relationship between students' learning outcome and teachers' instructional behaviours in many studies. Major studies among these studies are the "*Teacher Education and Development Study in Mathematics*" (TED-M), the Mathematical Knowledge for Teaching (MKT) study and the COACTIV study (Ball et al., 2005; Baumert et al., 2010; Tatto, Lerman, & Novotna, 2010). However, the concept of PCK was presented as a generic construct. Thus, the principles of PCK as explained by Shulman (1987) is the same for all domains of study as far as teaching is concerned. It can be agreed that some of these principles may be common to different domains of studies but the fact that each domain of study is unique in nature suggest that some PCK principles may be more peculiar to a specific domain of study.

Through an extensive interview of elementary Mathematics teachers from the United States and China on four questions which cover concepts of place-value and division of rational numbers and also involving modelling and representation, Ma (1999) (in Howe (1999)) propounded Profound Understanding of Fundamental Mathematics (PUFM) as a knowledge type which characterizes teachers' knowledge for teaching Mathematics. Ma identified this "knowledge package" to be more evident in the seventy-two Chinese elementary teachers she interviewed. Ma's (1999) PUFM goes beyond having a good command over the subject matter of Mathematics. It involves the ability to effectively communicate the subject matter of Mathematics to students.

It can be said that Ma's (1999) PUFM is similar to Shulman's (1987) PCK in that both of them involve transforming the subject matter into powerful pedagogical strategies that best suit students' learning. However, the two conceptualisations differ from each other on this basis: while Shulman's PCK is domain neutral construct, Ma's PUFM is limited to the field of Mathematics. Thus, Ma's PUFM is more content – specific than Shulman's PCK.

The point has to be made clear that earlier studies into the conceptualisation of teachers' knowledge provided qualitative information to describe the teachers' knowledge and their practices. Wilmot (2016) argues that such qualitative information cannot be overlooked in describing knowledge for teaching but in a subject like Mathematics, quantitative measures are essential to ensure that teachers have a good knowledge of the Mathematics content they are expected to teach.

In most states of the United States, prospective Mathematics teachers are to take a test in Mathematics after which they are certified upon their successful pass in the test. One of such tests which is used by most states as teachers' licensing examination is the PRAXIS. Despite the teacher licensing process put in place, the Mathematics achievement of high school students of the United States has been an issue of concern.

Knowing that the teacher's knowledge is key to the quality of students' achievement in Mathematics, the RAND Mathematics study panel proposed three areas of studies which will help improve teachers' knowledge for teaching. These include, clearly defining the mathematical knowledge needed to effectively teach Mathematics, devising means to readily make teachers' relevant and useable mathematical knowledge and the development of a valid and reliable tool to assess teachers' mathematical knowledge for teaching (RAND Mathematics Study Panel, 2003). In addition to the RAND recommendation, they singled out Algebra as the domain in Mathematics that requires further studies in order to ensure better proficiency in Algebra. Key among RAND's reason for proposing research studies in the domain of Algebra is the perverseness of Algebra throughout Mathematics and it being foundational to all other domains of Mathematics. As Grønmo (2018) rightly said, proficiency in algebra is a key determinant of one's success in Mathematics since it serves as the language of Mathematics.

To heed to the recommendation of RAND Mathematics study panel, two major studies took place within the first decade of the twenty first century, the

KAT study by Ferrini-Mundy, Floden, McCrory, Burrill, & Sandow, 2005; McCrory et al., 2012) and the Mathematics Knowledge for Teaching (MKT) project by (Ball et al., 2005; Hill et al., 2008; Hill et al., 2005). These studies did not only focus on reconceptualising teachers' knowledge for teaching but also developed reliable instruments to measure them.

One major difference between these two studies is that the MKT project team focused on conceptualising teachers' knowledge for teaching elementary school Mathematics while the KAT team focused on conceptualising teachers' knowledge for teaching at the high school level.

Based on already existing theories of teachers' knowledge MKT project team developed an extended conception of Shulman's conceptualisation, shedding more light on the Content Knowledge and PCK by breaking each into categories of knowledge. Knowledge types among their categorization of Content Knowledge are the Common Content Knowledge (CCK), Knowledge at the Mathematical Horizon (HCK) and Specialized Content Knowledge (SCK). CCK is the mathematical knowledge which is common to the teaching profession and other professions. HCK describes the knowledge of a teacher on how topics in Mathematics are related to each other in the curriculum. The type of knowledge used for the execution of the teaching task is SCK. (Hill et al., 2008). SCK encompasses the ability of a teacher to follow students' mathematical thinking, expound algorithms used in solving mathematical problems and judging the accuracy of students' solution. Thus, SCK is the distinctive knowledge that distinguishes the Mathematics teacher from other professions of Mathematics.

Again, in the MKT's conceptualisation was the elaboration of Shulman's (1986) PCK. PCK is a composition of three knowledge types: Knowledge of Content and Student (KCS), Knowledge of Content and Teaching (KCT) and Knowledge of Curriculum (KC) (Ball et al., 2005; Hill et al., 2005). KCS, as the name implies, is a combination of knowledge of the mathematical content and student. It is manifested through the teachers' ability to design lessons in Mathematics which students find stimulating by having a knowledge of students' characteristics. With the possession of KCS, algorithms which will enhance or impede students' understanding are catered for during the process of designing the lesson. In the process of designing the lesson or after designing the lesson, the ability of a teacher to select and decide the sequencing of mathematical tasks and examples so as to facilitate students' learning of the concept understudy requires the possession of KCT. The third subscale, KCS, described by the MKT researchers is analogous to what Shulman (1986) refers to as horizontal and vertical curriculum. Knowledge of Curriculum is the teachers' knowledge of how mathematics topics are related to one another within the scope of Mathematics beyond the specified grade level (horizontal curriculum) and how the mathematics concepts are integrated in topics outside Mathematics (vertical curriculum).

The KAT research team, through an extensive analysis of videos on teaching, analysis of content of Mathematics textbooks, review of researches on teachers' knowledge and teaching and interview with teachers created a framework of teachers' knowledge for teaching with high school algebra being the pivotal area of research. Their framework is categorized under two strands: knowledge of mathematical content and the application of mathematical knowledge in instruction. The mathematical content knowledge strand comprises knowledge of high algebra as stipulated in the high school curriculum (School Knowledge), knowledge of Advanced Algebra related to the concepts of algebra beyond the high school algebra (Advanced Knowledge) and knowledge of Algebra that is relevant to teaching (Teaching knowledge). The other strand of their framework highlights three uses of mathematical knowledge in teaching – Bridging, Decomposition and Trimming. Bridging involves the ability of the teacher to link the concept of high school Algebra to other related concepts and Advanced Algebra concepts. Decomposition involves the teacher's ability to explain algebraic concepts and rightly use of algorithms and procedures with reference to his knowledge in Advanced Algebra. Finally, Trimming entails the presentation of mathematical concepts in a way that suits the students' level of understanding without altering the underlying mathematical concept.

Building on the KAT project team's work, Wilmot et al. (2018) formulated an expanded version of the mathematical knowledge strand of the KAT framework. Their inspiration to study further into the KAT framework was from the work of Wilmot (2016) which aimed at validating the KAT framework. Though not all the three knowledge types were corroborated in his study, results from factor analysis revealed cross loading of items from Teaching knowledge and Advanced knowledge on one of the factors extracted from scree plot factor loadings. Even though this factor had not enough items to be described as a factor, the result inspired Wilmot (2016) to hypothesize that the mathematical knowledge types of the KAT framework can interlock to produce another complex knowledge type at the interlocking regions. Taking into consideration, recommendations made by Wilmot (2016), Wilmot et al. (2018) developed an instrument through the adaption of the original instrument from the KAT project. Factor analysis from their study corroborated fully all the hypothesized knowledge types by Wilmot (2016).

In Wilmot et al's (2018) study, two hundred and fifty-two in-service Mathematics teachers from forty senior high schools across the four categories of high schools by the Ghana Education Service (GES) in the Ashanti, Central and Western regions of Ghana were used. Moreover, a cross sectional survey which employed a multi-stage sampling technique was used. In each region, a simple random sampling was employed to select a district or municipality. Schools in these selected districts or municipality were put into strata using the GES categorization. Simple random sampling was then used to select schools in each strata and Mathematics teachers teaching either core or elective Mathematics in the selected schools became the participants for the study.

Though the KAT framework and its extension have provided a comprehensive characterization of teachers' knowledge for teaching high school Algebra, it is also important to understand the factors that characterize pre-service Mathematics teachers' knowledge for teaching high school Algebra.

The prime objective of this study is to explore the knowledge types that describe the knowledge of prospective Mathematics teachers for teaching high school Algebra. It is also to assess the level of knowledge in Algebra possessed by pre-service Mathematics teachers. In view of this, the instrument developed by Wilmot et al (2018) was adopted for this study and a cross sectional survey was used. However, in the case of this study, a cluster sampling technique was employed. This was done because unlike the senior high schools in Ghana, there is no special categorization of the teacher training universities and each teacher training university has diverse pre-service Mathematics teachers in terms of their knowledge [performance] and origin. Currently, there are three universities running Bachelor of Education in Mathematics programme that have up to third and final year students, two from the Central region and one from the Greater Accra region. These three teacher training universities are used as the clusters and a simple random sampling technique was used to select one of these three universities. The third and final year students reading the Bachelor of Education Mathematics programme in this selected universities formed the participants of the study.

**Pre-service teachers' knowledge for teaching high school Mathematics**

Literature on teacher education programmes argue that the structure and approach of such programmes contribute to teachers' knowledge, their practices and consequently, students' learning (Hill et al., 2005; Quinn, 1997; Vacc & Bright, 1999). A cross-national comparative studies into the structures and the organization of Mathematics teacher preparation programmes shows a vast difference in emphasis placed on Mathematics Content Knowledge (MCK) and Pedagogical Content Knowledge (PCK) of Shulman's (1986) conceptualisation of teachers' knowledge (Tatto et al., 2010). In their studies, Tatto et al. (2010) reported that much emphasis is placed on MCK in high school teacher education programmes while the opposite is true for the primary school teacher education programmes. However, much emphasis is placed on PCK in teacher education programmes for both the primary and high school teacher education.

Indeed, the existence of good structures may have a positive impact on the quality of prospective teachers who go through such educational programme. However, the knowledge base of prospective teachers who go through these structures counts most since it is their knowledge base on the subject matter they teach and other education related courses that influence their classroom practices and consequently affect students' achievement.

Goos (2013) investigated the relationship between MCK, PCK, Prior Mathematics experience and other demographic variables of prospective high school Mathematics teachers in Australia. In a number of stepwise regressions performed by using MCK and PCK as the outcome variables, MCK was the only predictor of PCK $(\beta = 0.30, t = 2.53, p < 0.014)$ while PCK $(\beta = 0.25, t = 2.04, p = 0.47)$ and prior level of mathematical experience $(\beta = 0.42, t = 3.41, p = 0.001)$ were the predictors of MCK. These findings reveal that MCK and PCK are two important knowledge types in the teaching of Mathematics and one may develop them in conjunction with each other while pursuing teacher education programme. Moreover, the fact that MCK is the only predictor of PCK supports the assertion of Byrne (1983) that it is the knowledge of the content being treated in conjunction with the application of the appropriate pedagogical strategies that makes a teacher effective in his teaching. Thus, PCK, as taught in teacher training institutions, should be treated as content specific construct.

Knowing that the students' mathematical achievement to is a reflection of teacher's mathematical knowledge (Hill et al., 2005), pre-service teachers' Mathematical Content Knowledge (MCK) and Mathematics Pedagogical Content Knowledge (MPCK) have been assessed across grade levels of learning. Analysis of data from these works reveals that prospective Mathematics teachers have limited MCK and MPCK (Depaepe et al., 2015;

Leong, Meng, Rahim, & Syrene, 2015; Wilburne & Long, 2010). Using elementary and lower secondary prospective Mathematics teachers Depaepe et al. (2015) assessed the MCK and MPCK on rational numbers. Using Shulman's (1987) conceptualisation of PCK, items that measured MPCK focused on assessing prospective teachers' knowledge on students' misconception and knowledge of instructional strategies and representations. The MCK items measured prospective teachers' knowledge on the concept of fraction and decimal numbers and the algorithmic operations (addition, subtraction, multiplication and division) that are involved in these concepts.

Using data on Malaysian prospective teachers who took part in the Teacher Education Study in Mathematics (TED-M) Leong et al. (2015) reported that 6.9% of the prospective secondary school Mathematics teachers fell within the higher level of MCK while 57.1% of them fell within the lower level of MCK. In all, the mean score of the prospective secondary teachers on the MCK items was 493 as compared to the international mean of 530. The same group of participants had a mean score of 472 as compared to the international mean of 520 on the MPCK items.

Even though results from these studies show that prospective mathematic teachers have low knowledge in MCK and MPCK, it appears from these studies that prospective teachers are more competent in the MCK than in MPCK. This suggests that the teacher training institutions are able to equip prospective teachers with the needed MCK as compared to the MPCK.

**The need to focus on Algebra**

Different sources have categorized Mathematics into different content areas. Prominent among these categorizations are arithmetic, Algebra, geometry, trigonometry and calculus. In Ghana, there are two Mathematics curricular – Core Mathematics and Elective Mathematics. In these two curricular, the Elective Mathematics covers Algebra; vectors and mechanics; coordinate geometry; logic; calculus; trigonometry; matrices and transformation; statistics and probability. The Core Mathematics curriculum covers plane geometry; numbers and numeration; mensuration; transformation Algebra; trigonometry; statistics and probability; and vectors in a plane (Ministry of Education, 2010a; 2010b).

Each of these content areas of Mathematics has specific content information that it focuses on. However, Algebra runs throughout all these content areas of Mathematics since it serves as a means of presenting the content information of these content areas even though it also has its fundamental content information that it focuses on. To Grønmo (2018), Algebra is the language of Mathematics and he further argues that just as one's success in a country is a function of his competency in the country's language, competency in Algebra is crucial to students' performance in Mathematics across all grade levels and to people across all professions. The RAND Mathematics Study Panel (2003) define Algebra as the basis of all branches of Mathematics since "it provides the tools [language and structure] for representing and analysing quantitative relationships, for modelling situations, for solving problems, and for stating and proving generalizations" (p. 44). Thus, Algebra is the bedrock of Mathematics on which all other branches of Mathematics are built.

In their recommendation for further studies on the teaching and learning of a specific domain of Mathematics to researchers, the RAND Mathematics Study Panel (2003) recommended Algebra for three reasons. First among their reasons is Algebra being fundamental to all branches of Mathematics and other discipline of studies like Physics, Engineering, and Commence. Second is students who lack competency in Algebra being disenfranchised to have access to educational and career opportunities and lastly Algebra being a requirement for graduating from high school.

Developers of both high school Mathematics curricular of Ghana esteemed the relevance of Algebra and its proficiency to students in other areas of studies and towards national development and for that matter included Algebra in both curricula even though these curricula differ in scope of content. Like the US counterparts, Ghanaian high school students are to pass the summative examination organized by West Africa Examinations Council (WAEC) on an integrative Mathematics curriculum [Core Mathematics] in order to have access to tertiary education. To pursue further studies in Mathematics and the sciences or any other programme that demand strong Mathematics background, students have to pass similar examination by the WAEC on the Elective Mathematics curriculum.

The Mathematics chief examiner of the WAEC has repeatedly reported weaknesses students demonstrate in Mathematics in the summative examination organized by the WAEC. Prominent among the weakness they have shown over the years is in the domain of Algebra (WAEC, 2006; 2012; 2014; 2015; 2016; 2017). Specific difficulties in Algebra that have been highlighted by the chief examiner over the years are inability to algebraically represent word problem, differentiate from first principle, simplify and solve equations involving fractions, logarithms, and trigonometric identities.

Since Algebra runs through all the other content areas of Mathematics in the Mathematics curricular and plays a gate-keeper role to students in high school, it is imperative to address students' difficulty in Algebra for when students are well equipped with proficiency in Algebra, they stand a higher chance of developing proficiency in other areas of Mathematics. Studies have also shown that students' performance in Mathematics is a function of the teachers' knowledge in the subject matter of Mathematics (Hill et al., 2005; RAND Mathematics Study Panel, 2003). Therefore, it is important to research into the algebraic knowledge possessed by pre-service teachers who will be deployed to teach Mathematics in the high schools and address their gaps in Algebra at the various training institutions before they are deployed to the high schools.

**The impact of field teaching experience on pre-service teachers' knowledge**

Intuitively, it is rationally sound that the more experience one is in his profession, the better the individual performs in his field of profession. Darling-Hammond (2000) asserted that one indicator of teachers' competency is his years of teaching experience. Numerous research findings have supported Darling-Hammond's (2000) assertion by concluding that teachers with more experience in teaching are more effective in terms of students' achievement than their counterpart with less experience in teaching (Clotfelter et al., 2007; Harris & Sass, 2011; Kane et al., 2008).

Studies have shown that Mathematics teachers learn through teaching experience (Klecker, 2002; Rosenholtz, 1986). As if to throw more light into these research findings, McCrory et al. (2012) in their description of the

Mathematics Teaching Knowledge in the KAT conceptualisation of teachers' knowledge asserted that this type of knowledge is "more readily available to an experienced teacher" (p. 599). Putnam (1987) further clarified this by asserting that experienced teachers have in store of knowledge of instructional activities and peculiar characteristics of students in their minds which give them the affordance to teach to students' understanding and to overcome students' misconceptions. Thus, teaching in the classroom context provides teacher with an additional knowledge which might not necessarily be taught to them in their training but also enhances their profession and places them at an advantage over their novice counterparts.

Recognizing the benefits of teaching experience and ensuring that pre-service teachers have a thorough understanding of the classroom situation, field teaching has become a vital component of the curriculum for most teacher training schools. These teacher trainees are to embark on either a semester or a full year teaching practice which is done either concurrently with coursework or on full time basis depending on the demand of the institution's curriculum. These teachers must organize, develop, and implement lessons as well as participate in other extracurricular activities during this period.

The impact of field teaching experience on pre-service teachers has been researched into from divergent viewpoints. One area that has researched on consistently on the impact of prospective teachers' field teaching experience is their teaching efficacy (Al-Awidi & Alghazo, 2012; Cheong, 2010; Flores, 2015; Logerwell, 2009; Moseley, Reinke, & Bookout, 2002; Schmidt, 2010; Yılmaz & Çavaş, 2008). These studies have shown a positive effect of pre-service teachers' field teaching on their teaching efficacy. Contrary to the positive effect field experience has on pre-service teachers' efficacy, Moseley et al's., (2002) work shows pre-service teachers' teaching efficacy was high prior to the teaching experience but dropped significantly after seven weeks of teaching. The measure of the effect of field experience on the attitudinal construct, teaching efficacy, is worthy since it describes pre-service teachers' confidence to effectively undertake the teaching act. The challenge is, the measure of the teaching efficacy of pre-service teachers may not be the true reflection of their knowledge on the teaching practices and of the subject matter.

Studies have shown divergent results concerning the effect of teaching practicum on prospective Mathematics teachers' knowledge (Philipp et al., 2007; Strawhecker, 2005). In Strawhecker's (2005) study, she explored the impact of different teacher preparation programmes on the content knowledge in Mathematical and Knowledge of Students and Content (KSC) subscale of PCK of pre-service teachers. Analysis of One-way ANOVA and post hoc test of the posttest revealed no significant difference in content knowledge of Mathematics among prospective teachers who were enrolled in either of these four programmes: concurrently taking Mathematics content course, method course and weekly field teaching experience (CMF group), concurrently taking Mathematics methods course and weekly field teaching experience (MF group), taking Mathematics methods course only (M-only group) and taking Mathematics content course only (C-only group). However, on the KSC subscale of PCK, results showed a significant difference between CMF group and M-only group, MF and M only group and MF and C only group. Also, the CMF and MF groups appeared to have similar level of knowledge in this construct. The fact that a noteworthy difference in PCK was detected between groups which participated in field teaching and those who took either content and method course only indicates that field teaching has the potential of enhancing pre-service teachers' PCK.

Contrary to Strawhecker's (2005) finding, in Philipp et al's. (2007) study, the Mathematics knowledge of prospective teachers who had indirect interaction with students underwent a positive change than their counterparts who had direct interaction with students in Mathematics lessons. Even though these studies have shown divergent effect of field teaching on prospective teachers' knowledge, the results give an impression that including field-base teaching experience in teacher preparation programmes as a means of providing them with the opportunity to experience the real classroom context in some way brings a positive gain in either the Mathematics content knowledge or the pedagogical content knowledge of pre-service teachers.

Moreover, with the advent of describing teachers' knowledge for teaching as a measurable construct by recent conceptualisation of teachers' knowledge (Ferrini-Mundy et al., 2005, McCrory et al., 2012; Wilmot et al., 2018), this research seeks to find out how field teaching experience affects prospective Mathematics teachers' knowledge across the various knowledge types of the Expanded KAT framework in the Ghanaian context.

**Conceptual Framework**

Results from factor analysis of data by Wilmot (2016) in his attempt to validate the KAT conceptualisation of mathematical knowledge for teaching served as the impetus to his formulation of the hypothesis: the three mathematical knowledge types in the KAT conceptualisation of mathematical knowledge for teaching interlock to form another complex type of knowledge that needs to be studied. The validated Expanded KAT framework by Wilmot et al. (2018) forms the conceptual framework that guides this study.

It is worth noting that the extended KAT framework acknowledges the three original knowledge types proposed by the KAT project team (See McCrory et al. (2012)) as valid knowledge that influences the teaching of Algebra. Also, by means of emphasizing the domain of Algebra, the knowledge types in the expanded KAT framework are qualified with Algebra. Figure 4 shows the expanded framework.

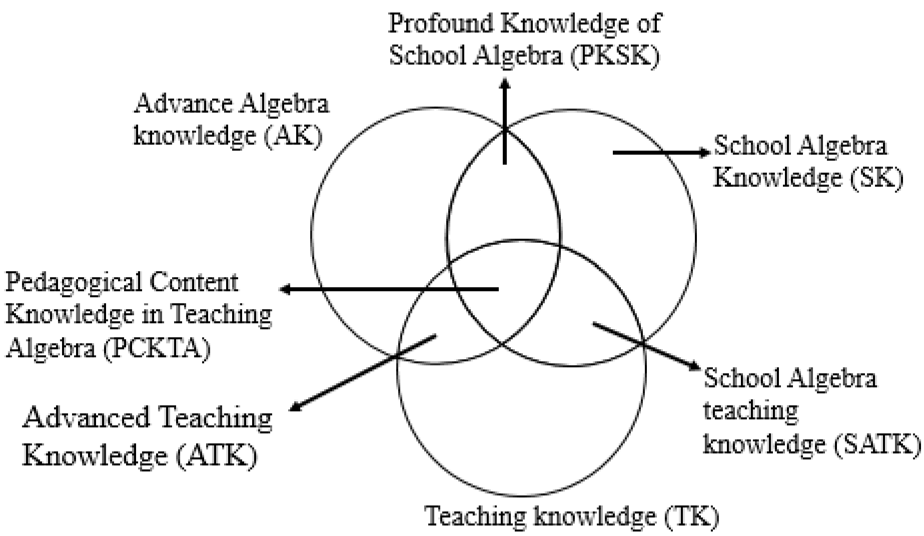



**Figure 4:The Expanded KAT framework (Wilmot, 2016; Wilmot et al., 2018)**



**School Algebra knowledge**

This is the knowledge of Algebra in the high school Mathematics curriculum. Unlike the United States, Ghana has two centralized Mathematics curricular – Core Mathematics and Elective Mathematics – which are integrative in nature. The Core Mathematics is a general Mathematics course for every Senior High School student while the Elective Mathematics is a selective Mathematics course which prepares students for advance Mathematics programmes or Mathematics related programmes at the tertiary level of education. Although these two courses differ in depth and scope of its content, both place much emphasis on proficiency in Algebra and its application to real life. Since teachers are to teach these concepts to students and help them in the acquisition of proficiency in Algebra, it is logically sound for teachers to possess this knowledge.

**Advanced Algebra knowledge**

This type of knowledge is the knowledge in algebra that teachers acquire from college level Mathematics. It is the knowledge that equips teachers with different viewpoints on Algebra beyond the scope of high school algebraic knowledge (McCrory et al., 2012). The KAT project team listed college level courses like Calculus, Real and Complex analysis, Linear Algebra, Abstract Algebra, Number theory and Mathematical modelling as the courses that afford teachers with the wider and deeper understanding of Algebra beyond high school algebraic knowledge. Basing on the characterization of advanced mathematical knowledge of Usiskin, Peressini, Marchisotto, and Stanley (2003), the KAT project team further described indicators to the possession of advanced algebraic knowledge as "knowing alternate definitions, extensions and generalization of familiar theorems, and a wide variety of application of high school Mathematics" (McCrory et al., 2012, p. 597).

Since teachers are to put their expertise in Mathematics into practice to ensure effective teaching which enhances students' understanding, possession of advanced knowledge of Algebra by teachers is very important. Possession of this knowledge by teachers affords them to connect and link the algebraic concept understudy to other related concepts of Algebra and their associated algorithms (*Bridging*), ability to explain the underlying concepts understudy and rightly use of related algorithms (*Decompressing*) and presentation of concepts to suit the students' level of understanding (*Trimming*). Thus, to effectively undertake the practices of bridging, decompressing and trimming in the teaching of Algebra, the teachers must have a good repertoire of advanced knowledge of Algebra.

**Mathematics teaching knowledge**

This is the mathematical knowledge that is useful in teaching but is not taught in a typical Mathematics class either at the secondary or the tertiary level. However, this knowledge may be taught to Mathematics teachers in their formal training to the teaching of Mathematics or can be gained through the practice of teaching. It entails pure mathematical knowledge for teaching that may fall within the concept of PCK of Shulman's conceptualisation (McCrory et al., 2012). It comprises knowledge that are readily available and more familiar to practicing teachers over a period of time. This type of knowledge can be likened to the *specialized content knowledge* in Hill et al.'s (2008) conceptualisation of Mathematical Knowledge for Teaching.

Thus, this is the knowledge of Mathematics which is peculiar to the teaching of Mathematics since it is not taught in pure Mathematics class. Furthermore, the sort of knowledge that distinguishes a Mathematics teacher from a mathematician is this knowledge.

**Profound knowledge of school Algebra**

This is an extension in knowledge of school Algebra. It entails deeper understanding of high school Algebra and algebraic content that precedes high school Algebra and those that proceed high school Algebra. Possession of profound knowledge of school Algebra is characterized by "alternate definitions, extensions and generalization of familiar theorems and wide variety of application of high school algebra" (Wilmot, 2016, p. 23).

**School Algebra teaching knowledge**

This category of knowledge equips teachers with ingenuity to the teaching of high school Algebra. It is the knowledge which offer teachers the skill to teach Algebra to the understanding of diverse group of learners (Wilmot, 2016). Possession of this type of knowledge plays a crucial role in deploying the practices – bridging, decompressing and trimming – in the teaching of high school Algebra.

**Advanced Algebra teaching knowledge**

Like the possession of the school Algebra teaching knowledge, a teacher who is equipped with advanced Algebra teaching knowledge will be able to teach advance Algebra when it becomes necessary to teach it. Advanced Algebra teaching knowledge is essential in engaging in the practices of bridging, trimming and decompressing when the need to teach advance content of Algebra (Wilmot, 2016).

**Pedagogical Content Knowledge in Algebra**

This is the PCK in the conceptualisation of Shulman (1986) which is a complex combination of content knowledge and pedagogical knowledge. However, unlike Shulman's PCK, this knowledge as expound in the expanded KAT framework is specific to the domain of Algebra.

# CHAPTER THREE

# RESEARCH METHODS

The prime purpose of this present study is to explore the knowledge types that characterize pre-service teachers' knowledge for teaching senior high school Algebra. This chapter expounds on the method employed in undertaking the study. The research design, the study area, the population, the sampling procedure, the research instrument, the data collection procedure, and data processing and analysis are all highlighted in this chapter.

## Research Design

This study aims at exploring the factors that characterise the knowledge of prospective Mathematics teachers for teaching senior high school Algebra and also to measure the level of algebraic knowledge of prospective Mathematics teachers. Consequently, tertiary Mathematics who are undergoing training to become teachers were engaged. These teachers' algebraic knowledge for teaching was assessed by using an adopted instrument which was developed by Wilmot et al. (2018). The instrument measures teachers' algebraic knowledge across the various knowledge types of the KAT framework: Advanced knowledge in Algebra, knowledge of high school algebraic content and knowledge on the teaching of Algebra. To conduct the study, procedures and techniques for a cross-sectional design were used to explore the knowledge kinds that describe these Mathematics teachers' teaching knowledge on Algebra at a specific time frame. These techniques were also followed to measure their algebraic knowledge for teaching.

A cross sectional research design was considered suitable for this study due to its ability to provide a "'snapshot' of the outcome" (Levin, 2006, p. 24), solicit and describe the characteristics of the participants of a study within a short period of time (Cohen, Manion, & Morrison, 2002; Creswell, 2012). Therefore, this study gathered information on the knowledge types that characterize prospective teachers' knowledge for teaching Algebra. In addition, the design is economical in that it helped in collecting current information on large number of prospective high school teachers' algebraic knowledge for teaching during the period specified to conduct the study (Creswell, 2012).

It is worth mentioning that because the data gathered on prospective teachers' algebraic knowledge for teaching Algebra at the high schools was within a specified period of time, the results of this study is highly susceptible to change when the same information is gathered at a different period of time. Thus, the result of the study cannot account for the changes that will occur in prospective teachers' algebraic knowledge for teaching Algebra at high schools after the study.

Despite the aforementioned weakness associated with the cross-sectional design, its ability to provide a representation of the population and its ability to study several variables within specified time frame far outweighs the weakness and thus, the cross-sectional design was deemed fit for the study.

**Study Area**

This study was conducted in the Central region of Ghana. This region was selected for the study because it has two major universities that train high school Mathematics teachers. Initially, it was proposed to include one university in the Greater Accra region but due to the outbreak of the COVID-19 pandemic, this university was closed at the time of collection of data within the partial reopen of school period. The university was closed because they resorted to online tutoring when the pandemic was at its peak in Ghana.

**Population**

The target population for this study is all prospective senior high school Mathematics teachers from the teacher training universities in the Central Region. These were Level 300 and Level 400 students who were reading Bachelor of Education in Mathematics at these universities in the region. The Level 300 and Level 400 students from this programme were deemed qualified for the study because they have taken enough advanced Mathematics content courses and courses that address the content of the senior high school Mathematics content and for that matter, have enough Mathematics content knowledge of which knowledge in Algebra is no exception. Also, these are students who have taken enough courses relating to the pedagogy and methods of teaching high school Mathematics. Hence, it is assumed that they are equipped with both the Mathematical content knowledge including Algebra and the knowledge for teaching the mathematical content.

Moreover, these Mathematics teachers in these training universities, in the second semester of their third year, had gone through On-Campus Teaching Practices (On-CTP). This platform is for the pre-service teachers to put into practice what they have learnt in their methods and pedagogy for teaching senior high school Mathematics courses. It is also the opportunity for these teachers to exhibit their expertise in other education related courses which address classroom practices such as classroom management, ways to motivate and reinforce students either positively or negatively. During this period, the pre-service Mathematics teachers are divided into groups where each member in the groups is given opportunities to teach their colleagues under the supervision of at least a faculty member every week. Thus, the On-CTP is meant to prepare pre-service teachers towards their deployment into the senior high schools.

At the first semester of their final year, these pre-service teachers are deployed to the senior high schools across Ghana to have what is termed as Off-Campus Teaching Practice (Off-CTP) where they teach Mathematics for a whole academic semester. They teach Mathematics under the regular supervision of trained teachers, mostly heads of department for Mathematics in the various schools. Also, faculty members from the respective training universities frequently go round to supervise these pre-service teachers at the various senior high schools of practice. Therefore, the third and final year students reading Bachelor of Education in Mathematics were deemed qualified for this study because they 1) have taken enough advance courses in Algebra and acquainted themselves with advance knowledge in Algebra. Some of these courses are Algebra and Trigonometry, Advanced Algebra and Calculus and Introduction to Abstract Algebra 2) have been taken through courses like the Secondary School Mathematic Curriculum which address concepts peculiar to the content of the High School Mathematics curriculum including Algebra. Hence, it is assumed they are well equipped with the knowledge of the High School Algebra 3) have taken Methods and Pedagogy courses and have also experienced teaching for at least a semester. As a result, it is anticipated that these future teachers are familiar with students' understanding and misconceptions, as well as effective teaching strategies for Algebra.

**Sampling Procedure**

The study employed the technique of cluster sampling to sample the participants of the study. Cluster sampling is a probability sampling technique whereby the target population is divided into subgroups with each subgroup having a variation of characteristics as the population (Ary, Jacobs, & Sorensen, 2010; Kothari, 2004).

In this study, the two universities in the Central Region of Ghana and one university in the Greater Accra Region of Ghana that run Bachelor of Education in Mathematics programme were used as clusters. These universities were used as clusters because they have pre-service teachers of Mathematics from all sixteen regions of Ghana. Also, the entry requirements into the Bachelor of Education in Mathematics programme offered by these universities are almost the same and offered almost the same courses with regards to courses that address Mathematics content, methods of teaching Mathematics and pedagogy of Mathematics. Though, there are variations in the structure of the Mathematics education programme in each of these universities, the variations are not too much. Therefore, on this basis, it can be said that the two universities in this region have similar variation in student teacher' knowledge for teaching.

In the process of sampling, one university out of the three universities was accessible to participate in the study due to the closure of schools to ensure the COVID-19 virus was contained. The Level 400 and Level 300 Mathematics education students in the available university were used as intact classes. Thus, every member in these classes of the selected university was included in the study.

In all, one hundred and sixty-four prospective teachers comprising one hundred and one Level 400 students and sixty-three Level 300 students participated in the study.

**Data Collection Instrument**

The instrument used in this study is an adopted instrument developed by Wilmot et al. (2018). This instrument is divided into two major sections. Section One solicited for the demographic information about respondents while the Section Two contained seventy-four multiple choice items on the School Algebra, Advanced Algebra and the Teaching knowledge types.

From a pilot study, Wilmot et al. (2018) found the reliability of the instrument to be 0.786 using the KR-20 formula which is statistically reliable. Also, the content validity of the instrument was also confirmed after a thorough evaluation by two professors from the University of Cape Coast's Department of Mathematics and ICT Education and three doctorate students who also teach Mathematics in senior high schools in Ghana.

Since the reliability and content validity of the instrument has already been established by the developers and the same instrument was adopted for this study, the reliability and validity were not tested in this recent study.



**Data Collection Procedure**

The prime purpose of this study is to explore the dominant factors that describe pre-service teachers' knowledge for teaching high school Algebra and also to measure the level of Algebra knowledge possessed by pre-service teachers.

The administration and collection of data were done in two phases. In Phase one, the instruments were administered to and collected from the final year perspective Mathematics teachers and this process was between the period of 6th July 2020 to 23rd July 2020. The second phase was between 20th September 2020 and 28th October 2020 within which the instruments were administered to and collected from these future teachers in their third year of study.

The researcher first visited the university that was sampled to participate in the study on the 7th July 2020 to issue permission letters to the Heads of Department of the pre-service teachers. This was a period within which tertiary institutions partially reopened during the COVID-19 pandemic in order for final years students in the various tertiary institutions to complete the final semester of the academic year.

Upon approving to take data from pre-service teachers from the department, the researcher sought for permission from some lecturers in the department to administer the instrument to the students during their lecture times. The researcher met the pre-service teachers at the agreed lecture time and venue with their lecturers. At the meeting, the purpose of the study and the significance of the study were explained to students and their consent was sought for. Later, the instrument was administered to them. The students at these times were preparing for end of semester examination as well. Therefore, they were allowed to take the instrument to their various halls and hostels to complete the items on the instrument. The researcher took the contact information of the pre-service teachers who consented to participate in the study to ensure they completed the test. The researcher from time to time called to remind them to complete the work. The final collection of instruments from respondents was done on 28th October 2020 by the researcher.

The major challenges faced in the process of data collection were the unavailability of students to respond to the instrument due to the closure of schools in the attempt to contain COVID-19 virus and also the unwillingness of the few available students to respond to the instruments.

**Data Processing and Analysis**

In every research, data analysis is purposefully to present the data gathered for the study in a well-organised and meaningful way in order to answer the research questions and to aid decision making. In order to answer the research questions and test the hypothesis that guide this study, the data analysis conducted have been categorized under the various research questions and hypothesis. Items on the second section of the instruments were scored right and wrong. Therefore, in the data entry process, 1 was used for items that have been correctly answered and 0 was used for items that have been wrongly answered by a respondent.

**Analysis of data for research question one**

The first research question is "What are the dominant factors in exploring prospective teachers' knowledge for teaching Algebra at the senior high schools?" To answer this question, Principal Component Analysis (PCA) of exploratory factor analysis was conducted to find out the number of factors that constitute prospective teachers' knowledge in Algebra with regard to the teaching of Algebra. PCA is a type of exploratory factor analysis which transforms a number of linearly related variables into a number of unrelated components (Field, 2013). To perform PCA, the Kaiser-Meyer-Olkin (KMO) test for sample adequacy and the Bartlett's Test of sphericity were conducted to ensure the sample size was adequate for the PCA and the correlation between the variables retained for PCA are statistically not equal to zero respectively. Also, the factor loading of each item was analysed in order to determine whether or not there exist other factors that account for prospective teachers' knowledge for teaching high school Algebra. The purpose for conducting this analysis is to find out whether the seven knowledge types of the expanded KAT framework can be confirmed for the case of pre-service teachers.

**Analysis of data for research question two**

Research question two is "How knowledgeable are pre-service Mathematics teachers to teach high school Algebra?" This research question sought to find out the amount of algebraic knowledge pre-service teachers possess in order to ascertain whether or not they are well equipped with algebraic knowledge before they are deployed to the senior high schools to teach. To answer this question, the score for individual items for each respondent was summed and the sum was expressed as a percentage of the total score. Finally, the mean score and standard deviation for all respondents was computed. The mean score represents the average level of algebraic knowledge possessed by pre-service teachers and the standard deviation represents, on average, how much each respondent's algebraic knowledge differs from the average level of algebraic knowledge.

**Analysis of data for research hypothesis**

The research hypothesis tries to determine whether there is a statistically significant difference in the means of pre-service teachers' level of Algebra for teaching knowledge between those who have field-teaching experience and those who have not. The hypothesis states, "There is no significant difference in teachers' knowledge for teaching Algebra between prospective teachers with off campus teaching experience and prospective teachers with on campus teaching experience."

To test this hypothesis, the sum of scores of each respondent was expressed as a percentage of one hundred. These scores were later subjected to the test for normality to test whether or not they are normally distributed. The results from the normality test indicated that the scores were not normally distributed. Therefore, the Mann-Whitney's non-parametric test for difference in means of two independent samples was conducted.

**Chapter Summary**

The prime purpose of this study was to investigate the dominant factors that describe pre-service Mathematics teachers' knowledge for teaching high school Algebra. To achieve this, a cross-section research designed was employed since the study was to be conducted within a specific time frame. The cluster sampling technique was used to sample one hundred and sixty-four pre-service teachers from the target population of this kind of Mathematics teachers in the Central Region. The study also adopted the Expanded KAT instrument developed by Wilmot et al (2018). Principal Component Analysis (PCA), Parallel Analysis (PA), mean, standard deviation and the Mann-Whiney test static were employed in the analyses of data.

# CHAPTER FOUR

# RESULTS AND DISCUSSION

This study aimed at exploring the underlying factors that explain pre-service teachers' knowledge for teaching senior high school Algebra and the level of knowledge these pre-service teachers possess in Algebra. The study employed the cross-sectional research design and the cluster sampling method. The study incorporated one hundred and sixty-four pre-service teachers who were either in their third or final year in their studies for Bachelor of Education in Mathematics.

The results and their interpretations have been presented and categorized according to research questions and hypothesis. In the same vein, the discussion of the results follows in similar categorization.

The chapter ends with a summary of the key findings from the study accompanied with their implications.

## Results from analysis

This section presents the results from the study with each result accompanied with its interpretation in the context of the study. The results are grouped under research questions and research hypothesis.

## Research question one

Research question one states "What are the dominant factors in exploring prospective teachers' knowledge for teaching Algebra at the senior high schools?" To answer this question, a Principal Component Analysis (PCA) of exploratory factor analysis was conducted to extract the underlying factors that underpin pre-service teachers' knowledge for teaching Algebra.

PCA is a type of exploratory factor analysis technique that finds the relationship between cluster of variables and group interrelated variables as factors. Variables that have low correlation with most of the other variables must be taken out of the exploratory factor analysis (Field, 2013). In view of this, since PCA looks for the relationship (correlation) between variables and the items on instrument for data collection were scored 1 for right response and 0 for wrong response, items that most respondents got wrong correlated weakly with the other items. Following Field's (2013) recommendation, items that seventy percent of the respondents wrongly responded to were taken out of the analysis since they have the potential to cause weak relationship among the other items. After taking such items out, forty-five items were retained for analysis. Table 1 lists the items and the percentages of respondents who answered them correctly or incorrectly.

**Table 1: Number of Items Responded Correctly or Wrongly**

| Question No. | Wrong response | | Correct response | |
|---|---|---|---|---|
| | F | % | F | % |
| Q1 * | 101 | 61.6 | 63 | 38.4 |
| Q2 * | 88 | 53.7 | 76 | 46.3 |
| Q3 * | 48 | 29.3 | 116 | 70.7 |
| Q4 * | 95 | 57.9 | 69 | 42.1 |
| Q5 | 147 | 89.6 | 17 | 10.4 |
| Q6 | 120 | 73.2 | 44 | 26.8 |
| Q7 * | 110 | 67.1 | 54 | 32.9 |
| Q8 | 118 | 72.0 | 46 | 28.0 |
| Q9 | 137 | 83.5 | 27 | 16.5 |
| Q10* | 89 | 54.3 | 75 | 45.7 |
| Q11* | 88 | 53.7 | 76 | 46.7 |

**Table 1, Continued**

| | Wrong response | | Correct response | |
|---|---|---|---|---|
| No. | F | % | F | % |
| Q9 | 137 | 83.5 | 27 | 16.5 |
| Q10 * | 89 | 54.3 | 75 | 45.7 |
| Q11 * | 88 | 53.7 | 76 | 46.3 |
| Q12 * | 95 | 57.9 | 69 | 42.1 |
| Q13 | 131 | 79.9 | 33 | 20.1 |
| Q14 * | 114 | 69.5 | 50 | 30.5 |
| Q15 * | 108 | 65.9 | 56 | 34.1 |
| Q16 * | 91 | 55.5 | 73 | 44.5 |
| Q17 | 143 | 87.2 | 21 | 12.8 |
| Q18 * | 83 | 50.6 | 81 | 49.4 |
| Q19 * | 88 | 53.7 | 76 | 46.3 |
| Q20 * | 61 | 37.2 | 103 | 62.8 |
| Q21 * | 106 | 64.6 | 58 | 35.4 |
| Q22 * | 77 | 47.0 | 87 | 53.0 |
| Q23 | 124 | 75.6 | 40 | 24.4 |
| Q24 * | 80 | 48.8 | 84 | 51.2 |
| Q25 * | 51 | 31.1 | 113 | 68.9 |
| Q26 | 142 | 86.6 | 22 | 13.4 |
| Q27 * | 113 | 68.9 | 51 | 31.1 |
| Q28 * | 63 | 38.4 | 101 | 61.6 |
| Q29 * | 52 | 31.7 | 112 | 68.3 |
| Q30 * | 42 | 25.6 | 122 | 74.4 |
| Q31 * | 73 | 44.5 | 91 | 55.5 |
| Q32 * | 66 | 40.2 | 98 | 59.8 |
| Q33 * | 87 | 53.0 | 77 | 47.0 |
| Q34 * | 69 | 42.1 | 95 | 57.9 |
| Q35 * | 85 | 51.8 | 79 | 48.2 |
| Q36 | 131 | 79.9 | 33 | 20.1 |
| Q37 * | 111 | 67.7 | 53 | 32.3 |
| Q38 | 127 | 77.4 | 37 | 22.6 |

**Table 1, continued**

| | Wrong response | | Correct response | |
|---|---|---|---|---|
| No. | F | % | F | % |
| Q39 * | 100 | 61.0 | 64 | 39.0 |
| Q40 * | 101 | 61.6 | 63 | 38.4 |
| Q41 | 132 | 80.5 | 32 | 19.5 |
| Q42 | 151 | 92.1 | 13 | 7.9 |
| Q43 * | 102 | 62.2 | 62 | 37.8 |
| Q44 | 121 | 73.8 | 43 | 26.2 |
| Q45 | 142 | 86.6 | 22 | 13.4 |
| Q46 | 135 | 82.3 | 29 | 17.7 |
| Q47 | 125 | 76.2 | 39 | 23.8 |
| Q48 | 119 | 72.6 | 45 | 27.4 |
| Q49 | 141 | 86.0 | 23 | 14.0 |
| Q50 | 119 | 72.6 | 45 | 27.4 |
| Q51 * | 80 | 48.8 | 84 | 51.2 |
| Q52 | 115 | 70.1 | 49 | 29.9 |
| Q53 * | 75 | 45.7 | 89 | 54.3 |
| Q54 * | 107 | 65.2 | 57 | 34.8 |
| Q55 * | 100 | 61.0 | 64 | 39.0 |
| Q56 * | 111 | 67.7 | 53 | 32.3 |
| Q57 * | 88 | 53.7 | 76 | 46.3 |
| Q58 | 118 | 72.0 | 46 | 28.0 |
| Q59 * | 86 | 52.4 | 78 | 47.6 |
| Q60 * | 102 | 62.2 | 62 | 37.8 |
| Q61 | 128 | 78.0 | 36 | 22.0 |
| Q62 | 130 | 79.3 | 34 | 20.7 |
| Q63 | 116 | 70.7 | 48 | 29.3 |
| Q64 * | 112 | 68.3 | 52 | 31.7 |
| Q65 * | 111 | 67.7 | 53 | 32.3 |
| Q66 * | 97 | 59.1 | 67 | 40.9 |
| Q67 * | 81 | 49.4 | 83 | 50.6 |
| Q68 * | 106 | 64.6 | 58 | 35.4 |

**Table 1, continued**

| | Wrong response | | Correct response | |
|---|---|---|---|---|
| No. | F | % | F | % |
| Q69 | 134 | 81.7 | 30 | 18.3 |
| Q70 * | 110 | 67.1 | 54 | 32.9 |
| Q71 | 140 | 85.4 | 24 | 14.6 |
| Q72 * | 92 | 56.1 | 72 | 43.9 |
| Q73 | 115 | 70.1 | 49 | 29.9 |
| Q74 | 124 | 75.6 | 40 | 24.4 |

* item was retained for analysis
Source: Field survey (2020).

After deleting items that have the potential to cause weak correlation, the next was to test for the adequacy of the sample size used for the factor analysis. Different authors have different "rule of thumb" for determining the adequacy of the sample size (Comrey & Lee, 1992; Kass & Tinsley, 1979; Nunnally, 1978; Tabachnick & Fidell, 2007). One which is common among these authors is that for the sample size to be adequate for factor analysis, there must be ten to fifteen participants per each variable. Thus, following this rule of thumb, there should have been at least four hundred and fifty participants for this study since forty-five items were retained for the analysis. Other empirical studies have also shown different findings contrary to the aforementioned authors (Guadagnoli & Velicer, 1988; MacCallum, Widaman, Zhang, & Hong, 1999). Guadagnoli and Velicer (1988) argued that the most important information in obtaining a suitable factor result is the absolute factor loading of a variable. In their finding, a factor having at least four factor loadings greater than .6 is suitable irrespective of the sample size. In addition to their findings, they asserted that the findings of a factor analysis are suitable and reliable when at least ten factor loadings are .4 or more when the sample size is at least one hundred and fifty.

To determine the suitability of sample size for factor analysis using a single statistic, Kaiser (1970) came out with the Kaiser-Meyer-Olkin (KMO) statistic. The KMO measure of sample adequacy is the ratio of the squared correlation between the items to the square partial correlation between the items. The KMO statistic ranges from 0 to 1 with KMO closer to 1 indicating sample adequacy for factor analysis. Hutcheson and Sofroniou (1999) categorized KMO statistic between .5 and .7 as inadequate, .7 and .8 as adequate, .8 and .9 as very adequate and greater than .9 as excellent. The KMO statistic was computed and Table 2 shows the KMO measure of sample adequacy and the Bartlett's Test of Sphericity.

**Table 2: KMO measure of sample adequacy and the Bartlett's Test of Sphericity**

| | | |
|---|---|---|
| Kaiser-Meyer-Olkin Measure of Sampling Adequacy | | .834 |
| Bartlett's Test of Sphericity | Approx. Chi-Square | 3422.163 |
| | Df | 990 |
| | Sig. | .000 |

Source: Field survey (2020)

Thus, following Hutcheson and Sofroniou (1999) categorization, the KMO statistic indicates the sample size for this study is very adequate for conducting exploratory factor analysis.

Moreover, the Bartlett's Test for sphericity from the test is also significant which means that the correlation between the items retained for the analysis are statistically different from zero. Satisfying both KMO test for sample adequacy and Bartlett's test for Sphericity means that the data is suited for exploratory factor analysis and the analysis' results are credible.

Table 3 from the exploratory factor analysis is utilized to determine the number of underlying components that account for pre-service teachers' understanding of Algebra instruction. Table 3 gives account of the eigenvalues and the percentage variance each factor contributes to these teachers' knowledge Algebra teaching. It also provides the cumulative percentage variance. The smaller the eigenvalue for a given factor, the smaller the variance explained by that factor. The Kaiser criterion was used in identifying the number of components. The Kaiser's criterion states that all factors (components as used in the table) with eigenvalues greater than 1 must be retained. Thus, following Kaiser's criterion, from Table 3, twelve factors explain prospective teachers' teaching knowledge in high school Algebra. The twelve factors as proposed by the Kaiser's criterion explain 65.725% of the knowledge pre-service teachers possess for teaching high school Algebra. Table 3 shows the eigenvalues of each component and their respective total variance explained.

**Table 3: The Total Variance Explained by Factors**

| | Initial Eigenvalues | | | Extraction Sums of Squared Loadings | | |
|---|---|---|---|---|---|---|
| Component | Eigenvalues | % of Variance | Cumulative % | Total | % of Variance | Cumulative % |
| 1 | 12.054 | 26.787 | 26.787 | 12.054 | 26.787 | 26.787 |
| 2 | 2.403 | 5.340 | 32.127 | 2.403 | 5.340 | 32.127 |
| 3 | 2.242 | 4.982 | 37.109 | 2.242 | 4.982 | 37.109 |
| 4 | 1.880 | 4.179 | 41.288 | 1.880 | 4.179 | 41.288 |
| 5 | 1.750 | 3.889 | 45.177 | 1.750 | 3.889 | 45.177 |
| 6 | 1.562 | 3.471 | 48.647 | 1.562 | 3.471 | 48.647 |
| 7 | 1.504 | 3.343 | 51.990 | 1.504 | 3.343 | 51.990 |

**Table 3, continued**

| | Initial Eigenvalues | | | Extraction Sums of Squared Loadings | | |
|---|---|---|---|---|---|---|
| Component | Eigenvalues | % of Variance | Cumulative % | Total | % of Variance | Cumulative % |
| 8 | 1.445 | 3.210 | 55.201 | 1.445 | 3.210 | 55.201 |
| 9 | 1.387 | 3.081 | 58.282 | 1.387 | 3.081 | 58.282 |
| 10 | 1.202 | 2.670 | 60.952 | 1.202 | 2.670 | 60.952 |
| 11 | 1.092 | 2.428 | 63.380 | 1.092 | 2.428 | 63.380 |
| 12 | 1.056 | 2.346 | 65.725 | 1.056 | 2.346 | 65.725 |
| 13 | .961 | 2.136 | 67.862 | | | |
| 14 | .930 | 2.067 | 69.929 | | | |
| 15 | .893 | 1.984 | 71.913 | | | |
| 16 | .850 | 1.889 | 73.802 | | | |
| 17 | .763 | 1.696 | 75.498 | | | |
| 18 | .742 | 1.650 | 77.148 | | | |
| 19 | .730 | 1.622 | 78.770 | | | |
| 20 | .695 | 1.544 | 80.314 | | | |
| 21 | .649 | 1.442 | 81.756 | | | |
| 22 | .631 | 1.402 | 83.158 | | | |
| 23 | .601 | 1.335 | 84.492 | | | |
| 24 | .567 | 1.261 | 85.753 | | | |
| 25 | .549 | 1.219 | 86.972 | | | |
| 26 | .542 | 1.205 | 88.177 | | | |
| 27 | .485 | 1.078 | 89.255 | | | |
| 28 | .469 | 1.043 | 90.298 | | | |
| 29 | .410 | .910 | 91.209 | | | |
| 30 | .395 | .877 | 92.086 | | | |
| 31 | .366 | .814 | 92.899 | | | |
| 32 | .354 | .786 | 93.686 | | | |
| 33 | .324 | .720 | 94.405 | | | |
| 34 | .303 | .674 | 95.080 | | | |
| 35 | .279 | .621 | 95.701 | | | |
| 36 | .267 | .594 | 96.294 | | | |
| 37 | .256 | .569 | 96.863 | | | |
| 38 | .235 | .521 | 97.384 | | | |
| 39 | .220 | .489 | 97.873 | | | |
| 40 | .203 | .451 | 98.324 | | | |
| 41 | .173 | .385 | 98.710 | | | |
| 42 | .159 | .354 | 99.063 | | | |
| 43 | .155 | .343 | 99.407 | | | |
| 44 | .146 | .326 | 99.732 | | | |

**Table 3, continued**

| Component | Initial Eigenvalues | | | Extraction Sums of Squared Loadings | | |
|---|---|---|---|---|---|---|
| | Eigenvalues | % of Variance | Cumulative % | Total | % of variance | Cumulative % |
| 45 | .120 | .268 | 100.000 | | | |

Extraction Method: Principal Component Analysis
Source: Field Survey (2020)

However, the conceptual framework of this study stipulates seven factors that account for knowledge for teaching high school Algebra. To further ascertain the number of factors that need to be retained, the scree plot was used. This is a graph of eigenvalues against the number of factors and it is used as a diagnostic tool in determining the number of factors to be retained in a PCA. The number of factors to be retained is found at the point of inflexion. This is the point beyond which the slope of the curve begins levels off. Figure 5 shows the scree plot.

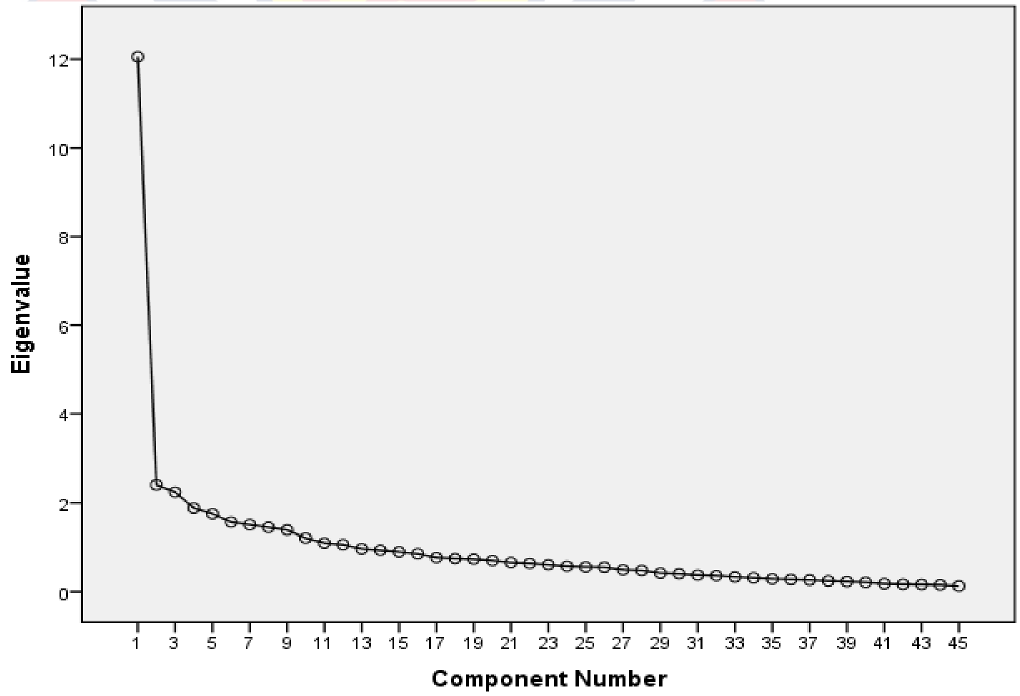



**Figure 5: Scree Plot from Principal Component Analysis (Field survey, 2020)**

From Figure 5, there are two points of inflexions. The first point of inflexion is found at the fourth factor indicating that three factors need to be extracted. The second point of inflexion at which a final plateau is reached is found at the tenth factor indicating nine factors need to be retained.

Relying only on the information provided by the scree plot gives room for the use of personal discretion to determine the number of components to retain. To avoid this, a further analysis called the Parallel Analysis (PA) was performed. This is a method of objectively determining the number of factors to be retained in a PCA. The PA works by generating random eigenvalues according to the number of variables and the sample size of the original data (Franklin, Gibson, Robertson, Pohlmann, & Fralish, 1995). The number of factors to be retained is obtained by comparing the randomly generated eigenvalues in the PA with the initial eigenvalues generated in the PCA. Where the eigenvalues in the PA is greater than that in the PCA, we maintain the preceding factors.

Table 4 shows the eigenvalues from both PA and PCA.

**Table 4: Eigenvalues for Parallel Analysis and PCA**

| Factors number | Eigenvalues from Parallel Analysis | Eigenvalues from PCA |
|---|---|---|
| 1 | 2.190924 | 12.054 |
| 2 | 2.050655 | 2.403 |
| 3 | 1.945066 | 2.242 |
| 4 | 1.857462 | 1.88 |
| 5 | 1.785724 | 1.75 |
| 6 | 1.711757 | 1.562 |
| 7 | 1.646162 | 1.504 |

**Table 4, continued**

| Factors number | Eigenvalues from Parallel Analysis | Eigenvalues from PCA |
|---|---|---|
| 8 | 1.584246 | 1.445 |
| 9 | 1.523574 | 1.387 |
| 10 | 1.468411 | 1.202 |
| 11 | 1.418456 | 1.092 |
| 12 | 1.368704 | 1.056 |
| 13 | 1.317626 | .961 |
| 14 | 1.274294 | .93 |
| 15 | 1.229584 | .893 |
| 16 | 1.182974 | .85 |
| 17 | 1.139984 | .763 |
| 18 | 1.09944 | .742 |
| 19 | 1.061727 | .73 |
| 20 | 1.025364 | .695 |
| 21 | .986665 | .649 |
| 22 | .945562 | .631 |
| 23 | .912693 | .601 |
| 24 | .876928 | .567 |
| 25 | .841449 | .549 |
| 26 | .812085 | .542 |
| 27 | .776634 | .485 |
| 28 | .747067 | .469 |
| 29 | .718435 | .41 |
| 30 | .685021 | .395 |
| 31 | .652354 | .366 |
| 32 | .624403 | .354 |
| 33 | .594642 | .324 |
| 34 | .563857 | .303 |
| 35 | .53673 | .279 |
| 36 | .509805 | .267 |

**Table 4, continued**

| Factors number | Eigenvalues from Parallel Analysis | Eigenvalues from PCA |
|---|---|---|
| 37 | .47985 | .256 |
| 38 | .452706 | .235 |
| 39 | .42567 | .22 |
| 40 | .402802 | .203 |
| 41 | .37434 | .173 |
| 42 | .345775 | .159 |
| 43 | .316737 | .155 |
| 44 | .287122 | .146 |
| 45 | .248536 | .12 |

Source: Field survey (2020)

To retain factors in PCA using PA, the factors in PCA with eigenvalues greater than eigenvalues in PA are retained (Franklin et al., 1995). From Table 4 the first four factors have their PCA eigenvalues being greater than that from PA. This means that the four factors must be retained from the PCA which indicates there are four factors that characterise the knowledge base of pre-service teachers for teaching high school Algebra.

To understand the nature of the extracted factors that explain the knowledge of pre-service teachers for teaching high school Algebra, the Rotated Component Matrix from the PCA was used. Table 5 gives the items that loaded uniquely on a factor with their respective factor loadings which have been sorted according to their size. Table 5 serves as a guide in the development of themes for naming extracted factors since it provides the items that loaded distinctly onto a component.

Table 5 shows the Rotated Component Matrix.

**Table 5: Rotated Component Matrix from the PCA**

| | Component | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| Q51 | .769 | | | | | | | | | | | |
| Q66 | .716 | | | | | | | | | | | |
| Q22 | .702 | | | | | | | | | | | |
| Q18 | .696 | | | | | | | | | | | |
| Q32 | .651 | | | | | | | | | | | |
| Q11 | .641 | | | | | | | | | | | |
| Q20 | .637 | | | | | | | | | | | |
| Q56 | .620 | | | | | | | | | | | |
| Q4 | .611 | | | | | | | | | | | |
| Q12 | .610 | | | | | | | | | | | |
| Q27 | .601 | | | | | | | | | | | |
| Q67 | .589 | | | | | | | | | | | |
| Q72 | .579 | | | | | | | | | | | |
| Q35 | .561 | | | | | | | | | | | |
| Q29 | .558 | | | | | | | | | | | |
| Q53 | .549 | | | | | | | | | | | |
| Q1 | .466 | | | | | | | | | | | |
| Q68 | .443 | | | | | | | | | | | |
| Q2 | | .700 | | | | | | | | | | |
| Q3 | | .630 | | | | | | | | | | |
| Q25 | .424 | .588 | | | | | | | | | | |
| Q33 | | .421 | | | | | | | | | | |
| Q15 | | | .716 | | | | | | | | | |
| Q21 | .429 | | .604 | | | | | | | | | |
| Q59 | | | .581 | | | | | | | | | |
| Q60 | | | .496 | | | | | | | | | |
| Q28 | | | | .786 | | | | | | | | |
| Q24 | | | | .563 | | | | | | | | |
| Q10 | | | | | .635 | | | | | | | |
| Q31 | | | | | .606 | | | | | | | |
| Q57 | | | | | .584 | | | | | | | |
| Q64 | | | | | | -.779 | | | | | | |
| Q54 | | | | | | | -.805 | | | | | |
| Q30 | | | | | | | .430 | | | | | |
| Q43 | | | | | | | | .790 | | | | |
| Q55 | | | | | | | | -.583 | | | | |
| Q39 | | | | | | | | | .402 | | | |
| Q40 | | | | | | | | | .811 | | | |

**Table 5, continued**

| | Component | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| Q37 | | | | | | | | | | -.718 | | |
| Q34 | | | | | | | | | | .597 | | |
| Q16 | | | | | | | | | | | .718 | |
| Q19 | | | | | | | | | | | .426 | |
| Q70 | .437 | | | | | | | | | | | -.596 |
| Q7 | | | | | | | | | | | | .487 |
| Q65 | | | | | | | | | | | | .438 |

Extraction Method: Principal Component Analysis.
Rotation Method: Varimax with Kaiser Normalization.
a. Rotation converged in 21 iterations.
Source: Field Survey (2020)

From Table 5, twenty items loaded on Component 1, four on Component 2, four on Component 3 and two on Component 4. Again, from Table 5, Item 25 loaded on both Component 1 and Component 2. Also, Item 21 loaded on both Component 1 and Component 3. Such items cannot be used to describe the nature of the factor extracted since they do not uniquely load on one component (Guadagnoli & Velicer, 1988). Thus, they are to be excluded. Table 6 shows the items categorization for each factor extracted from the PCA.

**Table 6: Item Categorization for each Factor**

| Component 1 | Component 2 | Component 3 | Component 4 |
|---|---|---|---|
| Q51 (TK) | Q02 (SK) | Q15 (SK) | Q28 (AK) |
| Q66 (TK) | Q03 (SK) | Q59 (TK) | Q24 (TK) |
| Q22 (AK) | Q33 (SK) | Q60 (SK) | |
| Q18 (SK) | | | |
| Q32 (SK) | | | |
| Q11 (SK) | | | |

**Table 6, continued**

| Component 1 | Component 2 | Component 3 | Component 4 |
|---|---|---|---|
| Q20 (SK) | | | |
| Q56 (TK) | | | |
| Q04 (AK) | | | |
| Q12 (AK) | | | |
| Q27 (AK) | | | |
| Q67 (TK) | | | |
| Q72 (AK) | | | |
| Q35 (TK) | | | |
| Q29 (TK) | | | |
| Q53 (SK) | | | |
| Q01 (SK) | | | |
| Q68 (AK) | | | |

TK-Teaching Knowledge AK- Advance Algebra Knowledge SK-School Algebra Knowledge

Source: Field survey (2020)

From Table 6, eighteen items loaded uniquely on Component 1, three items on Component 2 and 3 and two items loaded uniquely on Component 4. However, for an extracted component to be labelled as a factor, there must be at least three items loading uniquely on the component (O'Rourke & Hatcher, 2013). Based on this, Component 4 cannot be labelled as a factor. Therefore, pre-service Mathematics teachers' knowledge for teaching high school Algebra, from this study, is characterized by three factors.

Items that loaded on Component 1 are from all the three foundational knowledge types: knowledge of School algebraic content, knowledge on Advanced Algebra and knowledge in the teaching Algebra, as proposed by McCrory et al. (2012). Basing on the conceptual framework that underpins this study and the nature of items that loaded distinctly on this component, Component 1 is labelled the Pedagogical Content Knowledge in Teaching Algebra (PCKTA). Thus, the first Factor, from this study, that characterises the knowledge of these teachers is the Pedagogical content knowledge in teaching Algebra. Component 2 consists of only School Algebra knowledge items loading distinctly on it and it is christened the School Algebra knowledge. Component 3 has two items from School Algebra knowledge and one item from Algebra teaching knowledge. This component is labelled School Algebra Teaching knowledge.

Also, it is evident from Table 6 with reference to Components 1 and 3 that the knowledge of Algebra for teaching lies in a continuum. That is, the intersectional regions of the three knowledge types as propounded in the KAT conceptualisation cannot be fuzzy. For instance, Component 1 has six items each from School Algebra knowledge (SK), Advanced Algebra knowledge (AK) and Teaching knowledge (TK) loading on it and Component 3 has two items from SK and one TK item loading uniquely on it.

Therefore, the results from PCA together with the PA indicate that the algebraic knowledge of prospective high school teachers is characterized by three factors – the School Algebra knowledge which is the knowledge of algebraic content as stipulated in the senior high curriculum, School Algebra Teaching knowledge which is the knowledge that gives one the affordance to efficiently and effectively teach high school Algebra to enhance understanding among diverse learners and lastly Pedagogical content knowledge for teaching high school Algebra which is Shulman's (1986) Pedagogical Content Knowledge, specifically, in the domain of Algebra.

Figure 6 depicts the key characteristics of pre-service Mathematics teachers' mastery of senior high school Algebra.

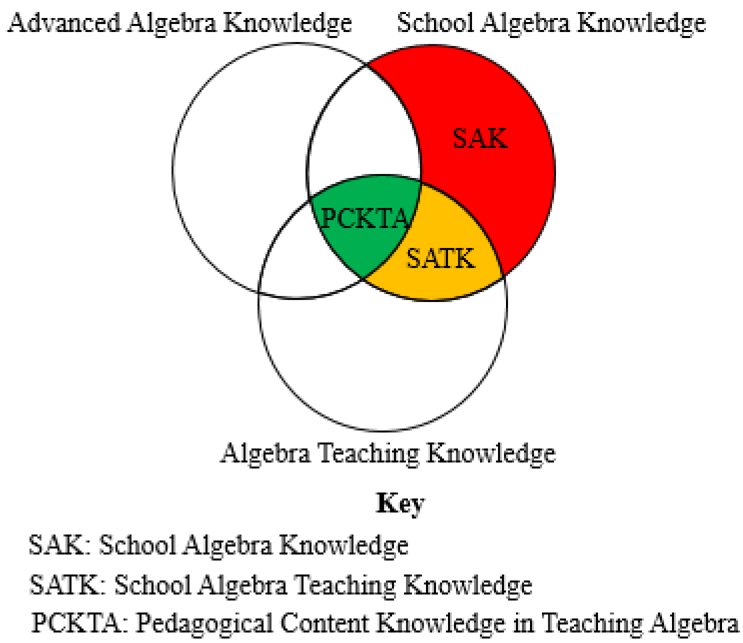



**Figure 6: Factors that characterize the knowledge of pre-service teachers for teaching senior high school Algebra**

Source: Field Survey (2020)

**Research question two**

Research question two sought to assess prospective teachers' algebraic knowledge across KAT framework. To achieve this, the sum of each respondent's score in all the items on the instrument were converted to a one-hundred-point scale. The mean and standard deviation were computed.

Table 7 shows the mean score of pre-service teachers' scores on the Algebra Teaching, Advanced Algebra and School Algebra Knowledge types. The mean and standard deviation of pre-service teachers' Knowledge of Algebra for Teaching (KAT) which is the sum of scores of all the items on the instrument was included in Table 7.

**Table 7: Mean and Standard Deviation of Pre-service teachers' score on KAT**

| | N | Mean | Std. Deviation |
|---|---|---|---|
| Algebra Teaching Knowledge | 164 | 36.1951 | 16.58918 |
| Advanced Algebra Knowledge | 164 | 30.2744 | 16.69147 |
| School Algebra Knowledge | 164 | 40.7696 | 21.16352 |
| Overall KAT | 164 | 36.3876 | 16.98857 |

Source: Field survey (2020)

From Table 7, the mean scores of pre-service teachers on all the strands of knowledge types of the KAT framework are: Algebra Teaching Knowledge (M= 36.1951, SD = 16.58918), Advanced Knowledge (M = 30. 2744, SD = 16.69147) and School Algebra Knowledge (M = 40.7696, SD = 21. 16352) are low since none of them is up to 50 which is half of the total score. This low score also translated in the overall knowledge of Algebra for Teaching (M = 36.3876, SD = 16.98857). The findings indicate that pre-service teachers possess low level of understanding in the TK, AK, SK and the overall KAT.

Though results from Table 7 indicate that pre-service teachers exhibited weaker level of knowledge, analysis of some individual items under each of these types of knowledge gives interesting revelation. Item 67 which happened to load uniquely on the Pedagogical Content Knowledge in Algebra is one the items. The item states;

Consider the following mathematical topics:

i. Composition of functions
ii. One-to-one functions
iii. Inverse functions
iv. Domain and range of functions

Which of the following orders could be used to teach these topics in a rigorous advanced algebra class?

**A.** ii, i, iii, iv
**B.** ii, iii, iv, i
**C.** iv, ii, iii, i
**D.** They can be taught in any order.

This item was designed to evaluate pre-service teachers' knowledge of the prerequisite knowledge for each of the topics listed. It was assumed that knowing the topic which is the prerequisite to the other will enable them to rightly sequence these topics when they are presented with the opportunity to teach these topics under functions.

Pre-service teachers demonstrated evidence of limited knowledge in this item. Figure 7 shows the performance of pre-service teachers in Item 67.

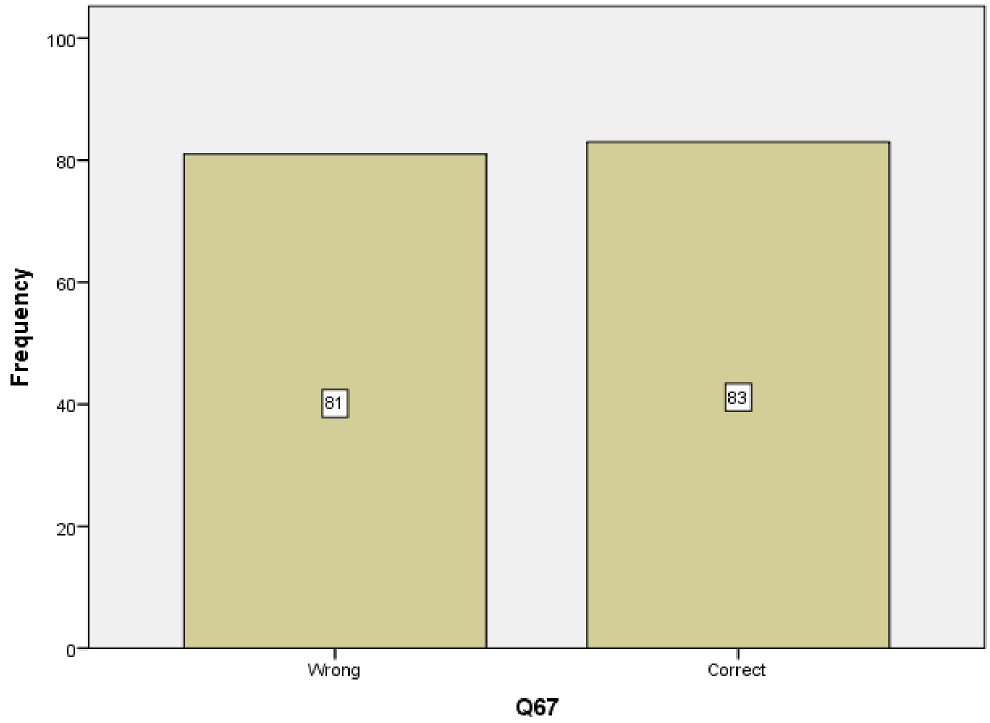



**Figure 7: Responses of Pre-service teachers on Item 67**

Source: Field survey (2020)

Item 67 loaded uniquely on the Pedagogical Content Knowledge in Algebra which pre-service teachers are expected to possess to teach Algebra. From Figure 7, 83 (50.6%) out the pre-service teachers answered this item correctly whiles 81 (49.4%) of them wrongly answered it.

The correct answer to Item 67 is option C because the domain and range of a function is a prerequisite knowledge to composition of functions, one-to-one functions and inverse of a function. For instance, given two functions $f$ and $h$, to compose $f$ of $h$ given $x$ is in the domain of $h$, then the image of $x$ under $h$, $h(x)$, must be in the domain of $f$. If $h(x)$ happens not to be in the domain of $f$, the composed function $f(h(x))$ is undefined. In order to grasp that the domain of a function becomes the range of the inverse of the same function, it is also necessary to understand what a function's domain and range are. Again, knowledge in the domain and range of a function is needed to understand that for a function to be one-to-one, each element in the domain maps unto only one element in the codomain. For this reason, the domain and range of functions must to taught first given the set of topics.

The next topic in the list of topics to be treated after the domain and range of functions is one-to-one function since a function can have an inverse only when the function is one-to-one. Therefore, composition of a function becomes the last topic to be considered in the list provided making the order in which these topics should be sequenced to be domain and range of a function, one-to-one functions, inverse of a function and composition of a function.

The results show almost equal number of pre-service teachers answered the item wrongly as correctly. The wrong responses of pre-service teachers to this item suggests that they possess a limited knowledge as to how these topics are related and which one needs to be treated in order to provide the prerequisite knowledge for the subsequent topic. It was expected the pre-service teachers exhibit a higher level of knowledge in this item since, at the time of data collection, they have been taken through the required courses that are to equip them the knowledge to rightly answer this item.

Another item which revealed a surprising result is Item 2. This item required pre-service teachers to algebraically write a model to represent a situation. The item states;

**Item 2.** Timothy's age in 15 years will be twice what it was 5 years ago. If $t$ represents Timothy's age now, write the equation that models this situation.

Item 2 loaded uniquely on the School Algebra Knowledge in this study. The School Algebra Knowledge is the knowledge of Algebra as stipulated in the senior high school integrated Mathematics curriculum. This is the content of Algebra that pre-service teachers are to teach when they are deployed to the senior high schools to teach. Surprisingly, most the pre-service teachers who answered this item got it wrong and this is alarming. Figure 8 shows the performance of pre-service teachers on Item 2.

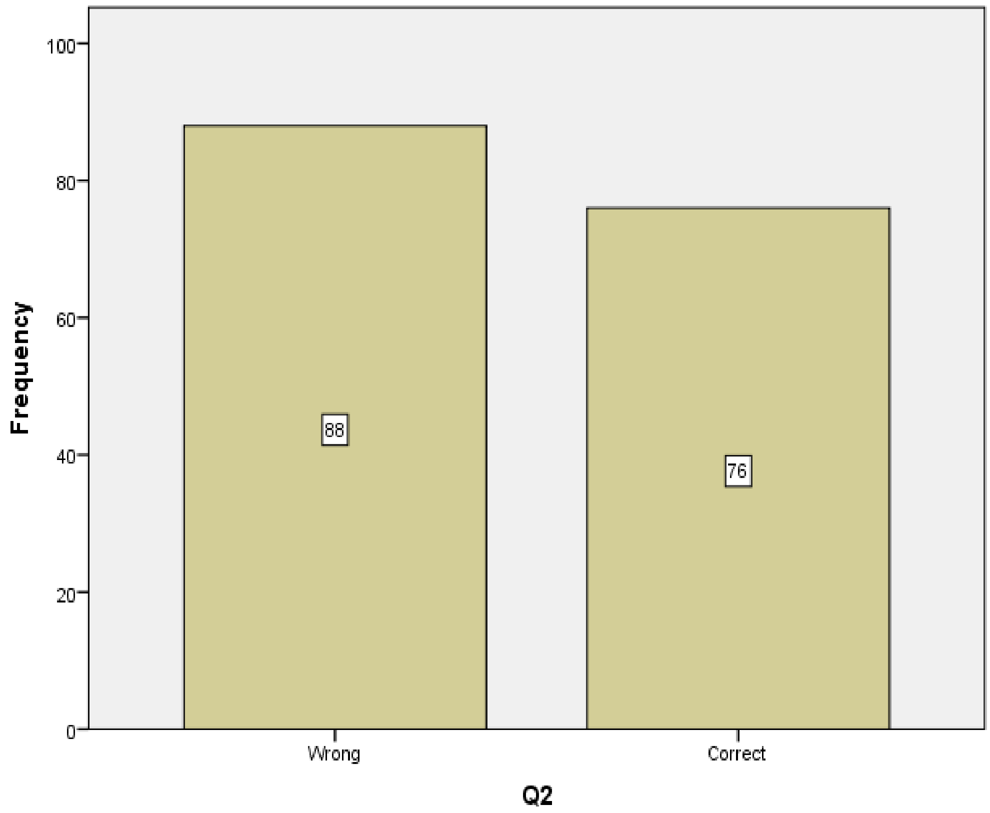
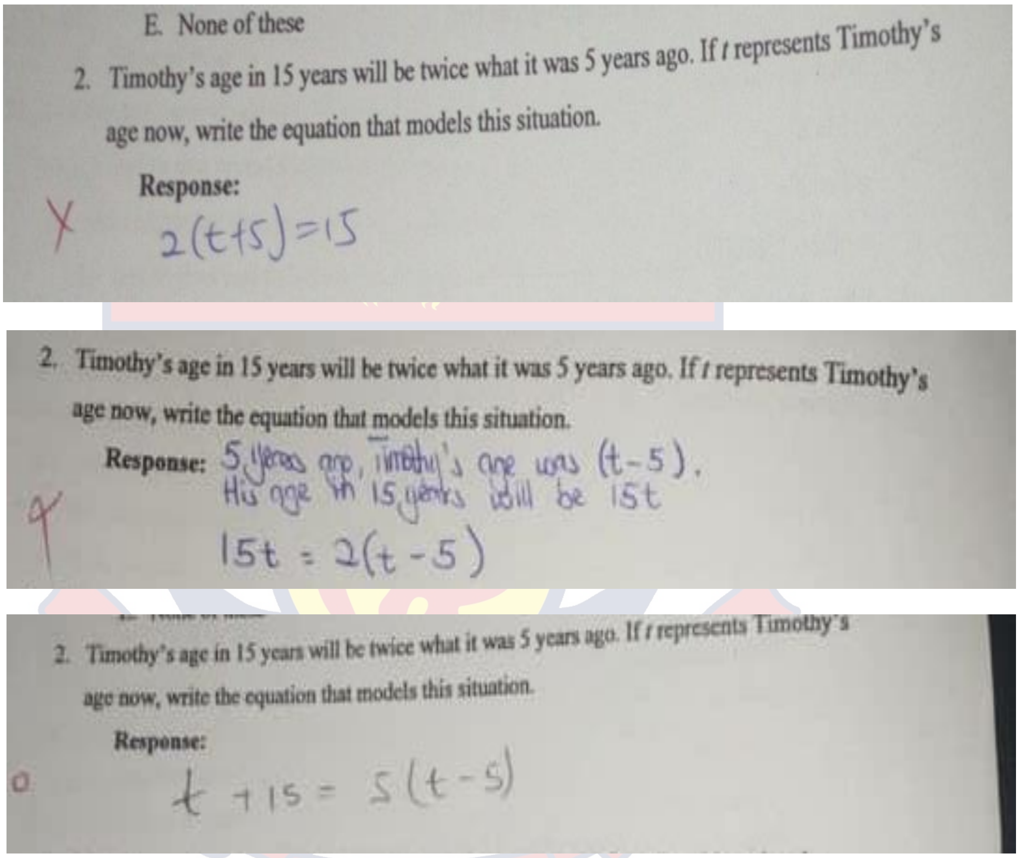



**Figure 8: Pre-service teachers' responses on Item 2**

Source: Field survey (2020)

Figure 8 shows that 88 (53.7%) out of 164 of the pre-service teachers wrongly answered this item. This suggests that, the majority of these teachers who participated in this study lack knowledge of modelling algebraic situation which is a knowledge they need to possess in order to teach senior high students how to algebraically represent situations with an equation.

Going through the equations written for this situation by pre-service teachers who participated in this study, it was revealed that the source of error in their answers to this item resides in either their difficulty in modelling the statement “in 15 years”, “twice”, “5 years ago” or the combination of any of these three statements. Figure 9 shows some of the solutions presented by pre-service teachers.


E. None of these

2. Timothy's age in 15 years will be twice what it was 5 years ago. If *t* represents Timothy's age now, write the equation that models this situation.

Response: $2(t+5)=15$

2. Timothy's age in 15 years will be twice what it was 5 years ago. If *t* represents Timothy's age now, write the equation that models this situation.

Response: 5 years ago, Timothy's age was $(t-5)$.
His age in 15 years will be $15t$
$15t = 2(t-5)$

2. Timothy's age in 15 years will be twice what it was 5 years ago. If *t* represents Timothy's age now, write the equation that models this situation.

Response: $t + 15 = 5(t-5)$



**Figure 9: Sample responses of pre-service teachers to Item 2**

Source: Field survey (2020)

To write the equation to model this situation, pre-service teachers were to model Timothy’s age in 15 years and 5 years ago given his present age to be $t$. In 15 years, Timothy’s age will be $t+15$ and his age 5 years ago will also be $t-5$. Timothy’s age in 15 years will be twice his age 5 years ago means his age in 15 years is two times his age 5 years ago. Hence the right equation expected to be written for this situation was $t + 15 = 2(t - 5)$.

The performance of pre-service teachers in Item 2 is alarming because, this is a concept they were taught at both the Junior High School (JHS) and at the Senior High School (SHS). Therefore, it was expected their understanding in modelling an algebraic situation should have been strengthened by this time where they are almost done with their training as teachers of senior high school Mathematics in the university.

Item 60 which was designated School Algebra item and loaded distinctly on the Factor 3 (School Algebra Teaching Knowledge) is one of the items that gave astonishing results. This question states;

Which relation is a function?

A. $\{(-1,3), (-2,6), (0,0), (-2,2)\}$

B. $\{(-2,-2), (0,0), (1,1), (2,2)\}$

C. $\{(4,0), (4,1), (4,2), (4,3)\}$

D. $\{(7,4), (8,8), (10,8), (10,10)\}$

E. $\{(7,-4), (8,-8), (-10,8), (-10,-10)\}$

Primarily, this item was to test pre-service Mathematics teachers' knowledge on differentiating between Relations, Mapping and Functions. In Mathematics, relation can exist between elements of two sets (the domain and codomain) with each element in the domain having or not having a corresponding image or images in the codomain. For a relation to be a Mapping, it becomes necessary for each element in the domain to have at least a corresponding image in the codomain. However, in Functions each element of the domain must have one and only one image in the codomain. By this definition of a function, no two ordered pair must have their first coordinate to be the same when the function is presented as ordered pairs. This makes the correct answer to Item 60 to be option B which is $\{(-2,-2), (0,0), (1,1), (2,2)\}$

Surprisingly, this item happened to be one that pre-service Mathematics teachers performed poorly on. Figure 10 shows the number of students who rightly answered Item 60 rightly and wrongly.

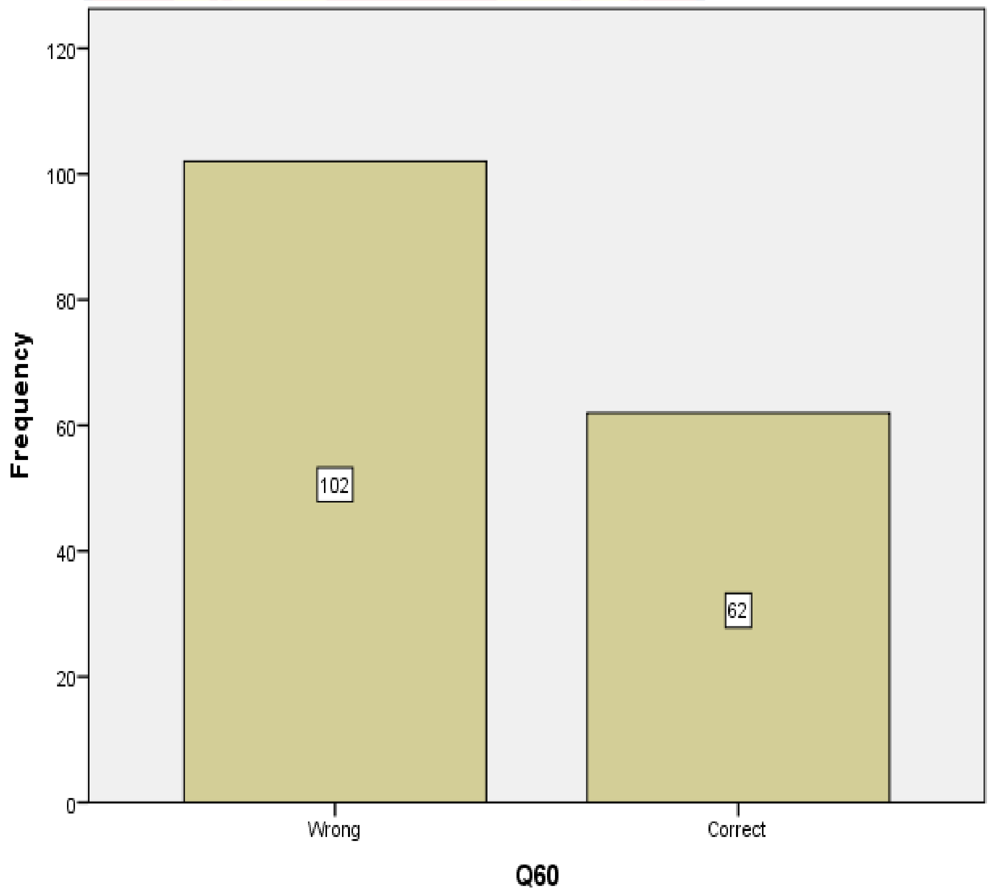



**Figure 10: Pre-service teachers' response to Item 60**

Source: Field survey (2020)

From Figure 10, 102 (62.20%) out of the 164 pre-service teachers got Item 60 wrong. This indicates that, even though pre-service Mathematics teachers frequently deal with functions, there is a gap in their knowledge with regards to basic knowledge in Relations, Mappings and Functions, particularly in identifying and differentiating them.

Also, Item 59 which is an item that loaded uniquely on Factor 3 (School Algebra Teaching knowledge) in this study revealed pre-service teachers' limited knowledge in the teaching of Algebra. This item states;

Which of the following (taken by itself) would give substantial help to a student who wants to expand $(x + y + z)^2$ ?

i. See what happens in an example, such as $(3 + 4 + 5)^2$.

ii. Use $(x + y + z)^2 = ((x + y) + z)^2$ and the expansion of $(a + b)^2$.

iii. Use the geometric model shown below.

| | $x$ | $y$ | $z$ |
|---|---|---|---|
| $x$ | $x^2$ | $xy$ | $xz$ |
| $y$ | $xy$ | $y^2$ | $yz$ |
| $z$ | $xz$ | $yz$ | $z^2$ |

A. ii only
B. iii only
C. i and ii only
D. ii and iii only
E. i, ii and iii

Item 59 aims to evaluate pre-service Mathematics teachers' knowledge of methods for improving students' grasp of the expansion of the square of an algebraic trinomial. Among the options provided are a squared numerical trinomial which will result in a single value, a squared algebraic trinomial which has been grouped into the square of two algebraic terms and a geometric model.

Figure 11 shows pre-service teachers' performance on Item 59.

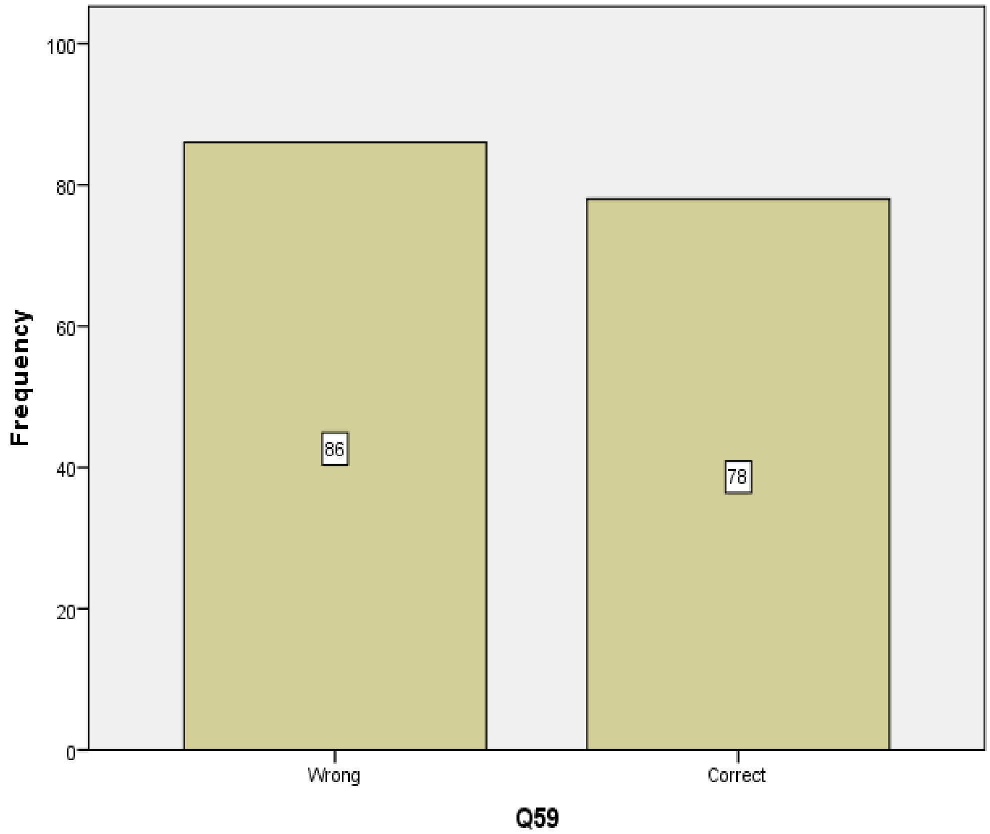



**Figure 11: Pre-service teachers' responses to Item 59**

Source: Field survey (2020)

From Figure 11, 78 (47.56%) of 164 pre-service teachers answered this item correctly and 86 (52.44%) of them answered the item wrongly. This indicates a gap in knowledge in pre-service teachers' knowledge in the appropriate ways to teach expansion of squared algebraic trinomials.

From the options presented in Item 59 as the approaches to be used to teach expansion of a squared trinomials, it is obvious that the geometric model which involves the use of Algebra tiles will be the first best option to use in order to enhance understanding among students. However, at the senior high school, with reference the Piaget's (1983) (as cited in Lefa, 2014) stages of cognitive development, students are at the Formal operational stage where students can reason logically and engage in abstract thinking. At this stage in cognitive development, the concrete materials are needed for the student to establish abstract relationship. Therefore, teaching students at this level does not only require the use of concrete materials but also projection into abstraction after the use of concrete evidence. Hence, in assisting students to expand the squared trinomial, it will be necessary to also use algebraic methods of expansion by grouping the trinomial into two terms like $(x + y + z)^2 = ((x+ y) + z)^2$. With students' prior knowledge in the expansion of squared binomials like $(a + b)^2$, the teacher can assist learners to understand the expansion of $(x + y + z)^2$ after the use of the concrete materials like Algebra tiles to engage in the geometric model. Therefore, the correct option for Item 59 is option D since substituting numerical figures into the squared trinomial for instance $(3 + 4 + 5)^2$ will only give a single figure and will not show the relationship between the individual terms of the trinomial.

Item 15 is another item which pre-service teachers performed poorly on. Item 15 states;

**Item 15.** The major sectorial angle of a circle with radius 14cm is 270°. If the sector is folded to form a cone, find the surface area of the cone.

A. 460.0 cm⍰2⍰

B. 460.8 $cm^2$

C. 461.0 $cm^2$

*D.* 461.8 $cm^2$

Figure 12 shows the performance of pre-service teachers on Item 15

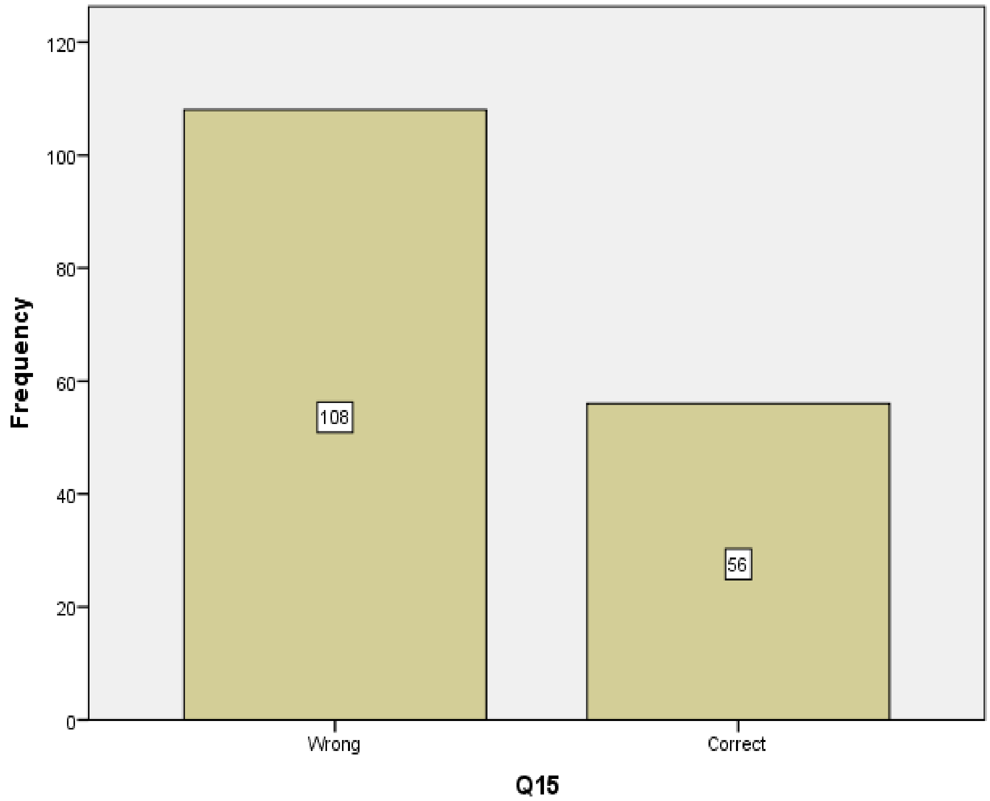



**Figure 12: Pre-service teachers' responses on Item 15**

Source: Field survey (2020)

From Figure 12, 56 representing 34.1% of the pre-service answered this item correctly. The main concept needed to correctly answer this item is the area of the sector from which the cone was formed is equal to surface area of a cone. Using this idea, the right answer to this item is option D. These teachers might have lacked the knowledge that the area of a sector is equal to the surface area of the resulting cone formed from the sector.

Item 15 loaded uniquely on the School Algebra Teaching Knowledge in this study. Possession of this knowledge, as described in the conceptual framework, "allows teachers to teach Algebra in a fluid manner to enhance understanding in diverse group of learners" (Wilmot, 2016, p. 23). Thus, a teacher with this type of knowledge type not understand the concepts of the high school Algebra but also is able to teach these concepts to the understanding of students. This, the teacher cannot achieve without delving into his cognitive kit tool of high school Algebra knowledge. Therefore, if pre-service teachers have difficulty in answering such items, then it calls for critical attention to be paid to the developing of teachers at the universities.

This study, therefore, has revealed that pre-service teachers of Mathematics who participated in the study possess low level of knowledge in the high school Algebra, the advanced Algebra and knowledge of how to teach these Algebra concepts. This weak knowledge was also showed in the analysis of some individual items. These results are alarming because majority of these pre-service teachers (102 out of 164) were at the final semester of their studies in their training as senior high school Mathematics teachers. This cohort of pre-service teachers has also undergone one academic semester field teaching for their Off Campus Teaching Practice (Off-CTP) and were expected to have demonstrated a good level of knowledge in Algebra and how to teach it. The remaining number of pre-service teachers (62 out of 164) were, at the time of this study, at the end of their third year of studies at the university. These teachers will be deployed to the senior high school to teach for one academic semester for their off-CTP. However, the third-year pre-service teachers were, at the time of this study, being taken through teaching practicum which was to prepare them for the off-CTP the following semester. In addition, both groups of pre-service teachers at the time of the study have taken all the required courses needed to equip them well enough to teach at the senior high school. Therefore, they were expected to have demonstrated a higher level of knowledge than revealed in this study.

**Research hypothesis**

The research hypothesis aimed at investigating whether there exists a remarkable difference in the sub-knowledge strands of the Knowledge of KAT and the overall KAT among pre-service teachers who have experience in Off-Campus Teaching Practice (Off-CTP) and those who have no experience in Off-CTP. The test for normality is very crucial in testing for the difference in means among two independent groups.

Table 8 gives information on the normality of scores from each sub-knowledge type of the KAT framework and the overall KAT.

**Table 8: Test of Normality for sub-knowledge types of the Knowledge of Algebra for Teaching**

| | Kolmogorov-Smirnov[a] | | | Shapiro-Wilk | | |
|---|---|---|---|---|---|---|
| | Statistic | Df | Sig. | Statistic | Df | Sig. |
| Teaching Knowledge | .116 | 164 | .000 | .951 | 164 | .000 |
| Advanced Algebra Knowledge | .112 | 164 | .000 | .962 | 164 | .000 |
| School Algebra Knowledge | .089 | 164 | .003 | .962 | 164 | .000 |
| Overall KAT | .130 | 164 | .000 | .936 | 164 | .000 |

a. Lilliefors Significance Correction
Source: Field survey (2020)

A critical look at Table 8 shows that the normality test for all the sub-knowledge types of the knowledge of Algebra for teaching and the overall KAT are significant (p-value = .000). This means that the scores for TK, AK, SK and the overall KAT are not normally distributed. In view of this, the Mann-Whitney non-parametric test for difference in mean among two independent samples was conducted.

Table 9 displays the descriptive statistics for each of the sub-knowledge types.

**Table 9: Descriptive Statistics for the Sub-knowledge Types**

| | Have you done your teaching practice? | N | Mean Rank | Sum of Ranks |
|---|---|---|---|---|
| Teaching Knowledge | Yes | 102 | 100.58 | 10259.00 |
| | No | 62 | 52.76 | 3271.00 |
| | Total | 164 | | |
| Advanced Algebra Knowledge | Yes | 102 | 100.93 | 10294.50 |
| | No | 62 | 52.19 | 3235.50 |
| | Total | 164 | | |
| School Algebra Knowledge | Yes | 102 | 102.42 | 10446.50 |
| | No | 62 | 49.73 | 3083.50 |
| | Total | 164 | | |
| Overall KAT | Yes | 102 | 102.78 | 10483.50 |
| | No | 62 | 49.14 | 3046.50 |
| | Total | 164 | | |

Source: Field survey (2020)

From Table 9, the mean ranking of scores for pre-service who have experienced field teaching in all sub-knowledge types are higher than their counterparts who have not experienced field teaching and consequently this difference exhibited in the overall KAT. This suggests that pre-service teachers who have experienced field-teaching are more knowledgeable in Algebra Teaching Knowledge, School Knowledge and Advanced Knowledge of algebra as compared to pre-service teachers who have not undergone field teaching. However, the descriptive information from Table 9 cannot be solely relied upon to conclude that the difference in knowledge level between these two categories of pre-service teachers is significant. Table 10 shows the Mann-Whitney's non-parametric test statistics for difference in means for two independent samples.

**Table 10: Mann-Whitney's non-parametric test statistics for difference in means for two independent samples**

| | Algebra Teaching Knowledge | Advanced Algebra Knowledge | School Algebra Knowledge | Overall KAT |
|---|---|---|---|---|
| Mann-Whitney U | 1318.000 | 1282.500 | 1130.500 | 1093.500 |
| Z | -6.271 | -6.402 | -6.899 | -7.019 |
| Asymp. Sig. (2-tailed) | .000 | .000 | .000 | .000 |

a. Grouping Variable: Have you done your teaching practice?

Source: Field survey (2020)

Results from Table 10 shows the differences in knowledge among pre-service teachers who have experience in field teaching and pre-service teachers who have no experience in field teaching for all the three knowledge types – Algebra Teaching Knowledge (Z = -6.271, p-value = .000), Advance Algebra Knowledge (Z = -6.402, p-value = .000) and School knowledge (Z = -6.899, p-value = .000) are significant. This consequently ended in a significant difference in the overall KAT (Z = -7.019, p-value = .000) between pre-service teachers with field teaching experience and those without field teaching experience. Referring to information from Table 9 and Table 10 it can be concluded that pre-service teachers with field teaching experience are more knowledgeable in Algebra Teaching knowledge, Advance Algebra knowledge, School Algebra knowledge and the overall KAT.

**Discussion of results**

This section provides the discussion of the findings and groups them under research questions and research hypothesis.

**Research question one**

It is important to know the level of knowledge pre-service Mathematics teachers possess in Mathematics before they are deployed to the field to teach. However, it will be prudent to understand the characterization of the knowledge they possess for teaching. The purpose for this concern is to explore the characterization of pre-service Mathematics teachers' knowledge for teaching high school Algebra to aid professional training both at teacher training institutions and on the job.

Results from this study revealed that out of the seven knowledge types hypothesized in the Expanded KAT framework, high school pre-service Mathematics teachers' knowledge for teaching high school Algebra is characterized by three factors: SK, School Algebra teaching knowledge (SATK) and the PCK in teaching Algebra (PCKTA).

The result partly agrees with the original KAT conceptualisation by McCrory et al. (2012) and partly agrees with the Expanded KAT conceptualisation by Wilmot et al. (2018). The part that is consistent with the original KAT conceptualisation of teachers' knowledge for teaching Algebra is the School Algebra knowledge. This is the knowledge of high school Algebra which pre-service Mathematics teachers are expected to teach at the high school when deployed to the field. It is logically sound that pre-service Mathematics teachers' knowledge for teaching high school Algebra, from this present study, is characterized by this knowledge type in that, to be able to teach Algebra at the senior high school level, pre-service teacher must possess this knowledge. This is the type of knowledge which is also referred to as the Common Content Knowledge in the Mathematical Knowledge for Teaching (MKT) conceptualisation of teachers' knowledge (Hill et al., 2008). Hill et al. (2008) qualified this type of knowledge with common because, to them, this type of knowledge is also possessed by experts in Mathematics who are not teachers. In other words, the School Algebra knowledge can be possessed by the counterparts of pre-service Mathematics teachers who majored in the mathematical sciences.

The other part of the result from this study which is consistent with the Expanded KAT conceptualisation of the teachers' knowledge for teaching is the Pedagogical Content Knowledge in Teaching Algebra and the School Algebra teaching knowledge. These affirm the fact that the three knowledge types proposed in the original KAT conceptualisation lie in a continuum in that the interlocking regions of these knowledge types: School knowledge, Advance knowledge and Teaching knowledge, cannot be fuzzy (Wilmot, 2016).

The School Algebra teaching knowledge is a blend of the knowledge of the School Algebra knowledge and the Teaching knowledge. According to Wilmot (2016), possession of this knowledge type enables one to teach Algebra to the understanding of diverse categories of learners. Wilmot (2016) further asserted the other strand of the original KAT conceptualisation which involves the use of mathematical knowledge in teaching – Bridging, Trimming and Decompression – can be achieved by the possession of the School Algebra teaching knowledge.

It is important to also note that if School Algebra teaching knowledge is characterized by a teacher engaging in Bridging, Trimming and Decompression of algebraic knowledge, then it involves the use of algebraic knowledge coupled with teaching strategies that enhance understanding among learners. Therefore, it is within the School Algebra teaching knowledge of the Expanded KAT conceptualisation that Ma's (1999) Profound Understanding of Fundamental Mathematics (PUFM) conceptualisation (in Howe (1999)) can be situated. Ma's (1999) PUFM is characterized by the ability of the teacher to effectively communicate the concepts of Mathematics to learners. The difference between these two knowledge types is that Ma's (1999) PUFM is generic to Mathematics whiles Wilmot et al's (2018) School Algebra Teaching is specific to the domain of Algebra.

Therefore, pre-service teachers' algebraic knowledge for teaching being characterized by the School Algebra Teaching knowledge from this present study is an indication that, pre-service teachers are to some extent well equipped to teach Algebra at the high school.

Finally, the other component of the characterization of pre-service teachers' knowledge for Algebra teaching, from this study, is the PCK in teaching Algebra which also partly corroborates the finding of Wilmot et al. (2018) in their expanded KAT conceptualisation. This knowledge type, according to Wilmot (2016), is the content specific version of Shulman's (1986) PCK which is a complex combination of the knowledge of the content and general pedagogical knowledge. The only difference between the Pedagogical Content Knowledge in teaching Algebra (PCKTA) in the Expanded KAT conceptualisation and PCK in Shulman's conceptualisation is that PCKTA is uniquely limited to the teaching of Algebra.

One point that is obvious from the results is the inconsistency in the factors in the Expanded KAT framework and that which were confirmed in this study. Wilmot et al. (2018) found out, in the Expanded KAT conceptualisation, that Mathematics teachers' algebraic knowledge for teaching is characterised by seven factors while the results from this study shows that pre-service teachers' algebraic knowledge for teaching is characterized by three factors. Among the factors in the Expanded KAT conceptualisation that were not found in this study are the Advanced Algebra Knowledge (AAK), Profound Knowledge of School Algebra (PKSA), Teaching Knowledge (TK) and the Advanced Algebra Teaching Knowledge (AATK). The major reasons that could account for the absence of these knowledge types in the characterisation of pre-service teachers' knowledge for teaching high school Algebra, in this study, are pre-service teachers' limited experience in the teaching of the high school Algebra and difference in sampling and sample size.

Wilmot et al's (2018) study incorporated 252 senior high school Mathematics teachers with varied teaching experience in terms of the number of years for teaching either the Core Mathematics or Elective Mathematics or combination of the two. However, in the case of this study pre-service Mathematics teachers who were at either their third or final year of their training participated in the study. At the time of data collection, the third-year pre-service teachers had not embarked on any official internship programme (Off-Campus Teaching Practice) at the senior high schools while the final year pre-service teachers had experienced a semester long teaching of either of the two Ghanaian Mathematics curricula or both at a senior high school. The following explains how pre-service teachers' limited experience in the teaching of high school Algebra could have contribute to the absence of the four factors.

Teachers with Profound Knowledge of School Algebra have a greater comprehension of the topic of School Algebra which is manifested by the "possession of alternate definitions, extensions and generalizations of familiar theorems, and wide variety of applications of high school Algebra" (Wilmot, 2016, p. 23). Moreover, teachers' years of teaching experience is a determinant of their competence (Darling-Hammond, 2000). Therefore, it is expected that if these future teachers are later deployed to the field permanently to teach high school Algebra, they will gradually gain the competency in the formulation of alternative definitions, extending and generalizing algebraic concepts. Again, inferring from Darling-Hammond's (2000) assertion, as pre-service teachers spend more time on the field to teach high school Algebra, they develop the competency in having numerous application of the high school Algebra. Thus, the knowledge of pre-service teachers' not being characterized by the Profound Knowledge of School Algebra in expanded KAT conceptualisation can be attributed to their limited teaching experience in the teaching of high school Algebra.

Another knowledge type in the Expanded KAT conceptualisation that was not found in this study is the Teaching Knowledge. In the original KAT conceptualisation, McCrory et al. (2012) described this category of knowledge as the composition of knowledge that are more available to experienced teachers such as their thinking about Mathematics and their interpretation to students' mathematical language. Similarly, Putnam (1987) asserted that experienced teachers have knowledge of past students in their mind. The knowledge of past students gives the experienced teacher fair knowledge of the misconception pupils are more likely to bring with them to class. This knowledge of past students makes the teacher more competent in handling the subject matter in a manner that addresses students perceived misconceptions. It also adds to the competence of the teacher his ability to interpret students' mathematical solutions. Thus, when these pre-service Mathematics teachers gain more classroom experience through teaching, they become more equipped with mathematical ideas that facilitate teaching and also address students' need. Therefore, the knowledge of pre-service teachers not being characterized by the Teaching knowledge in the Expanded KAT conceptualisation can be associated with their limited time of interaction with students with regard the teaching of high school Algebra.

Moreover, the Advanced Algebra Knowledge and the Advanced Algebra Teaching knowledge which were also not found in this study can also be attributed to pre-service teachers' low level of teaching experience. The Advanced Algebra knowledge is the knowledge of Algebra beyond high school Algebra and the Advanced Algebra Teaching knowledge is the knowledge that gives teachers the ability to fluidly teach Advanced algebraic concepts to the understanding of diverse students. As teachers teach, there are times they have to relate the high school Algebra to higher concepts of Algebra or project the school Algebra into Advanced Algebra (engaging in *Bridging*). Therefore, as teachers put their knowledge into use while teaching by engaging in Bridging, their knowledge in Advanced Algebra and the teaching of Advanced Algebra will be strengthened.

Therefore, absence of PKSA, TK, AAK and AATK in the characterisation of pre-service Mathematics teachers' knowledge, as found in this study, is as a result of their limited teaching experience. If pre-service Mathematics teachers are given enough time to practise teaching through internship programmes their knowledge in these four knowledge types will be strengthened and consequently, their knowledge for teaching high school Algebra will be characterised by these types of knowledge.

Another key reason why the characterization of pre-service Mathematics teachers' knowledge for teaching high school Algebra did not conform with the Expanded KAT conceptualisation is the difference in sampling techniques and sample size.

In their study, Wilmot et al. (2018) employed a multi-stage sampling technique to sample 252 Mathematics teachers from 40 senior high schools across the Ghana Education Service (GES) categorisation (Categories A, B, C and D) of high schools in the Ashanti, Central and Western Regions of Ghana. This form of sampling technique resulted in getting teachers with varied characteristics in terms of the resources the teachers are exposed to since the GES categorisation is based on the facilities a school has. On the subject of availability of school facilities, Oni (1992) as cited in Owoeye and Olatunde Yara (2011) asserted that adequate availability of facilities to an organisation promotes efficiency and productivity of the organisation. Hence, the availability of facilities that promote the learning of Mathematics such as computer laboratories equipped with computers with mathematical software such as GeoGebra facilitates teaching of Algebra which at the long run, also finetunes the teachers' knowledge in Algebra.

This present study resorted to a cluster sampling technique since there is no categorisation of the universities that train pre-service senior high school Mathematics teachers. The variation of pre-service Mathematics teachers in each university is unique in that each university has a unique variation of performance of pre-service teachers and also each university has a unique structure for the Mathematics Education programme with regards to content. In the process of sampling for this study, each university was used as a cluster and only one of the universities was sampled for its third and final year pre-service teachers to participate in the study. Hence, the pre-service Mathematics teachers used in this study had the intra variation within the university but lacks the inter variation of pre-service teachers from different universities.

Another contributing factor that might have caused the difference in the factors in the Expanded KAT conceptualisation of teachers' knowledge and that which were confirmed in this study is the difference in sample size. Due to the sampling technique used in this study, 164 pre-service teachers were accessible to participate in this recent study as against the 252 teachers who participated in Wilmot et al's (2018) study.

Though KMO statistic (.834) indicated that the 164 pre-service teachers was adequate for conducting an exploratory factor analysis, the sample size of 164 pre-service teachers from only one university may not be a good representation of the population. Hence, aside the difference in the characterisation of participants in the two studies in terms of level of teaching experience, the smaller sample size used in this study and the lack of variation of pre-service teachers from different universities may have accounted for the difference in the factors confirmed in this study and that in the Expanded KAT conceptualisation.

**Research question two**

According to the findings of this study pre-service teachers possess low level of the overall KAT. This low level of knowledge is also evidence in the other three sub-knowledge types of KAT; Algebra Teaching knowledge (M = 36.1951, SD = 16.58918), Advanced Algebra knowledge (M = 30.2744, SD = 16.69147) and School Algebra knowledge (M = 40.7696, 21.16352). The basis for classifying their level of knowledge in the three sub-knowledge types of the KAT to be low is that, the mean scores of items that measure the aforementioned types of knowledge are not even up to fifty percent of the total score of one hundred for each knowledge type.

Results from this study on pre-service teachers' knowledge is consistent with the findings of other studies which found that pre-service Mathematics teachers possess limited knowledge in Mathematics (Depaepe et al., 2015; Leong et al., 2015; Wilburne & Long, 2010).

Even though pre-service teachers' knowledge in all the three sub-knowledge types in the KAT are low, it is evidenced from the results that despite the rigorous Mathematics content courses which are enriched with Algebra taken by pre-service teachers at the universities, their knowledge in Advanced Algebra cannot be compared to their knowledge in high school Algebra. The difference in knowledge on Advance Algebra and School Algebra can be attributed to the advance Mathematics courses that are taken by pre-service teachers at the universities. The advanced knowledge of Mathematics as described in the original KAT framework offers teachers “perspective on the trajectory and growth of mathematical ideas beyond school Algebra” (McCrory et al., 2012, p. 597). Therefore, it can be said that the advanced Mathematics course such as Introductory Algebra, Advanced Algebra, Precalculus and Advanced Calculus pre-service teachers took at the universities strengthen their understanding of the algebraic concepts they learned at the senior high school and also finetune their understanding of those high school algebraic concepts which they had difficulties in understanding while they were at senior high school. Hence, the advanced Mathematics pre-service teachers studied at the universities enriched their knowledge in high school Algebra.

Also, the findings from this present study indicate that pre-service teachers are more knowledgeable in the algebraic content to be taught (School Algebra Knowledge) than in the knowledge of Mathematics for teaching (Teaching Knowledge). Other international studies have reported similar findings that pre-service teachers are more competent in the Mathematics content courses than in their PCK (Depaepe et al., 2015; Leong et al., 2015). Even though the Mathematics for Teaching Knowledge, as described in the conceptual framework, is not purely pedagogical knowledge. It is the mathematical knowledge needed to present and explain mathematical ideas, rules and procedures and also examine the students’ solutions and responses to a mathematical problem (Ball et al., 2005). Therefore, pre-service teachers having weaker knowledge on this type of knowledge is alarming since weaker knowledge in this knowledge type has the potential to create lapses in undertaking their teaching of Algebra tasks.

It is also important to note that, even though pre-service Mathematics teachers demonstrated low level of knowledge in both the senior high school Algebra content and how to teach them, the marginal difference in mean between School Algebra Knowledge (M = 40.7696, SD = 21.16352) and School Algebra Teaching Knowledge (M = 36.1957, SD = 16.58918) brings to notice that they have relatively the same level of knowledge in the high school Algebra concepts and how to teach these concepts. The reason may reside in the fact that the training institutions for pre-service teachers place relatively equal emphasis on the content to be taught and how to effectively teach these contents to enhance students' performance.

**Research hypothesis**

Results from this study indicate a significant difference in knowledge in all the types of knowledge [Algebra Teaching Knowledge, Advanced Algebra Knowledge and School Algebra Knowledge] among pre-service Mathematics teachers with the difference being in favour of pre-service Mathematics teachers with Off-CTP experience.

The effect of teaching experience on pre-service teachers' knowledge has been discussed by different authors. Strawhecker (2005), in her study, found no significance difference for the mathematical content between pre-service Mathematics teachers with field-teaching experience and those with non-field teaching experience. However, she found a significant difference in Knowledge of Students and Content (KSC) subscale of PCK between pre-service Mathematics teacher with field teaching experience and non-field teaching experience group. From Strawhecker's (2005) study, field-teaching experience has the capacity to improve these future teachers' knowledge in the Mathematics PCK than the Mathematics content. On the issue that field-teaching experience has no influence on the teachers' content knowledge, a qualitative study by Lucus (2006) also reported that teachers in the service and pre-service teachers demonstrated similar knowledge in composition of functions. Intuitively, it will be expected that in-service teachers who have had adequate teaching experience would have demonstrated in-depth knowledge of composition of function and its related PCK.

This present study, however, in relation to the content knowledge of Algebra shows a significant difference between pre-service teachers with field teaching experience and their counterparts without field teaching experience. From the results, pre-service Mathematics teachers with Off-CTP experience are more knowledgeable in both the content of high school Algebra and advanced Algebra they learned in the course of their training than their colleagues without Off-CTP experience.

The result from this current study is also somewhat consistent with literature on the grounds of the existing difference in Algebra Teaching Knowledge, an equivalence to the PCK, between pre-service teachers with field-teaching experience and non-field teaching experience pre-service teachers. Therefore, this study, based on the data at hand, takes a stance to the argument as to whether or not field teaching experience affect knowledge for teaching and concludes that Off-CTP experience positively affects the KAT of pre-service teachers.

**Summary of the key findings**

Previous researches have conceptualised the knowledge instructors need to teach effectively in different ways. Some of these conceptualisations have been domain-neutral and others are domain-specific. The advent of recent research has called for a content-specific conceptualisation of teachers' knowledge. The forebearers in this line of research have been the KAT project team (see McCrory et al. (2012)) and the Expanded KAT team members (see Wilmot et al. (2018)) who focused on conceptualising teachers' knowledge in Algebra. This study explored the extent to which the Expanded KAT conceptualisation can be realized among pre-service Mathematics teachers. It is in the light of the finding in this study the following implications are made.

It was found in this study that pre-service Mathematics teachers' knowledge for teaching Algebra is characterized by three factors: SAK, SATK and PCKTA. The finding neither fully corroborates the KAT conceptualisation nor the Expanded KAT conceptualisation of teachers' knowledge for teaching. However, the fact that two out of the four interlocking regions of the Expanded KAT conceptualisation were corroborated affirms that the interlocking regions of the primary knowledge types in the KAT conceptualisation cannot be fuzzy. This implies that pre-service Mathematics teachers blend two or more of these primary knowledge types in order to effectively teach high school Algebra.

It was also found from this study that pre-service Mathematics teachers possess low level of algebraic knowledge. This shows these teachers, by the time they are deployed to the field to teach Mathematics, have insufficient amount of algebraic knowledge. Again, these teachers demonstrated the same level of knowledge in both the content of Algebra and how to teach these contents. This implies that the training institutions place the same level of emphasis on the content of Mathematics to be taught and the methodological courses which equip these teachers the knowledge of methods of presenting these contents.

In addition, the study revealed pre-service teachers who have field teaching experience out performed their counterparts who have no field teaching experience in both the knowledge of the content of Algebra and knowledge of how to teach the content of Algebra. This means that field teaching coupled with the training of pre-service Mathematics teachers has the capacity to improve their knowledge base in Algebra.

# CHAPTER FIVE

## SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

Despite the numerous conceptualisations of teachers' knowledge, the characterization of knowledge for teaching of pre-service teachers has not been clearly understood. This research looked into the different sorts of knowledge that pre-service Mathematics teachers have for teaching senior high school Algebra. That is to find out the characterization of Mathematics student teachers' teaching knowledge in high school Algebra. It is also aimed at measuring the level of knowledge they possess in Algebra and the teaching of Algebra. The reason for concentrating on Algebra is pervasiveness of Algebra in all other domains in Mathematics.

The study employed the cross-sectional research design and the cluster sampling technique in data collection. One hundred and sixty-four pre-service Mathematics teachers reading the Bachelor of Education in Mathematics programme at Level 300 and Level 400 participated in the study with 102 of them being Level 400 students and 62 being Level 300 students. The Level 400 students had experience in field teaching while the Level 300 students had no experience in field teaching. Data on pre-service teachers' knowledge for teaching high school Algebra was collected by adopting the instrument developed by Wilmot et al. (2018). A Principal Component Analysis of Exploratory factor analysis was employed in extracting the factors that characterize pre-service Mathematics teachers' knowledge for teaching high school Algebra. The mean and standard deviation were used to describe the level of KAT these pre-service Mathematics teachers possess and the Mann-Whitney's test was used to test for the difference in knowledge between pre-service teachers with field-teaching experience and those without field-teaching experience.

**Summary**

Three components were discovered to describe pre-service teachers' competence for teaching high school Algebra: PCKTA, SAK and SATK.

It was also discovered that these teachers have a limited expertise of Algebra and the teaching of it. This level of knowledge was evidenced in the three knowledge types of the KAT framework. They also exhibit almost the same level of knowledge in the content of School Algebra and knowledge of how to teach the school Algebra content.

Again, pre-service teachers with field teaching experience are more knowledgeable in School Algebra, Advanced Algebra, and Teaching knowledge than their counterparts without field teaching experience, according to the research hypothesis' results.

**Conclusion**

Based on the findings from this study, the following conclusions are made.

SAK, SATK, and PCKTA Expertise in Teaching Algebra were confirmed to be described the knowledge of pre-service teachers for teaching Algebra in relation to research question one. Based on this finding, it can be inferred that the characterizations of pre-service Mathematics teachers' teaching expertise are the fundamental necessity for teaching high school Algebra.

Though pre-service Mathematics teachers possess the basic knowledge types for teaching high Algebra, it was also found that pre-service Mathematics teachers possess low level of knowledge of Algebra for teaching. This low level of knowledge was also exhibited in the Advanced knowledge of Algebra, School Algebra knowledge and the Teaching knowledge. This is alarming because, these pre-service teachers have been exposed to enough Mathematics content courses and methodology courses and were expected to exhibit a good level of knowledge in these three types of knowledge especially in the School Algebra and Teaching knowledge. From this, it can be concluded that pre-service Mathematics teachers, at the time they about to be deployed to the high schools, possess weak understanding of Algebra content and the teaching of Algebra.

Furthermore, the study found that pre-service Mathematics instructors with field teaching experience are more aware about Algebra subject and how to teach it. This suggests that field-teaching experience has a positive impact on the knowledge of pre-service teachers. This means that by providing more opportunities for pre-service Mathematics teachers to participate in internship programs, they will have a better chance of improving their expertise of teaching Algebra and the Algebra subject they teach.

**Recommendations**

Based on the finding from this study, the following are recommended

1. It turned out from the study that pre-service teachers possess low level of knowledge in Algebra and the teaching of Algebra as well. Therefore, it is recommended that the Mathematics education departments should make the necessary reformation to strengthen their students' knowledge in Algebra. The reformation can be reconsidering the content of the courses these prospective teachers

take or mounting more courses that will help strengthen their knowledge in Algebra.

2. Aside, the National Teaching Council's teacher license exam, it is suggested that the Ghana Education Service, in partnership with the various heads of senior high schools, give a specific mentorship session for newly hired teachers. This is to ensure that, under the supervision of their assigned mentors, these pre-service teachers are gradually prepared to have a good command of high school mathematics and the teaching of it.
3. In addition, it was found that prospective teachers with field teaching experience are more knowledgeable in all three knowledge kinds specified in the original KAT framework. As a result, it is suggested that pre-service Mathematics instructors be encouraged to participate in field experiences.

**Suggestions for Further Research**

1. Further study should be conducted to involve more student teachers of Mathematics in order to get a good representation of the population since this study could not reach more of these teachers due to closure of school as a result of the COVID-19 pandemic.
2. Further study on pre-service teachers' knowledge for teaching should incorporate qualitative data since this study only concentrated on using quantitative data. This will help get a better understanding of the characterisation of pre-service Mathematics teachers' knowledge for teaching high Algebra.

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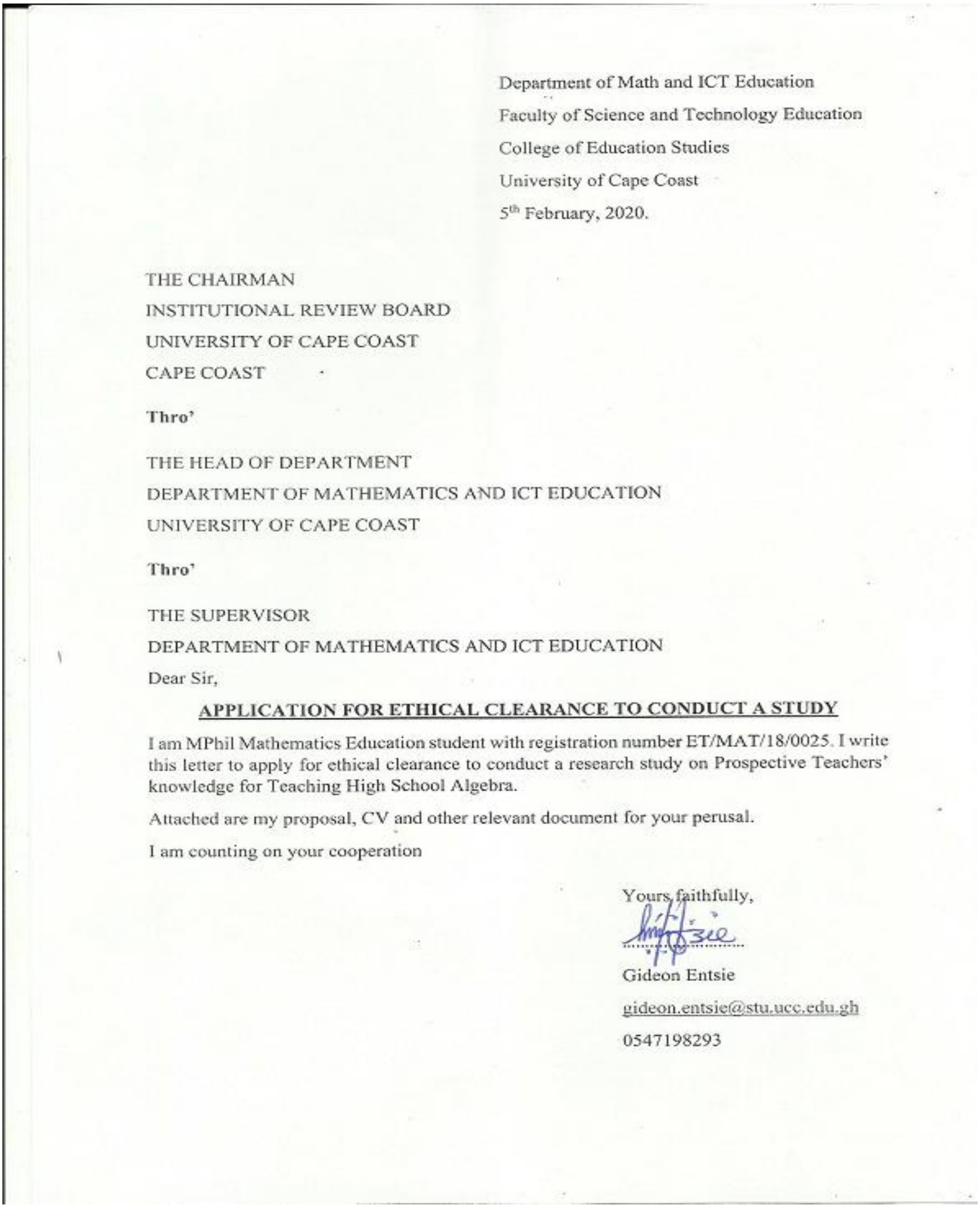
# APPENDICES

## APPENDIX A

## LETTER FOR ETHICAL CLEARANCE



Department of Math and ICT Education
Faculty of Science and Technology Education
College of Education Studies
University of Cape Coast
5th February, 2020.

THE CHAIRMAN
INSTITUTIONAL REVIEW BOARD
UNIVERSITY OF CAPE COAST
CAPE COAST

**Thro'**

THE HEAD OF DEPARTMENT
DEPARTMENT OF MATHEMATICS AND ICT EDUCATION
UNIVERSITY OF CAPE COAST

**Thro'**

THE SUPERVISOR
DEPARTMENT OF MATHEMATICS AND ICT EDUCATION

Dear Sir,

**APPLICATION FOR ETHICAL CLEARANCE TO CONDUCT A STUDY**

I am MPhil Mathematics Education student with registration number ET/MAT/18/0025. I write this letter to apply for ethical clearance to conduct a research study on Prospective Teachers' knowledge for Teaching High School Algebra.

Attached are my proposal, CV and other relevant document for your perusal.

I am counting on your cooperation

Yours faithfully,

Gideon Entsie
gideon.entsie@stu.ucc.edu.gh
0547198293

# APPENDIX B

## COVER LETTER FOR ETHICAL CLEARANCE

**UNIVERSITY OF CAPE COAST**
**COLLEGE OF EDUCATION STUDIES**
**FACULTY OF SCIENCE AND TECHNOLOGY EDUCATION**
**DEPARTMENT OF MATHEMATICS AND I.C.T EDUCATION**

Telephone: 0332096951
Telex: 2552, UCC, GH
Telegrams & Cables: University, Cape Coast
Email: dmicte@ucc.edu.gh

University Post Office
Cape Coast, Ghana

Your Ref:

Our Ref: DMICTE/P.3/V.1/049

Date: 20th February, 2020

The Director
Institutional Review Board
University of Cape Coast
Cape Coast

Dear Sir,

**REQUEST FOR ETHICAL CLEARANCE**

I write to introduce the bearer of this letter, Mr. Gideon Entsie, with registration number ET/MDP/18/0025 an MPhil (Mathematics Education) student of the Department of Mathematics and ICT Education, College of Education Studies, University of Cape Coast.

As part of the requirement for the award of a master's degree, he is required to undertake a research on the topic **"PROSPECTIVE TEACHERS' KNOWLEDGE FOR TEACHING HIGH SCHOOL ALGEBRA".**

I would be grateful if you could give him the necessary assistance he may need.

Thanks for your usual support.

Yours faithfully,

Dr Kofi Ayebi-Arthur
**HEAD**

# APPENDIX C

## ETHICAL CLEARANCE LETTER FROM IRB, UCC

UNIVERSITY OF CAPE COAST

INSTITUTIONAL REVIEW BOARD SECRETARIAT

TEL: 0558093143 / 0508878309/ 0244207814
E-MAIL: irb@ucc.edu.gh
C/O Directorate of Research, Innovation and Consultancy

OUR REF: UCC/IRB/A/2016/713
YOUR REF:
OMB NO: 0990-0279
IORG #: IORG0009096

26TH JUNE, 2020

Mr. Gideon Entsie
Department of Mathematics and ICT Education
University of Cape Coast

Dear Mr. Entsie,

**ETHICAL CLEARANCE – ID (UCCIRB/CES/2020/15)**

The University of Cape Coast Institutional Review Board (UCCIRB) has granted **Provisional Approval** for the implementation of your research protocol **Prospective Teachers' Knowledge for Teaching High School Algebra.** This approval is valid from 26th June, 2020 to 25th June, 2021. You may apply for a renewal subject to submission of all the required documents that will be prescribed by the UCCIRB.

Please note that any modification to the project must be submitted to the UCCIRB for review and approval before its implementation. You are required to submit periodic review of the protocol to the Board and a final full review to the UCCIRB on completion of the research. The UCCIRB may observe or cause to be observed procedures and records of the research during and after implementation.

You are also required to report all serious adverse events related to this study to the UCCIRB within seven days verbally and fourteen days in writing.

Always quote the protocol identification number in all future correspondence with us in relation to this protocol.

Yours faithfully,

Samuel Asiedu Owusu, PhD
**UCCIRB Administrator**

ADMINISTRATOR
INSTITUTIONAL REVIEW BOARD
UNIVERSITY OF CAPE COAST

# APPENDIX D

## COVER LETTER FOR RESEARCH VISIT

UNIVERSITY OF CAPE COAST
COLLEGE OF EDUCATION STUDIES
FACULTY OF SCIENCE AND TECHNOLOGY EDUCATION
DEPARTMENT OF MATHEMATICS AND I.C.T EDUCATION

Telephone: 0332096951
Telex: 2552, UCC, GH
Telegrams & Cables: University, Cape Coast
Email: dmicte@ucc.edu.gh

University Post Office
Cape Coast, Ghana

Your Ref:

Our Ref: DMICTE/P.3/V.1/074

Date: 6th July, 2020

**TO WHOM IT MAY CONCERN**

Dear Sir/Madam,

**RESEARCH VISIT**

I write as the Head of Department to introduce our student Mr. Gideon Entsie, with registration number ET/MDP/18/0025 an MPhil (Mathematics Education) student of the Department of Mathematics and ICT Education, College of Education Studies, University of Cape Coast.

As part of the requirement for the award of a master's degree, he is required to undertake a research on the topic "**PROSPECTIVE TEACHERS' KNOWLEDGE FOR TEACHING HIGH SCHOOL ALGEBRA**".

I would be grateful if you could give him the necessary assistance he may need.

Thanks for your usual support.

Yours faithfully,

Dr. Kofi Ayebi-Arthur
**HEAD**

# APPENDIX E

## RESEARCH INSTRUMENT

**UNIVERSITY OF CAPE COAST**

**COLLEGE OF EDUCATION STUDIES**

**FACULTY OF SCIENCE AND TECHNOLOGY EDUCATION**

**DEPARTMENT OF MATHEMATICS AND ICT EDUCATION**

Dear respondent,

This instrument is for collection of data for MPhil thesis on Prospective teachers' knowledge for teaching high school algebra. I am, by this means, assuring you of maximum confidentiality in that, your responses will be used solely for academic work. Thank you for agreeing to participate in this study.

### PART I: RESPONDENT'S BACKGROUND INFORMATION

At this section, you are to check the appropriate box

1. What is your sex?

□ Male

□ Female

□ others (Specify)____________________

2. What level are you presently at the University or College?

□ Level 300

□ Level 400

3. Have you done your Off-Campus teaching Practice at the University level?

□ Yes

□ No

4. Did you teach Mathematics before coming to the University?

□ Yes (If Yes, answer question 5)

□ No (If No, Skip question 5)

5. For how many years did you teach mathematics before coming to the University?

Specify__________________

### PART II: ASSESSMENT QUESTIONS

Instructions

This instrument contains 73 multiple-choice questions about knowledge for teaching algebra. You have 135 minutes to answer these questions. You may use a calculator if you choose.

In this booklet, each multiple-choice question has only one right answer. Please circle the correct answer for the multiple-choice questions, and write all your responses to the free-response questions.

1. A seafood restaurant has a dinner combo plate. For the plate, you can choose two entrees from six different choices. Then you can choose between baked potato, rice, mashed potatoes, or coleslaw. Last, you choose between soup and salad. How many possible dinner combo plates are available?

   A. 120

   B. 48

   C. 240

   D. 12

   E. None of these

2. Timothy's age in 15 years will be twice what it was 5 years ago. If *t* represents Timothy's age now, write the equation that models this situation.

**Response:**

3. Find the number that must divide each term in the equation $5x^2 + 2x = 20$ so that the equation can be solved by completing the square.

**Response:**

4. A small company invested ¢2,000.00 by putting part of it into a municipal bond fund that earned 4.5% annual simple interest and the remainder in a corporate bond fund that earned 9.5% annual simple interest. If the company earned ¢1,500.00 annually from the investments, how much was in the municipal bond fund?

A. ¢8,000.00

B. ¢10,000.00

C. ¢9,000.00

D. ¢7,000.00

E. None of these

5. A gramophone record rotates 220 times during the performance of a certain tune, the smallest and largest radii of the record being 6.25cm and 12.75cm respectively, the circular paths being equidistant. Calculate the total distance traversed on the record by the needle.

A. 131.3m

B. 130.3m

C. 220.3m

D. 220.0m

6. A circular ball is thrown downward with an initial velocity 5 feet per second from a bridge located in a water which is 220 feet above the water. How long will it take for the ball to hit the water? Round your answer to the nearest 0.01 second.

[ Take $g = 10ms^{-2}$]

A. 5.62 seconds

B. 6.21 seconds

C. 5.26 seconds

D. 6.15 seconds

**7.** Given a set D whose elements are the odd integers, positive and negative (zero is not an odd integer). Which of the following operations when applied to any pair of elements will yield only elements of D?

i. Addition

ii. Multiplication

iii. Division

iv. Finding the arithmetic mean

The correct answer is

A. i and ii only

B. ii and iv only

C. ii, iii, and iv only

D. ii and iii only

E. ii only

8. A particle moves in a straight line with uniform acceleration. At time 2s, the particle is 10m from its starting point and at 4s the particle is 40m from its starting point. Find the velocity of the particle when it is 160m from its starting point.

A. 5 m/s

B. 25 m/s

C. 40 m/s

D. 45 m/s

9. A and B begin work together. A's initial salary is GH¢200.00 a year and he has an annual increment of GH¢20.00. B is paid at first at the rate of GH¢80.00 a year and has an increment of GH¢8.00 every half-year. At the end of how many years will B have received more money than A?

A. 5 years

B. 5.5 years

C. 6 years

D. 6.5 years

10. A carpet installer decides to replace carpets in some offices on a university campus and uses the formula Cost = 350 + 1.6A, where A is the number of square feet of carpet to be replaced, to determine the cost. In how large an office can the carpet be replaced for ¢9,600.00?

A. $5775.00\ ft^2$

B. $5781.25\ ft^2$

C. $5871.00\ ft^2$

D. $5817.25\ ft^2$

11. Which of the following is a false statement?

A. 2, 3, 9/2, 27/4… $2(3/2)^{n-1}$… is a geometric sequence with common ratio 3/2.

B. 5, 2, -1… -3n+5… is an arithmetic sequence with common difference 5.

C. If (Yarkwah) is a sequence, then $S_n = \dots$ Is the nth partial sum of the sequence.

D. Two terms of a sequence can be equal.

E. None of these

12. A rectangular piece of cardboard measures 35inches by 30inches. An open box is formed by cutting four squares that measure x inches on a side from the corners of the cardboard and then folding up the sides. Determine the volume of the box in terms of $x$.

A. $4x^3 - 130x^2 + 1050x$

B. $4x^3 + 130x^2 + 1050x$

C. $4x^3 - 130x^2 + 1050$

D. $4x^2 - 130x + 1050$

13. Ice forms on a refrigerator ice-box at the rate of (4-0.6t) g per minute after t minutes. If initially, there is no ice on the box, find the mass of ice formed in 5 minutes.

A. 5g

B. 17g

C. 26g

D. 35g

14. Find the number of terms of the sequence $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots$ that must be taken so that the difference between the sum and 2 is less than $10^{-3}$.

A. 10

B. 11

C. 12

D. 13

15. The major sectorial angle of a circle with radius 14cm is $270^0$.If the sector is folded to form a cone, find the surface area of the cone.

A. $460.0cm^2$

B. $460.8cm^2$

C. $461.0cm^2$

D. $461.8cm^2$

16. In how many ways can the fraction $\frac{1}{2}$ be written as a sum of two positive fractions with numerator equal to 1 and denominator a natural number?

A. 0

B. 1

C. 2

D. 4

E. More than 4

17. A desktop screen measures 60inches diagonally and its aspect ratio is 16 to 9. This means that the ratio of the width of the screen to the height of the screen is 16 to 9. Find the width and height of the screen. Round to the nearest tenth of an inch.

    A. height = 3.27inches, width = 29.4inches

    B. height = 3.27inches, width = 52.3inches

    C. height = 29.4inches, width = 52.3inches

    D. height = 52.3inches, width = 29.4inches

    E. height = 52.0inches, width = 29.0inches

18. If $p:q$ and $r:s$ are two equal ratios and (q≠0, s≠0) then

    A. p=r and q=s

    B. pr = qs

    C. p+r = q+s

    D. p-r = q—s

    E. ps = qr

19. A cup of hot tea is heated to $180^0$F and placed in a room that maintains a temperature of $60^0$F. The temperature of the tea after t minutes is given by $T(t) = 60+120e^{-0.038t}$. Find the temperature, to the nearest degree, of the tea 5 minutes after it is placed in the room.

    A. $637.6^0$F

    B. $159.2^0$F

    C. $72.2^0$F

    D. $60.1^0$F

    E. None of these

20. Solve algebraically: $\log_3(x-4) = 2$

    A. x = 10

    B. x = 18

    C. x = 729

    D. x = 13

    E. None of these

21. The major sectorial angle of a circle with radius 14cm is $270^0$.If the sector is folded to form a cone, find the surface area of the cone.
    A. $460.0cm^2$
    B. $460.8cm^2$
    C. $461.0cm^2$
    D. $461.8cm^2$
22. Which of the following is a true statement?
    A. The solution of the matrix equation AX = B, is X = $A^{-1}$B, provided $A^{-1}$ exists.
    B. $\begin{bmatrix} p & q \\ r & s \end{bmatrix}$ and $\begin{bmatrix} p & q \\ r & s \end{bmatrix}$ are inverses.
    C. A singular matrix is a matrix that has a multiplicative inverse.
    D. All matrices have an inverse.
    E. None of these
23. A farmer wishes to make a rectangular hen-run of area $50m^2$ against a wall which is to serve as one of the boundaries. Find the smallest length of wire netting required for the other three sides.
    A. 5m
    B. 10m
    C. 11m
    D. 20m
24. Given that a + b = c where a, b, and c are integers and a is positive, which one of the following statements is true?

A. a is always greater than c
B. a is always less than c
C. b is always less than c
D. c is never zero
E. c – a is always positive.

25. What is the conclusion of this statement? $If\ x^2 = 4, then\ x = -2\ or\ x = 2.$
    A. $x^2 = 4$
    B. $x = 2$
    C. $x = -2$

D. $x = -2 \ or \ x = 2$

26. Kwame's average driving speed for a 4-hour trip was 45 miles per hour. During the first 3 hours he drove 40 miles per hour. What was his average speed for the last hour of his trip?
    A. 50 miles per hour
    B. 60 miles per hour
    C. 65 miles per hour
    D. 70 miles per hour
27. One pipe can fill a tank in 20 minutes, while another takes 30 minutes to fill the same tank. How long would it take the two pipes together to fill the tank?
    A. 50 min
    B. 25 min
    C. 15 min
    D. 12 min
28. Which statement best explains why there is no real solution to the quadratic equation $2x^2 + x + 7 = 0$?

A. The value of $1^2 - 4.2.7$ is positive.

B. The value of $1^2 - 4.2.7$ is equal to 0.

C. The value of $1^2 - 4.2.7$ is negative.

D. The value of $1^2 - 4.2.7$ is not a perfect square.

29. Four steps to derive the quadratic formula are shown below:

i. $x^2 + \frac{bx}{a} = \frac{-c}{a}$

ii. $\left(x + \frac{b}{2a}\right)^2 = \frac{b^2-4ac}{4a^2}$

iii. $x = \pm\sqrt{\frac{b^2-4ac}{4a^2}} - \frac{b}{2a}$

iv. $x^2 + \frac{bx}{a} + \left(\frac{b}{2a}\right)^2 = \frac{-c}{a} + \left(\frac{b}{2a}\right)^2$

What is the correct order for these steps?

A. i, iv, ii, iii

B. i, iii, iv, ii

C. ii, iv, I, iii

D. ii, iii, I, iv

30. Kofi's solution to an equation is shown below:

Given: $n + 8(n + 20) = 110$

Step 1: $n + 8n + 20 = 110$

Step 2: $9n + 20 = 110$

Step 3: $9n = 110 - 20$

Step 4: $9n = 90$

Step 5: $\frac{9n}{9} = \frac{90}{9}$

Step 6: $n = 10$

Which statement about Kofi's solution is true?

A. Kofi's solution is correct

B. Kofi made a mistake in step 1

C. Kofi made a mistake in step 3

D. Kofi made a mistake in step 5

31. Araba Atta correctly solved the equation $x^2 + 4x = 6$ by completing the square. Which equation is part of her solution?

A. $(x + 2)^2 = 8$

B. $(x + 2)^2 = 10$

C. $(x + 4)^2 = 10$

D. $(x + 4)^2 = 22$

32. Which of the following is a valid conclusion to the statement ''If a student is a high school band member, then the student is a good musician''?

A. All good musicians are high school band members.

B. A student is a high school member band member.

C. All students are good musicians

D. All high school band members are good musicians.

33. The equation of line *l* is $6x + 5y = 3$, and the equation of the line *q* is $5x - 6y = 0$. Which statement about the two lines is true?

A. Lines *l* and *q* have the same y-intercept

B. Lines *l* and *q* are parallel

C. Lines *l* and *q* have the same x-intercept

D. Lines $l$ and $q$ are perpendicular

34. John's solution to an equation is shown below:

Given: $x^2 + 5x + 6 = 0$

Step 1: $(x + 2)(x + 3) = 0$

Step 2: $x + 2 = 0 \; or \; x + 3 = 0$

Step 3: $x = -2 \; or \; x = -3$

Which property of real numbers did John use for Step 2:

A. multiplication property of equality

B. zero product property of multiplication

C. commutative property of multiplication

D. distributive property of multiplication over addition

35. When is this statement true?

The opposite of a number is less than the original number.

A. This statement is never true.

B. This statement is always true.

C. This statement is true for positive numbers.

D. This statement is true for negative numbers.

36. Kwame solved the equation $\frac{1}{x-5} = \frac{5}{12x-60}$.

Step 1: He factored the denominator in the expression on the right side of the equation and obtained $\frac{1}{x-5} = \frac{5}{12(x-5)}$.

Step 2: He multiplied both sides by $x - 5$ and obtained $1 = \frac{5}{12}$.

Conclusion: The solution set is the empty set.

A. The conclusion is correct.

B. The conclusion is wrong because we cannot multiply both sides by $x - 5$.

C. The conclusion is wrong because another procedure produces a conclusion different from the one obtained.

D. The conclusion is wrong because if we 'cross multiply' by the common denominator we obtain a different solution.

E. There is some other reason why the solution is wrong.

37. Students in Mr. Carson's class were learning to verify the equivalence of expressions. He asked his class to explain why the expressions a –

(b + c) and a – b – c are equivalent. Some of the answers given by students are listed below.

Which of the following statements comes closest to explaining why a – (b + c) and a – b – c are equivalent? (Mark ONE answer.)

A. They're the same because we know that a – (b + c) doesn't equal a – b + c, so it must equal a – b – c.

B. They're equivalent because if you substitute in numbers, like a=10, b=2, and c=5, then you get 3 for both expressions.

C. They're equal because of the associative property. We know that a – (b + c) equals (a – b) – c which equals a – b – c.

D. They're equivalent because what you do to one side you must always do to the other.

38. The set of nonnegative rational numbers with the operations of addition and multiplication has one of the following characteristics:

A. It is not closed under one of these operations

B. More than one of its elements does not have an inverse for the operation of multiplication.

C. Zero is not a member of this set

D. The distributive law of multiplication over addition does not hold

E. None of the above is a characteristic of the given set

39. Susan was trying to solve the equation $2x^2 = 6x$.

First, she divided both sides by 2 to get $x^2 = 3x$

Then she divided both sides by $x$ to get $x = 3$

Gustavo said, "You can't divide both sides by $x$." Susan responded, "If you can divide both sides by 2, why can't you divide by $x$?" They asked their teacher to explain.

Which of the following explanations is correct?

A. Since $x$ is a variable it can vary, you may not be dividing both sides by the same number.

B. You can't cancel $x$ because it does not represent a real number.

C. You can only divide by whole numbers when solving equations.
D. It is better to take the square root of both sides after dividing by 2, that way you won't have to worry about dividing by $x$.
E. If you divide both sides by $x$, then you might be dividing by 0, and would miss the solution $x = 0$.

40. In a first-year elective mathematics class, which of the following is **NOT** an appropriate way to introduce the concept of slope of a line?
A. Talk about the rate of change of a graph of a line on an interval.
B. Talk about speed as distance divided by time.
C. Toss a ball in the air and use a motion detector to graph its trajectory.
D. Apply the formula $slope = \frac{rise}{run}$ to several points in the plane.
E. Discuss the meaning of $m$ in the graphs of several equations of the form $y = mx + b$.

41. Consider the statement below.
*For all $a, b \in S$, if $ab = 0$, then either $a = 0$ or $b = 0$.*
For which of the following sets S is the above statement true?
i. the set of real numbers
ii. the set of complex numbers
iii. the set of 2x2 matrices with real number entries
A. i only
B. ii only
C. iii only
D. i and ii only
E. i, ii and iii

42. Some students were asked to prove that the following statement is true:
*When you multiply any 3 consecutive whole numbers, your answer is always a multiple of 6.*

Below are proofs offered by three of them.

***Kate's answer***

A multiple of 6 must have factors of 3 and 2.

If you have three consecutive numbers, one will be a multiple of 3 as every third number is in the three times table.

Also, at least one number will be even and all even numbers are multiples of 2.

If you multiply the three consecutive numbers together the answer must have at least one factor of 3 and one factor of 2.

***Leon's answer***

$1 \times 2 \times 3 = 6$

$2 \times 3 \times 4 = 24 = 6 \times 4$

$4 \times 5 \times 6 = 120 = 6 \times 20$

$6 \times 7 \times 8 = 336 = 6 \times 56$

***Maria's answer***

$n$ is any whole number

$n \times (n+1) \times (n+2) = (n^2 + n) \times (n+2)$

$= n^3 + n^2 + 2n^2 + 2n$

Canceling the $n$'s gives $1 + 1 + 2 + 2 = 6$

Which are valid proofs?

**A.** Kate's only

**B.** Maria's only

**C.** Kate's and Leon's

**D.** Leon's and Maria's

**E.** Kate's and Maria's

43. The statement 'For all whole numbers, if to the product of two consecutive whole numbers we add the larger number, the result is equal to the square of the larger number' can be expressed symbolically as: For all whole numbers n

A. $n^2 + 1 = n(n-1) + n + 1$

B. $(n+1)^2 = n^2 + 2n + 1$

C. $n^2 = n(n-1) + n$

D. $(n+1)n = n^2 + n$

E. $(n-1)^2 + 2n = n^2 + 1$

44. The polynomial $p^2 - p - 6$ can be factored into $(p-3)(p+2)$. If natural numbers are substituted in place of $p$, which one of the following statements is true about the set of numbers obtained?

A. Some numbers will be odd

B. The number zero does not appear

C. None of the numbers will be prime

D. All of the numbers will be less than 100

E. None of the above statements is correct

45. Let $f(x) = \log_2 x^2$. Which of the following functions have the same graph as $y = f(x)$?

i. $y = 2\log_2 x$

ii. $y = 2\log_2 |x|$

iii. $y = 2|\log_2 x|$

**A.** i only

**B.** ii only

**C.** iii only

**D.** i and ii only

**E.** i, ii, and iii

46. Students are given the following problem:

Find the number of the real roots of the equation $9^x - 3^x - 6 = 0$

Peter denotes $y = 3^x$ and gets the equation $y^2 - y - 6 = 0$, which has 2 different roots. He concludes that the given equation also has 2 different roots.

Which of the following is true about Peter's solution?

A. Peter's conclusion and his arguments are correct.

B. Peter's original approach to the problem (substitution of $y = 3^x$) is not correct.

C. Peter factors wrong.

D. The quadratic equation $y^2 - y - 6 = 0$ does not have 2 different roots.

E. Peter does not take into account the range of the function $y = 3^x$.

47. Which of the following can be represented by areas of rectangles?

i. The equivalence of fractions and percents, e.g. $\frac{3}{5} = 60\%$

ii. The distributive property of multiplication over addition: For all real numbers a, b, and c, we have $a(b + c) = ab + ac$

iii. The expansion of the square of a binomial: $(a + b)^2 = a^2 + 2ab + b^2$

A. ii only

B. i and ii only

C. i and iii only

D. ii and iii only

E. i, ii, and iii

48. A student is asked to give an example of a graph of a function $y = f(x)$ that passes through the points A and B (see Figure 1). The student gives the answer shown in Figure 2. When asked if there is another answer the student says: "No, this is the only function."

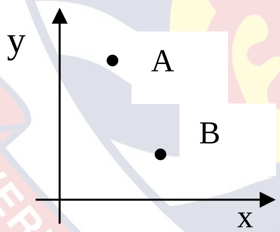



*Figure 1*

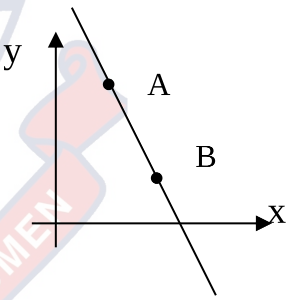



*Figure 2*

Which of the following best evaluates the student's answer of "No" to the second question?

A. The student is right, because that is the only way a line will pass through both points.

B. The student is right, because this function is of the form $f(x) = mx + b$.

C. The student is right, because his graph passes the vertical line test.

D. The student is wrong, because graphing is not an appropriate way to solve this problem.

E. The student is wrong, because there are infinitely many functions that pass through points A and B.

49. A textbook contains the following theorem:

*If line $l_1$ has slope $m_1$ and line $l_2$ has slope $m_2$ then $l_1 \perp l_2$ if and only if*

*$m_1 \cdot m_2 = -1$ (i.e. "slopes of perpendicular lines are negative reciprocals").*

*(McDougal Littell, Algebra 2)*

Three teachers were discussing whether or not this statement generalizes to all lines in the Cartesian plane.

Mrs. Allen: The statement of the theorem is incomplete: it doesn't provide for the pair of lines where one is horizontal and one is vertical. Such lines are perpendicular.

Mr. Brown: The statement is fine: a horizontal line has slope 0 and a vertical line has slope ∞ and it's OK to think of 0 times ∞ as –1.

Ms. Corelli: The statement is fine; horizontal and vertical lines are not perpendicular.

Whose comment(s) is/are correct?

A. Mrs. Allen only
B. Mr. Brown only
C. Ms. Corelli only
D. Mr. Brown and Ms. Corelli.
E. None are correct.

50. Consider the statement below.

*For all $a, b \in S$, if $ab = 0$, then either $a = 0$ or $b = 0$.*

For which of the following sets S is the above statement true?

i. the set of real numbers
ii. the set of complex numbers
iii. the set of 2x2 matrices with real number entries

A. i only
B. ii only
C. iii only

D. i and ii only

E. i, ii and iii

51. Mr. Nkrumah asked his algebra students to divide $x^2 - 4$ by $x + 2$. Eric said, "I have an easy method, Mr. Nkrumah. I just divide the $x^2$ by $x$ and the 4 by the 2. I get $x - 2$, which is correct." Mr. Nkrumah is not surprised by this as he had seen students do this before. What did he know? (Mark one answer.)

A. He knew that Eric's method was wrong, even though he happened to get the right answer for this problem.

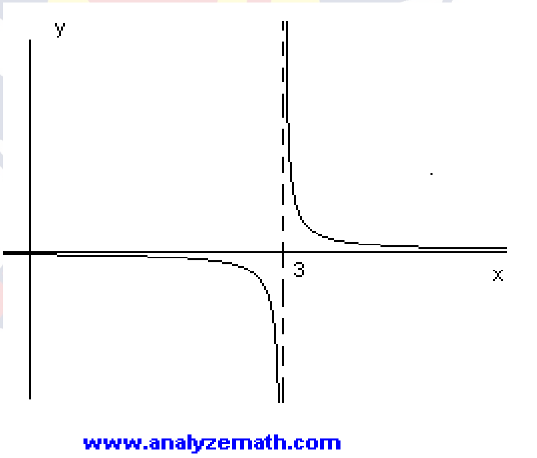
B. He knew that Eric's answer was actually wrong.

C. He knew that Eric's method was right, but that for many algebraic fraction division problems this would produce a messy answer.

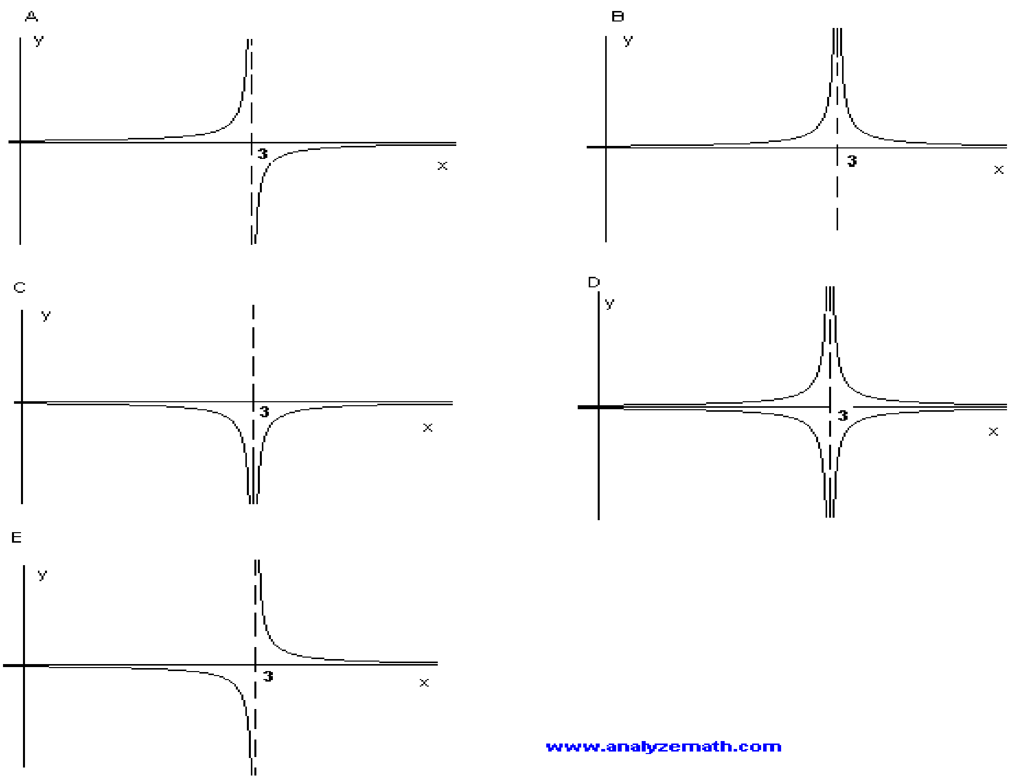
D. He knew that Eric's method only works for some algebraic fractions.

E. I'm not sure.

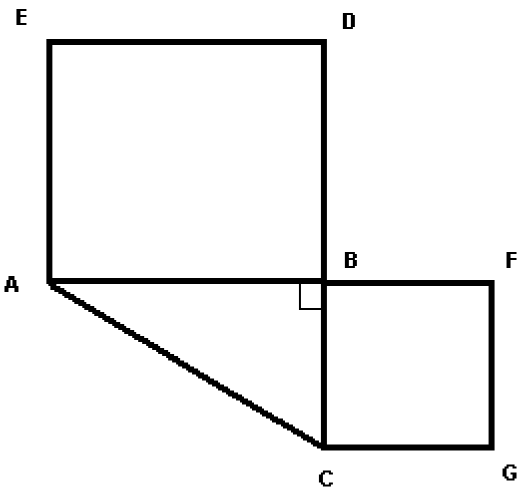
52. The graph of y = 2/(x - 3) is shown below




Among the following, which is the best possible graphical representation of y = -2/|x - 3|




53. In the figure below ABC is a right triangle. ABDE is a square of area 200 square inches and BCGF is a square of 100 square inches. What is the length, in inches, of AC?




www.analyzemath.com

A)10√3

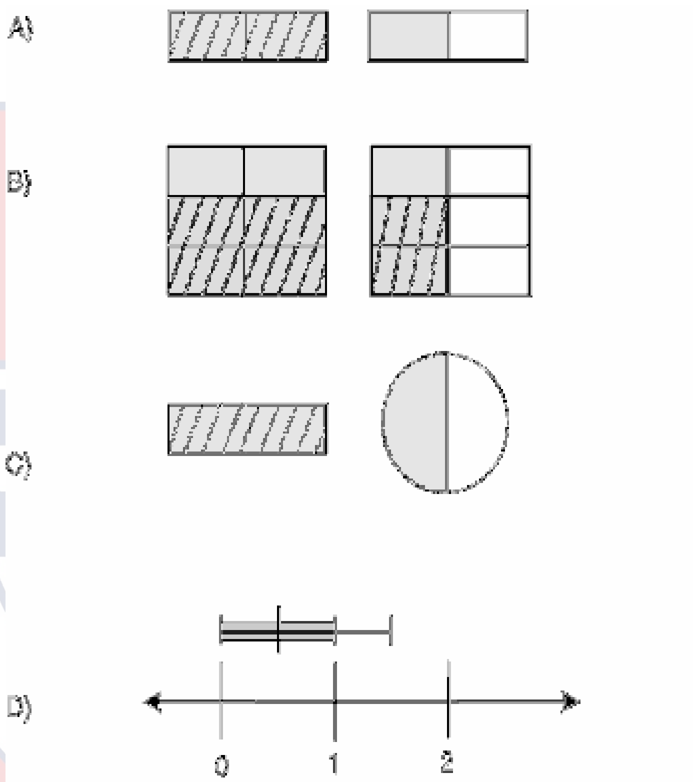
B)10√2

C)300

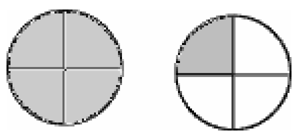
D)10

E) 15

54. At a professional development workshop, teachers were learning about different ways to represent multiplication of fractions problems. The leader also helped them to become aware of examples that do not represent multiplication of fractions appropriately. Which model below cannot be used to show that $1\frac{1}{2} \times \frac{2}{3} = 1$? (Mark ONE answer.)




55. Mrs. Johnson thinks it is important to vary the whole when she teaches fractions. For example, she might use five dollars to be the whole, or ten students, or a single rectangle. On one particular day, she uses the whole as a picture of two pizzas. What fraction of the two pizzas is she illustrating below? (Mark ONE answer.)



A. $\frac{5}{4}$

B. $\frac{5}{3}$

C. $\frac{5}{8}$

D. $\frac{1}{4}$

56. Mr. Fitzgerald has been helping his students learn how to compare decimals. He is trying to devise an assignment that shows him whether his students know how to correctly put a list of decimals in order of size. Which of the following sets of numbers will best suit that purpose?

A. .5 7 .01 11.4

B. .60 2.53 3.14 .45

C. .6 4.25 .565 2.5

D. Any of these would work well for this purpose. They all require the students to read and interpret decimals.

57. If $f(x) = ax^3 + bx^2 + cx + d$, what is the slope of the line tangent to this curve at $x$ = 2?

A. 8$a$ + 4$b$ + 2$c$

B. 8$a$ + 4$b$ + 2$c$ + $d$

C. 12$a$ + 4$b$ + $c$

D. 12$a$ + 4$b$ + $c$ + $d$

58. In a first-year elective mathematics class, which of the following is **NOT** an appropriate way to introduce the concept of slope of a line?

A. Talk about the rate of change of a graph of a line on an interval.

B. Talk about speed as distance divided by time.

C. Toss a ball in the air and use a motion detector to graph its trajectory.

D. Apply the formula $slope = \frac{rise}{run}$ to several points in the plane.

E. Discuss the meaning of $m$ in the graphs of several equations of the form $y = mx + b$.

59. Which of the following (taken by itself) would give substantial help to a student who wants to expand $(x+ y + z)^2$ ?

i. See what happens in an example, such as $(3 + 4 + 5)^2$ .

ii. Use $(x + y + z)^2 = ((x+ y) + z)^2$ and the expansion of $(a + b)^2$ .

iii. Use the geometric model shown below.

| | $x$ | $y$ | $z$ |
|---|---|---|---|
| $x$ | $x^2$ | $xy$ | $xz$ |
| $y$ | $xy$ | $y^2$ | $yz$ |
| $z$ | $xz$ | $yz$ | $z^2$ |

A. ii only
B. iii only
C. i and ii only
D. ii and iii only
E. i, ii and iii

60. Which relation is a function?

A. $\{(-1,3),(-2,6),(0,0),(-2,2)\}$

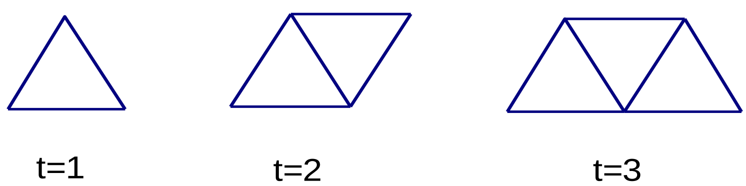
B. $\{(-2,-2),(0,0),(1,1),(2,2)\}$

C. $\{(4,0),(4,1),(4,2),(4,3)\}$

D. $\{(7,4),(8,8),(10,8),(10,10)\}$

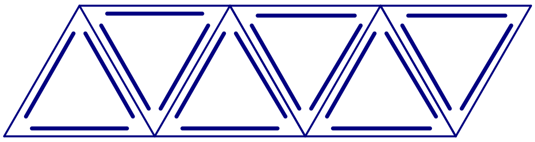
E. $\{(7,-4),(8,-8),(-10,8),(-10,-10)\}$

61. Amy is building a sequence of geometric figures with toothpicks, by following a specific pattern (making triangles up and down alternatively). Below are the pictures of the first three figures she builds. Variable $t$ denotes the position of a figure in the sequence.




In finding a mathematical description of the pattern, Amy explains her thinking by saying:

*"First, I use three sticks for each triangle:*

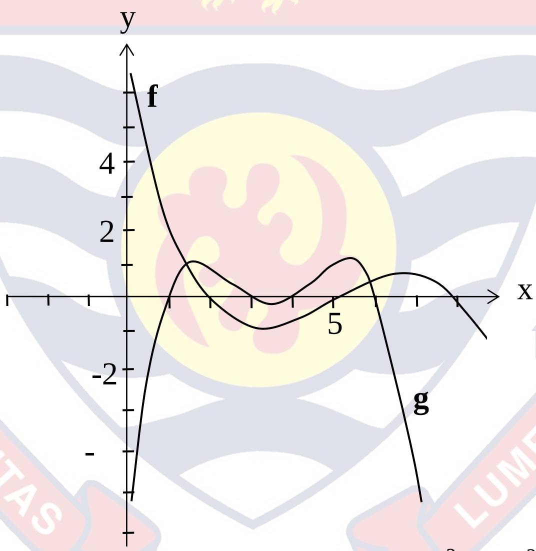


*But then I see that I am counting one stick twice for each of the triangles except the last one, so I have to take those away*."

If $f$ represents the total number of toothpicks used in a picture, which of the following equivalent formulas most closely matches Amy's explanation?

**A.** $f = 2t + 1$
**B.** $f = 2(t + 1) - 1$
**C.** $f = 3t - (t - 1)$
**D.** $f = 3t + 1 - t$

62. The graphs of two real-valued functions, $f$ and $g$, are shown below. Each mark on the axes represents one unit.




How many solutions does the equation $f^3 - 2f^2g + fg^2 = 0$ have on the interval [0, 8]?

A. 2
B. 3
C. 5
D. 6
E. 7

63. Which of the following is **NOT** true about the concept of absolute value?

A. Absolute value can be used to find the distance between two points on the number line.

B. The graph of the absolute value function $f(x) = |x|$ has no points below the horizontal axis in the Cartesian coordinate plane.

C. For all real numbers, $\sqrt{x^2} = |x|$.

D. For all real numbers, $\sqrt[3]{x^3} = |x|$.

64. Students were asked to solve the following problem.

*Is it possible to have a polynomial of degree 10 of the form*

$P(x) = x^{10} + a_9x^9 + \ldots + a_1x + 6$ *with 10 distinct integer roots?*

Which of the following is the most acceptable response to the question?

A. Yes, because every polynomial of degree *n* has *n* roots.

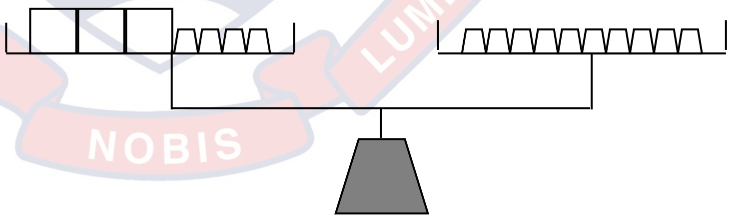
B. Yes, $P(x) = (x+1)^6(x-1)^2(x-2)(x+3)$.

C. Yes, $P(x) = (x-1)^2(x+1)(x-2)(x+2)(x-3)(x+3)^2(x-6)(x+6)$.

D. No, because the only possible integer solutions to $P(x) = 0$ are ±1, ±2, ±3, ±6 (i.e. there are only eight factors of 6).

E. No, because $x^{10} + 6 = 0$ has some solutions that are not integers.

65. Some textbooks suggest that teachers use a pan balance to represent mathematical sentences. For instance, if B represents the weight of each box pictured below (in ounces), and represents a one-kilogram weight, the balance pictured below represents the equation 3B + 4 = 10



Ms. Clarke is preparing to teach a unit on solving linear sentences. If X represents the weight of a given box, which of the following sentences can **NOT** be represented by a pan balance?

A. 13 = 4X + 5

B. 3X + 10 = 4

C. 3X + 3 = 2X + 15

D. 9 + 6X < 21

66. Currently, Germany has a law against creating new surnames for newborns by combining the parents' surnames with hyphens. A language expert explains why hyphenation is not a good idea for naming:

*If a double-named boy grew up to marry and have children with a double-named woman, those children could have four names, and their children could have eight, and their children could have 16… The bureaucracy shudders.*

(Excerpt from the front page of *The Wall Street Journal*, Wednesday, October 12, 2005)

For which of the following topics could the situation described by the expert be used as an introduction?

A. Direct variations
B. Linear functions
C. Quadratic functions
D. Exponential growth

67. Consider the following mathematical topics:

i. Composition of functions
ii. One-to-one functions
iii. Inverse functions
iv. Domain and range of functions

Which of the following orders could be used to teach these topics in a rigorous advanced algebra class?

**A.** ii, i, iii, iv
**B.** ii, iii, iv, i
**C.** iv, ii, iii, i
**D.** They can be taught in any order.

68. Mr. Matheson asked students to solve the following system of equations:

$$\begin{cases} 2x + y = 3 \\ 4x + 2y = 6 \end{cases}$$

Orlando wrote:

$$(-2)(2x + y) = 3(-2)$$

So

$$-4x - 2y = -6$$

$$4x + 2y = 6$$

$$0 = 0$$

This system doesn't have a solution.

Which of the following is true about Orlando's response?

**A.** Orlando's solution and reasoning are correct.

**B.** Orlando made an arithmetic error.

**C.** You cannot add equations.

**D.** Orlando drew the wrong conclusion from $0 = 0$.

**E.** None of the above

69. Which of the following questions that involve the equation $2x^2 - 3 = x + 1$ can be answered by graphing?

i. Determine how many real solutions the equation $2x^2 - 3 = x + 1$ has.

ii. Find the exact coordinates of the point(s) where the functions $f(x) = 2x^2 - 3$ and $g(x) = x + 1$ intersect.

iii. Determine the exact values of the solutions to the equation $2x^2 - 3 = x + 1$.

**A.** i only

**B.** ii only

**C.** iii only

**D.** i and ii

**E.** i, ii, and iii

70. When both sides of an equation reduce to the same number for certain values of the unknown number, the equation is said to be

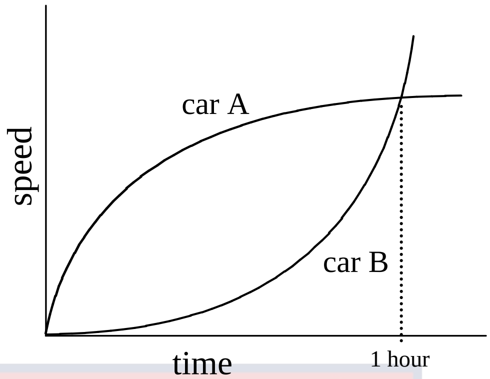
A. literal

B. satisfied

C. substituted

D. transitive

E. unsatisfied




71. The given graph represents speed vs. time for two cars. (Assume the cars start from the same position and are traveling in the same direction.) Use this information and the graph above to answer the question that follows.

    What is the relationship between the ***position*** of car A and car B at $t$ = 1 hour?

    A. The cars are at the same position.

    B. Car A is ahead of car B.

    C. Car B is passing car A.

    D. Car A and car B are colliding.

    E. The cars are at the same position and car B is passing car A.

72. Kwamena is taking medications for a recent illness. Every 6 hours he takes an antibiotic, every 4 hours he takes a pain reliever, and every 3 hours he drinks a glass of water. If he starts this regime at 10 am, at what time will he be taking both medicines and a glass of water?

    A. 12:00 noon

    B. 4:00 pm

    C. 6:00 pm

    D. 10:00pm

    E. None of these

73. As a teacher, how would you view student errors and misconceptions? I would see student replies that reveal a misconception as [choose one]

A. more important than correct ones, as they provide an opportunity to extend learning for that student and for others in the class who may share the same misconception.
B. to be avoided at all cost.
C. needing to be immediately countered by the teacher's intervention about what the correct solution is.
D. useful for assessing student ability.

74. The Assistant headmaster who teaches mathematics students are working on the following problem:

Is 371 a prime number?

As he walks around the room looking at their papers, he sees many different ways to solve this problem. Which solution method is correct? (Mark ONE answer.).

A. Check to see whether 371 is divisible by 2, 3, 4, 5, 6, 7, 8, or 9.
B. Break 371 into 3 and 71; they are both prime, so 371 must also be prime.
C. Check to see whether 371 is divisible by any prime number less than 20.
D. Break 371 into 37 and 1; they are both prime, so 371 must also be prime

**THANK YOU FOR YOUR TIME**

# APPENDIX F

## REQUEST FOR INSTRUMENT

University of Cape Coast

Faculty of Science and Technology Education

Department of Mathematics and ICT Education

Cape Coast

October 3, 2019.

Dr. Christopher Yarkwah

University of Cape Coast

Faculty of Science and Technology Education

Department of Mathematics and ICT Education

Dear Sir,

**PERMISSION TO USE INSTRUMENT FOR RESEARCH STUDY**

I am a postgraduate (MPhil) student from the Department of Mathematics and ICT Education of the University of Cape Coast. Currently, I am working on my thesis with the topic Pre-service Mathematics Teachers' knowledge for teaching senior high school Algebra. This research study is aimed at exploring the factors that characterise pre-service mathematics teachers for teaching senior high school algebra.

Reading your article titled "*Conceptualisation of teachers' knowledge in domain specific and measurable terms: Validation of the expanded KAT framework*", your instrument will be of good help to me in achieving the objectives of my research. Therefore, I write this letter to request for your permission to use your instrument for my research study.

I am counting on your cooperation.

Yours faithfully

Gideon Entsie

# APPENDIX G

## RESPONSE TO REQUEST FOR INSTRUMENT

Department of Mathematics and ICT Education
Faculty of Science and Technology Education
College of Education Studies
University of Cape Coast
Cape Coast
10th 0ctober, 2019.

Dear Mr. Entsie,

**RE: PERMISSION TO USE INSTRUMENT FOR RESEARCH STUDY**

With reference to your letter dated 3rd October, 2019, I wish to grant you permission to use my instrument for your research work.

Please make sure no changes are made to the instrument and no publication of any kind regarding the instrument is made without my prior permission.

Wishing you all the very best in your research work.

Thanks and best regards.

Cheers!

Sincerely,

Christopher Yarkwah (Ph.D.)

*Lecturer*

*Department of Mathematics and ICT Education*

*Faculty of Science and Technology Education*

*College of Education Studies*

*University of Cape Coast*

*Cape Coast, Ghana*

*West Africa*

*Email: cyarkwah@ucc.edu.gh*

*Tel: +233-209-209-147/+233-244-973-780*