ZDM (2019) 51:419–432
https://doi.org/10.1007/s11858-019-01025-z

ORIGINAL ARTICLE

Identity resources and mathematics teaching identity: an exploratory study

Forster D. Ntow[1,2] · **Jill Adler**[1]

Accepted: 8 January 2019 / Published online: 21 January 2019

Abstract

Previous studies have reported the influence of professional development (PD) on participating teachers' identities. However, *what* goes on in PDs, *how* and *why* they shape particular identities require further investigation. This study contributes in this direction by drawing on the notions of *practice-linked identities* and *identity resources* to examine how two teachers' mathematics teaching identities developed following their interactions with the resources offered in a particular PD. We argue that their developing mathematics teaching identities appeared to be linked to their backgrounds and initial motivations for joining the PD, which in turn influenced their selective interaction with resources. Implications for research and PD are discussed.

Keywords Practice-linked identity resources · Professional development · Mathematics teachers identity

1 Introduction

We have learned a great deal about teachers' learning and identity development through professional development (PD) (e.g. Goldsmith et al. 2014). However, we do not know enough about *what* happens in PD settings (Sztajn et al. 2017), and *how* and *why* such contexts of learning shape teachers' mathematics teaching identities, MTIs. Hodges and Hodge (2017), in their study of identity development through pre-service mathematics teacher education, called for "more studies that examine the detailed resources that contribute to the development of different kinds of personal identities in relation to mathematics teaching" (p. 116). In this paper we seek to contribute to research on PD and mathematics teaching identities (MTIs) by examining the resources made available in the Wits Maths Connect Secondary (WMCS) PD (the *what* of PD), *how* teachers identified with the particular resources offered and *why* particular identities formed. We focus on two purposefully selected teachers who participated in WMCS, yet reflect interesting similarities and differences in terms of how the identity resources offered in the PD influenced their MTIs.

Elsewhere (Adler 2017), we have described our orientation to mathematics as a network of scientific, connected and hierarchic concepts (Vygotsky 1978) and to teaching and PD as social practices. Also, we view teachers' developing MTIs as a function of access to and interaction with particular resources made available in the practice (Lave and Wenger 1991). With this framing, our study of MTIs required an investigation into the particular resources WMCS made available, how teachers interacted with these resources and why this might have led to differing MTIs. To this end, we have drawn analytically on Nasir and Cooks' (2009) notions of "practice-linked identities" and "identity resources"; and their distinction between ideational, material and relational resources (each elaborated later). This is in line with Adler's (2000) conceptualisation of resources in teaching and teacher education as extending beyond the material to include socio-cultural resources like knowledge, language and time. Our research questions are:

1. What ideational, material and relational resources were made available in the WMCS PD?
2. What are the influences of the practice-linked identity resources offered in a particular PD on teachers' developing MTIs? How do they
   (a) talk about selective resources? and

✉ Jill Adler
jill.adler@wits.ac.za

Forster D. Ntow
fntow@ucc.edu.gh

1 University of the Witwatersrand, Johannesburg, South Africa

2 University of Cape Coast, Cape Coast, Ghana

(b) use them (enactments) in their mathematics teaching?

2 Locating the study

Identity as a lens to study both students' and teachers' learning continues to gain traction in mathematics education (Graven and Lerman 2014; Sfard and Prusak 2005) leading to major reviews to understand its potentials and challenges, and to suggest directions for future research (Darragh 2016; Lutovac and Kaasila 2017). While potential lies in 'identity' offering researchers a lens to either zoom in and/or zoom out on both the social and individual components of learning (Lerman 2001), how this complexity is to be studied is debated. There is agreement that clear operational definitions of identity are missing in much of the literature on identity, and thus calls for greater clarity. At the same time there is disagreement on theoretical and related methodological approaches. Darragh (2016) argues for a sociological orientation to identity as performance rather than a psychological orientation to identity as acquisition. Lutovac and Kassila (2017), with a focus on MTI, argue against such polarisation, calling for the use of methodologies that are versatile and employ multiple data sources in order to provide a holistic view of MTIs and so illuminate the complexity of identity as at once personal and situated. This complexity is reinforced in the more recent review by Radovic, et al (2018), who avoided including studies of MTI as a professional identity is more complex than the developing identities of mathematics learners. We agree, particularly as a professional identity includes a disciplinary identity as well as a teaching identity, and their amalgam in an MTI. Our approach thus aligns with Lutovic and Kaasila, and does so with what we hope is clarity of definition, coherence in theory and method, and with an eye to both the mathematical and teaching.

The complexity of professional learning and identity formation is well documented. For example, Battey and Franke (2008, p. 129) analysed "teacher learning and classroom practices" following their participation in a PD. Similarly, Gresalfi and Cobb (2011) documented the norms a group of teachers had to negotiate following their participation in communities with different visions of high-quality mathematics instruction. They argued that teachers' motivation for affiliating with PD norms for effective teaching was "a critical condition" for improving their teaching (p. 270).

Additionally, PD often seeks substantive changes in teaching practices. Much of the research on learning from PD has focused on PD programmes where, for example, the concern was with the integration of technology into teaching (Goos 2005), or with using cognitively demanding tasks productively (Gresalfi and Cobb 2011), or learning about and being responsive to students' thinking (Battey and Franke 2008). These studies have highlighted the challenges teachers face in engaging with ruptures to their current practices as promoted in the PD. However, we have not come across studies of MTI development focusing on PD that aims to impact the quality of instruction while remaining relatively close to current classroom practices such as the study reported in this paper. What might we learn about MTI development through participation in PD that seeks change through continuity and strengthening rather than 'rupture'? And, the broader question persists: What does it mean to study identity development across these different contexts of practice?

Gresalfi and Cobb (2011) point out a reliance on narratives without particular attention to classroom practice in identity studies reviewed, commenting that "…these lines of work foreground and separate what is believed or recounted from the contexts in which beliefs or stories are constructed". This critique is especially valid with Borko (2004) arguing that: "To understand teacher learning, we must study it within these multiple contexts, taking into account both the individual teacher-learners and the social systems in which they are participants" (p. 4). In line with the call by Lutovac and Kaasila for multiple sources of data, there is thus a need for studies that focus on the continuum of practices, from initial motivations, through the PD itself and into actual practice, an approach we take in this study.

In a review by Stazjn et al. (2017) on mathematics teacher PD that linked it to instructional quality and student learning, they described the progress that has been made in illuminating the complexity and situatedness of PD and its impact, notably in particular case studies. However, they observed that there is an emphasis on impact in the research reviewed without simultaneous illumination of what is offered in the PD. As we suggested earlier what is offered in PD, in both its mathematics and teaching dimensions, is a critical component of understanding mathematics teacher learning from PD, and so MTI. Hence our interest in what is made available in PD, specifically the ideational, material and relational resources offered, and how participants identify with these offerings as they talk about the PD, their motivations and learning in it, and their enactment of valued practices.

3 Identity and identity resources

Implicit in all the above is a view of MTI as relational. As Lave and Wenger argue, a theory of social practice such as WMCS, "emphasises the relational interdependency of agent and world, activity, meaning, cognition, learning and knowing" (1991, p. 50). We define MTI as a relationship with specific practices, where the process of identification involves an interaction between the person and the set of resources made available in the learning community and contexts of practice.

A recurrent feature of most PDs is an offering of ideas (in our terms resources) about how to improve teaching (Kennedy 2016), together with ideas about the content i.e. mathematics being taught, and supporting mathematics and teaching materials. In addition, a common understanding of learning in PD is that it is collaboratory, leading PD providers to work on building communities of inquiry (Jaworski 2006). These resources are what Nasir and Cooks (2009) refer to as ideational, material and relational resources respectively. We view these resources as constituting the *what* of any PD that, being a social practice, will shape *how* and *why* particular MTIs emerge. All PD and teacher development programmes make particular resources available with the assumption that they are valuable artefacts of the teaching profession, be these mathematics or mathematics teaching focused. We have thus found synergy with Nasir and Cooks' (2009) notion of *identity resources and the practice-linked identities these afford*.

Nasir and Cook (2009) define ideational resources as "ideas about oneself and one's relationship to and place in the practice and the world, as well as ideas about what is valued and what is good"; material resources "means the way in which the physical environment, its organisation and the artefacts in it support one's sense of connection to the practice" and relational resources as the "positive relationship with others in the context that can increase connection to the practice" (p. 44). Their framework was developed to study the practice of athletics, a practice which is quite different from the practices of teaching and teacher education. For example, (1) it is difficult to determine whether an individual teacher has an inbound or peripheral trajectory since indicators for competence are situated, bound up with the complex relationship between culture and pedagogy (Alexander 2000); and (2) in PD, the learning context may be outside their professional teaching context. We nevertheless found this conceptualisation analytically useful as: (1) both practices value particular ideas and these serve as expectations of how a competent member could and should act, or in Adler's (2000) terms, become knowledge-able); (2) in both contexts, access to particular material resources is crucial in participants' learning. For athletes, access to spikes is crucial in who becomes a hurdler (Nasir and Cooks 2009) while in the teaching profession access to curriculum materials like textbooks are vital for effective mathematics teaching (Remillard 2005). (3) In both contexts, participants' learning takes place alongside other professionals, such as coaches/educators, other participants, or in communities of practice (Wenger 1998) where positive relationships with others can increase connections to the practice.

4 Context and the professional development programme as research setting

The WMCS is a research-linked development project working with secondary mathematics teachers in disadvantaged and underperforming schools in selected school districts in the wider metropolitan area of Johannesburg, South Africa - schools that have been appropriately described as "schools for the poor" (Shalem and Hoadley 2009). Here, overcrowded classrooms tend to produce instructional practices with extensive teacher talk, limiting opportunities for learners to talk publicly or with other learners beyond whole class chorusing of one-word answers. Notwithstanding the massive increase in qualification levels of teachers post-apartheid, poor learning outcomes in mathematics persist, together with evidence of inadequate teacher knowledge and limited practices (e.g. Bansilal et al. 2014). State interventions include recurring curriculum reforms, with increasing prescription in an attempt to address concerns of inadequate curriculum coverage across many schools, and to support teachers. Recently, Grades 8 and 9 mathematics teachers have been required to follow a weekly annual teaching plan (ATP). In the province where the WMCS is located, teachers have daily lesson plans in line with the ATP and their use of these monitored by district officials.

The WMCS intervention provided a 16 contact-day mathematics for teaching course (over 8 by 2-days), named Transition Maths 1 (TM1). It was aimed at lower secondary teachers (Grades 8–10) teaching in low income communities and schools as described above. It extended over a full academic year, i.e. 10 months, making available *time* (in PD sessions) *over time* (over a full year) for teachers to engage with mathematical knowledge and aspects of teaching practice. The main focus (75% of the time) was on mathematics. The goal was for participating teachers to strengthen their relationship with mathematics by revisiting, deepening and/or extending their knowledge of the mathematics they teach in school (Pournara et al 2015). An underlying assumption was that increased mathematical competence and confidence was an important first step to improving teaching. The remaining 25% of time was structured by a Mathematics Teaching Framework (MTF), (see Adler and Ronda 2015), constituted by four mathematics teaching practices elaborated later.

5 Methodology

5.1 Participants

We focus on two teachers, Patricia and Thulelah (pseudonyms), who participated in the 2016 TM1 course. They were part of a cohort of 48 teachers. They were selected as telling cases (Bohning and Hale 1998). Both were teaching in Grade 9, relatively inexperienced (less than 4 years) and in schools where poor mathematics performance was a major concern. But they were also different. Patricia had no experience either of schooling or living in a disadvantaged community, or of teaching in the lower secondary grades. She was also teaching children whose cultural background and linguistic resources were different from hers. Thulelah, on the other hand, grew up in a disadvantaged community. She had direct experience of such schooling as a learner and now as a teacher. She also shared the linguistic resources of many of her learners.[1] In addition, Patricia had a relatively strong mathematics background, obtaining high marks in the developmentally oriented WMCS mathematics entrance test (Pournara et al 2015). Thulelah, like many others teaching mathematics in Grades 8 and 9 in schools in poorer communities, came with a weaker mathematics base, evidenced in her test results.

Another difference was their motivation for joining WMCS, as indicated in the entry survey completed by 40 of the 48 teachers in the 2016 cohort. The survey focused on teachers' reasons for joining the PD and what they hoped to learn. Patricia's motivation for joining the PD was to learn ways of teaching better in her context. This was evident in her responses to the question of "What two key things do you hope to learn from the course?" She foregrounded teaching in both: "how to teach maths effectively in an under resourced school" and "how to fill gaps my learners may have without moving too far away from the ATP and wasting too much time". Thulelah hoped: "to teach effectively … to produce a well-timed lesson … (and) manage a large group of learners so most of them understand and pass maths"; and "have deeper understanding of the subject". While both mentioned constraints of time and interest in their learners, their orientations to these appear different. *Patricia points to what is lacking in the context* (resources and learner knowledge), while *Thulelah points to what is needed from her* (a better timed lesson that includes all learners, and deeper mathematical understanding). Thulelah thus articulated a clearer vision of what she herself needed to come to know (deeper mathematics) and be able to do (meet her learner needs within the given curriculum). These articulations together with their different mathematics background suggested that their developing MTIs through participation in the course would be interesting to explore.

5.2 Data sources

To enable our investigation of both mathematics and mathematics teaching resources, we focused on one topic – functions. In response to Lutovac and Kaasila's (2017) call for use of multiple data sources that offer a broad view of what goes in the study contexts, we collected three sets of data: (1) video recordings and materials (powerpoint slides, handouts etc) of TM1 sessions 3 and 4 on functions - these would communicate what was valued specifically in relation to functions (17.5 h) and the teaching of these (6.5 h). (2) Video recordings (and transcriptions) of two consecutive lessons on linear functions taught by each teacher. These lessons were to provide insight into whether and how they used resources made available in the course. (3) Individual semi-structured reflective interviews immediately after each lesson observation. The interview guide was organised around themes, provoking narratives such as: (a) the teacher's goals for each lesson and whether she felt she succeeded with these; (b) what, if anything, was different about her teaching of functions relative to previous years, and why. The interview ended with explicit discussion of the teacher's valuing of the teaching ideational resources in the MTF. The interview and observation took place in September 2016 to coincide with the teaching of functions in the school calendar. While this was before the TM1 course ended, seven of the eight 2-day TM1 sessions had taken place.

5.3 Data analysis

Examination of the PowerPoint presentations, in conjunction with video recordings demonstrated that the slides in TM1 captured valued ideas about mathematics (functions) as well as about mathematics teaching and the elements of the MTF, and the main tasks and activities offered. We carried out a content analysis on all the slides from TM1 sessions 1, 3 and 4 and thus of the 'what' of functions in TM1. For example, Fig. 1, slide 24 from TM1 Session 3 shows a valuing of not only different representations of functions, but their connectedness, reinforced in the activity for teachers in slide 25.

Notwithstanding the limitations of excluding actual transcripts of interactions amongst teachers, and between them and the lecturer we hold that the slides identified from the sessions capture the ideational resources made available.

We carried out a narrative analysis of *the transcripts of the two interviews* with an extended utterance as unit of analysis. We identified and coded narratives where the teacher

[1] In urban areas, mathematics classes are multilingual – learners come with varied main languages. The language of learning and teaching in all secondary schools is English, unlike Thulelah, not all teachers will be multilingual themselves.

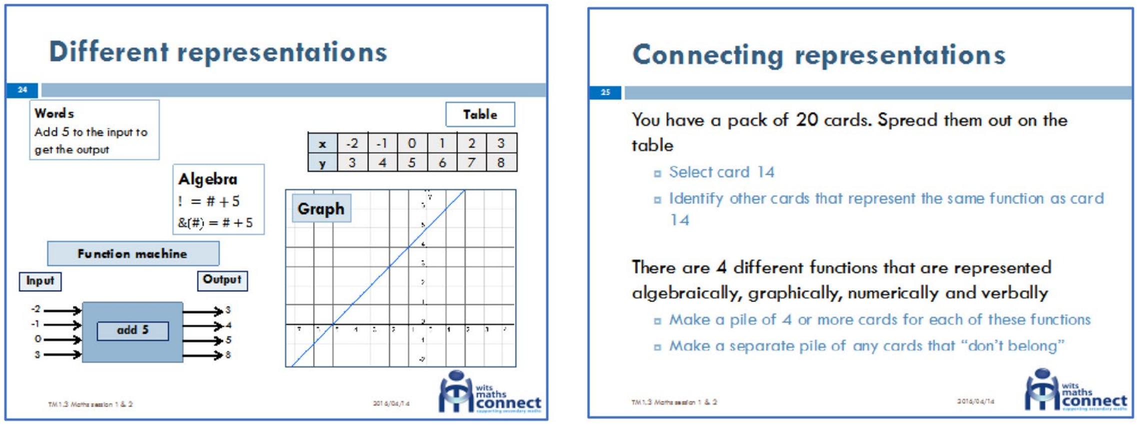

Fig. 1 Slides 24 and 25 from TM1.3 in April 2016

Table 1 Analytic coding related to the three resources offered

Theme	Codes	Utterance/indicator	P	T	Total
Ideational resources_ mathematics - functions (as scientific knowledge)	Defined A function is a relationship between input and output	The idea of a function, like an x coordinate should have only one y value…to classify them as a function or a non-function (P)	3	5	8
	Different representations functions as connected representations	I learnt in that course that linking the different representations enhances the understanding (P)	2	17	19
	Generality Parameters have meaning and generality	I used the idea of the constant and the increasing and the decreasing to show them that if m is positive, the graph slopes as if it is increasing, and if m is negative, it's like a decreasing function (P)	4	0	4
Ideational resource_ teaching (as intentional and goal directed)	Intentional_examples Deliberate selection and sequencing of examples	What do I want them to achieve? Yes, I know this is my goal but are they going to achieve that goal if I give them this list of examples? (P)	9	5	14
	Intentional_learner Opening up space for learners to contribute to identify misconception	They are voicing what they understand. So I can pick up from there the misconception that she's saying this but actually she should be saying that, so that I can help in that (T)	0	6	6
	Intentional—goal directed Need for lesson goal	Obviously, before you do anything, you have to know where you are going. So this is the goal, what do I want to reach? (T)	3	7	10
	Intentional—talking mathematically	The language sticks out because the way we say things even amongst ourselves to each other, I think we need to change that as well (P)	7	3	10
Material resource	Documents, software, teaching materials; textbooks etc	You can change your approach because Maths Connect, they give us these nice activities too. So you can actually incorporate that with your classes as well (T)	2	1	3
Relational resource	People	I think what's different about it is the people that I'm with. They have experience of being in schools like this and their knowledge is different (P)	4	2	6

P Patricia, *T* Thulela

talked about herself in relation to the specific resources offered namely material resources, ideational resources (ideas about mathematics and ideas about how to teach), and relational (interactions with others). Similar or related utterances were collected into broader themes, as summarised in Table 1 together with codes and their indicators. We have

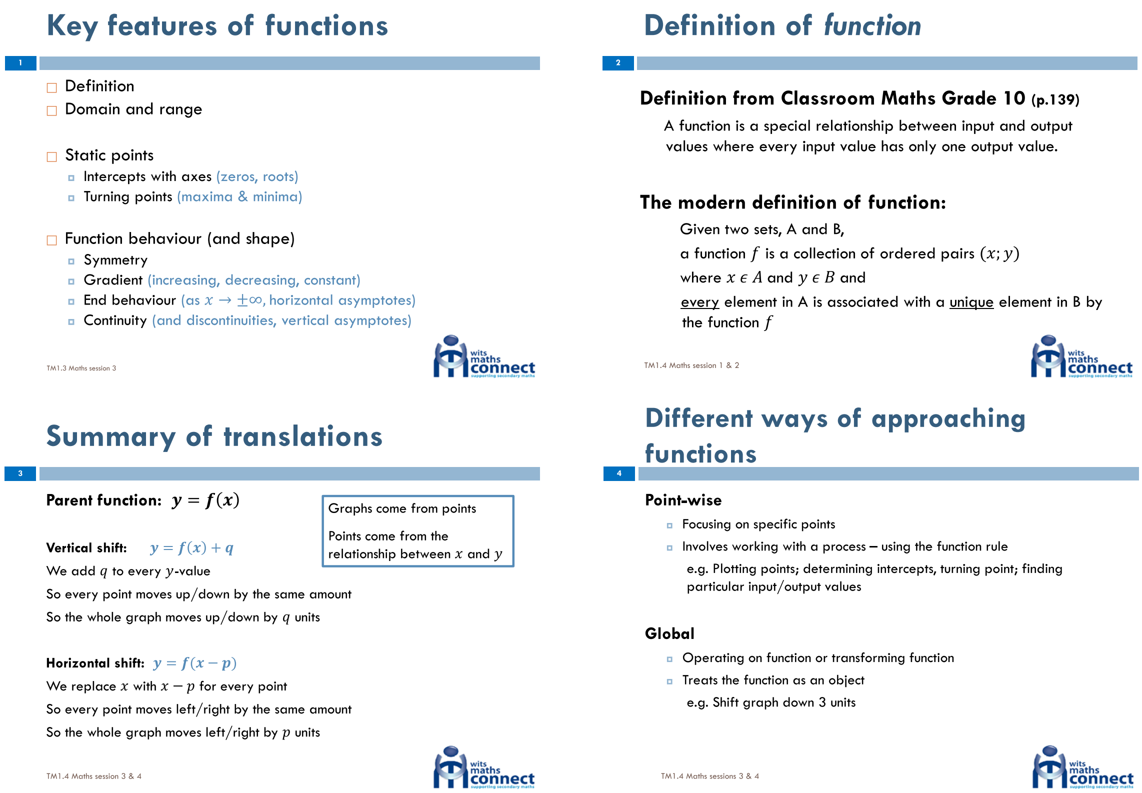

Fig. 2 Slides from TM1.3 and TM1.4 on functions

included the tallies of utterances from our analysis here and return to discuss these in the results section.

We used *the video tapes of lessons* as supporting data sources to evidence take-up at the level of enactment. We have selected one excerpt for each teacher that enables us to illustrate their enactment in relation to emphases in their interview narratives.

6 What practice-based resources were offered in the WMCS PD?

6.1 Ideational resources

Ideational resources were made available two ways – ideas about mathematics in the mathematics focused sessions; and then specific ideas about components of mathematics teaching within the MTF.

Intended *mathematics ideational resources* were embedded in tasks presented to teachers. As evident in Fig. 2, key ideas about functions included: (1) key features – domain, range, static point, behaviour (2) definition in school and more formally (3) function transformations (4) pointwise and global approaches to functions and (5) different representations of functions and their connections. *Each communicates ideas about functions*. In the first TM1 session (TM1.1) a repeated refrain was "Does it always work, and if so why? (TM 1.1) and thus a valuing of reasoning mathematically.

As illustrated in Fig. 3, the MTF drew attention to (1) articulating a lesson goal; (2) choosing and using examples; (3) providing explanations and justifications; and (4) setting up appropriate learner activity all in relation to the lesson goal. For WMCS, a 'good' lesson would reflect coherence among the lesson goal, examples and associated tasks and the mathematical words used and how ideas were justified.

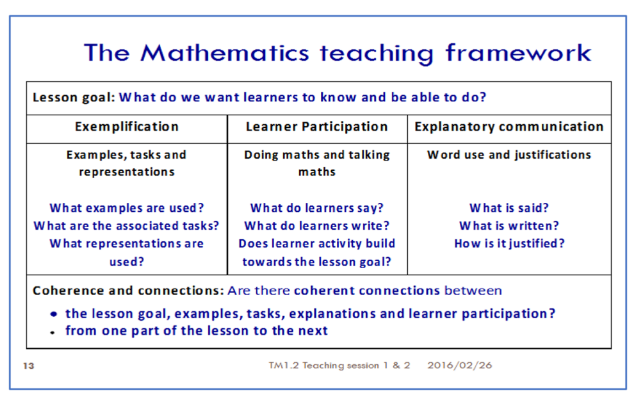

Fig. 3 Slide projection of WMCS Mathematics Teaching Framework (MTF)

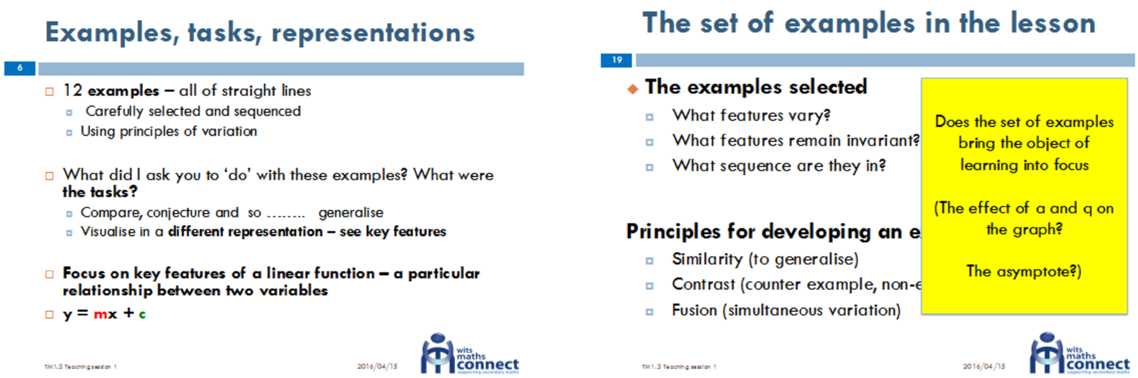

Fig. 4 Slides 6 and 19 from TM1.3 session on teaching

This would be characterised by more thoughtful selections of examples and tasks that invited learner participation, and by mathematical explanations that focus explicitly on the mathematics that the teacher intended learners to learn.

As such, the MTF provided the participating teachers with ideas about 'good' mathematics teaching with emphasis on mathematical coherence, deliberately remaining close to current teaching practices and curriculum demands so as to be possible to implement. In concert with the mathematics sessions in TM1, valued mathematics taught would enable learners to reason and justify, and appreciate generality. The MTF was introduced and its rationale discussed in TM Session 1.

> This framework will be examined in close detail through each of TM 1.1 to 1.8. ... our goal is to make explicit for teachers why MTF is focused on lesson goal (or object of learning), exemplification, learner participation, and explanatory communication. So we hope that you'll find that interesting and that by the end of the year it will be a tool for you.

The MTF also projected teaching practice requiring deliberate planning. In TM1 Session 3 the teaching sessions focused on exemplification with the slides highlighting ideas offered in terms of strategies for choosing examples drawing on the language of variation (Watson and Mason 2006) (Fig. 4). Session 4 focused on learner participation, grounded in the idea that learning mathematics entailed "learning to talk, and talking to learn" (TM1.4).

6.2 Material resources

With respect to *material resources*, time was set aside to introduce teachers to the use of Geogebra since most of them had no access to such software previously. It was also used extensively in the functions sessions by the lecturer. Textbooks served as reference materials in sessions and for teachers to consult for their own learning and do assignments. These included Grades 9, 10 and 11 textbooks prescribed in school, and a pre-calculus textbook (where, for example, the formal definition of function could be found). Packs of cards with different functions representations was a *material resource* used and as teachers worked collaboratively at a table. A particular resource teachers worked on in Session 4 and then in later sessions on Trigonometric functions was an A2 size laminated grid, on which a Cartesian Plane could be drawn and graphing done. The grid was also intended as a resource teachers could use in school – as most would not have grids on their chalkboards.

6.3 Relational resources

Relational resources were made available in the way the TM1 sessions were organised. There was one lecturer with four to five teachers sitting at one table with ten tables spread across the large teaching room. Teachers spent large amounts of time working on tasks collaboratively with others at their table. In each session there were at least three graduate or postdoctoral fellows who interacted with teachers at particular tables while they were working on tasks. Through the year, additional time was made available for teachers to meet with one of the project members if they desired additional support with mathematics or mathematics teaching tasks. As such, the teachers had access to numerous and different layers of relational resources to further increase their opportunities to learn.

If these were the identity resources made available, which of them did Patricia and Thulelah talk about and act with and how?

7 Focal Teachers' Identification with WMCS Identity resources

The description of the 'what' in the TM1 course projects an emphasis on mathematical and mathematics teaching *ideas*, with *material and relational resources* supporting the learning of ideas. It is thus not surprising that both teachers spoke most about ideational resources as reflected in Table 1. Their emphases within these in their reflective interviews point to their preferences. Both teachers identified strongly with being teachers of mathematics, and having a good understanding of it. Patricia said: "I think it's your understanding [of the subject] as an educator". In her view, a teacher of mathematics, "should be able to answer learners' questions, even from a higher grade.", and this requires a deeper understanding of the content that goes beyond the "good scores" she got as a student. Thulelah valued learning from TM1 because "we are dealing with the misconceptions for both teachers and learners". She perceived both a personal and professional need in terms of her own and learners' misconceptions and difficulties with topics such as functions. With respect specifically to functions, Thulelah was clearly taken by the significance of different representations (17 utterances). Patricia reflected across definitions, representations and the generality of parameters, the latter being absent in Thulelah's reflections. Both talked about being deliberate in their teaching, with respect to clear goals, example selection, and language use. Yet is it only Thulelah who talked intentionally 'opening up space for learners to contribute to identify misconception' (see Table 1). We now move on to more detailed description and analysis of each teacher' talk of and enactment with identity resources offered in the PD.

7.1 Patricia's enactment and talk of resources related to teaching mathematics

Patricia's lesson goals were 1): "to recap the terminology because according to CAPS, a major part of functions is recapping and revisiting all of the grade eight terminology" (e.g. co-ordinates, quadrants, points, input and output) and 2) "to show them how the gradient affects the steepness of a graph." Prior to the selected episode from the first lesson, Patricia reminded her learners that they had learned to "draw graphs" on the "Cartesian plane." She then invited learners to describe the Cartesian plane in terms of "how it looks" and how the axes are "labelled". She also had learners call out, in chorus, whether the signs of the co-ordinates of points in each of the four quadrants were positive or negative. She then wrote a set of six ordered pairs shown in Fig. 5 on the board in one go, *copying all from a prepared worksheet that she continually referred to through the lesson*. The associated task was for learners to identify the quadrant in which each point lay. She demonstrated how to do this with number 6, the point (2; 1). She then asked the learners to complete the remaining examples "very quickly". She walked around parts of the classroom looking at what learners were doing, and noticed that most were unsure of numbers 4 and 5. She wrote the relevant quadrants onto the board for 1, 2 and 3. The excerpt begins at this point.

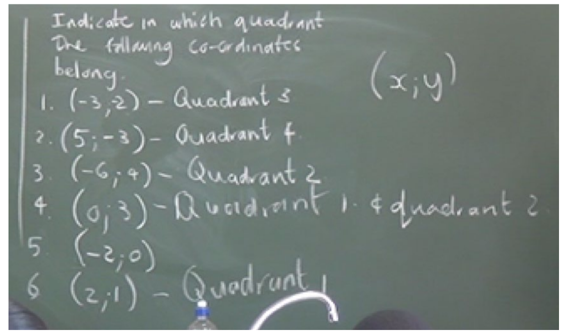

Fig. 5 Excerpt from Patricia's board work

7.2 Excerpt 1: (13:01–16:45)

T: Now would anyone like to give us the answer to number 4 .... Anyone who would like to try? Yes, [L1]?

L1: Number four its quadrant one.

T: Quadrant one. Okay let's see if she is correct. So if you have your ruler, if we look at our x values our x value is zero. Right I know it's a bit messy, but you know how to do this. So here our x value is minus one, here our x value is one. Here our x value is zero. So it's on our Y axis. And our y value is positive three. So it's this point here *(pointing to the particular points on the Cartesian plane on the board as she speaks)*. Where the two dotted lines or broken lines meet is our point. If you look at this point, is it in quadrant one or is it in quadrant two?

L2: One.

T: But Zero has no value. Yes, [L3] tell us …

L3: Because three is in *y* axis.

T: And if you look here (pointing to quadrants 1 and 2) our y values are positive and our y values are positive, so it could be in quadrant one, or it could be quadrant two. But zero has no value, it is neither positive, nor is it negative, so it is in between quadrant one and quadrant two. Yes?

And she wrote this onto the board.

We have selected this excerpt as it typifies Patricia's teaching through the two lessons, and enables us to discuss her identification with resources related to teaching mathematics. Here, as with other tasks, the example set suggests a careful selection. Four of the points can be located in particular and different quadrants, (1, 2, 3 and 6). Examples 4 and 5 are special cases as these points lie on the axes. The inclusion of 4 and 5 was purposeful, yet *she* ultimately asserted the conclusion.

Other tasks through the lessons were to identify functions as tables of values and later as she moved to the linear function, to identify how the gradient changed as the co-efficient of x in a set of linear functions changed from ½ to 1, 2 and 3 while the constant was kept invariant. Here too, she stated the generality of the parameters and not her learners.

In the interview Patricia talked repeatedly about examples as critical to achieving her goals for teaching and something that prior to WMCS she hardly thought of:

> We don't think about stuff this way. Yes, I know I have to give them examples … but am I really thinking about the sequence? Yes, I know this is my goal but are they going to achieve that goal if I give them this list of examples?

and later.

> I don't think I would have thought about examples the way I have [sic]. I would have not thought about the sequence. That's for sure. I never really thought about sequence before.

Patricia identified strongly with mathematics teaching as an intentional act where example selection is purposeful. She showed an appreciation of variation amidst invariance, an idea stressed in relation to example sets together with the notion of contrast. Referring to the task of identifying functions later in the lesson, she said:

> Also, in terms of examples, we have to show them what it's not in order to show them what it is. I think I tried to do that with the function thing, to show them that x is equal to 2 is not a function. So I was thinking about that when I was doing that lesson, let me not lie.

As indicated above, Patricia asked learners to justify 'why', as with L3: "tell us why it's not in between? Tell us why". She wanted her learners ask why, justify their thinking and 'debate':

> Well, when we did the Cartesian plane … when I put it in between, and I purposely did that, I gave them a point in between quadrant one and quadrant two, knowing that everyone would say it's quadrant one – like I wanted them to ask me why.

She lamented that she was not able to follow through with this across the lessons. On a later episode where there was confusion with tables of values she said:

> I really messed up the explanatory part … [but felt she] got them back eventually today [second lesson]. … Sometimes what they say I may not understand. Sometimes they don't understand the language I'm using but in my head it sounds beautiful. Like I know I'm explaining it beautifully but they don't get that. So, that for me is very difficult, to be honest.

She elaborated why she had difficulties communicating with her learners, saying that her lessons were carried out in a science laboratory and therefore perceived it as not conducive to dialogue:

> Like yesterday, when they were like "zero is positive," that is a debate for them. They can go on debating about that till they get to that point of "After all of the conflict, I agree, zero is neutral". I would love a class where I could do that.

but:

> If … you look at my classroom, I'm using a lab, it's difficult to walk through and help kids the way I want to help them. Like I teach life sciences and my classes are like 30 to 40; it's easier to work with them individually. Here I can't do that in this classroom setting. It's not conducive to learning.

Exacerbating this challenge was the time such communication necessarily took and her commitment to completing the ATP since one of her motivations for joining WMCS was to support her learners without "… moving too far away from the ATP…". She thus offered example sets that could provoke discussion but ultimately closed these providing the answers.

Patricia also talked about her identification with the mathematics offered in TM1, valuing its difference from her school days. She referred specifically to connected representations and to the formal definition of function. e.g.:

> I understood the one-to-one and one-to-many, like I got that when my teacher said that when I was in school but the actual definition of the function, I was never given up until that point (TM1).

And later.

> … [and] linking them because I learnt in the course that linking the different representations enhances the understanding.

On material resources, Patricia said she consulted "lots of textbooks" and other curriculum resources in order to plan her lessons, reflecting her purposiveness. In the two lessons we observed, other material resources offered in the course were not visible. She nevertheless talked about Geogebra and card sorting activities in the interview, suggesting that these were good for her personally but not professionally. She continually referred to her contextual conditions:

> In an ideal world you would have amazing resources like we do at Wits Maths. Like we have the projector, we have Geogebra, like it's so easy and I can do that if I had those things. … But then you come to a school like this and there was no electricity, there was no photocopy machine and … So that was very challenging for me.

She also talked of her appreciation of the relational resources in the course:

> What's different about it is the people that I'm with. They have experience of being in schools like this and their knowledge is different … I feel that I'm learning from the people around me because when I was doing my undergrad everyone was fresh. No one had experience of being in like rural schools. … Now, being in an environment where we as educators can share – like if I sit in my group I can say I don't have this to do this and they'll say: maybe you should try this and use this textbook and use this activity. That aspect I think I value and I can use it.

Patricia, we argue, identified particularly with the ideational resources offered in both the mathematics and teaching sessions as reflected in Table 1. She learned and developed an identity of a teacher who is intentional in her example selection and task designs, consulting textbooks and other curriculum resources to plan her lessons. She integrated more purposeful sets of examples and clearer lesson goals while maintaining her responsibility to the ATP. While purposeful, her practice-linked MTI is also constrained. Mediating mathematical ideas remained a struggle. For example, she did not invite her learners into extended participation explaining that her classroom setting and the demands of the ATP were prohibitive. Also, her valuing of having other teachers (relational resource) with experience of teaching in such schools as made available in the course did not appear to carry over into support for her struggles in her classroom; this despite her initial motivations for joining WMCS being to teach better in her context.

7.3 Thulelah's enactment and talk of resources related to teaching mathematics

Thulelah talked about her lesson objectives in terms of what she expected her learners to gain. For example the first lesson: "I expected them to learn on how to find the output using the rule"; and for the second lesson: "to know or to recognise even when you are given a graph, you can actually get those tools [different representations] taught".

The extract is from the second lesson that began with a review of homework. The seven tasks selected from the textbook required that learners: (a) found the rule and equation for a given function; (b) calculated the output for a given input; (c) calculated the input for a given output and; (d) determined the equation for a function presented using a flow diagram. She invited answers from her learners including different answers to the same task. In some instances, she invited learners to the board to present their work. When Thulelah obtained contradictory answers she moved to the board and demonstrated (with interaction) the correct answer. The review of the homework was cut short because she perceived the learners to be "slow". As she integrated deliberate attention to connecting tables of values, equations and input–output diagrams and inviting contributions from learners, time efficiency difficulties emerged. The excerpt begins at this point.

7.4 Excerpt 2: (16:25–25:09)

> T: Can we all see that graph?... So you are given this [a graph with some specific points], you can produce, produce a table, number 1, number 2 find a rule, number 3, give an equation. So you are supposed to have three things. Can I give you at least six minutes? I know it's a lot, six minutes is a lot.

She had prepared the graph $y = x$ on the WMCS grid. She pasted this up and wrote the tasks on the board.

Learners began working in their workbooks some alone, others in pairs and she moved around the classroom observing what learners were doing. She noticed that some were copying the graph and so urged them "to get going", moved to the board to draw their attention to the points she had marked on the graph.

> Let me enlarge the points, maybe probably some of you don't see them. Let me enlarge the points, okay? We are talking about that one, that one. Right those ones. Do we know how to draw a graph? What do we need to draw a graph? Or, or what is a point, what makes a point? One point, what makes a point? How did I get this point here?

She moved back to observe if learners were able to progress and soon said:

> T: Are we lost. Okay give me some questions then.

> Ls: Yes.

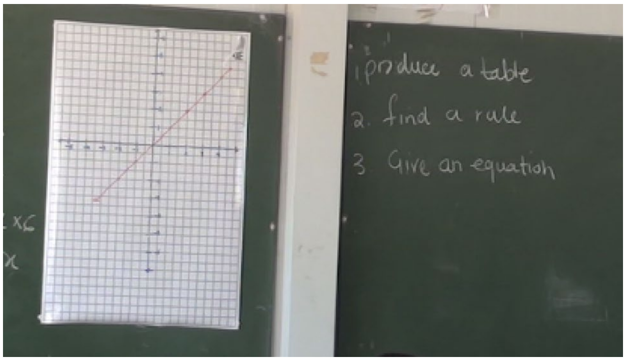

Fig. 6 Excerpt from Thulelah's board work

> T: If you have a question, give me a question why you are lost? Or tell me what you don't understand. And remember, remember in grade eight you were doing prediction and functions, right?

As with Patricia, we selected this episode as it illuminated the kind of teaching Thulelah was engaged in through the two lessons, and enabled us to discuss her identification with resources related to teaching mathematics. As noted, she came with an already prepared example on the grid supplied by WMCS for the second lesson. She walked to the various tables to see what learners were doing. She then moved back to the board to provide further scaffolding when she realised most of the learners were stuck. Thulelah eventually realised the nature of learners' challenges but time was not on her side. The selected episode ended with the majority of learners unable to complete all the tasks. Figure 6 brings to the fore Thulelah's identification with ideas about functions and the value attached to connecting representations. During the interview she commented:

> Because it's been a week thing, they know how to use the flow diagram and then from the flow diagram they can move to the rule, the rule they can move to equation. So today the main thing was for them to know or to recognise even when you are given a graph, you can actually get those tools.
>
> She explained further:
>
> Well, what I've noticed, we need to work and by working it means you need to think. So if I give them – that activity, it was a good one. If I give them that, they need to think deeply on what they have done during the week. So they needed to connect what they have learnt during the week to do that activity.

Thulelah explicitly referenced WMCS as the source of this idea and her identification with choosing examples as purposeful. She asserted:

> Well, before I joined Wits Connect, I didn't see the reason in considering the exercises. Then I would say "Page – number one" – you know, just give them before thinking. But now I just go through and I think "Okay, this one can inform the other one, the other one can go there," so I can easily give them what I want them to gain.

In addition, she also identified herself as a mathematics teacher with a better grasp of the content. She asserted:

> For the functions that we did at Maths Connect, functions are a hard topic to deal with. Not really hard but you need to understand how these numbers connect with each other and how these numbers get you a graph".

Another ideational resource related to language and need for definition. As such, she started the revision with a focus on what "input" and "output" meant "because those two words are important."

She sought to enact this idea in her class and explained the difficulty she faced:

> I think they didn't get that actually a graph has a point and [that] point is made up of an input and output. I wanted them to see "Okay, these numbers are my output and those numbers are my input". You know, what I feel like is, these kids don't talk and if they're not talking, it's hard for you to help.

Her comment is consistent with her motivation for joining WMCS; to learn how to support her learners' understanding through identifying their "misconceptions" and on providing space for her learners to voice "what they understand". In the selected episode, Thulelah did not immediately tell her learners what to do after noticing that they could not make sufficient progress. Instead, she made several attempts at scaffolding the task such as being explicit in what a point is on graph (i.e. "Let me enlarge the points, maybe probably some of you don't see them") and finding out rather belatedly, whether they knew "what makes a point". All these efforts did not yield the intended results leading her to comment after the lesson: "What they say is important because that informs you on what they understand. So if they can't say anything, then it's a problem".

From her post lesson interview, we gain insight into a deliberate attempt in her selection of material resources offered. She referred to her use of the textbook at the end of the lesson to assign homework. While she found the warm up activities done before each TM1 mathematics session, for example, "interesting", she noted:

> If we can have that in schools where you start with a little game, it would be nice but it's not because they are looking at where are you in the annual teaching

plan (ATP). These kids they can't use their time effectively. Time is a problem.

Thulelah appeared selective in her practice, determining what was possible within her classroom context and her school obligation of finishing the ATP. She used the WMCS grid to draw the graph and so link representations in the lesson. She spoke only briefly of the WMCS environment thus hinting but not emphasising relational resources offered: "its busy, interesting, as well, it's conducive, you can learn".

In summary, Thulelah identified with particular ideational resources offered in both the mathematics and teaching sessions, and in line with her motivation to deepen her mathematical knowledge and to teach effectively within her school context. She was purposeful in her choice of examples to promote better understanding of the concept of functions. Her learners were repeatedly invited to actively participate in the lessons so that they could offer something to guide her teaching, and she worked hard to scaffold their learning.

7.5 Looking across their talk and enactments

Both teachers identified with the idea that mathematics is a connected system. In response to the question of what had changed (would they not have done) about their teaching of functions, both talked about the idea of linking different representations. Patricia emphasised their purpose: "I don't think linking them because I learnt in that course that linking the different representations enhances the understanding". Thulelah, focused on her teaching, saying that she was "proud" of herself for being able to enact this particular ideational resource in her classroom despite the observed difficulties in mediating her learners' learning.

Both teachers' MTIs were purposeful. Patricia focused on key features of functions and chose example sets to bring these into focus. Thulelah focused on connecting representations and was successful until she introduced the graph. Both struggled to meet their goals within the time constraints of a lesson. It is here where nuanced differences in their developing MTIs emerge.

Patricia could not shift from ultimately telling learners what she had wanted them to notice, and while lamenting this, reflected her need to comply with the ATP and being thwarted by her contextual constraints. Thulelah, taught in ways she espoused in her motivation for joining WMCS notwithstanding her classroom constraints.

Another difference is how they talked and acted with the material resources offered them that both appreciated. Aside from textbooks, Patricia did not use other material resources in her teaching. She talked extensively about what she could not do due to the lack of these resources e.g. enabling learners to generalise the effect of the slope of a line by making it visual. Thulelah, on the other hand, appeared to be more positively selective in deciding the WMCS material resources that crossed into her classroom space, focusing on what was feasible within her school constraints, e.g. bringing in the grid but deciding not to use card sorting activity despite having copies of these.

In summary, both valued what was offered in the PD and articulated what were central challenges they faced in their work as teachers in schools where supporting assets for their work are limiting (Shalem and Hoadley 2009). While Patricia talked explicitly across the various identity resources offered and her valuing of these, she emphasised her constraints and commitment to the ATP. We suggest that her developing MTI was *purposeful yet reactive*. Thulelah emphasised her concerns for engaging and eliciting learner contributions and consciously selected from the array of resources, what was feasible in her classroom. We suggest that Thulelah, in contrast, was developing a *purposeful and proactive* MTI.

8 Discussion and conclusion

In this paper we discussed the developing practice-linked (MTIs) of two teachers, using Nasir and Cooks' (2009) constructs of ideational, material and relational identity resources. These enabled us to describe the *what* of the WMCS PD, in terms of its mathematics and mathematics teaching resources. As noted earlier, the former have typically not been foregrounded in previous research. Yet this is critical in making sense of how and why teachers learn in and from PD. This productive use of Nasir and Cooks also illustrates benefits of drawing on advances in identity research outside of mathematics education (Lutovac and Kaasila 2017).

A second contribution is the insight into how ideational resources influenced each teacher, hence their similarities and differences. For instance, both identified with teaching as deliberate and intentional, with having clear goals and planned examples and tasks. However, with regards to ideas about learner participation, Thulelah talked and enacted teaching in ways that involved her learners. Patricia, it appears, valued the various resources offered and talked about them but could not enact such in her classroom. Our study thus confirmed that to understand teachers' learning, and their developing identities, requires a focus on the multiple contexts of practice (Borko 2004). By parsing narratives around teachers' MTI in terms of identity resources we observed that teachers may value and identify strongly with the identity resources offered, and yet act differently with these resources.

Insights into the what and how, of course, do not transparently convey why MTIs form as they do. Our inclusion of the teachers' motivations for joining the PD so as to explore links with their learning and practice-linked identities (Gresalfi and Cobb 2011) is apposite. Thulelah's expressed a desire to be able to teach in ways that promoted her learners' understanding of mathematics. Examining her MTI therefore was valuable in illuminating the influence of the ideational resources related to functions as linking multiple representations and to teaching as an intentional act which should manifest in purposeful selection of examples and related tasks. Her narration showed how she worked to manage her contextual issues while integrating resources she valued in the PD. Patricia integrated more coherent example sets related to key features of functions. Her motivation to be more effective in her context and the location of this difficulty outside of herself perhaps explains why she could not interact more productively with her learners. What is further interesting here was her appreciation of the relational resources available in the PD, particularly her lack of experience teaching in such school systems and cultural difference. This did not cross over into her school. Unfortunately, we did not explore why colleagues in her school were not similarly useful resources for working with her constraints.

Notwithstanding the differences in their motivations and identifications, two aspects of the PD and our identity study emerge. Firstly, the PD itself did not offer support in the context of school teaching itself. In addition, while the PD intentionally offered teaching ideas 'close' to teachers' current practices so as to enable enactment their adaptations are not trivial. Secondly, while multiple sources of data have supported the study (Lutovac and Kaasila), these were not extended over time. More extended study over time would as was done by Nasir and Cooks (2009), we suggest, would offer more insight into how identification with material and relational resources in the context of school constraints evolve. This will help indicate whether and how their reactive and proactive MTIs as we have described will continue to develop and change. Here is a methodological challenge in taking this work forward. Although mindful of this limitation in the reported study, we believe that the findings provide a useful snapshot of what a focus on resources in PD offers to identity research.

Theoretically there are also potentials and issues with studying practice-linked identity resources. Initially, we categorised the MTF as a material resource because teachers had it in printed form that was referred to and worked with in each session. However, it was clearly the ideas in the MTF that were 'resources' for mathematics teaching. This is perhaps the most significant way in which the identity resources offered by Nasir and Cooks (2009) differ when recontextualised from the practice of athletics into mathematics teacher education. As we have argued, however, a focus on ideational resources, and we would go further to suggest that in mathematics teacher education, the analytic separation of mathematical and mathematics teaching ideational resources, was productive for describing the what of the PD.

In conclusion we have presented a study of what happens in PDs in terms of the resources made available and how teachers identify with these. We offer this as one productive way to take forward identity research in mathematics teacher education.

Acknowledgements This work is based on the research supported by the South African Research Chairs Initiative of the Department of Science and Technology and National Research Foundation (Grant No. 71218). Any opinion, finding and conclusion or recommendation expressed in this material is that of the author(s) and the NRF does not accept any liability in this regard. The authors thank reviewers for critical feedback, and extend gratitude to the teachers who generously opened themselves and their classrooms and so contribute to this research.

References

Adler, J. (2000). Conceptualising resources as a theme for teacher education. *Journal of Mathematics Teacher Education, 3*(1996), 205–224.

Adler, J. (2017). Mathematics discourse in instruction (MDI): A discursive resource as boundary object across practices. In G. Kaiser (Ed.), *13th International Congress on Mathematical Education, ICME-13* (Proceedings of the Monographs, pp. 125–143). Hamburg: Springer.

Adler, J., & Ronda, E. (2015). A framework for describing mathematics discourse in instruction and interpreting differences in teaching. *African Journal of Research in Mathematics, Science and Technology Education, 19*(3), 237–254.

Alexander, R. (2000). Culture and Pedagogy: International Comparisons in Primary Education. Oxford: Blackwell.

Battey, D., & Franke, M. L. (2008). Transforming identities: Understanding teachers across professional development and classroom practice. *Teacher Education Quarterly, 106*, 127–149.

Bohning, G., & Hale, L. (1998). Images of self-confidence and the change-of-career prospective elementary science teacher. *Journal of Elementary Science Education, 10*(1), 39–59.

Borko, H. (2004). Professional development and teacher learning: Mapping the terrain. *Educational Researchers, 33*, 3–15.

Darragh, L. (2016). Identity research in mathematics education. *Educational Studies in Mathematics*, 1–15.

development. In J. Cai (Ed.). *First Compendium for Research in Mathematics Education*. Reston: National Council of Teachers of Mathematics.

Goldsmith, L. T., Doerr, H. M., & Lewis, C. C. (2014). Mathematics teachers' learning: A conceptual framework and synthesis of research. *Journal of Mathematics Teacher Education, 17*, 5–36.

Graven, M. H., & Lerman, S. (2014). Mathematics teacher identity. In S. Lerman (Ed.), *Encyclopedia of mathematics education* (pp. 434–437). Dordrecht:Springer.

Gresalfi, M. S., & Cobb, P. (2011). Negotiating identities for mathematics teaching in the context of professional development. *Journal for Research in Mathematics Education, 42*(3), 270–304.

Hodges, T. E., & Hodge, L. L. (2017). Unpacking personal identities for teaching mathematics within the context of prospective teacher education. *Journal of Mathematics Teacher Education, 20*(2), 101–118.

Jaworski, B. (2006). Theory and practice in mathematics teaching development: Critical inquiry as a mode of learning in teaching. *Journal of Mathematics Teacher Education, 9*(2), 187–211.

Kennedy, M. M. (2016). How does professional development improve teaching? *Review of Educational Research, 86*(4), 945–980.

Lave, J., & Wenger, E. (1991). *Situated learning: Legitimate peripheral participation*. Cambridge: Cambridge University Press.

Lerman, S. (2001). Cultural, discursive psychology: a sociocultural approach to studying the teaching and learning of mathematics. *Educational Studies in Mathematics, 46*, 87–113.

Lutovac, S., & Kaasila, R. (2017). Future directions in research on mathematics-related reacher tdentity. *International Journal of Science and Mathematics Education, 16*(4), 759–776.

Nasir, N. S., & Cooks, J. (2009). Becoming a hurdler: How learning settings afford identities. *Anthropology & Education Quarterly, 40*(1), 41–61.

Radovic, D., Black, L., Williams, J., & Salas, C. E. (2018). Towards conceptual coherence in the research on mathematics learner identity: a systematic review of the literature. *Educational Studies in Mathematics, 99*, 21–42.

Remillard, J. T. (2005). Examining key concepts in research on teachers' use of mathematics curricula. *Review of Educational Research, 75*(2), 211–246.

Sfard, A., & Prusak, A. (2005). Telling identities: In search of an analytic tool for investigating learning as a culturally shaped activity. *Educational Researcher, 34*(4), 14–22.

Shalem, Y., & Hoadley, U. (2009). The dual economy of schooling and teacher morale in South Africa. *International Studies in Sociology of Education, 19*(2), 119–134.

Sztajn, P., Borko, H., & Smith, T. (2017). Research on mathematics professional development. In J. Cai (Ed.), *Compendium for research in mathematics education* (pp. 793–823). Reston, VA: National Council of Teachers of Mathematics.

Vygotsky, L. S. (1978). *Mind in society: The development of higher psychological processes*. Cambridge: Harvard University Press.

Watson, A., & Mason, J. (2006). Seeing an exercise as a single mathematical object: Using variation to structure sense-making. *Mathematical Thinking and Learning, 8*(2), 113–147.

Wenger, E. (1998). *Communities of Practice: Learning, Meaning, and Identity*. New York: Cambridge University Press.